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GATE 2024

प्रचण्ड Batch

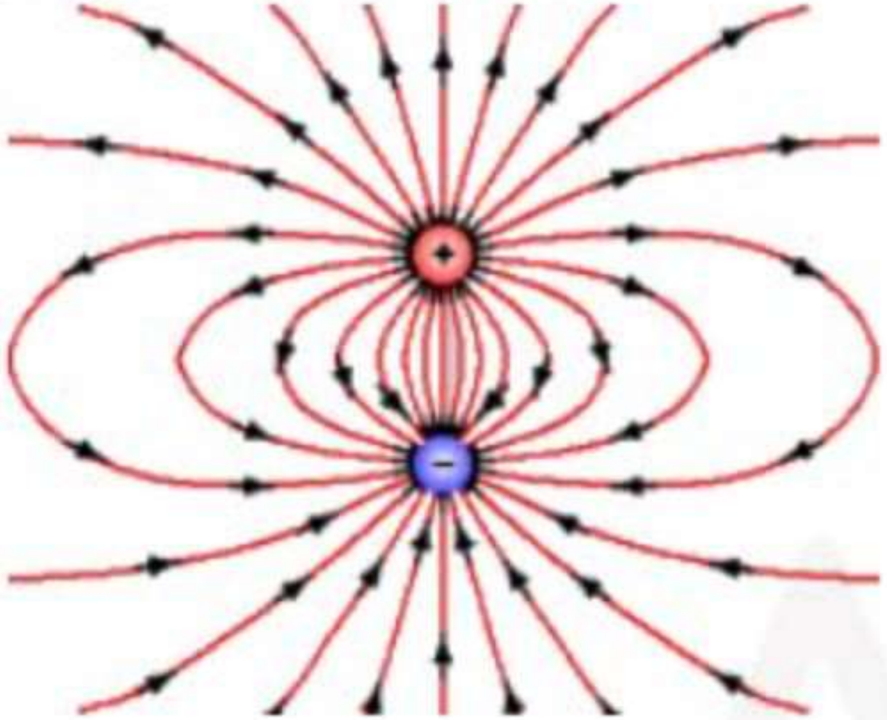
Electromagnetic Field Theory

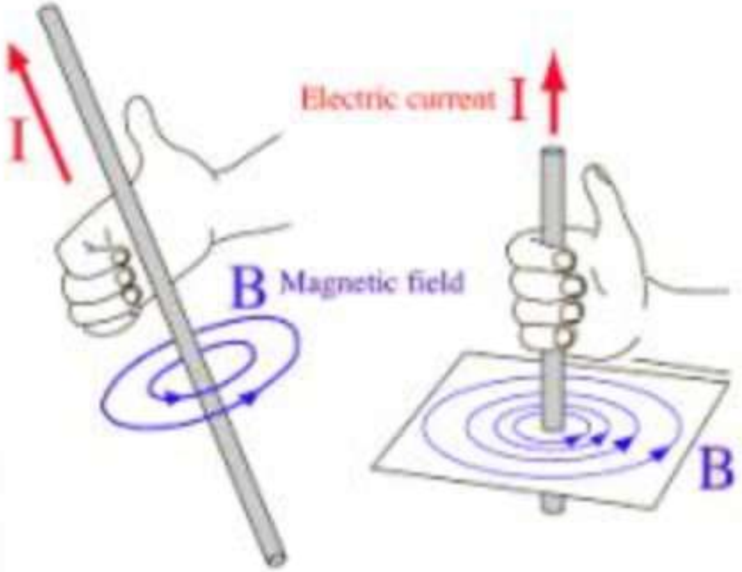
BASICS OF VECTOR CALCULUS

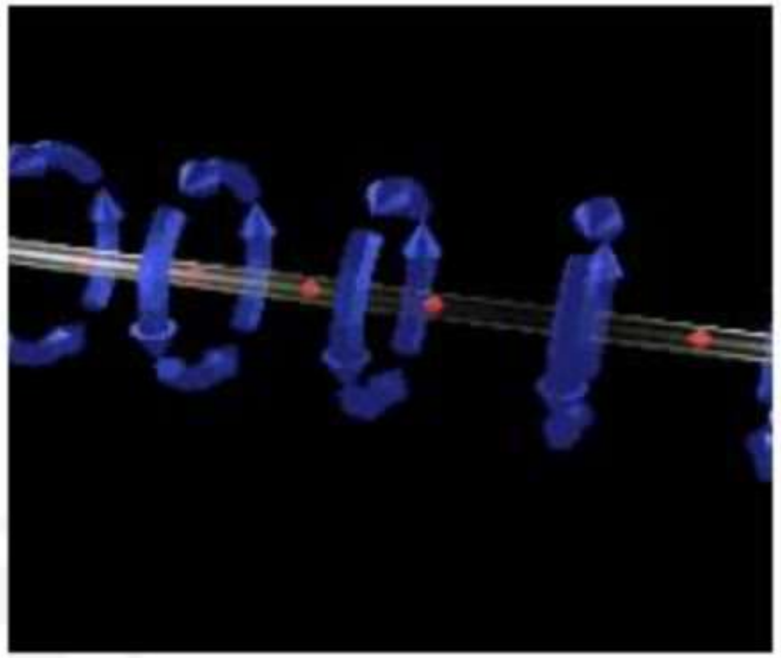
LEC-02

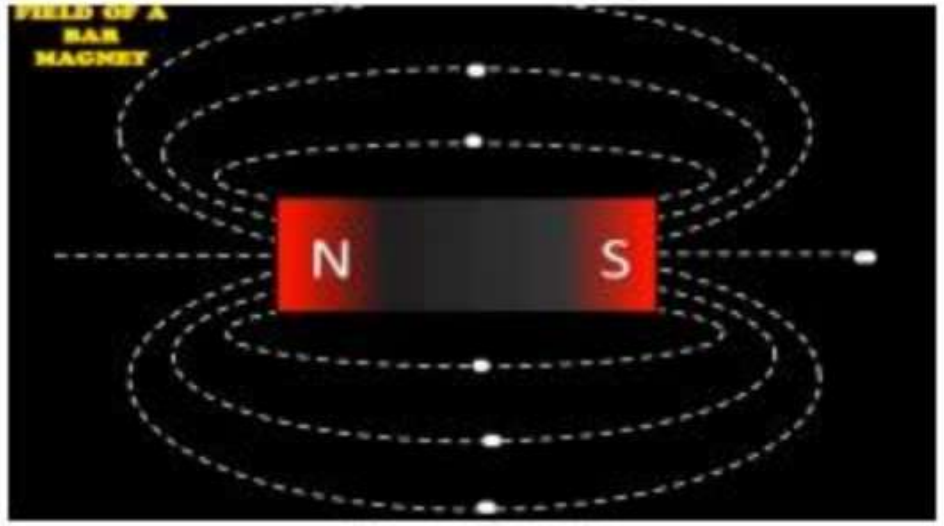
Electronics & Communication

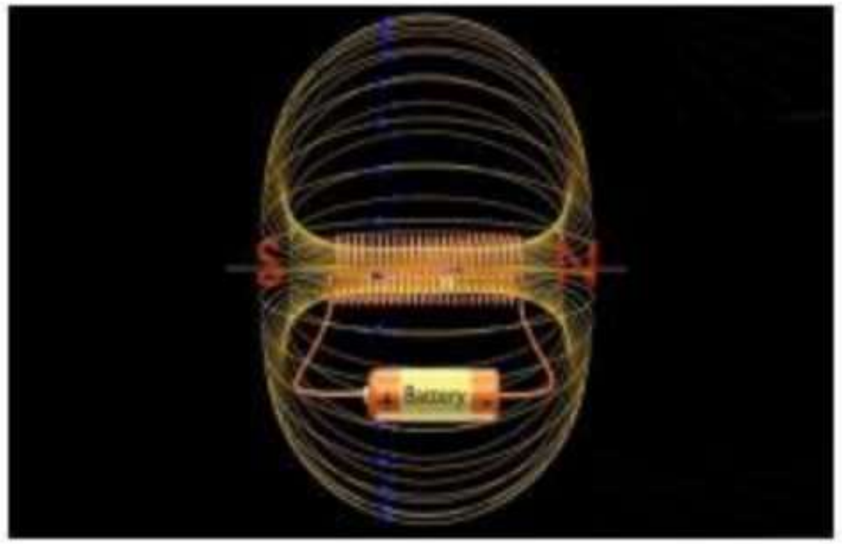
Recap

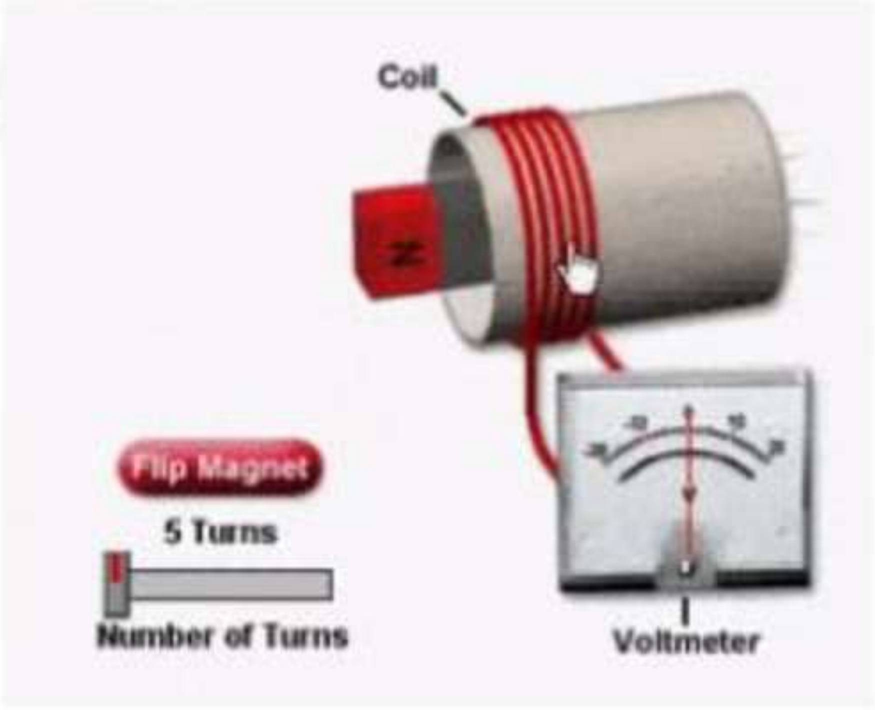



I
Electric current I
B Magnetic field
B

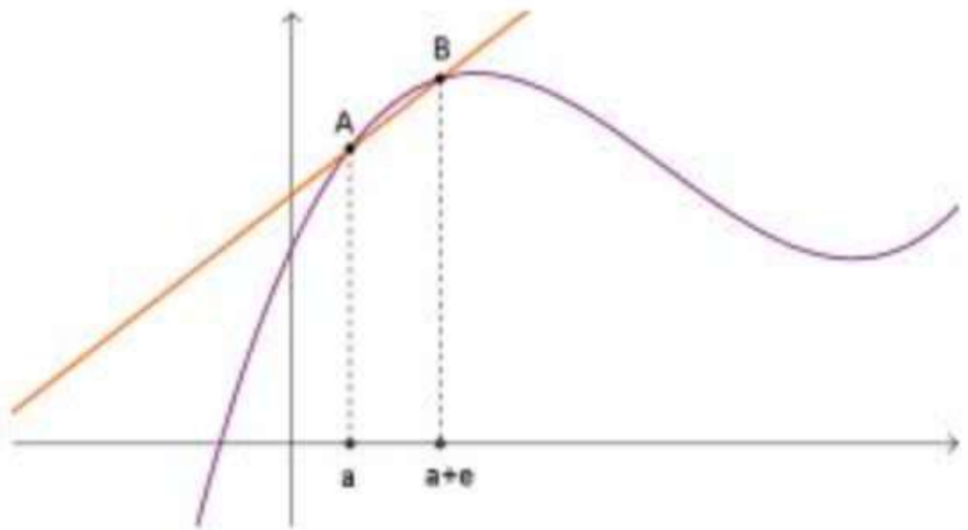



FIELD OF A BAR MAGNET
N
S

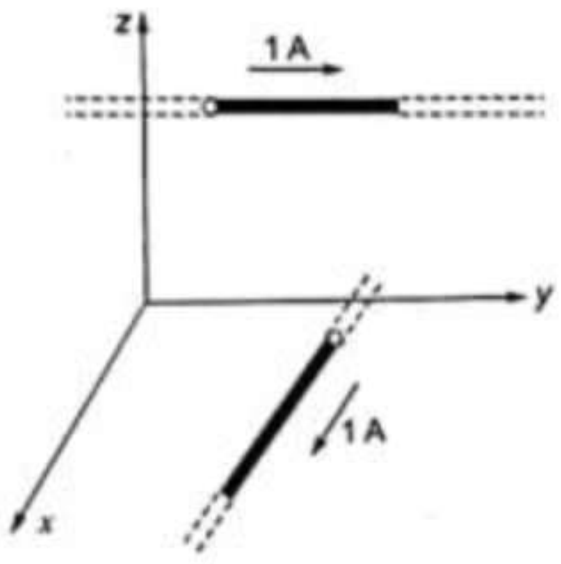



Coil
Flip Magnet
5 Turns
Number of Turns
Voltmeter

Recap



Q:1 Two infinitely long wires carrying current are as shown in the figure below. One wire is in the y - z plane and parallel to the y - axis. The other wire is in the x - y plane and parallel to the x - axis. Which components of the resulting magnetic field are non - zero at the origin?



