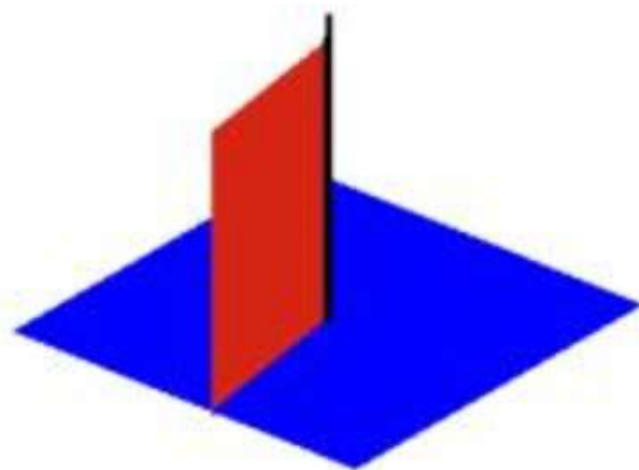
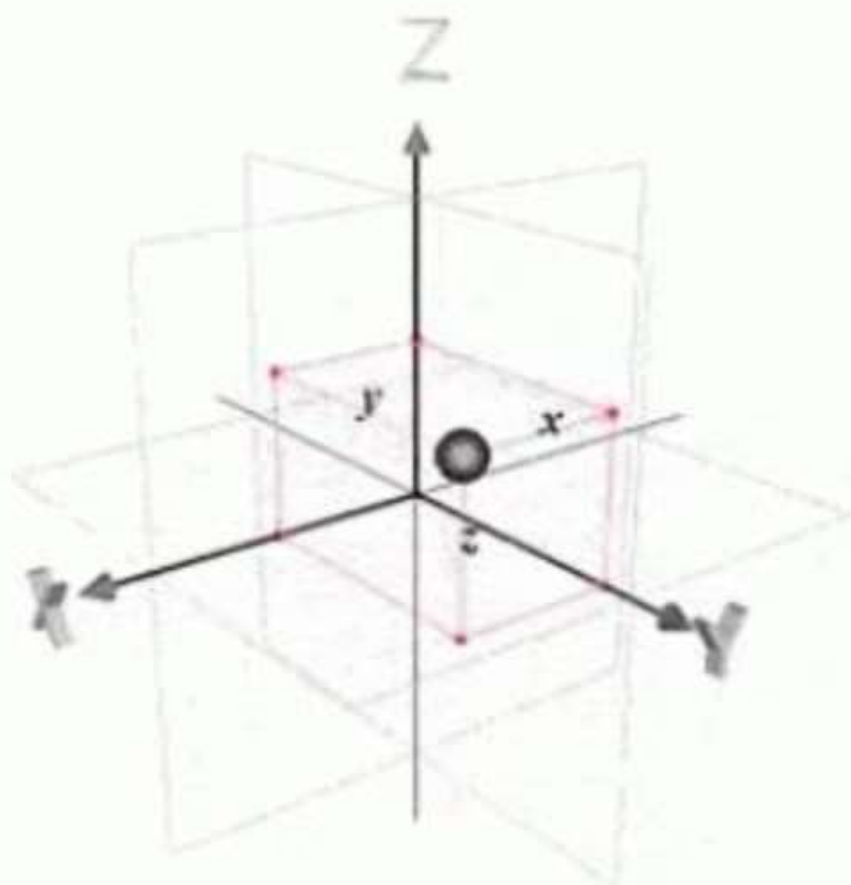
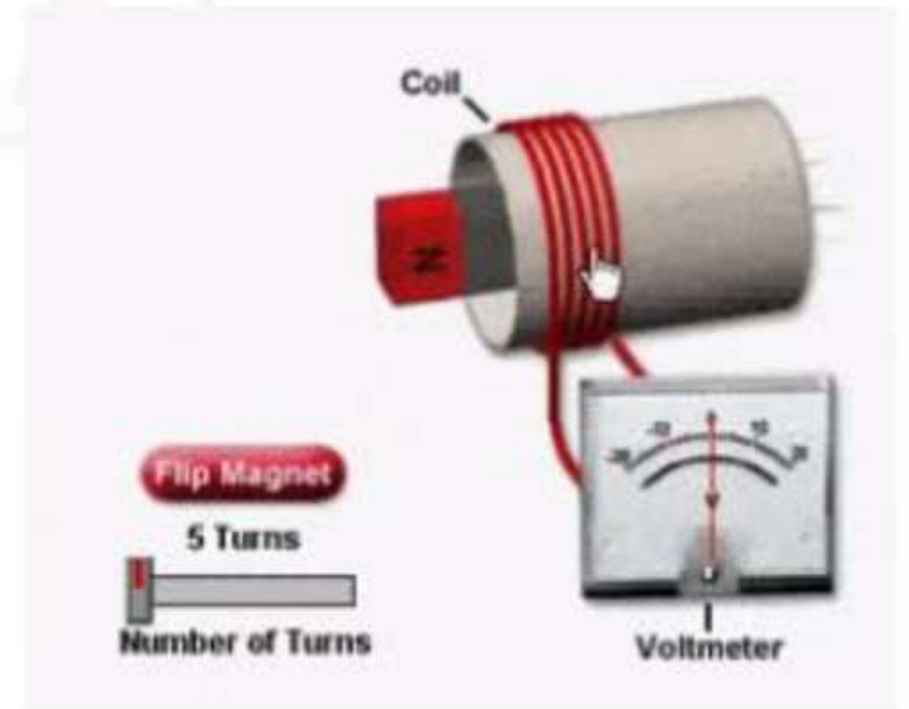
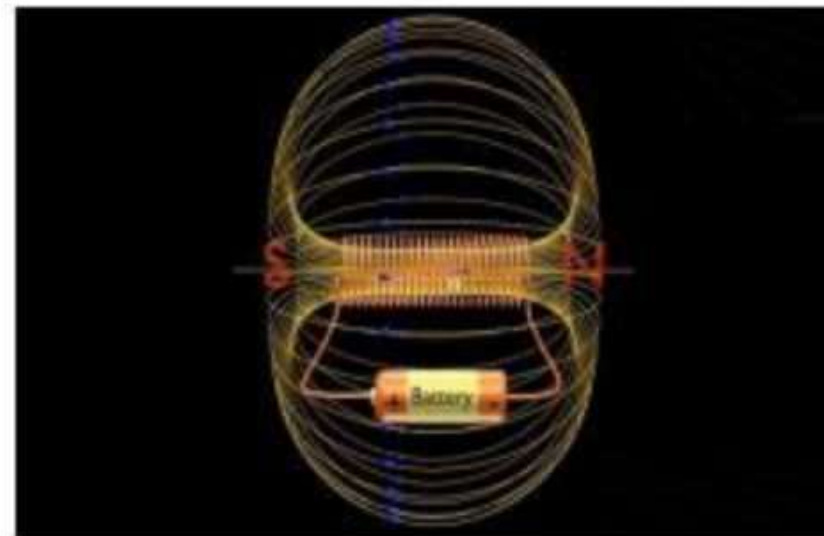
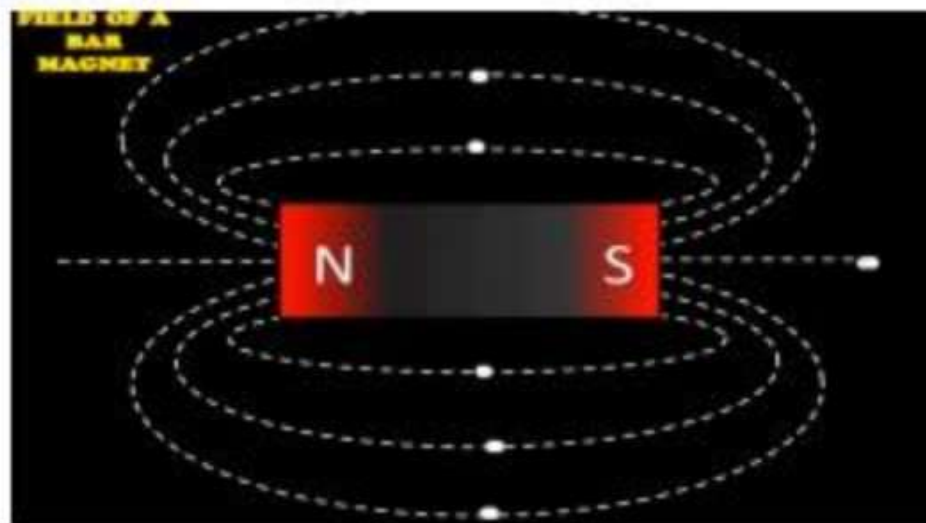
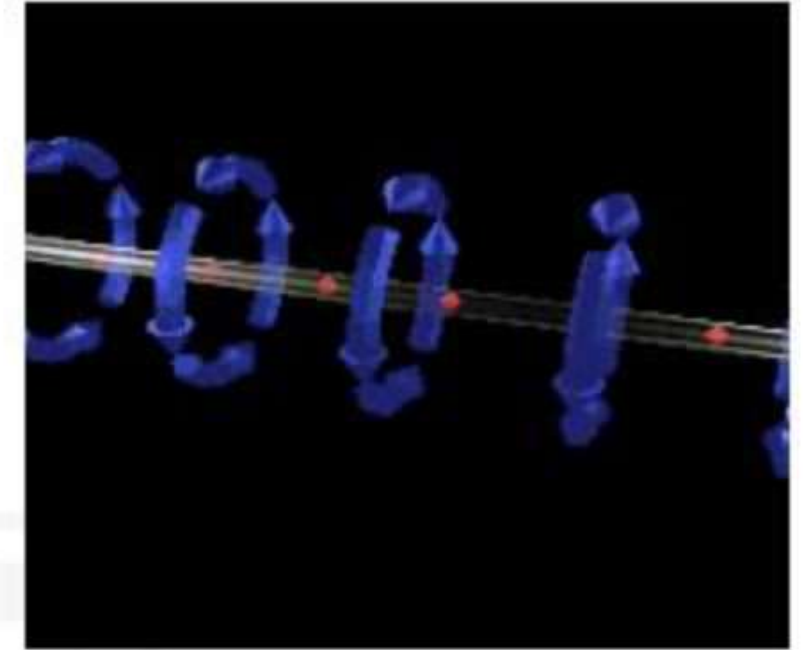
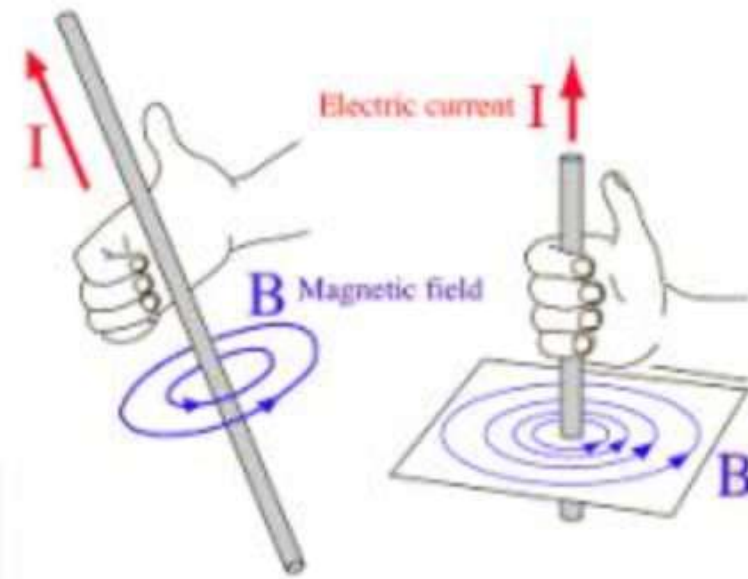
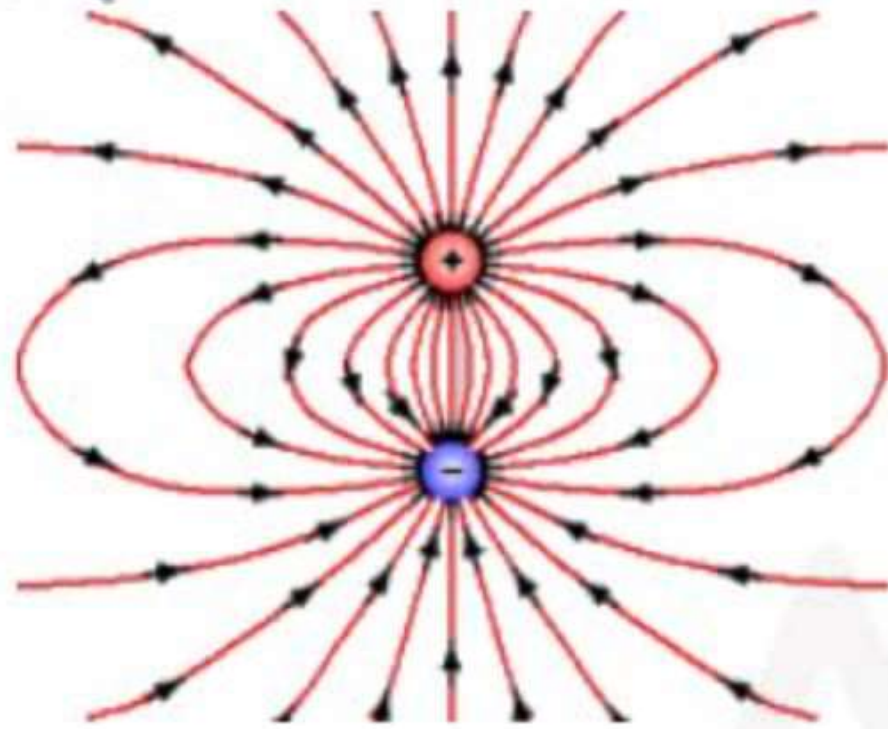