(a) x, y, z components

(b) x, y components

(c) y, z components

(d) x, z components

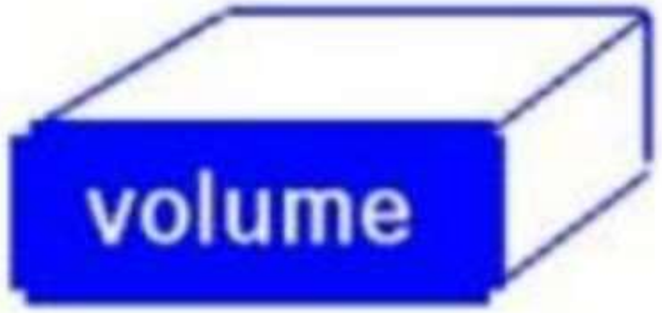
A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.

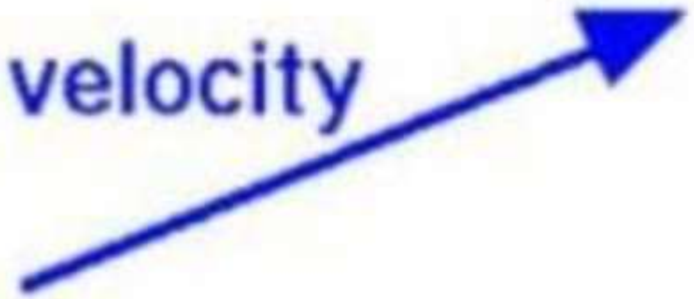
Scalar Quantities

length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power

Vector Quantities

displacement
velocity
acceleration
momentum
force
lift , drag , thrust
weight





Vector Calculus

1. Basics → position vector, magnitude, unit vector, dot product, cross product, projection.

2. Coordinate Systems → ① Cartesian C.S.
② Cylindrical C.S.
③ Spherical C.S.

3. Vector Integrals ① line integral → closed line ② Surface integral → closed surface
③ Volume integral.

4. Vector Differentials del operator, gradient, divergence, curl, laplacian.

Vector

$\vec{A}$ = magnitude × direction

$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

Basics of vector calculus

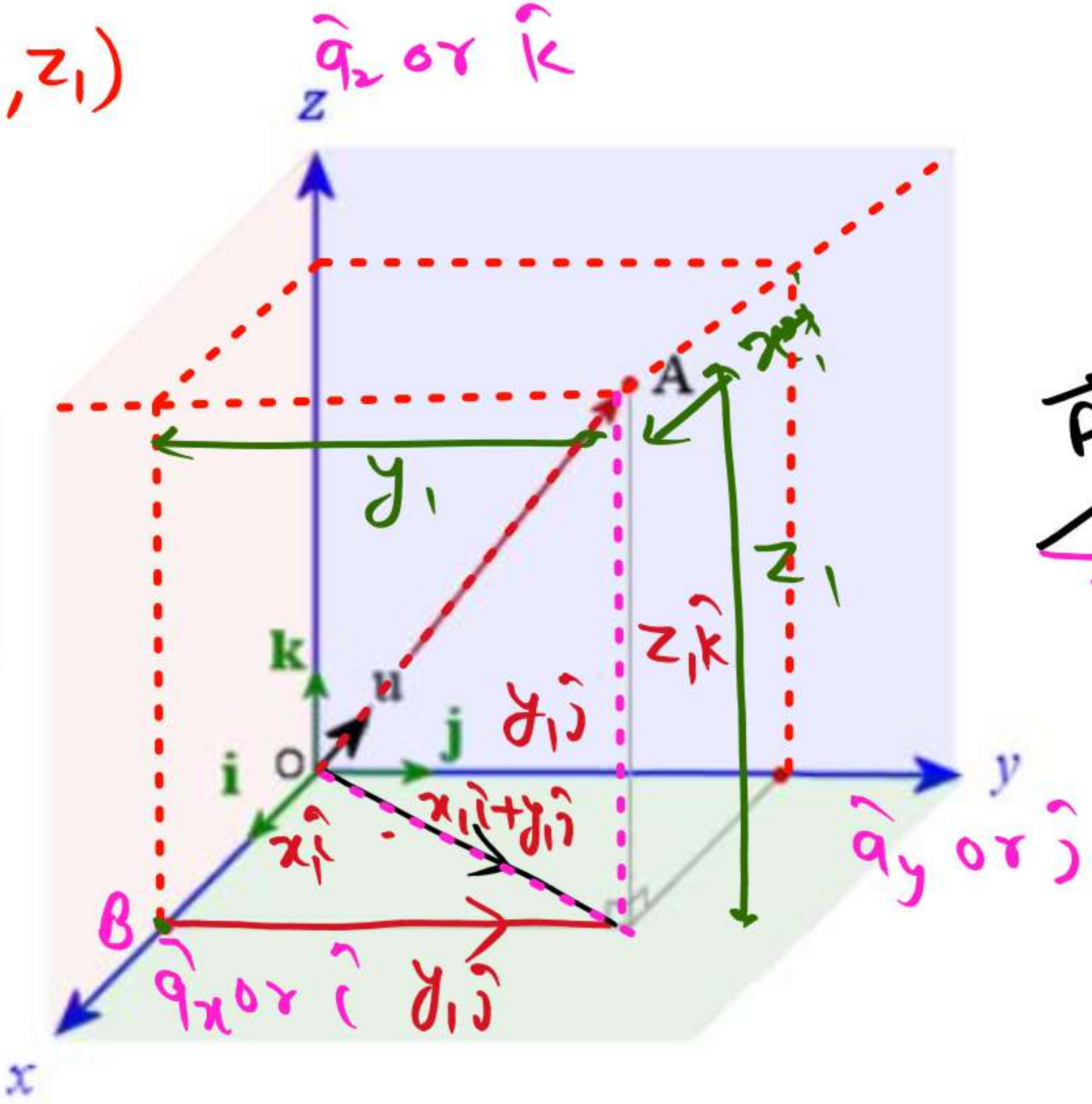
Position vector

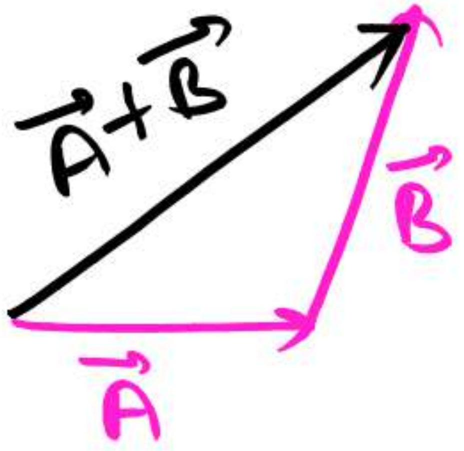
$A(x_1, y_1, z_1)$

$$\vec{OA} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

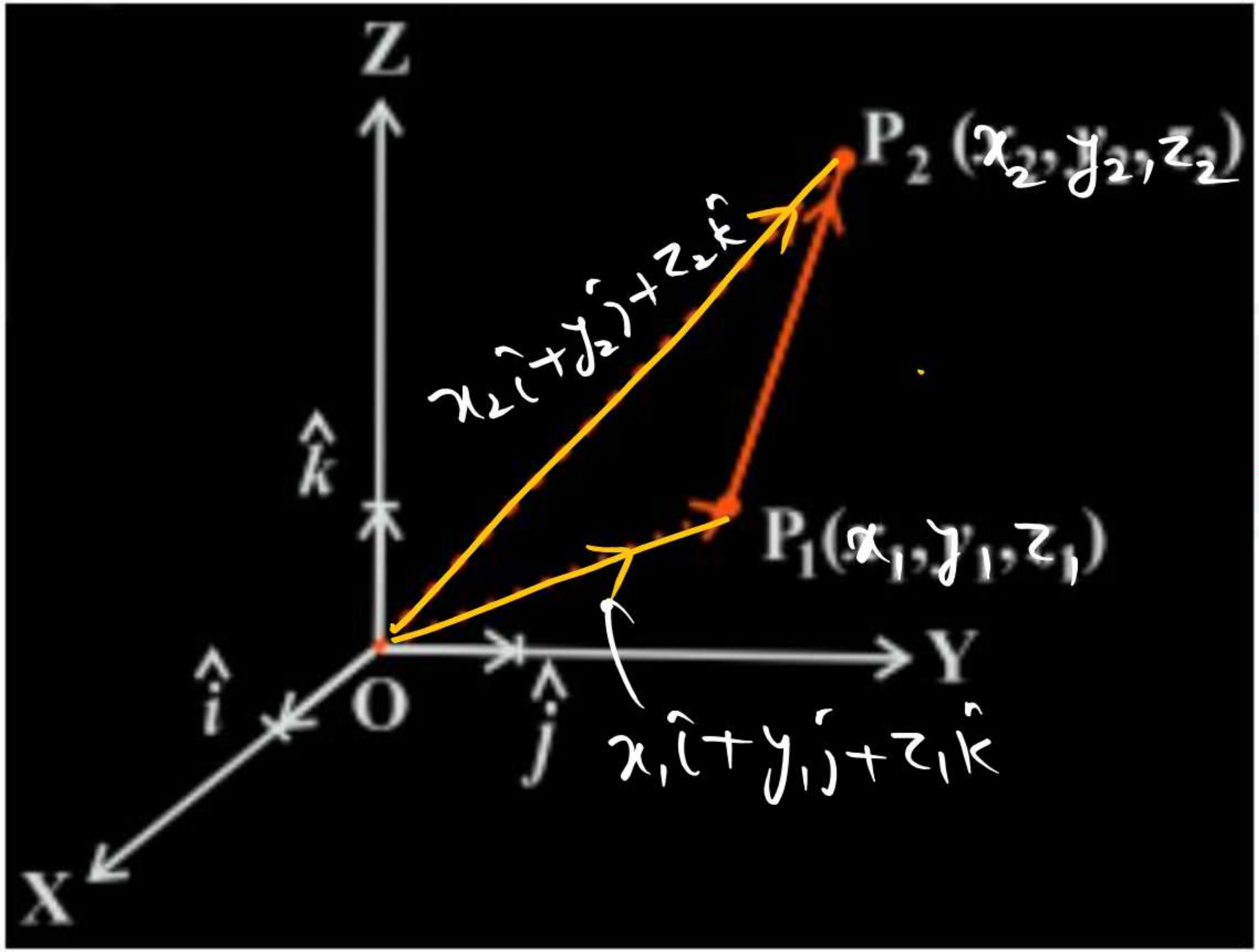
$B(-3, 2, 1)$

$$\vec{OB} = -3\hat{i} + 2\hat{j} + \hat{k}$$





Vector between two points



$$\overrightarrow{P_1P_2} = \vec{A}$$

$$\overrightarrow{OP_1} + \vec{A} = \overrightarrow{OP_2}$$

$$\vec{A} = \overrightarrow{OP_2} - \overrightarrow{OP_1}$$

$$\vec{A} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\vec{A} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