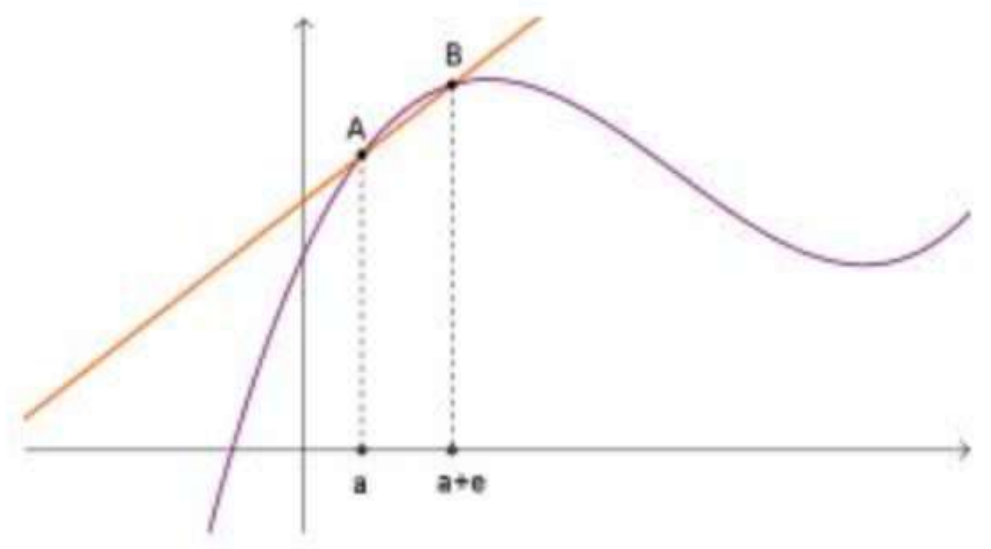
1. Triple Products and its Applications
2. Coordinate systems



Recap



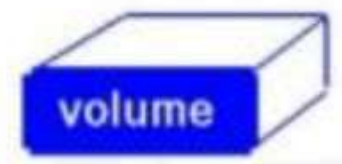
Recap



A scalar quantity has only **magnitude**.
 A vector quantity has both **magnitude** and **direction**.

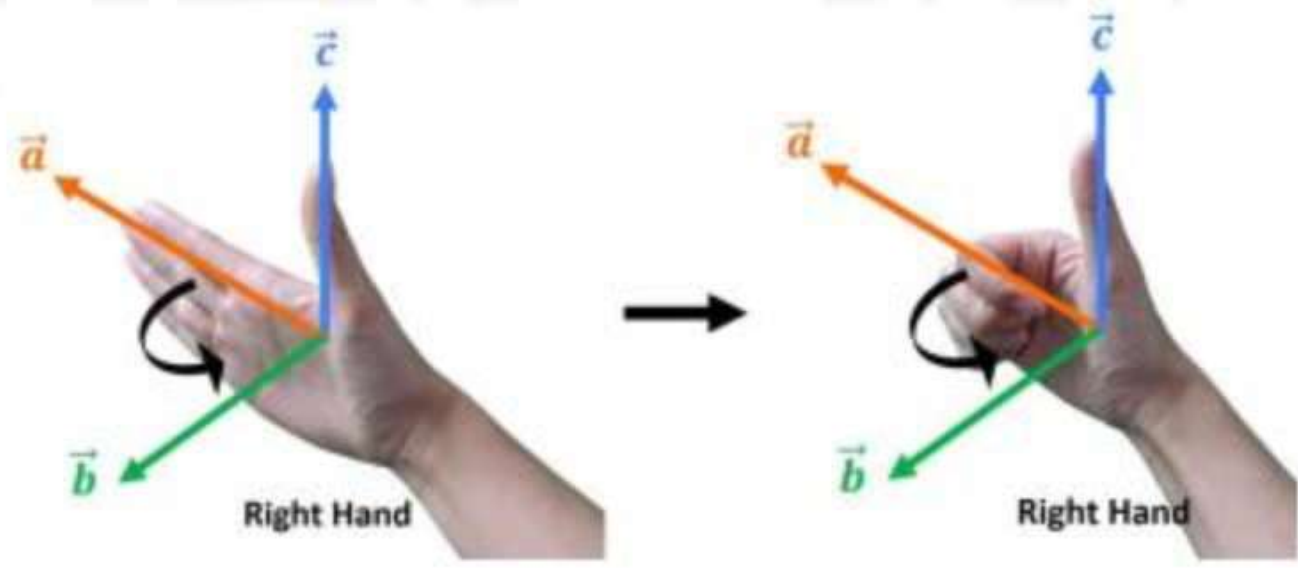
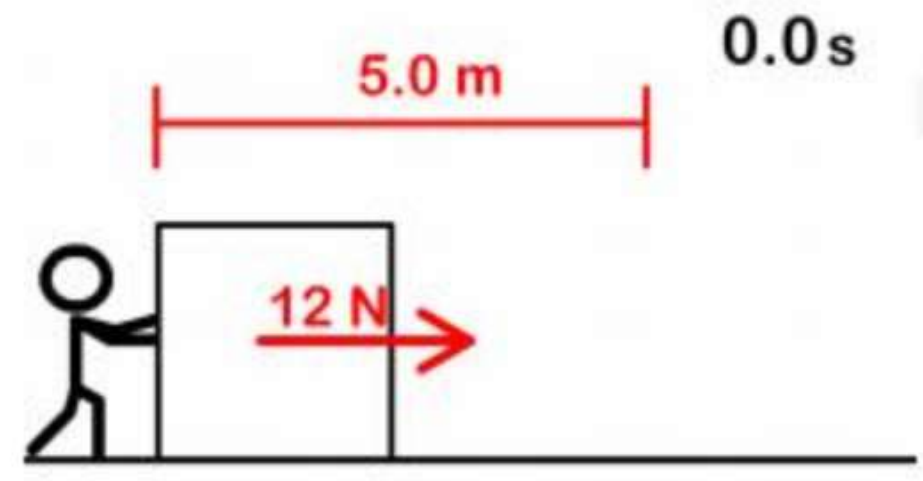
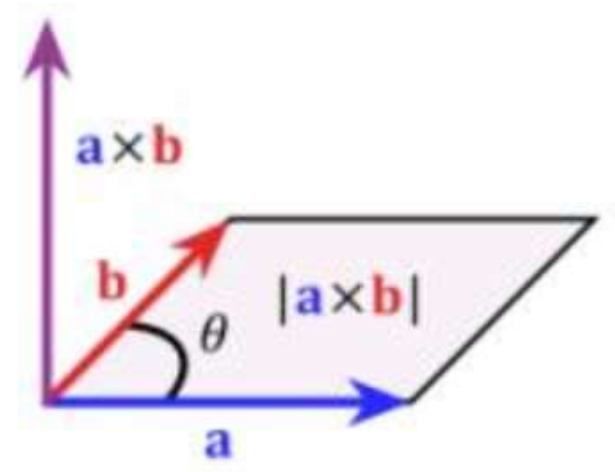
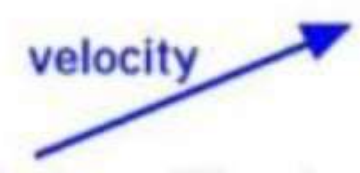
Scalar Quantities

- length, area, volume
- speed
- mass, density
- pressure
- temperature
- energy, entropy
- work, power



Vector Quantities

- displacement
- velocity
- acceleration
- momentum
- force
- lift, drag, thrust
- weight



Recap

Projection of a vector on other vector/Parallel component

$\vec{a} \cdot \vec{b} = ab \cos \theta$
 $= \underline{a \cos \theta} b$
 Projection = $a \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
 $\vec{a} = \vec{a}_1 + \vec{a}_2$
 $\vec{a}_1 =$ parallel component of \vec{a} in the direction \vec{b}
 $\vec{a}_2 =$ perpendicular component of \vec{a} to the \vec{b}
 $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = |\vec{a}| \cos \theta$

← Applications of DOT Product

Applications of DOT Product →

$a b \sin \theta = |\vec{a} \times \vec{b}| = \text{Area of parallelogram corresponding to } \vec{A} \& \vec{B}.$
 Area of triangle corresponding to $\vec{A} \& \vec{B}$
 $= \frac{1}{2} |\vec{A} \times \vec{B}| = ab \sin \theta$

Q. If \vec{a} and \vec{b} are two arbitrary vectors with magnitudes a and b , respectively, $|\vec{a} \times \vec{b}|^2$ will be equal to

(a) $a^2b^2 - (\vec{a} \cdot \vec{b})^2$
 (b) $ab - \vec{a} \cdot \vec{b}$
 (c) $a^2b^2 + (\vec{a} \cdot \vec{b})^2$
 (d) $ab + \vec{a} \cdot \vec{b}$

$\vec{a} \times \vec{b} = ab \sin \theta \hat{a}_n$
 $|\vec{a} \times \vec{b}| = ab \sin \theta$
 $|\vec{a} \times \vec{b}|^2 = a^2 b^2 \sin^2 \theta$ Ans
 $\vec{a} \cdot \vec{b} = ab \cos \theta$ $\sin^2 \theta = 1 - \cos^2 \theta$
 $|\vec{a} \times \vec{b}|^2 = a^2 b^2 (1 - \cos^2 \theta) = a^2 b^2 - a^2 b^2 \cos^2 \theta$
 $= a^2 b^2 - (\vec{a} \cdot \vec{b})^2$

Q. The angle between two unit - magnitude coplanar vectors P(0.866, 0.500, 0) and Q(0.259, 0.966, 0) will be

(a) 0°
 (b) 30°
 (c) 45°
 (d) 60°

$\theta = \cos^{-1} \left(\frac{0.866 \times 0.259 + 0.500 \times 0.966}{\sqrt{0.866^2 + 0.500^2} \sqrt{0.259^2 + 0.966^2}} \right)$
 $= \cos^{-1} \left(\frac{0.707}{1 \times 1} \right) = \cos^{-1} \left(\frac{1}{\sqrt{2}} \right) = 45^\circ$

$A(2, -3, 2)$
 $\vec{OA} = 2\hat{i} - 3\hat{j} + 2\hat{k}$
 $|\vec{OA}| = \sqrt{2^2 + (-3)^2 + 2^2} = \sqrt{17}$

Number of Questions covered-9

Q. For the parallelogram OPQR shown in the sketch, $\vec{OP} = a\hat{i} + b\hat{j}$ and $\vec{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is.