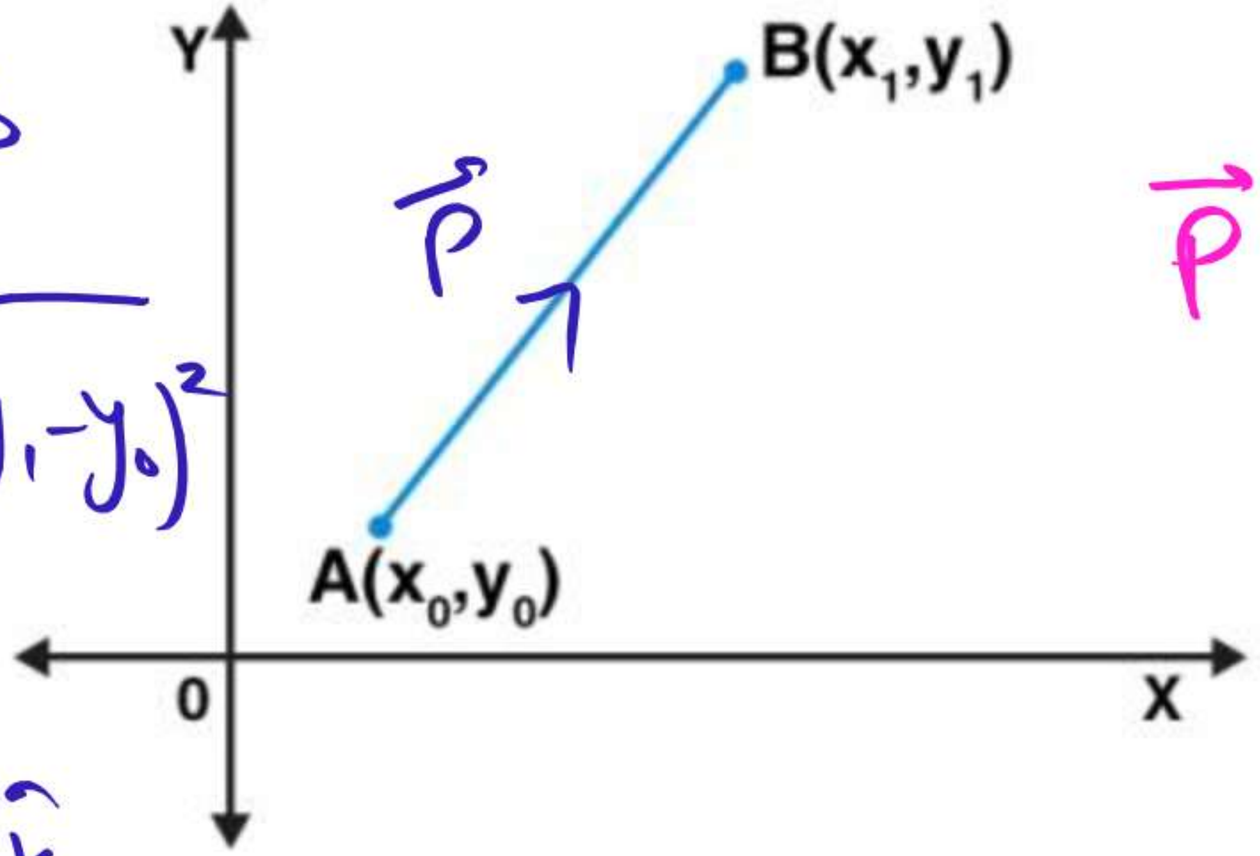
$A(-3,1,4)$, $B(3,-1,2)$

$\overrightarrow{BA} = 6\hat{i} - 2\hat{j} - 2\hat{k}$

Magnitude of vector

$|\vec{P}|$ = distance b/w A & B

$$= \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$



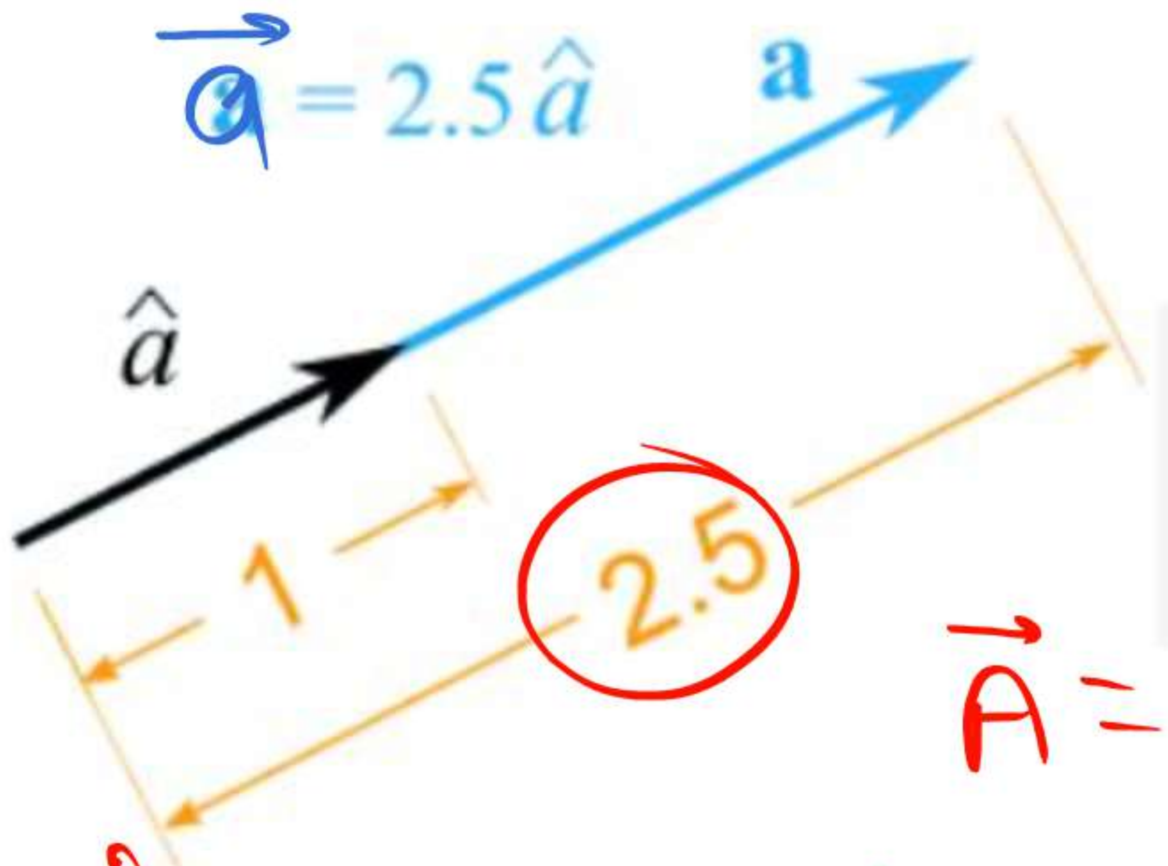
$$\vec{P} = \vec{AB} = (x_1 - x_0)\hat{i} + (y_1 - y_0)\hat{j}$$

$$\vec{P} = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2}$$

$$\vec{A} = 3\hat{i} + \hat{j} - 2\hat{k}$$

$$|\vec{A}| = \sqrt{(3)^2 + (1)^2 + (-2)^2} = \sqrt{9 + 1 + 4} = \sqrt{14}$$

Unit Vector/Direction of a vector:- Unit vector of a vector $\vec{\mathbf{A}}$ is the vector with magnitude 1 and direction same of $\vec{\mathbf{A}}$.



$\vec{A} = |\vec{A}|$

$\hat{a}$ = magnitud x direct

$\hat{a}$ = 1 x direction

vector   unit vector

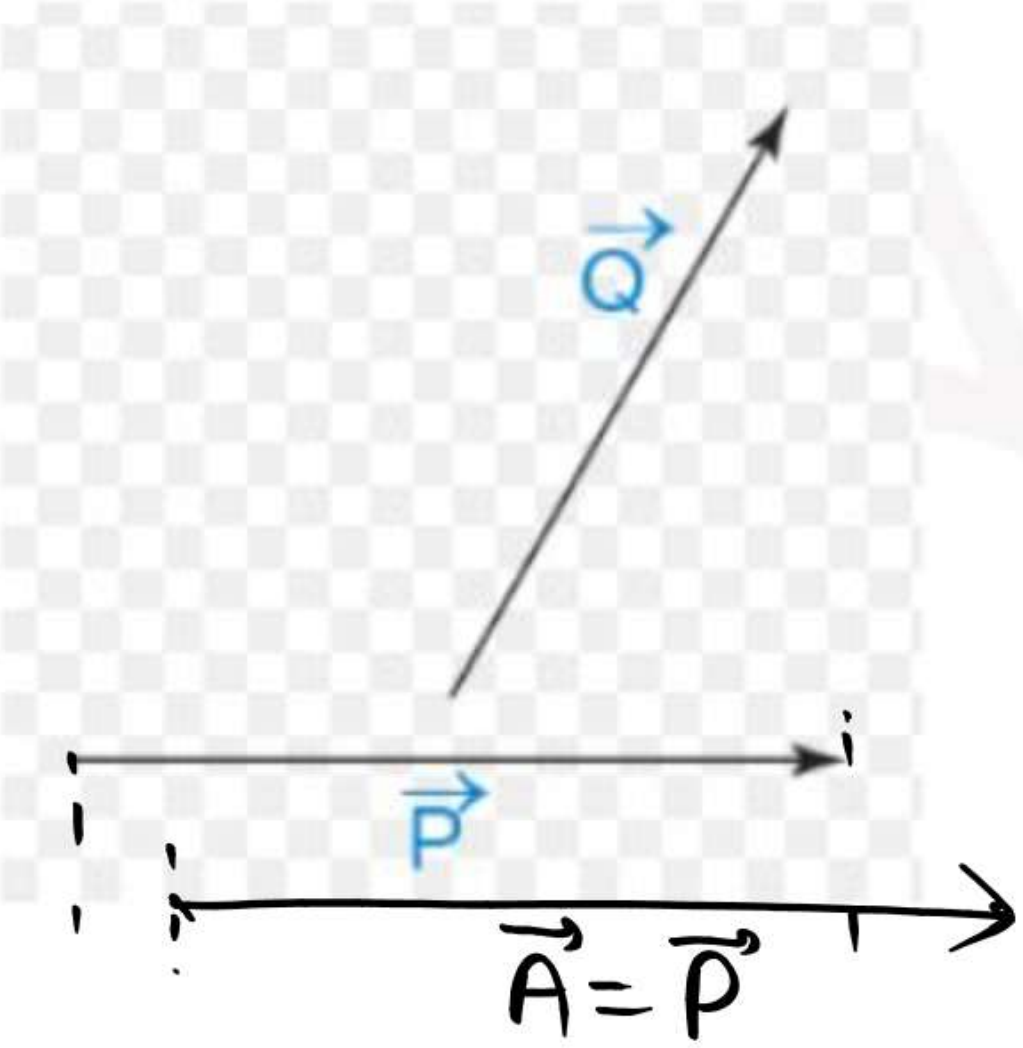
$\vec{A} \xrightarrow{\text{unit vector}} \hat{a}_A$ or $\hat{A}$

$* \quad \vec{A} = |\vec{A}|\,\hat{a}_A$

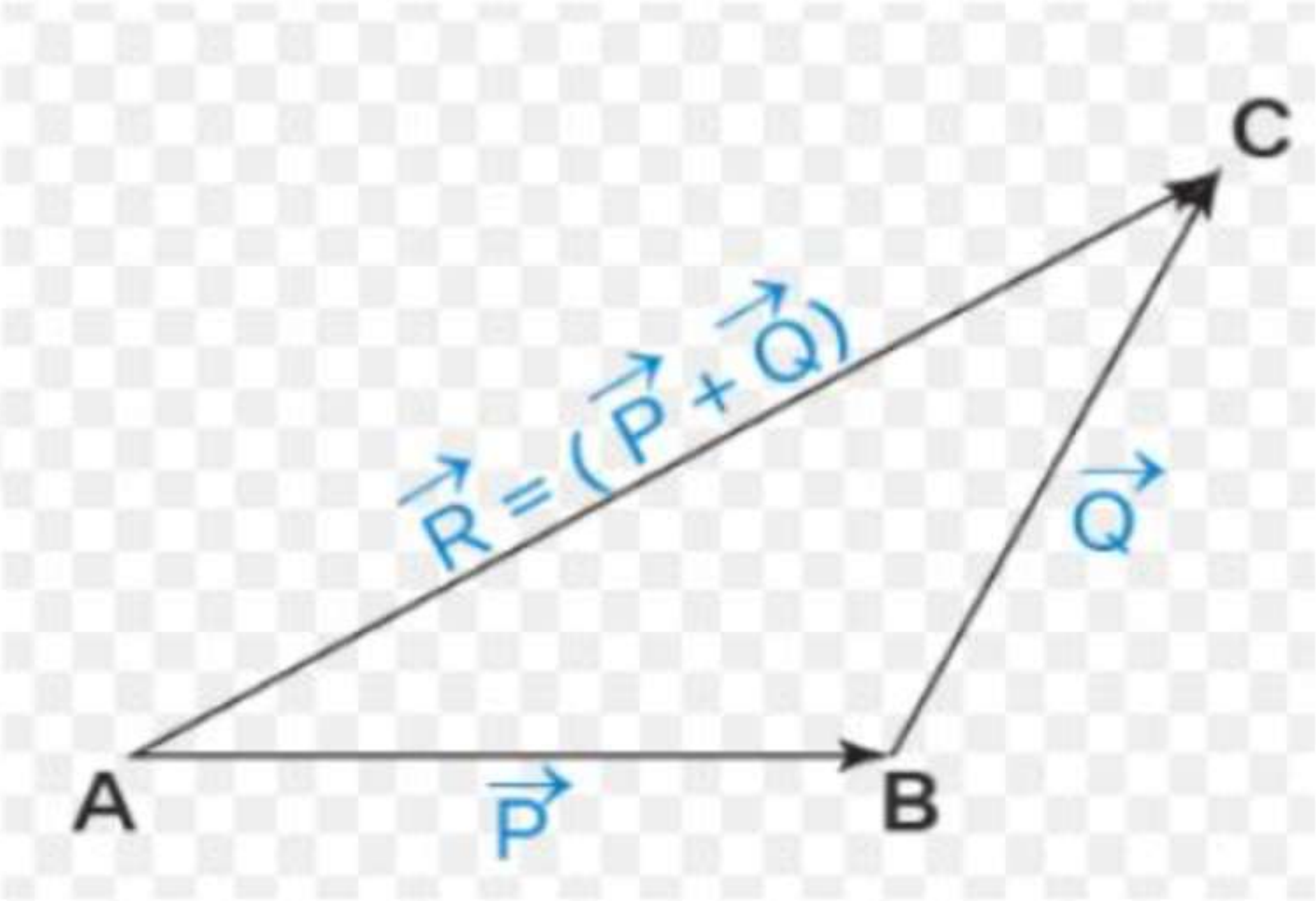
$\Rightarrow \hat{a}_A = \dfrac{\vec{A}}{|\vec{A}|}$