(a) $ad - bc$
 (b) $ac + bd$
 (c) $ad + bc$
 (d) $ab - cd$

$\vec{A} \times \vec{B} = \vec{OP} \times \vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = (ad - bc)\hat{k}$

$\text{Area} = |\vec{A} \times \vec{B}| = |ad - bc|$

Q. P, Q and R are three points having coordinates (3, -2, -1), (1, 3, 4), (2, 1, -2) in XYZ space, then the distance from point P to plane OQR (O being the origin of coordinate system) is given by

$\vec{OQ} \times \vec{OR} = \vec{X}$

$\vec{OQ} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{OR} = 2\hat{i} + \hat{j} - 2\hat{k}$

$\vec{OP} \times \vec{X} = ?$

$\text{Distance} = \frac{|\vec{OP} \cdot \vec{X}|}{|\vec{X}|}$

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AAI ATC
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 2 mark - 336

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GATE 2024



प्रचण्ड Batch

Electromagnetic Field Theory

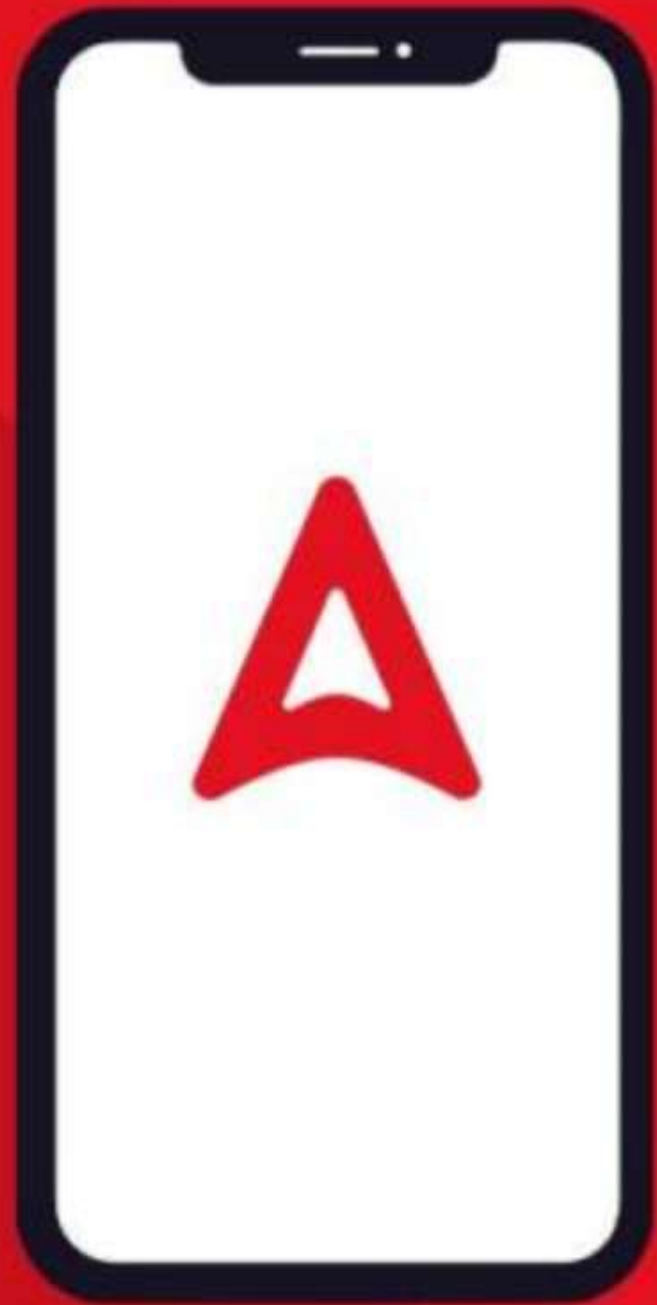
COORDINATE SYSTEMS

LEC-03

EE & ECE



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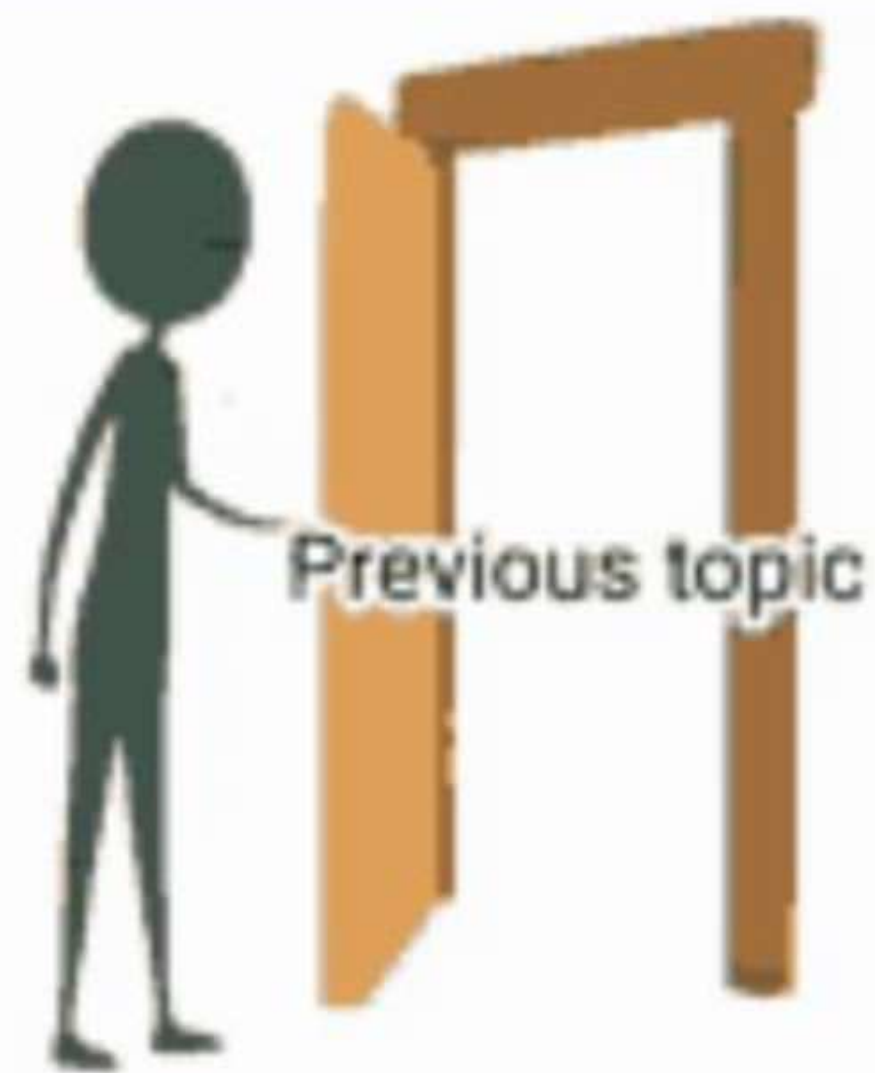
Power Capsule



Notes & Articles



Videos

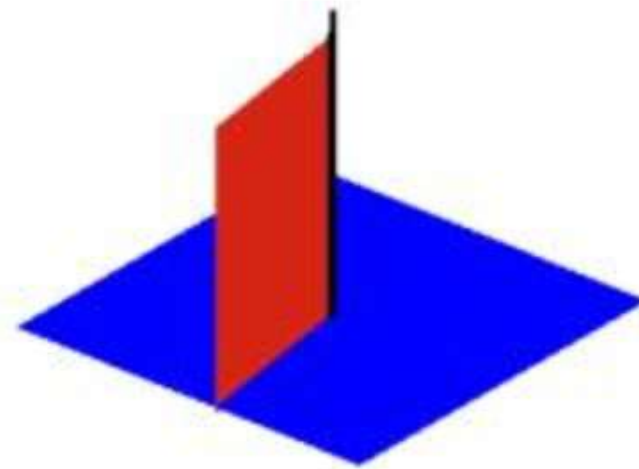
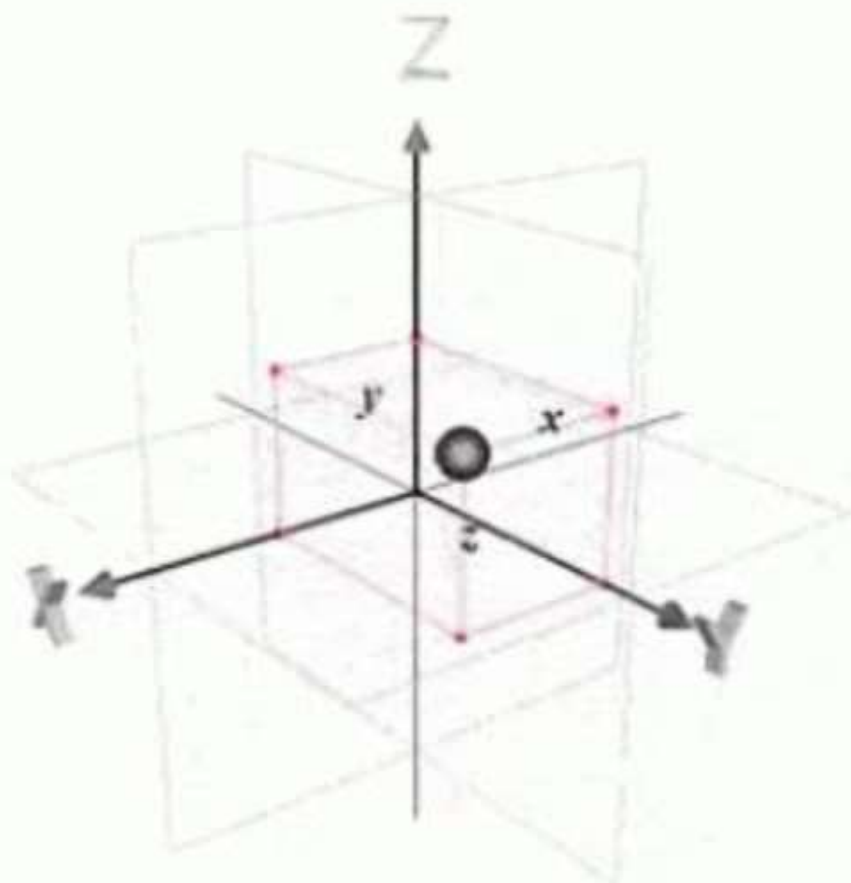


- 1. Basic introduction of Fields**
- 2. Vectors, Scalars and Tensors**
- 3. Position vector and vector between points**
- 4. Magnitude and direction of vector**
- 5. Dot and cross products and its applications**

247



- 1. Triple Products and its Applications**
- 2. Coordinate systems**



Triple Product:- $\vec{A}, \vec{B}, \vec{C}$

\times 1. $\vec{A} \cdot \vec{B} \cdot \vec{C} =$ invalid

$\vec{A} \cdot \vec{B} \rightarrow$ Scalar

\checkmark 2. $(\vec{A} \times \vec{B}) \times \vec{C} =$ Vector tripple product $5\vec{A}$

\times 3. $(\vec{A} \cdot \vec{B}) \times \vec{C} =$ invalid

\checkmark 4. $(\vec{A} \times \vec{B}) \cdot \vec{C} =$ Scalar tripple product.

$(\vec{A} \times \vec{B}) \cdot \vec{C}$

$\vec{A} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + \hat{k}$

$\vec{C} = 3\hat{i} + \hat{j} - \hat{k}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix} = -3\hat{i} - \hat{j} + 5\hat{k}$

$(\vec{A} \times \vec{B}) \cdot \vec{C} = 9 - 1 - 5 = 3$

* if $(\vec{A} \times \vec{B}) \cdot \vec{C} = 0$
then \vec{A}, \vec{B} & \vec{C} are called coplaner vector.

$$(\vec{A} \times \vec{B}) \times \vec{C} = \text{BAC-CAB rule}$$

$$(\vec{A} \times \vec{B}) \times \vec{C} = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

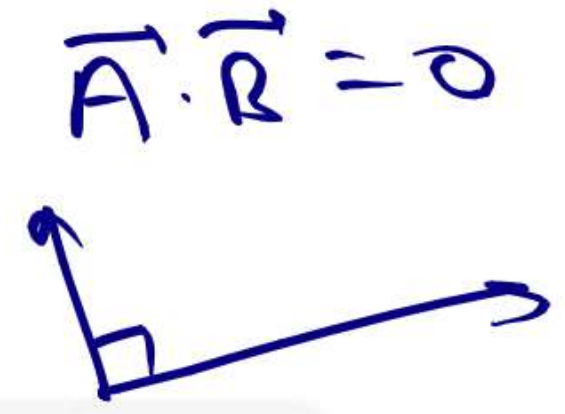
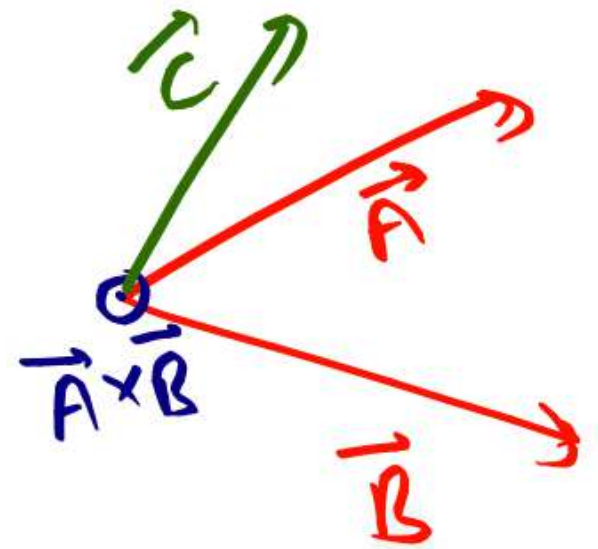
$$\vec{P} \times \vec{Q} \times \vec{R} = \vec{Q}(\vec{P} \cdot \vec{R}) - \vec{R}(\vec{P} \cdot \vec{Q})$$

$$\vec{X} \times \vec{M} \times \vec{D} = \vec{M}(\vec{X} \cdot \vec{D}) - \vec{D}(\vec{X} \cdot \vec{M})$$

(Q: $\nabla \times \nabla \times \vec{P}$ is -

$$\nabla(\nabla \cdot \vec{P}) - \vec{P}(\nabla \cdot \nabla) \Rightarrow \nabla(\nabla \cdot \vec{P}) - \nabla^2 \vec{P}$$

Coplanar Vectors



if

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = 0$$

$\Rightarrow \vec{A} \times \vec{B}$ & \vec{C} are normal to each other.

& $(\vec{A} \times \vec{B})$ is always normal to both \vec{A} & \vec{B} .

$\Rightarrow \vec{A}, \vec{B}$ & \vec{C} are in the same plane.

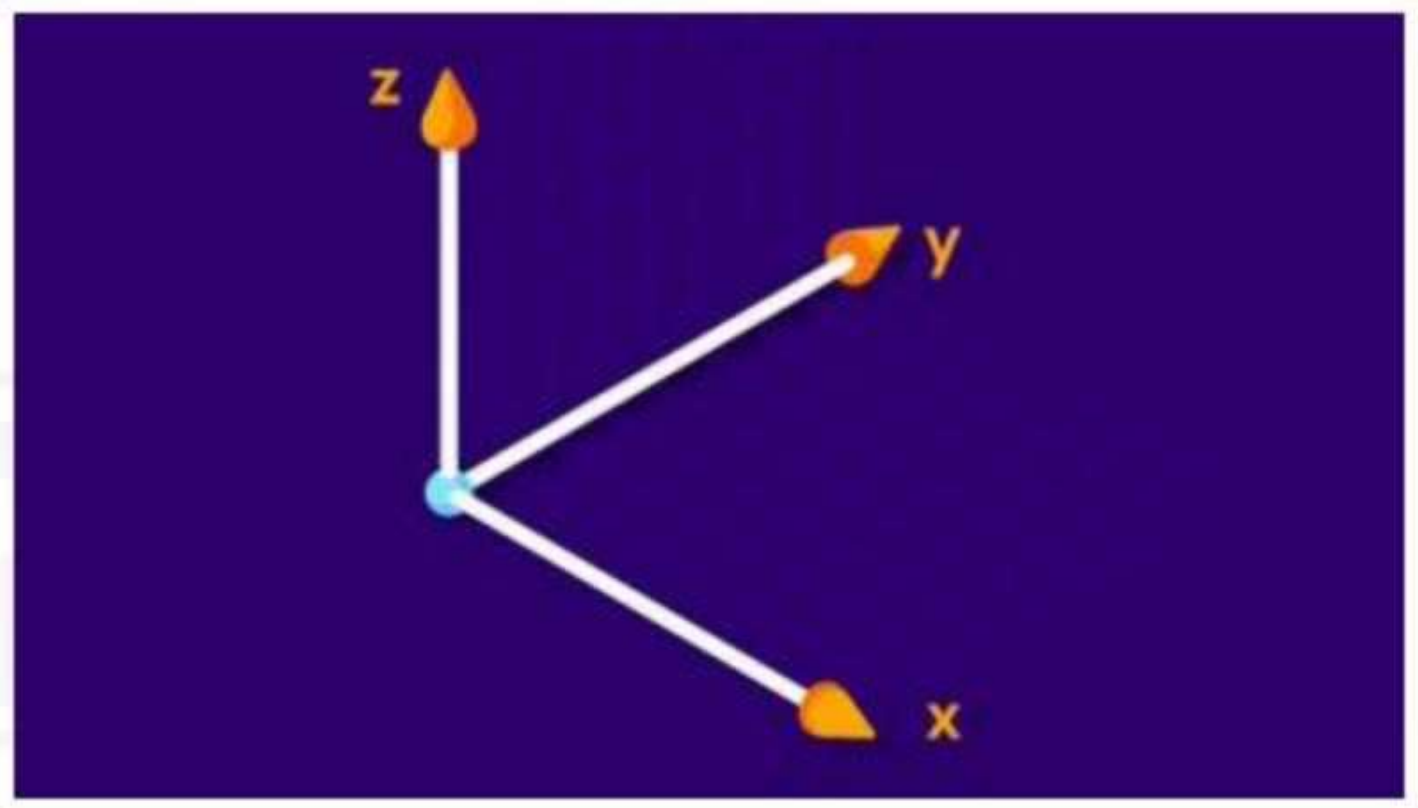
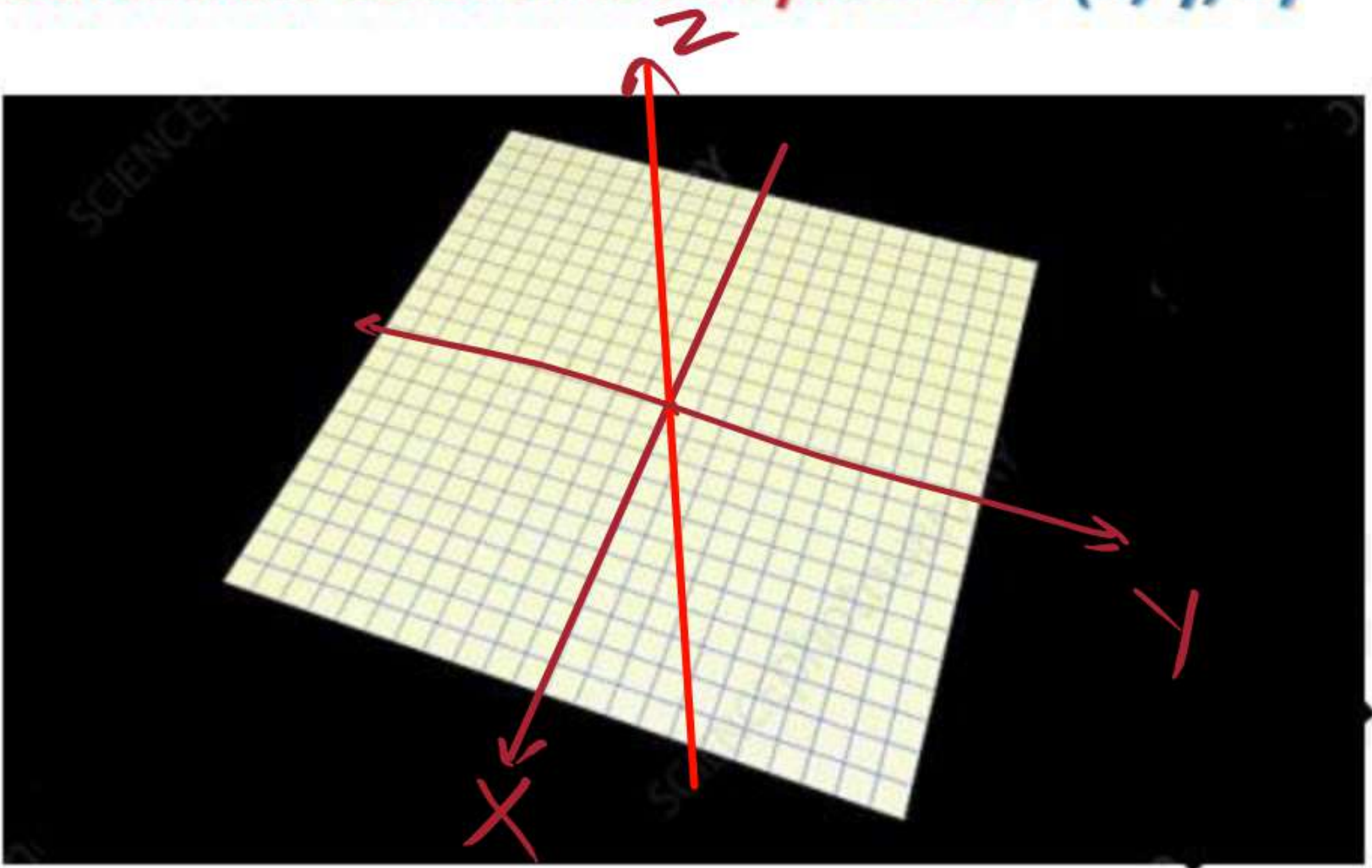
$$(\vec{A} \times \vec{B}) \cdot \vec{C} = 0 \rightarrow \text{coplanar vector.}$$