Vector additions

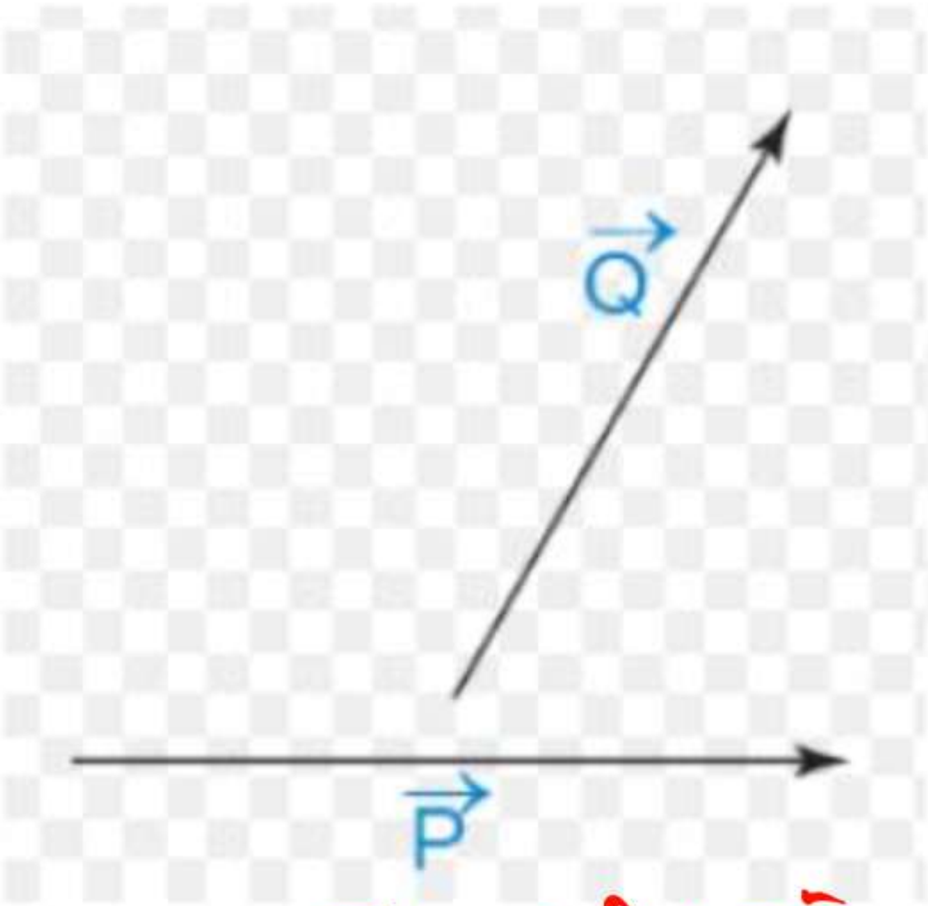
Triangle Rule

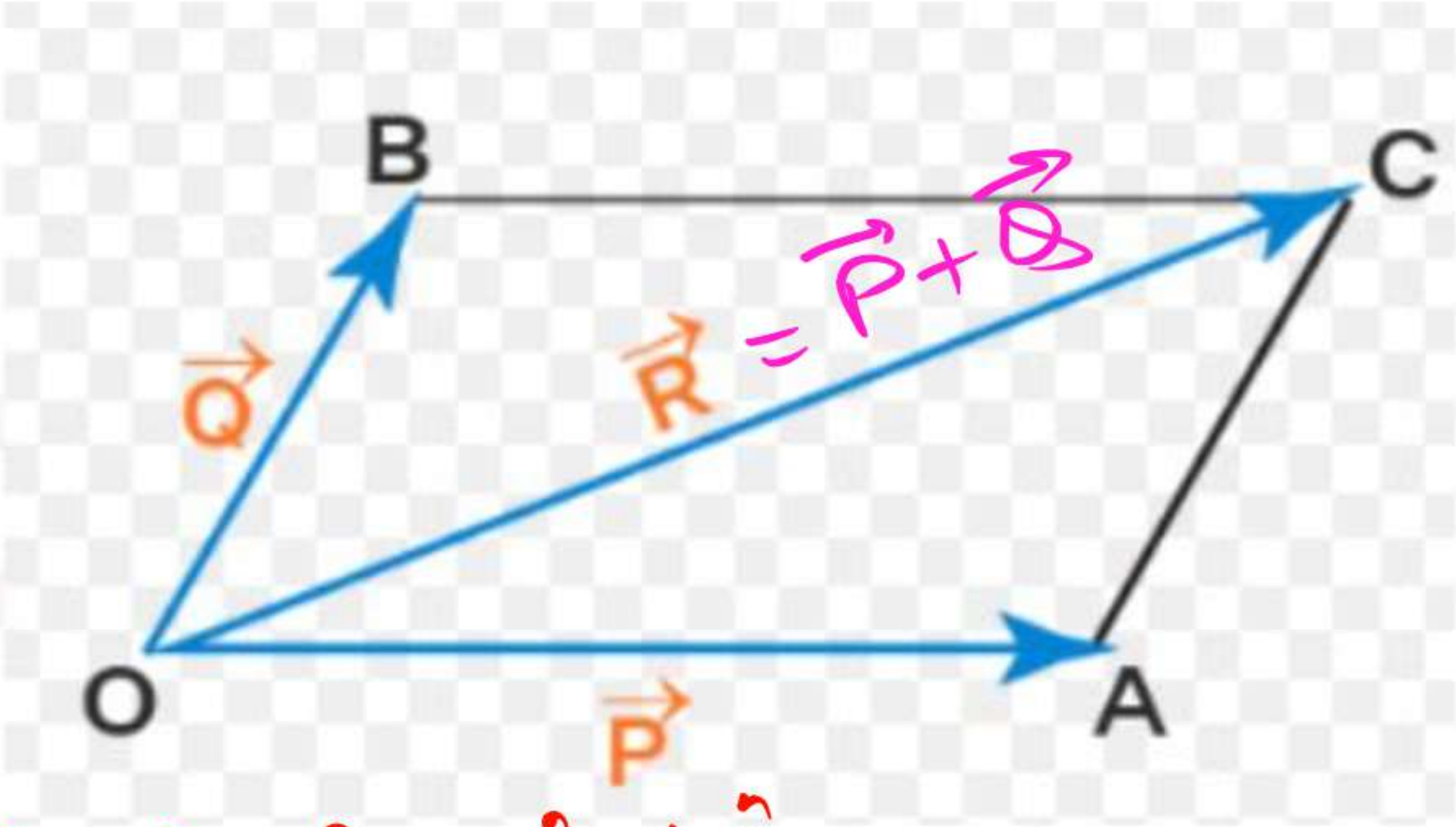


$\Rightarrow$



Parallelogram Rule





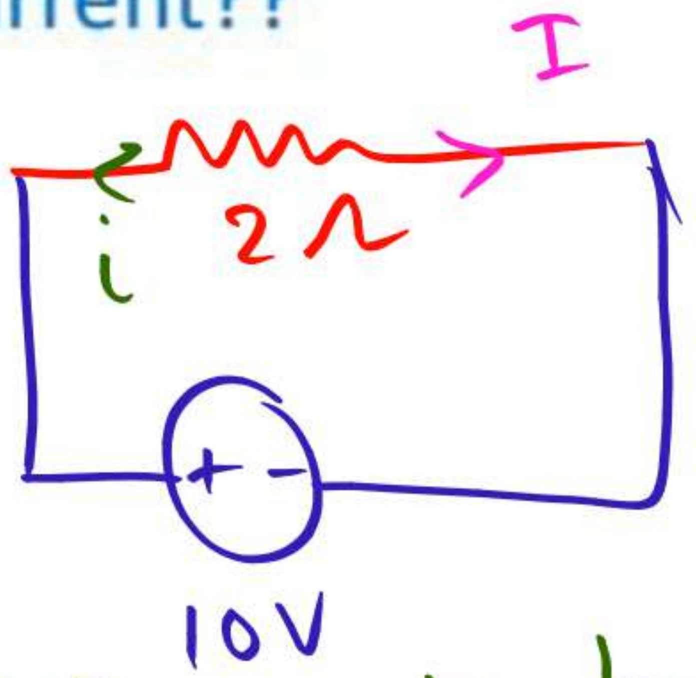
$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{B} = -\hat{i} + 2\hat{j} - 3\hat{k}$

$\vec{A} + \vec{B} = \hat{i} + 5\hat{j} - 4\hat{k}$

$\vec{A} - \vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$

Current??

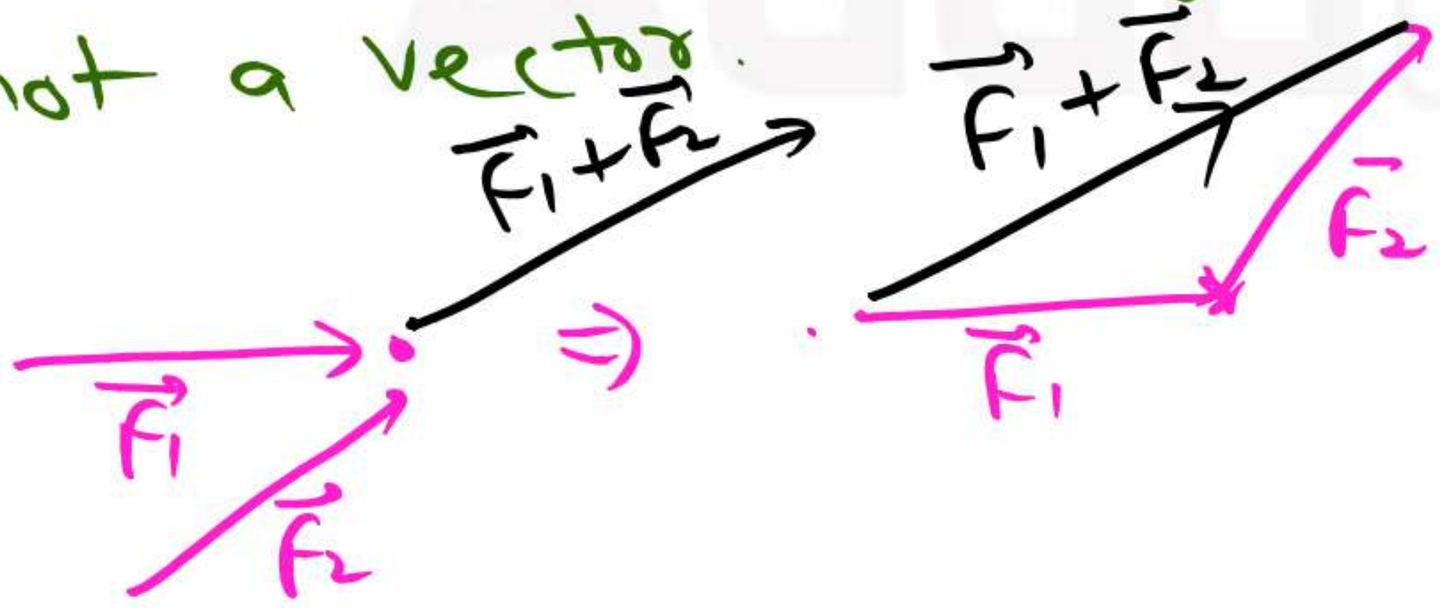


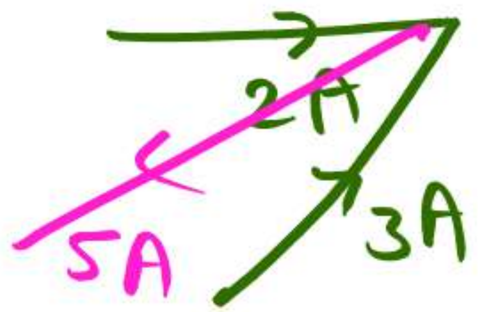
$I = \frac{V}{R} = 5A$

$i = -5A$

$I = 2\hat{i} - 3\hat{j}$ Amp ✗

* Current has both magnitude and direction but it is not a vector.





Tensors: → Physical Quantities which have both magnitude and direction but does not follows vector addition rules are called **Tensors**.

e.g. Current.