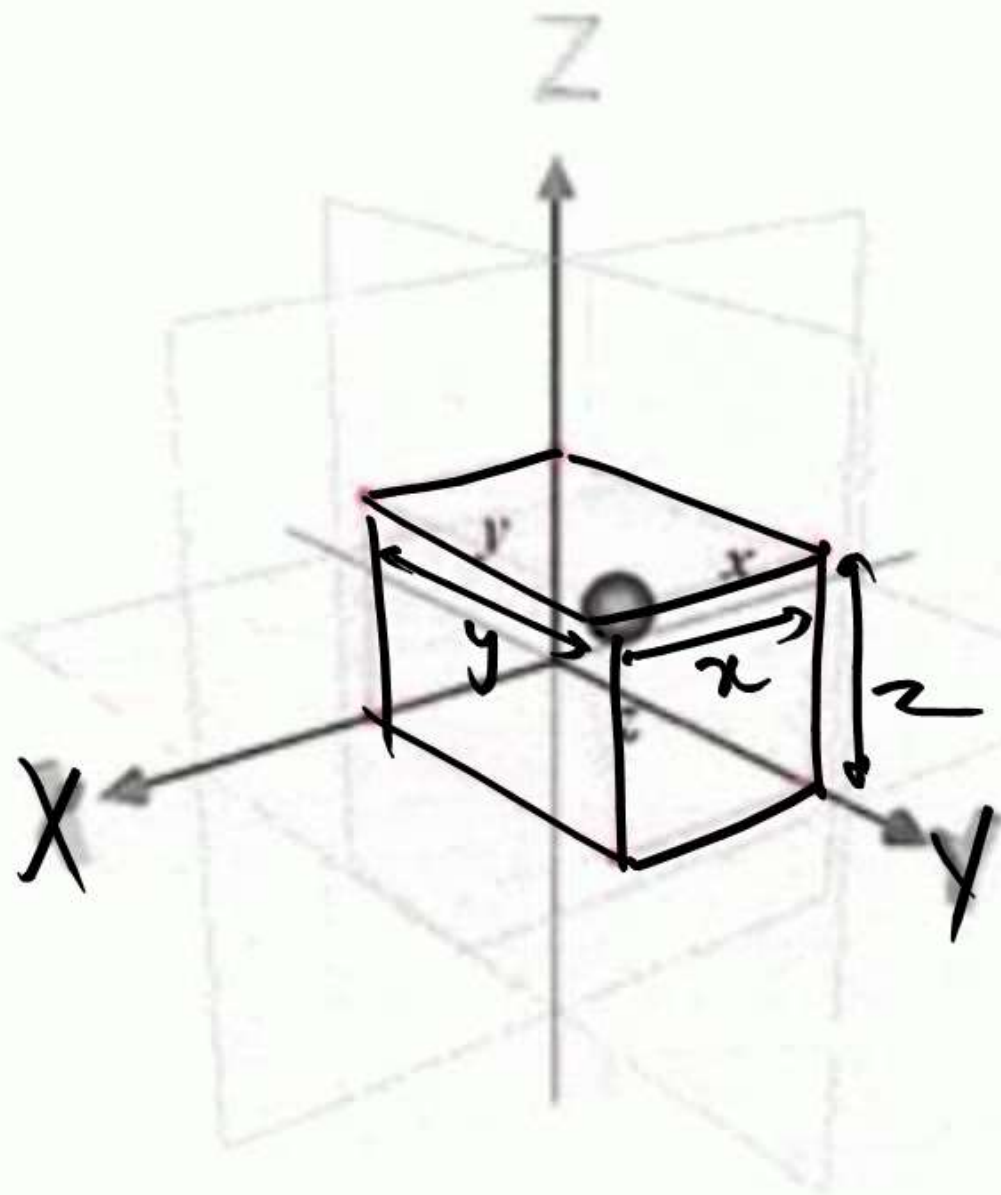
\searrow scalar

$$(\vec{A} \times \vec{B}) \times \vec{C} \rightarrow \text{vector}$$

$$\vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Cartesian Coordinate systems $P(x, y, z)$





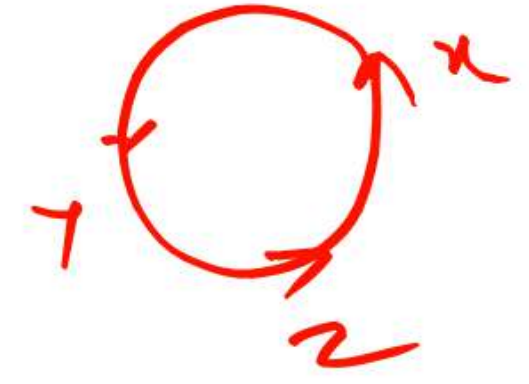
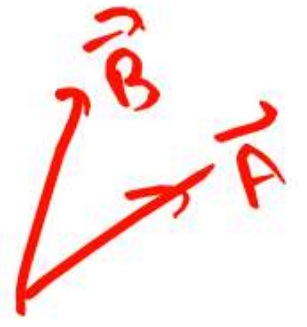
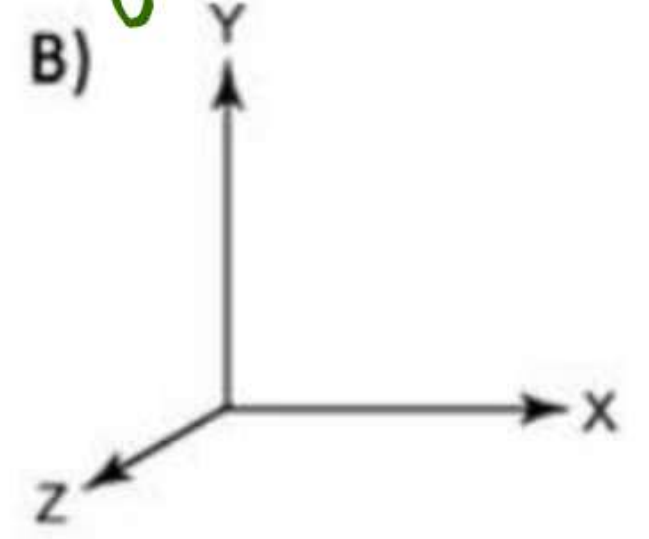
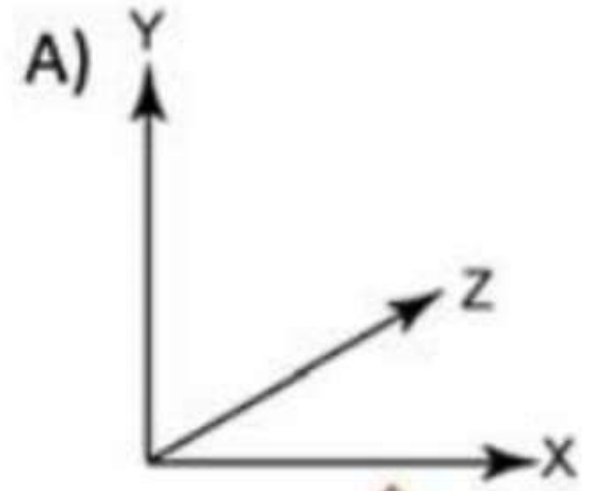
$$\underline{-\infty < x < \infty}$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$