Dot Product

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \rightarrow \underline{\text{Scalor}}$$

$$= AB\cos\theta$$

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{B} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

$|\vec{A}| = \sqrt{4+1+1} = \sqrt{6}$

$|\vec{B}| = \sqrt{9+4+4} = \sqrt{17}$

$\vec{A}\cdot\vec{B} = ??$

$(2\hat{i} - \hat{j} + \hat{k})\cdot(3\hat{i} - 2\hat{j} + 2\hat{k})$

$= 2\hat{i}\cdot 3\hat{i} + 2\hat{i}\cdot(-2\hat{j}) + 2\hat{i}\cdot 2\hat{k} + \ldots$

$= 6 + 2 + 2$

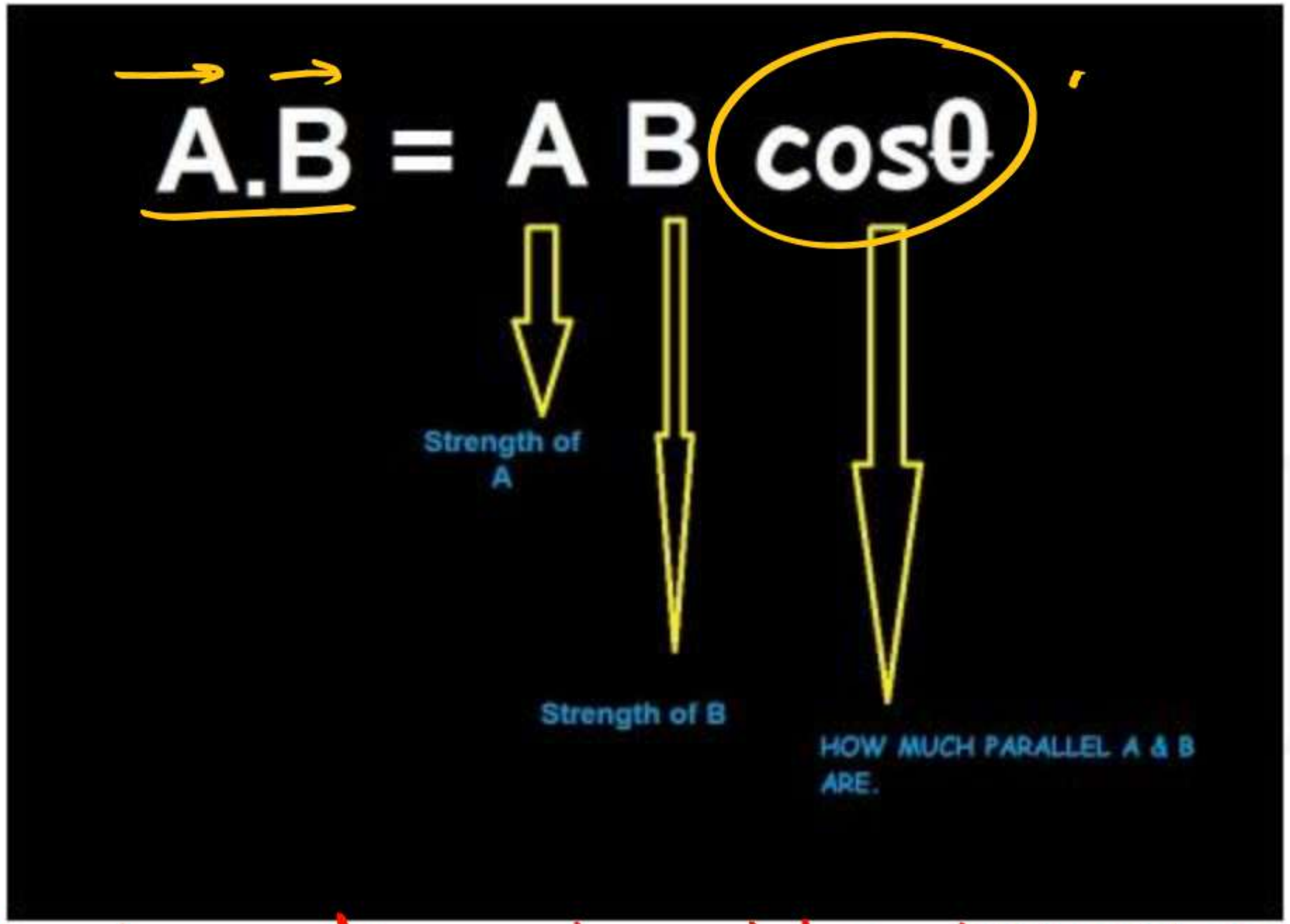
$= 8$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = 0 = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k}$

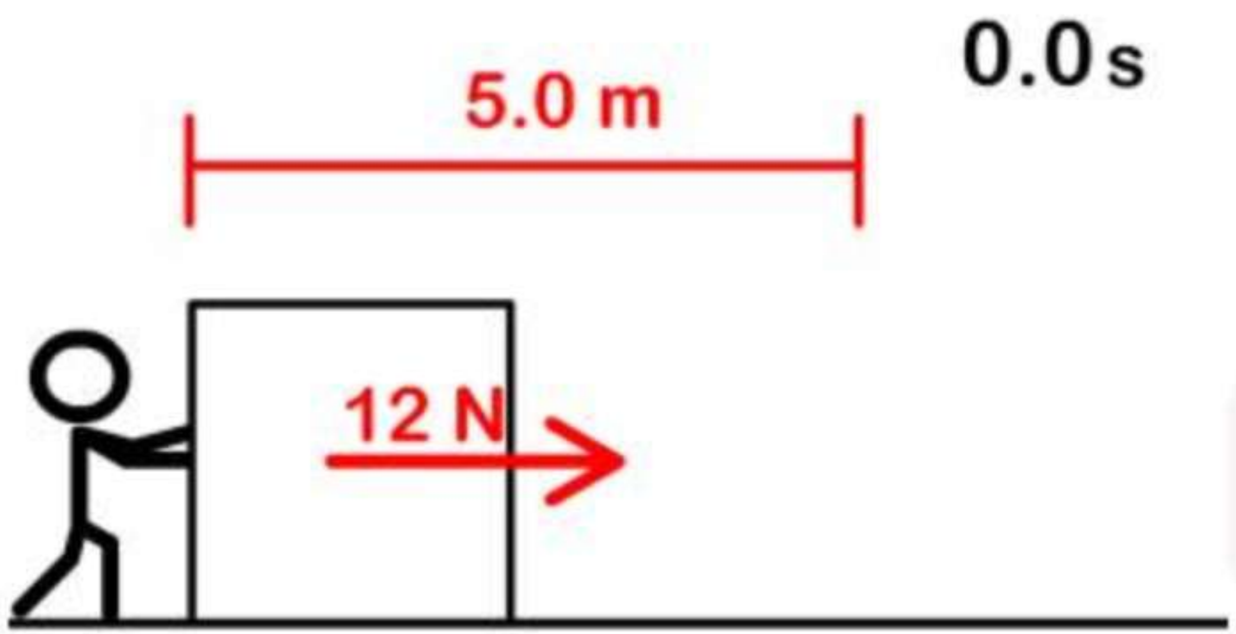
$$\boxed{\Rightarrow \theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)}$$

Dot Product



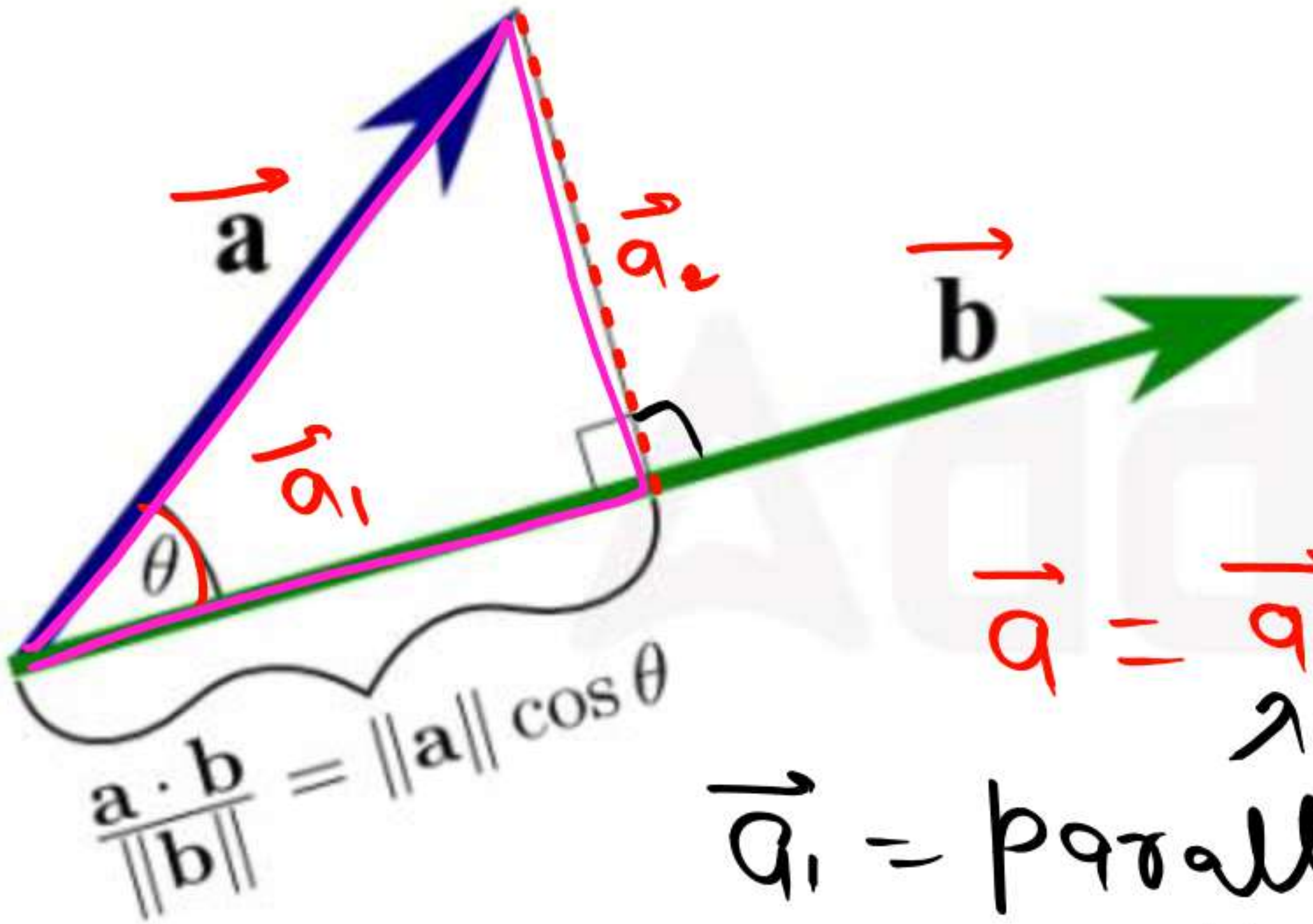
dot product value is the measure of how much two vectors has their strength as well as how samely they are oriented.

Dot Product Physical Significance



e.g. work done = $\vec{\text{Force}} \cdot \vec{\text{displacement}}$

Projection of a vector on other vector/Parallel component



$\vec{a}\cdot\vec{b} = ab\cos\theta$

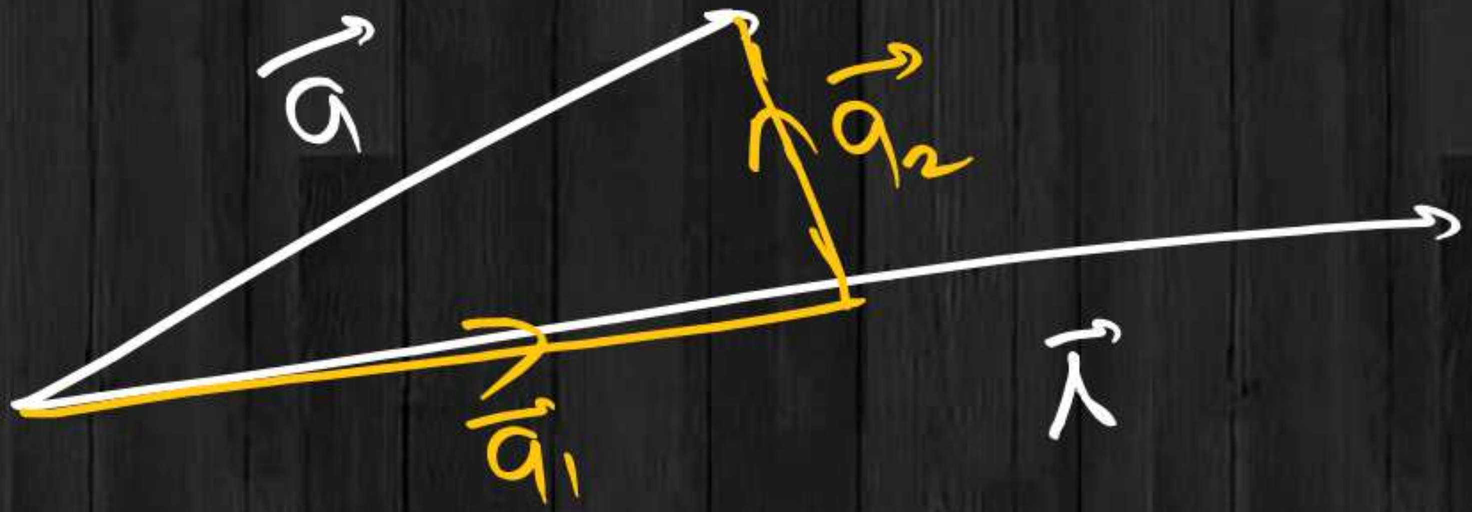
$= \underline{a\cos\theta}\ b$

projection $= a\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

$\vec{a} = \vec{a}_1 + \vec{a}_2$

$\vec{a}_1$ = parallel component of $\vec{a}$ in the direction $\vec{b}$

$\vec{a}_2$ = perpendicular component of $\vec{a}$ to the $\vec{b}$



$\vec{a}_1 = |\vec{a}_1|$ direction

$= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\,\hat{a}_b$

Q:2 If $\vec{A} = 2\hat{i}+\hat{j}+4\hat{k}$ & $\vec{B} = -\hat{i}+2\hat{k}$ then find

1. parallel component of $\vec{B}$ to the $\vec{A}$.
2. perpendicular component of $\vec{B}$ to the $\vec{A}$

① parallel component $= \dfrac{\vec{B}\cdot\vec{A}}{|\vec{A}|}\,\hat{a}_A$

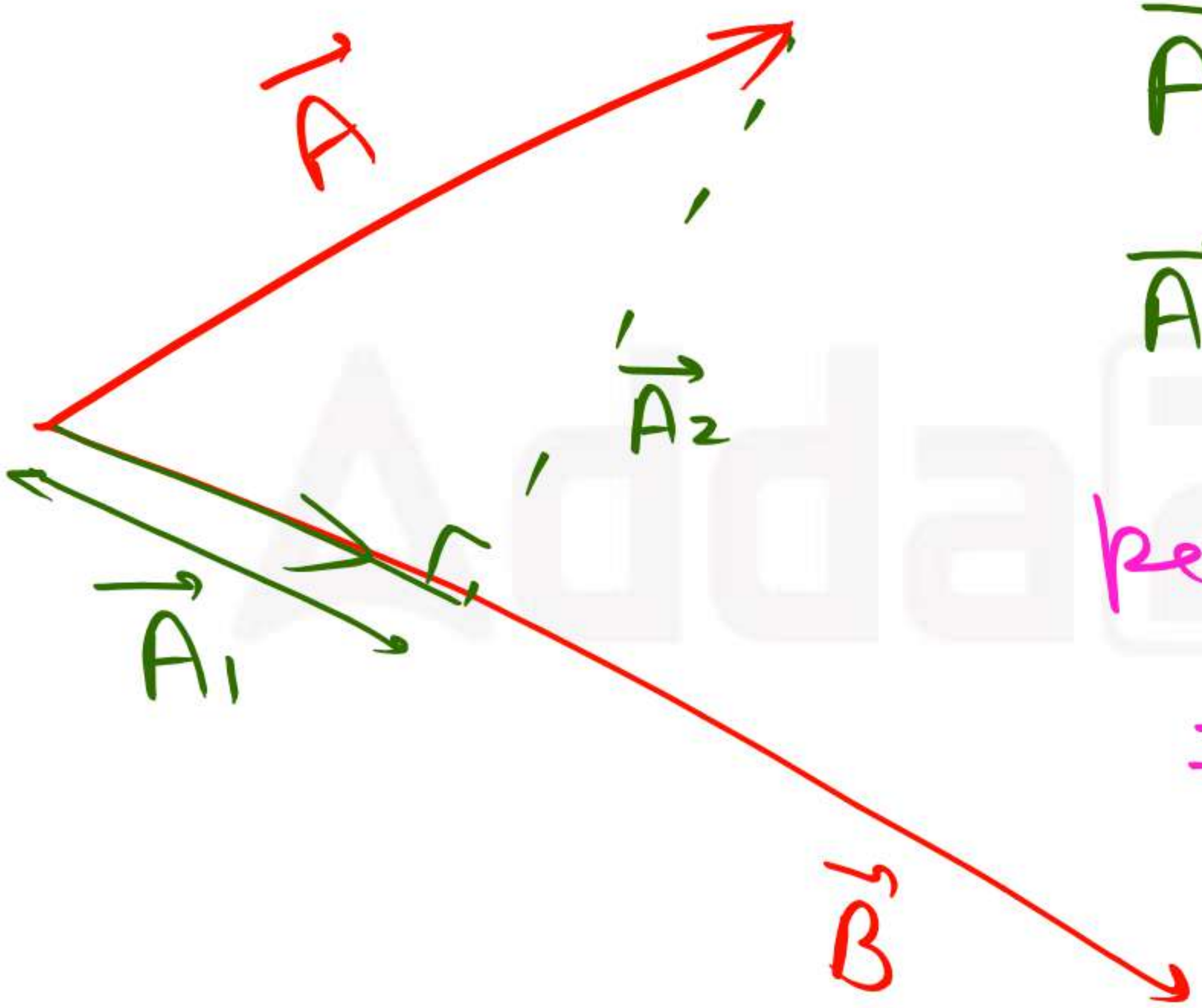
$$= \left(\frac{-2+8}{\sqrt{21}}\right)\left(\frac{2\hat{i}+\hat{j}+4\hat{k}}{\sqrt{21}}\right)$$

$$= \frac{6\,(2\hat{i}+\hat{j}+4\hat{k})}{21}$$

② perpendicular component $= \left(-\hat{i}+2\hat{k}\right) - \left(\frac{4}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{8}{7}\hat{k}\right)$

$$= -\frac{11}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Perpendicular component

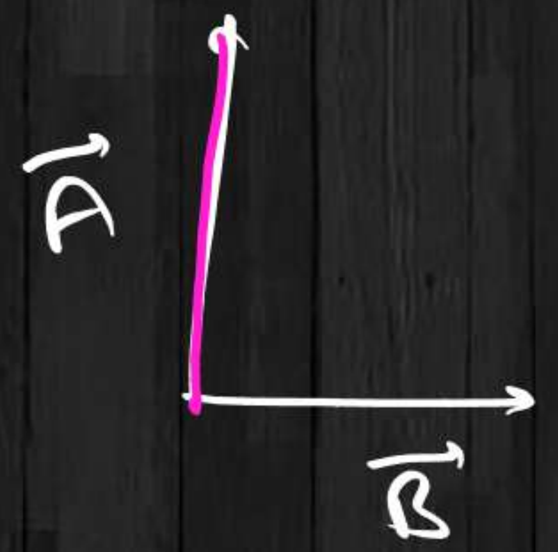


$$\vec{A} = \vec{A_1} + \vec{A_2}$$

$$\vec{A_2} = \vec{A} - \vec{A_1}$$

perpendicular component = Vector - parallel component.

* If $\vec{A} \cdot \vec{B} = 0$



$AB\cos\theta = 0$

$\cos\theta = 0$

$\Rightarrow \theta = 90^\circ$

$\Rightarrow$ $\vec{A}$ & $\vec{B}$ are perpenduculer to each other.

Q:3 if $\vec{A}$ & $\vec{B}$ are perpendicular then parallel Component of $\vec{A}$ to the $\vec{B}$ is _

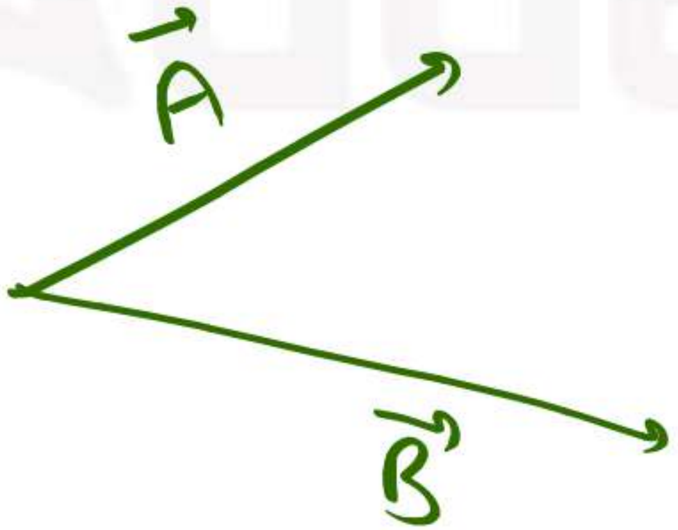
parallel component $= \dfrac{\vec{A} \cdot \vec{B}}{|\vec{B}|}\hat{a}_B = 0$ $\Rightarrow$ Null vector

An

Cross Product

$\vec{A} \times \vec{B} = \underbrace{|\vec{A}||\vec{B}| \sin\theta}_{\text{magnitude}} \; \underset{\text{direction}}{\hat{a}_n} \longrightarrow$ Vector

* Here $\hat{a}_n$ is unit vector in the direction normal to both $\vec{A}$ & $\vec{B}$.



* $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

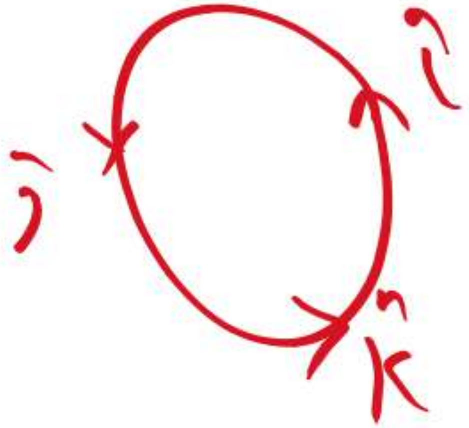
Cross Product

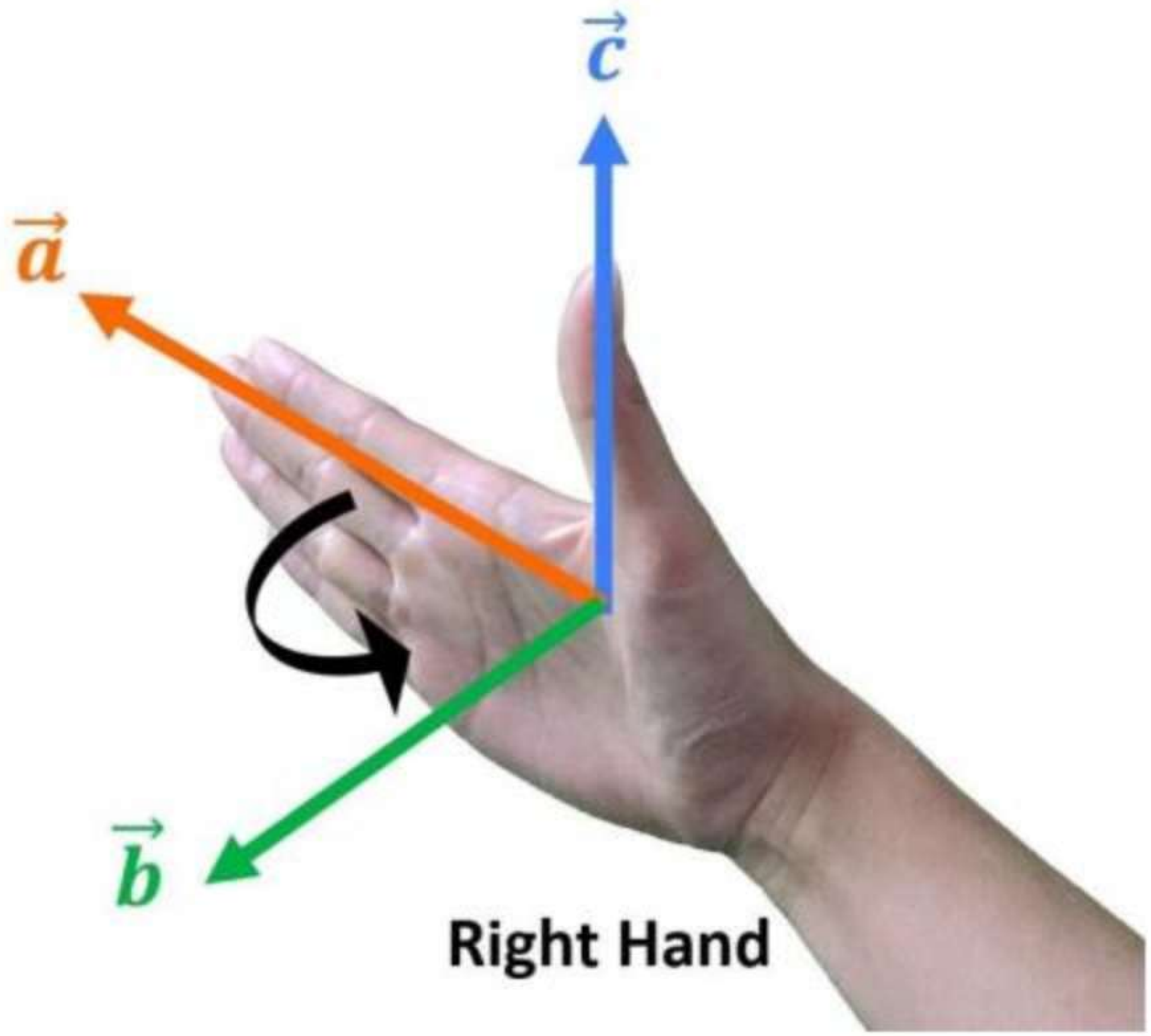
$\vec{A} = 3\hat{i} - 2\hat{j} + \hat{k}$, $\quad \vec{B} = \hat{i} + 2\hat{j} - \hat{k}$