Left hand and right hand cartesian coordinate systems

By default



Right hand
Co-ordinate
System

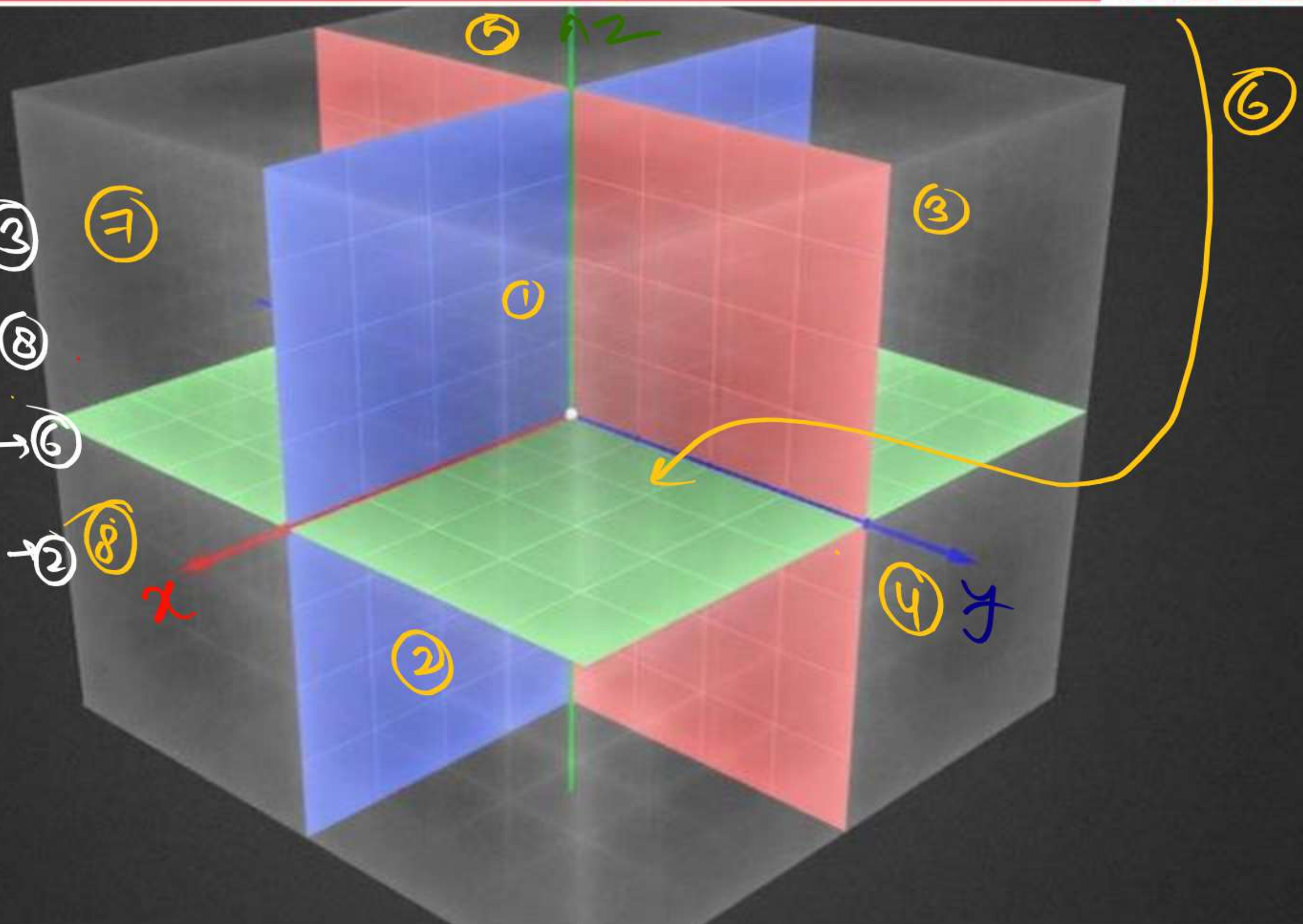
Points

$A(-3, 1, 5) \rightarrow \textcircled{3}$

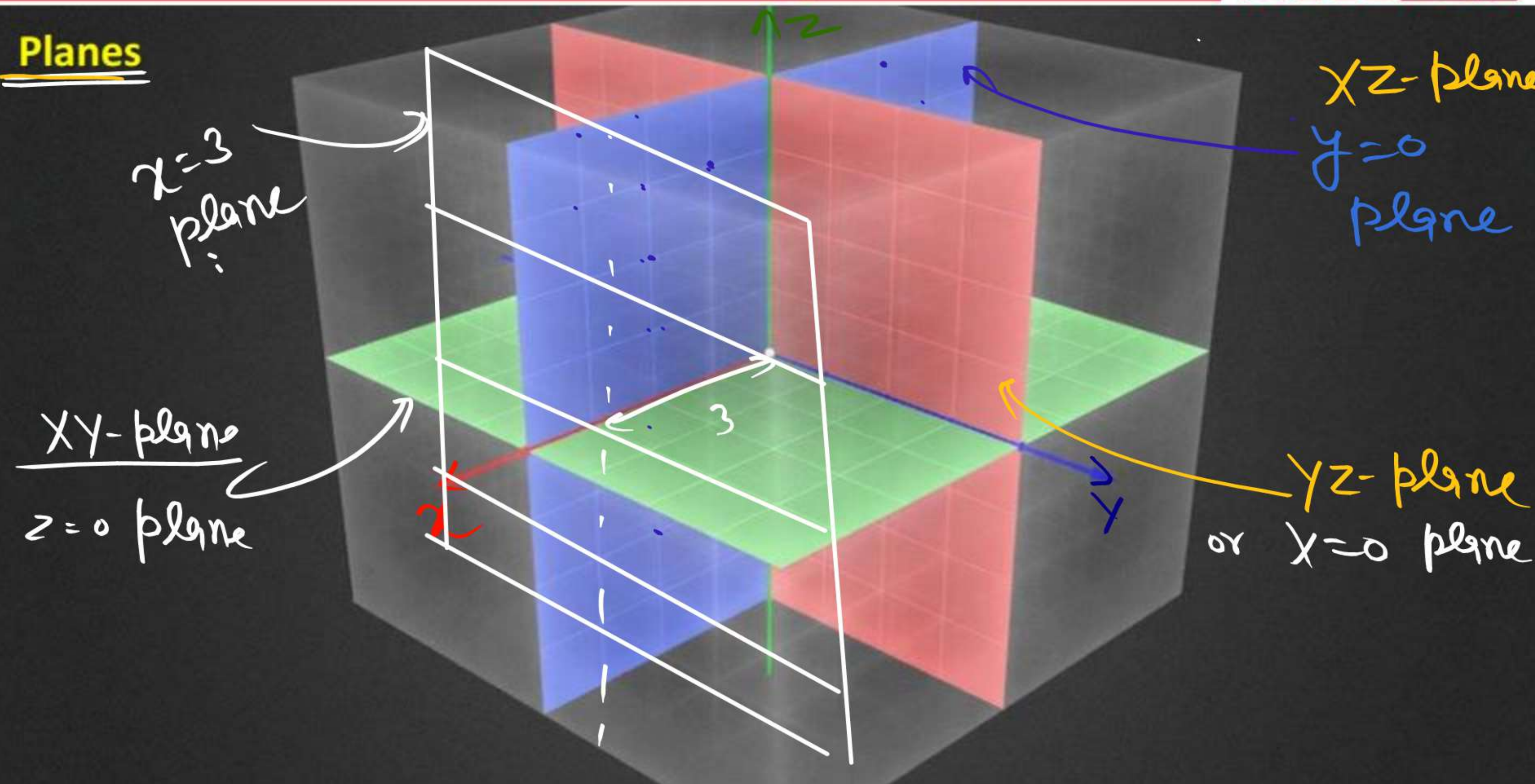
$B(2, -1, -3) \rightarrow \textcircled{8}$

$C(-2, -4, -1) \rightarrow \textcircled{6}$

$D(2, 1, -4) \rightarrow \textcircled{2}$



Planes



$x = \text{constant}$

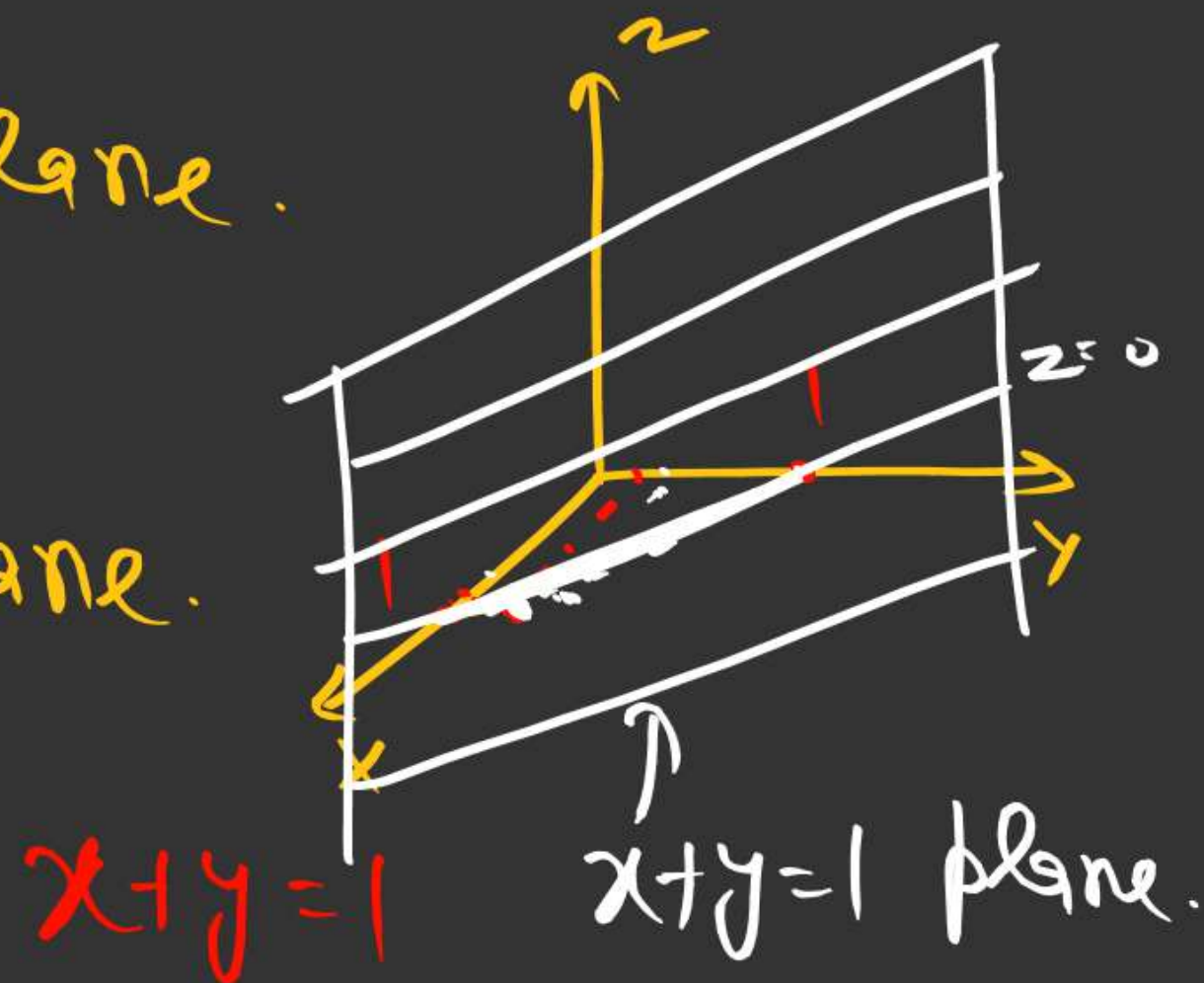
plane parallel to YZ -plane ($x=0$) plane.

$y = \text{constant}$

plane parallel to XZ -plane.

$z = \text{constant}$

plane parallel to XY -plane.



Lines

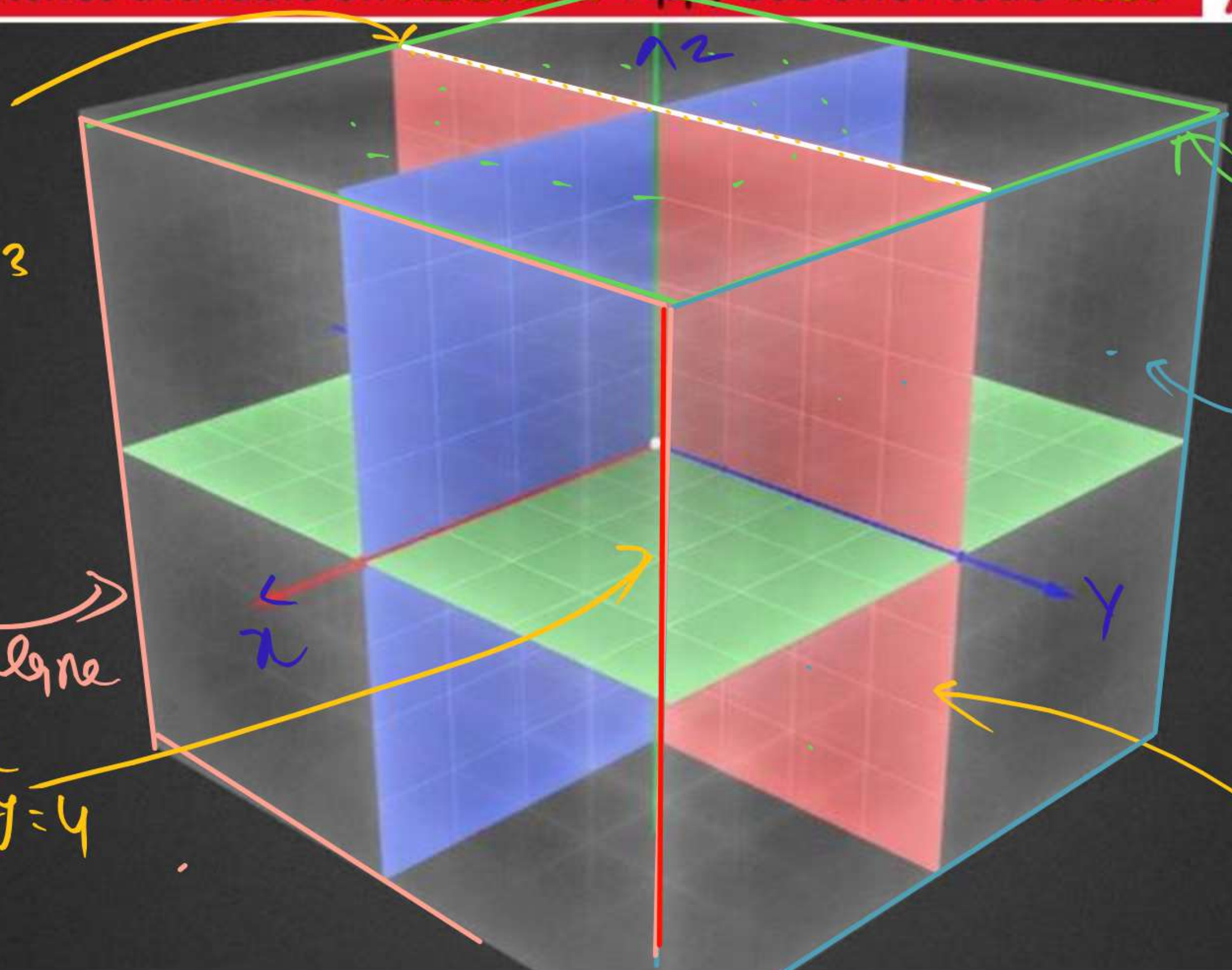
line
 $x=0 \& z=3$

$x=5$ plane
line
 $x=5 \& y=4$

$z=3$ plane

$y=4$ plane

$x=0$ plane



$$x=r \text{ \& } y=k$$

→ line parallel to z-axis.