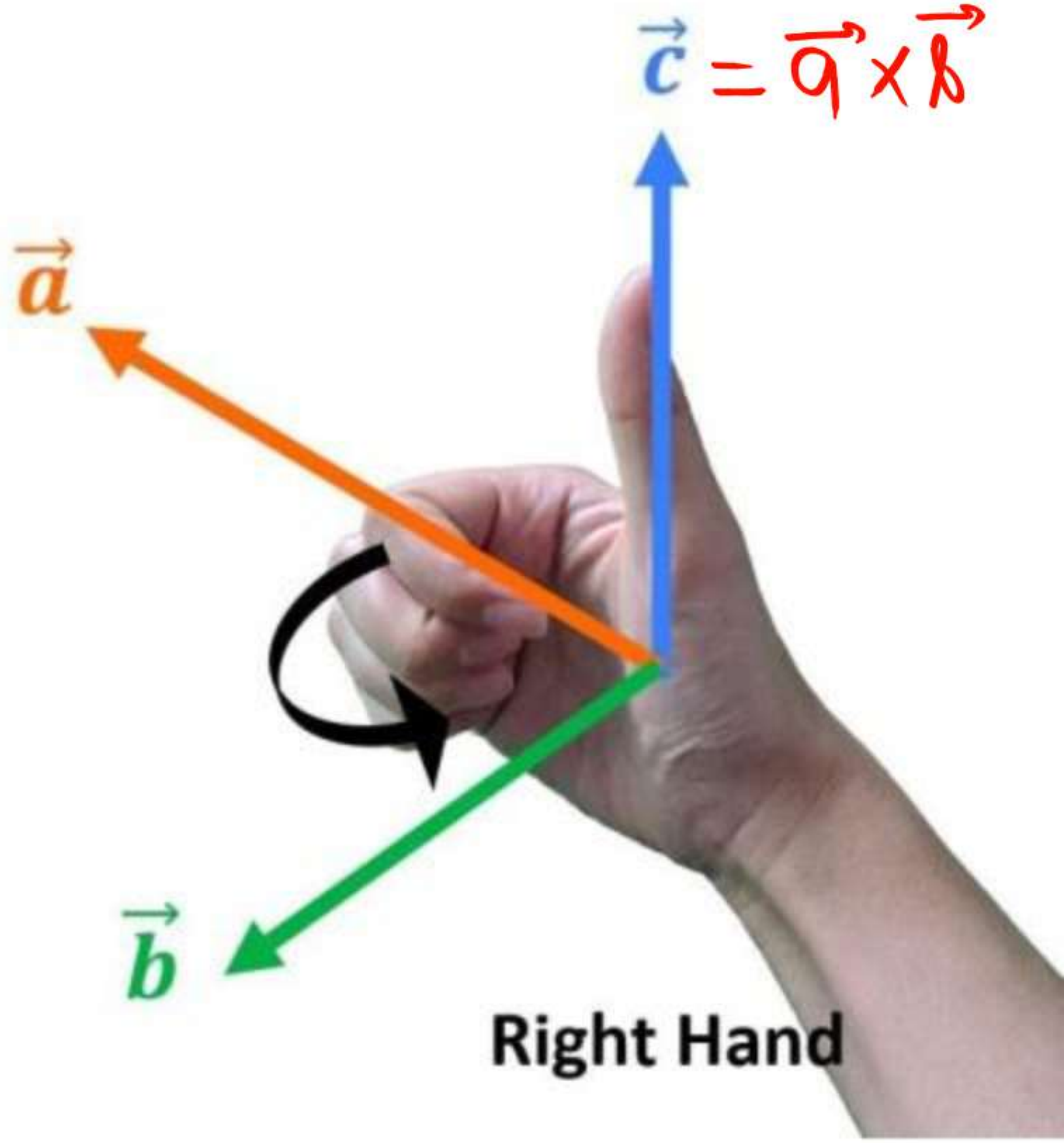
$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 1 \\ 1 & 2 & -1 \end{vmatrix} = 0\hat{i} - 7\hat{j} + 8\hat{k}$$

* $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
* $\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$




$\vec{c}$
$\vec{a}$
$\vec{b}$
Right Hand

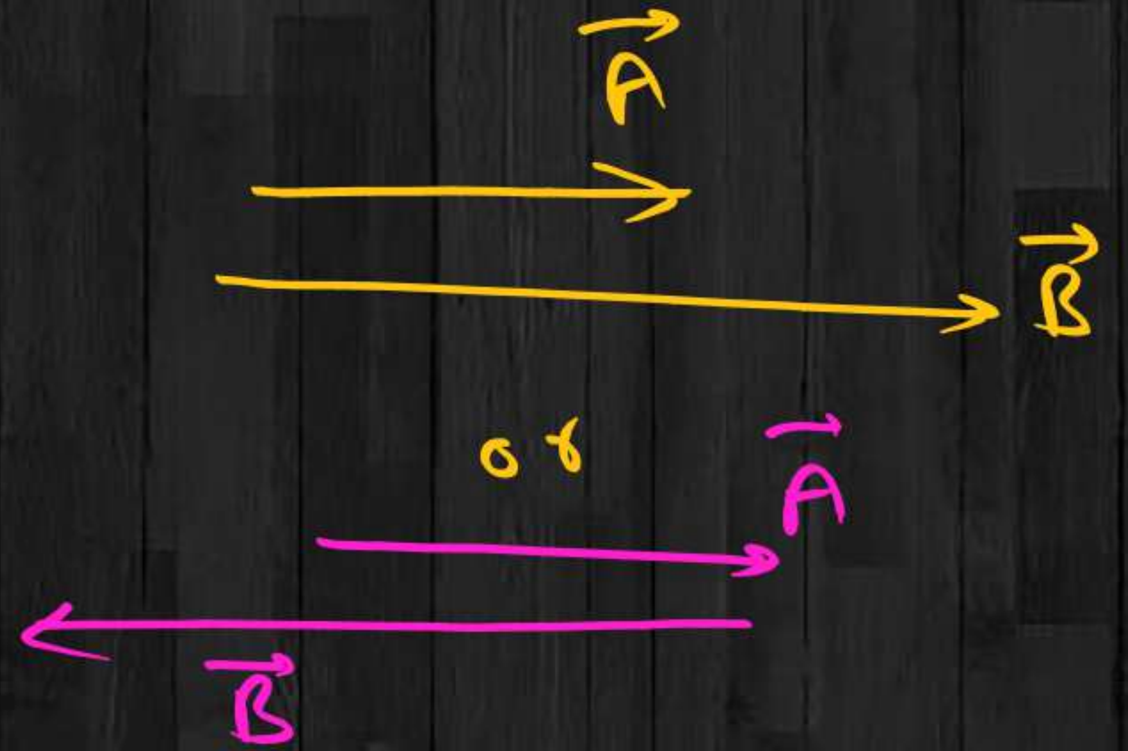

$\vec{c} = \vec{a} \times \vec{b}$
$\vec{a}$
$\vec{b}$
Right Hand

* If $\vec{A} \times \vec{B} = 0$

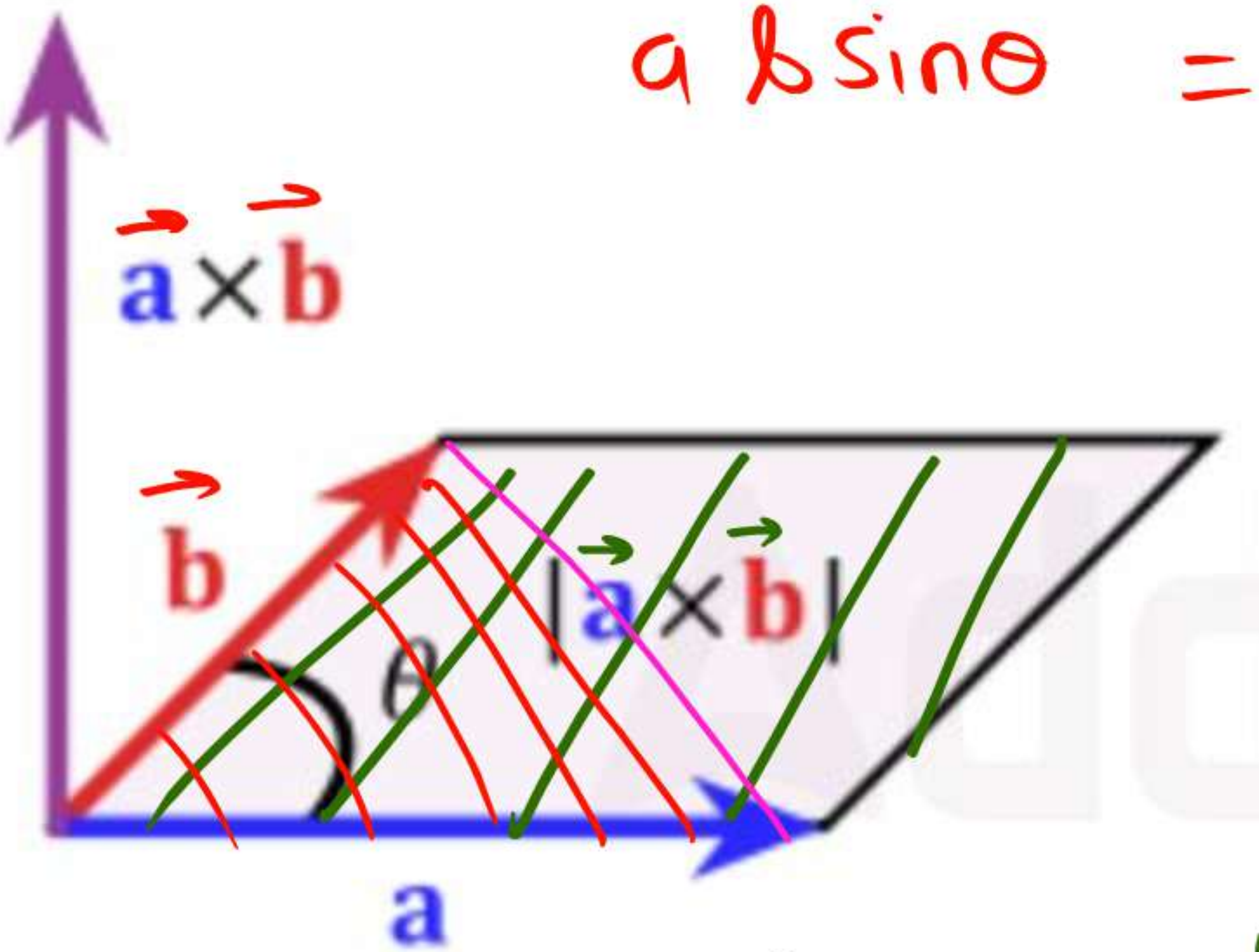
$$AB\sin\theta \, \hat{a}_n = 0$$

$$\Rightarrow \sin\theta = 0$$

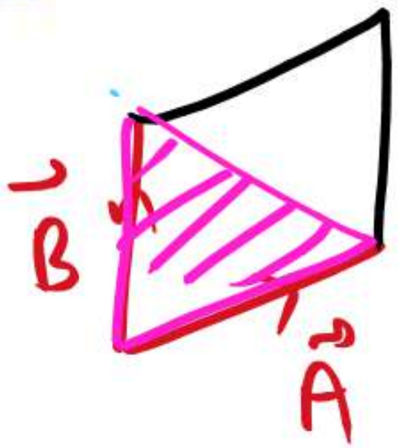
$$\theta = 0^\circ \text{ or } 180^\circ$$



$\Rightarrow$ $\vec{A}$ & $\vec{B}$ are parallel to each other or $\vec{A}$ & $\vec{B}$ are colinear.



$ab\sin\theta = |\vec{a} \times \vec{b}|$ = Area of parallelo-gram corresponding to $\vec{A}$ & $\vec{B}$.



Area of triangle corresponding to $\vec{A}$ & $\vec{B}$

$= \frac{1}{2}|\vec{A} \times \vec{B}| = ab\sin\theta$

Q:4 The inner (dot) product of two non-zero vectors $\bar{P}$ and $\vec{Q}$ is zero. The angle (degrees) between the two vectors is

(a) 0

(b) 30

(c) 90

(d) 120

$\vec{A}\cdot\vec{B} = AB\cos\theta = 0$

$\Rightarrow \cos\theta = 0$

$\theta = 90^\circ$

Q.5 If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $|\vec{a} \times \vec{b}|^2$ will be equal to

(a) $a^2b^2 - (\vec{a} \cdot \vec{b})^2$

(b) $ab - \vec{a} \cdot \vec{b}$

(c) $a^2b^2 + (\vec{a} \cdot \vec{b})^2$

(d) $ab + \vec{a} \cdot \vec{b}$

$\vec{a} \times \vec{b} = ab\sin\theta\, \hat{a}_n$

$|\vec{a} \times \vec{b}| = ab\sin\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2\sin^2\theta$ Ans

$\vec{a} \cdot \vec{b} = ab\cos\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2(1-\cos^2\theta) = a^2b^2 - a^2b^2\cos^2\theta$

$= a^2b^2 - (\vec{a} \cdot \vec{b})^2$

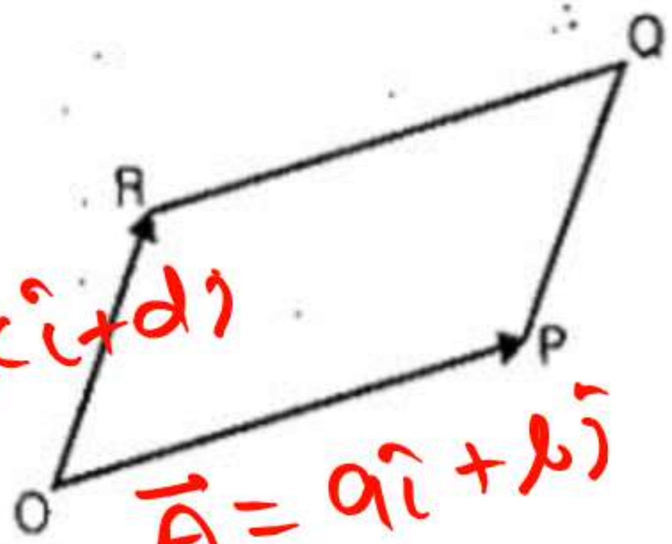
Q.5 For the parallelogram OPQR shown in the sketch, $\overline{OP} = a\hat{i} + b\hat{j}$ and $\overline{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is.

(a) ad - bc

(b) ac + bd

(c) ad + bc

(d) ab - cd



$\vec{B} = c\hat{i} + d\hat{j}$

$\vec{A} = a\hat{i} + b\hat{j}$

$$\text{Area} = |\vec{A} \times \vec{B}|$$

$$\vec{A} \times \vec{B} = \vec{OP} \times \vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = (ad - bc)\hat{k}$$

$$\text{Area} = ad - bc$$

Q.7 The angle between two unit - magnitude coplanar vectors P(0.866, 0.500, 0) and Q(0.259, 0.966, 0) will be

(a) 0°

(b) 30°

(c) 45°

(d) 60°

$A(2,-3,2)$

$\vec{OA} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\theta = \cos^{-1}\left(\frac{\vec{P}\cdot\vec{Q}}{PQ}\right) = \cos^{-1}\left(0.866 \times 0.259 + 0.5 \times 0.966\right)$

$|\vec{P}| = \sqrt{(0.866)^2 + (0.5)^2}$

$= \cos^{-1}(0.707)$

$0.707 = \frac{1}{\sqrt{2}}$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$

Q:8 The area of a triangle formed by the tips of vectors $\vec{a}, \bar{b}$ and $\vec{c}$ is

(a) $\frac{1}{2}(\bar{a}-\vec{b})\cdot(\vec{a}-\bar{c})$

(b) $\frac{1}{2}|(\bar{a}-\bar{b}) \times (\ddot{a}-\bar{c})|$

(c) $\frac{1}{2}|\bar{a} \times \bar{b} \times \bar{c}|$

(d) $\frac{1}{2}(\bar{a} \times \vec{b})\cdot\bar{c}$

E.M.F.T.

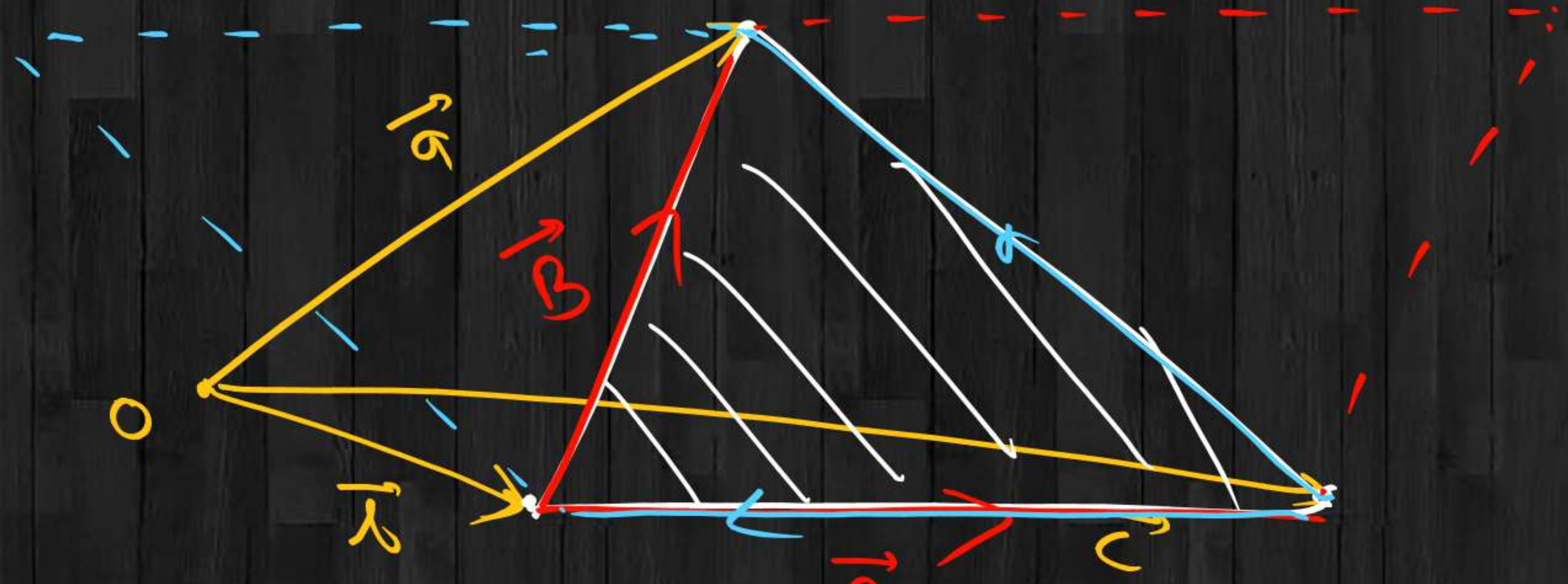
Th, Fr, Sa $\rightarrow$ 9 P.M.

Maths

Sa, Sun $\rightarrow$ 3 P.M.

6 P.M.

9 P.M



Area of triangle $= \frac{1}{2}|\vec{A} \times \vec{B}|$

$$\vec{A} = \vec{c} - \vec{b} \quad , \quad \vec{B} = \vec{a} - \vec{b}$$

$$\text{Area} = \frac{1}{2}\left|(\vec{c} - \vec{b}) \times (\vec{a} - \vec{b})\right|$$

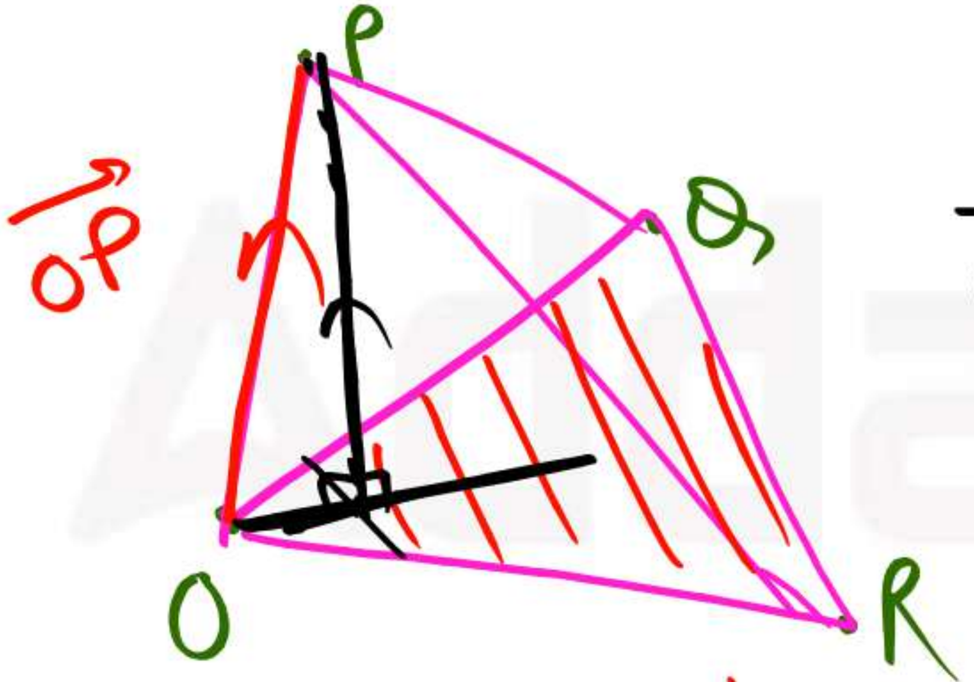
Q.9 If P, Q and R are three points having coordinates (3, - 2, - 1), (1, 3, 4), (2, 1, - 2) in XYZ space, then the distance from point P to plane OQR (O being the origin of the coordinate system) is given by

(a) 3

(b) 5

(c) 7

(d) 9



$\vec{OQ} \times \vec{OR} = \vec{X}$

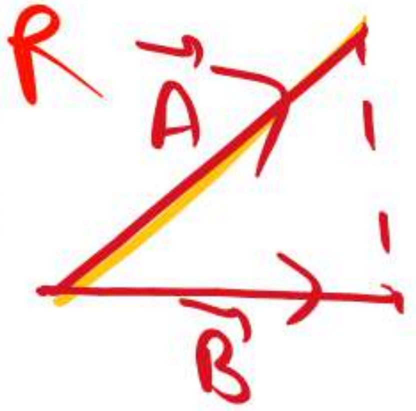
GATE

AAI ATC

1 marks - 1.98

2 marks - 3.96

$\vec{X}$ = normal vector to plane OQR



$\vec{OQ} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{OR} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{X} = \vec{OQ} \times \vec{OR} = ?$

$$= \vec{OQ} \times \vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix} = -10\hat{i} + 10\hat{j} - 5\hat{k}$$

$$\vec{OP} = 3\hat{i} - 2\hat{j} - \hat{k}$$

$$\frac{\vec{OP} \cdot \vec{X}}{|\vec{X}|} = \frac{-30 - 20 + 5}{\sqrt{100 + 100 + 25}} = \frac{-45}{15} = \underline{-3}$$

distance $= 3$

GATE 2024

3 P.M. → Sat & Sunday

6 P.M

9 PM →

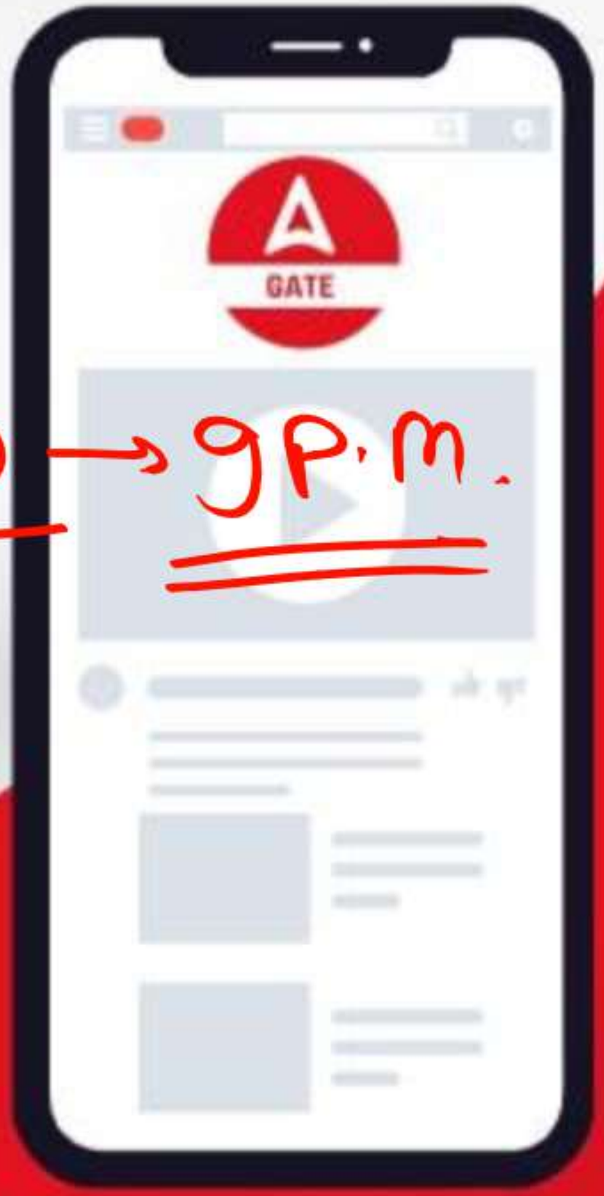
प्रचण्ड Batch

Engineering Mathematics

LINEAR ALGEBRA

OPERATIONS AND CLASSIFICATIONS OF MATRICES

Thursday → 9 p.m.



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