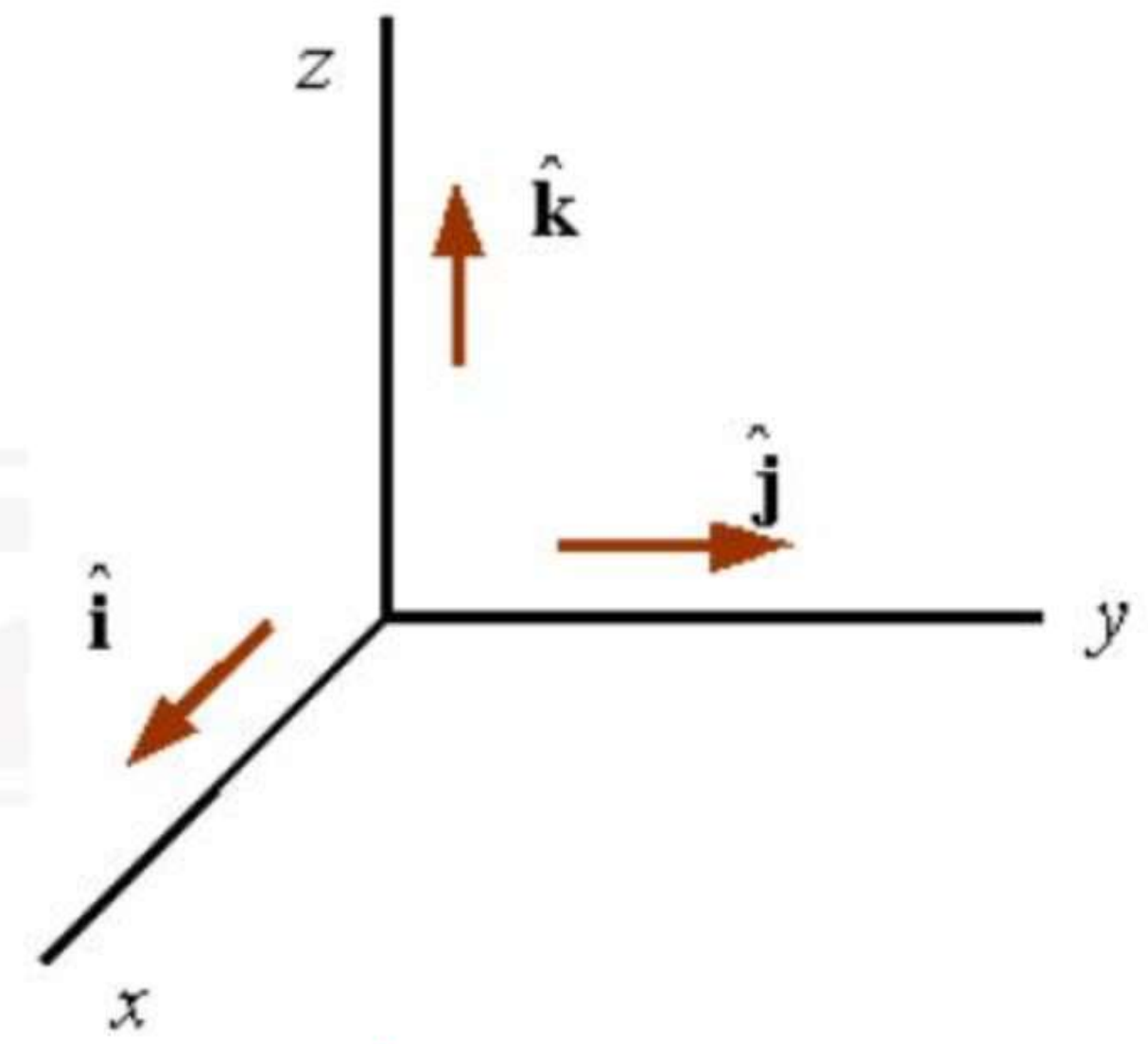
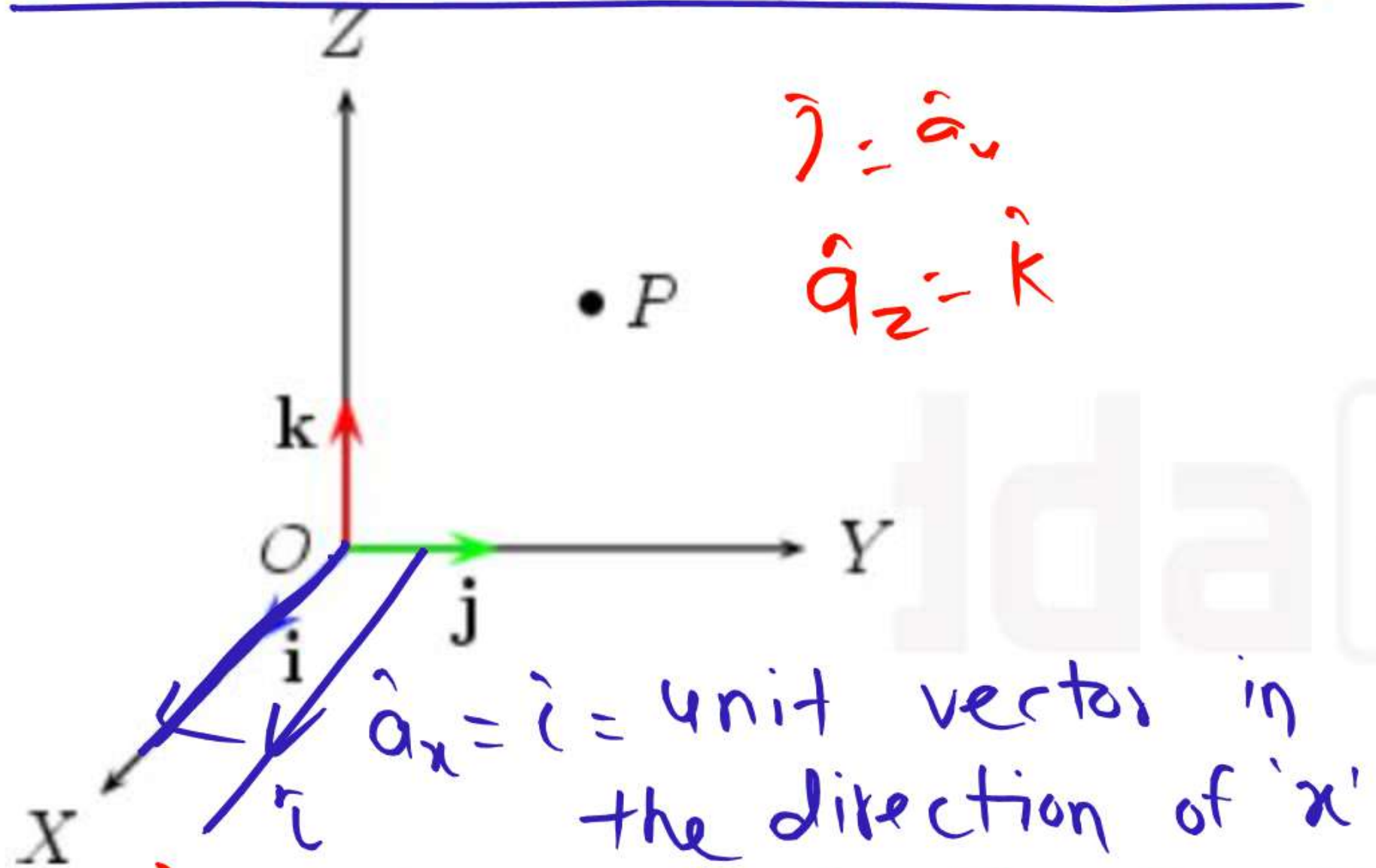
$$x=p \text{ \& } z=m$$

→ line parallel to y-axis.

$$y=q, z=n$$

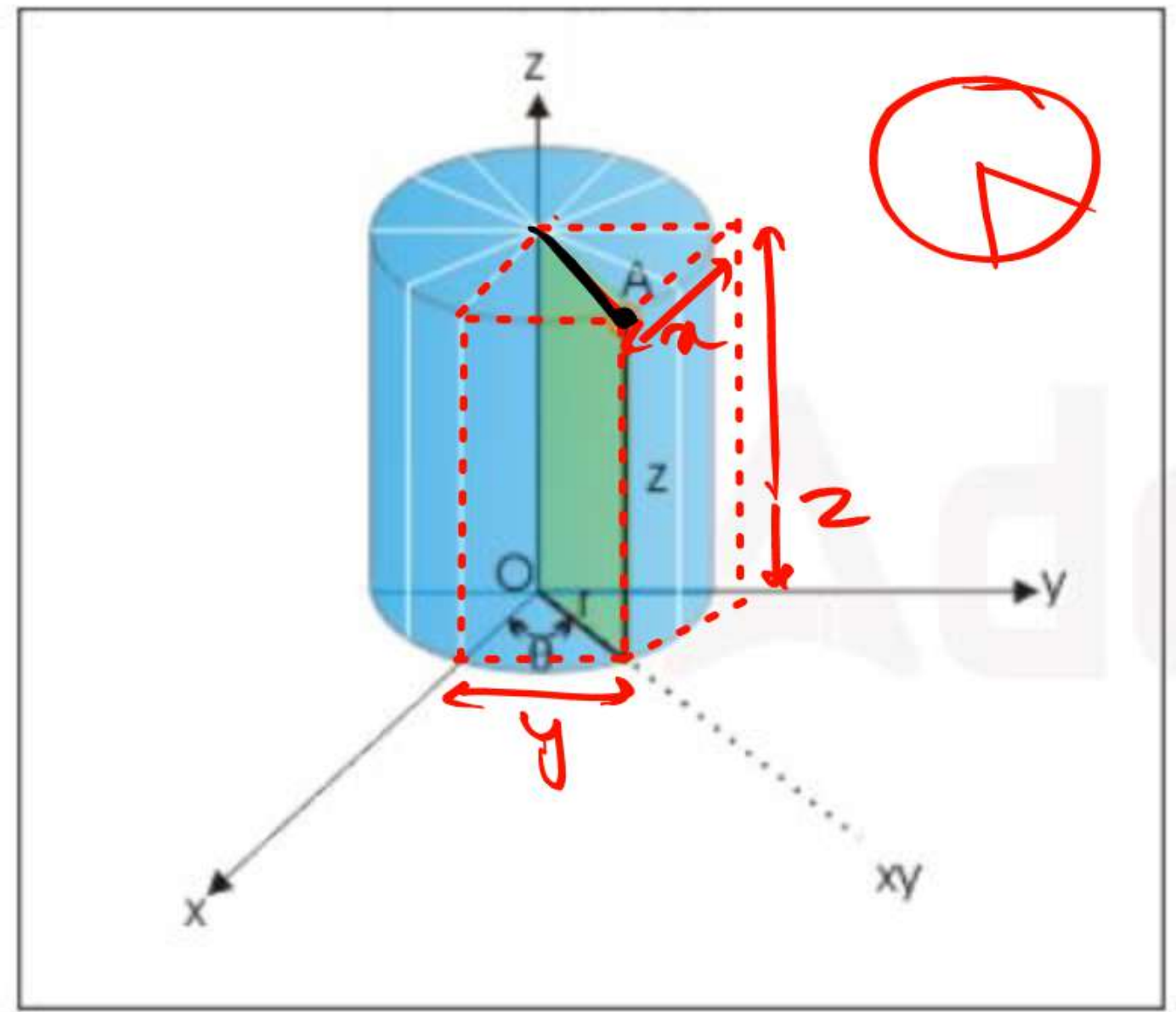
⇒ line parallel to x-axis.

Unit vectors in cartesian coordinate systems



$\hat{a}_x \rightarrow$ unit vector in the direction of increment of x with y & z fixed.

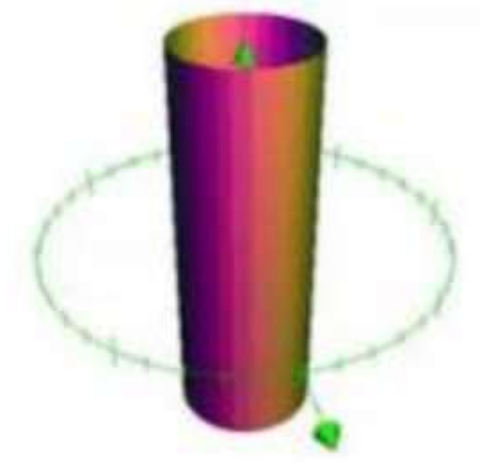
Cylindrical Coordinate systems $P(\rho, \phi, z)$ or ' r '



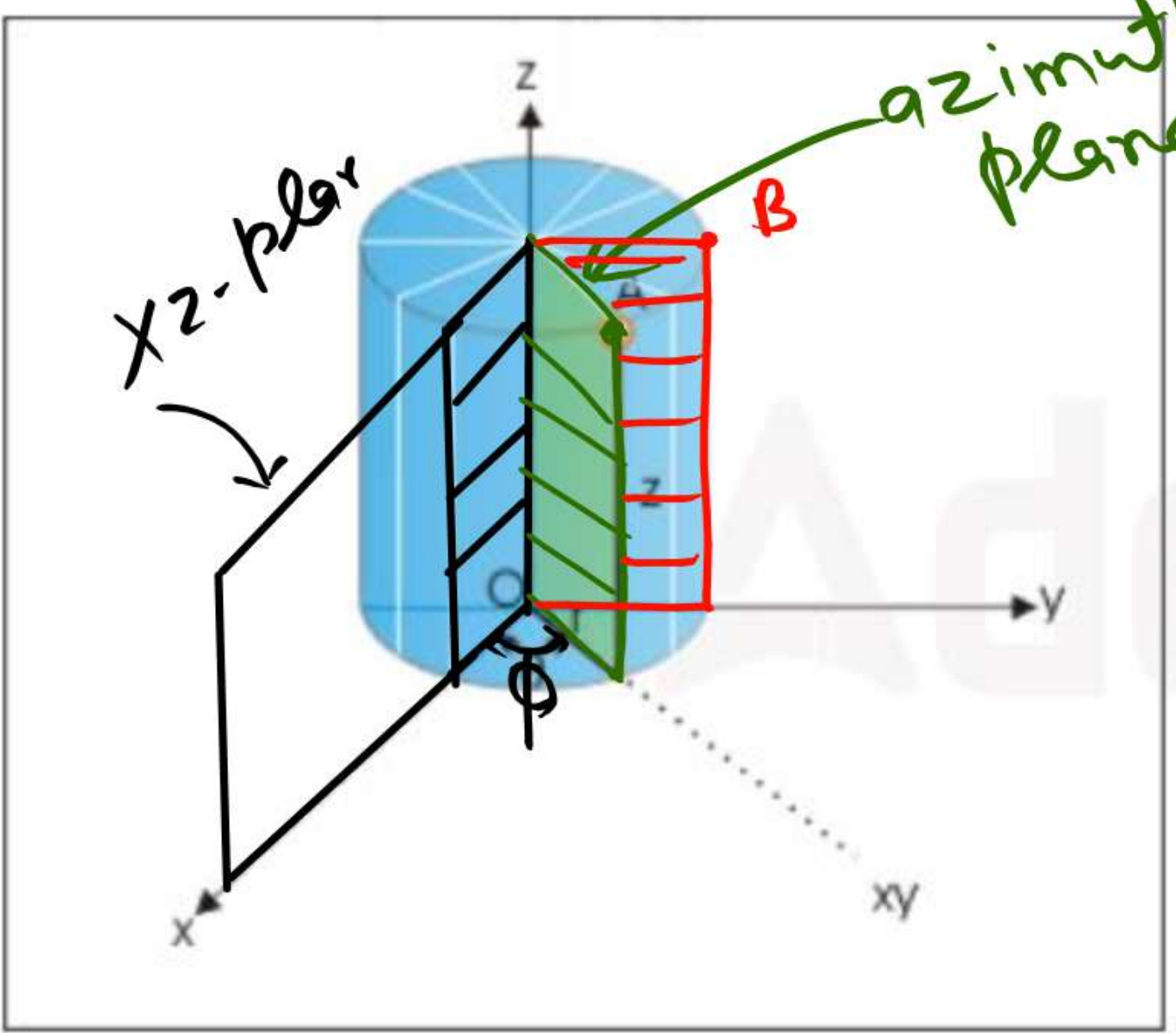
✓ ρ = radius of cylinder passing through the point and centered around z axis

↳ It is perpendicular distance of point from z-axis.

$$0 \leq \rho < \infty$$



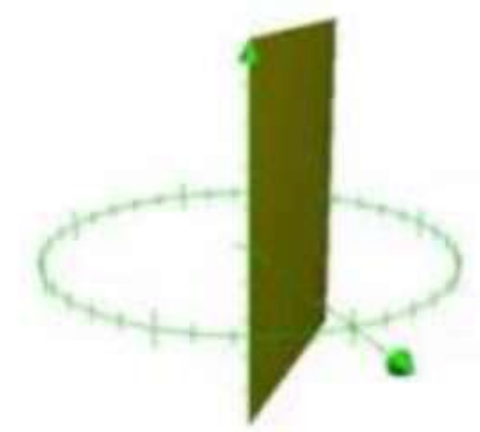
Cylindrical Coordinate systems $P(\rho, \phi, z)$



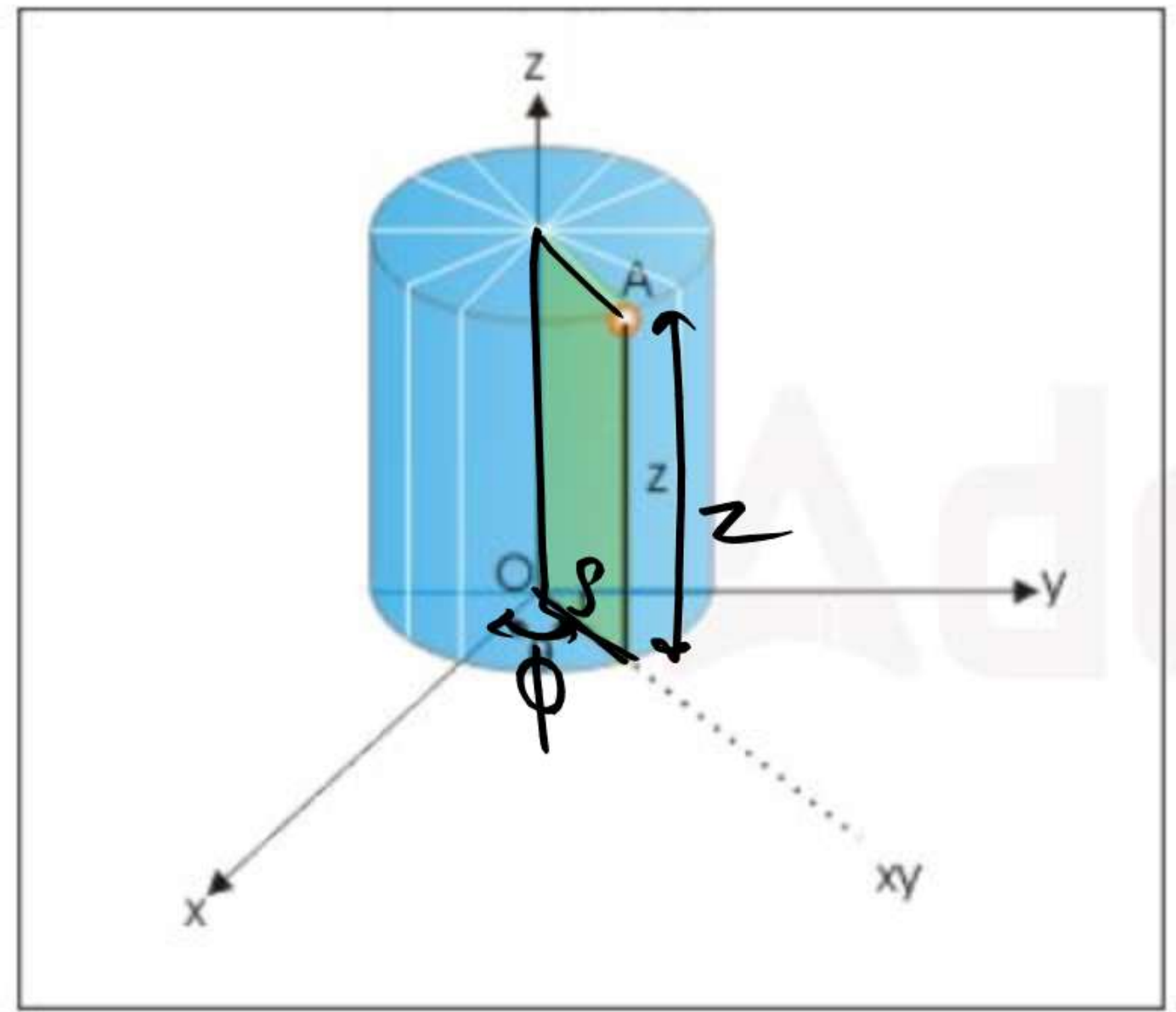
ϕ = Azimuthal angle

Angle of the azimuthal plane from XZ-plane

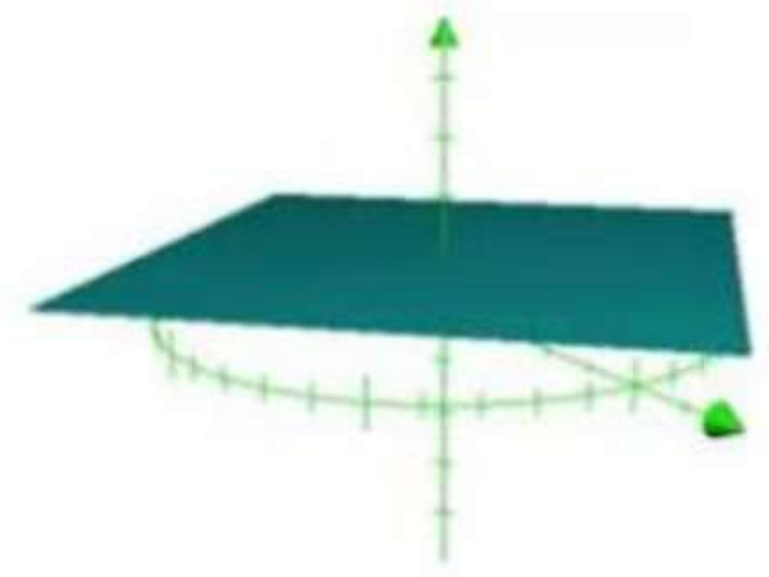
Azimuthal plane:- Imaginary plane passing from Z-axis and point of observation



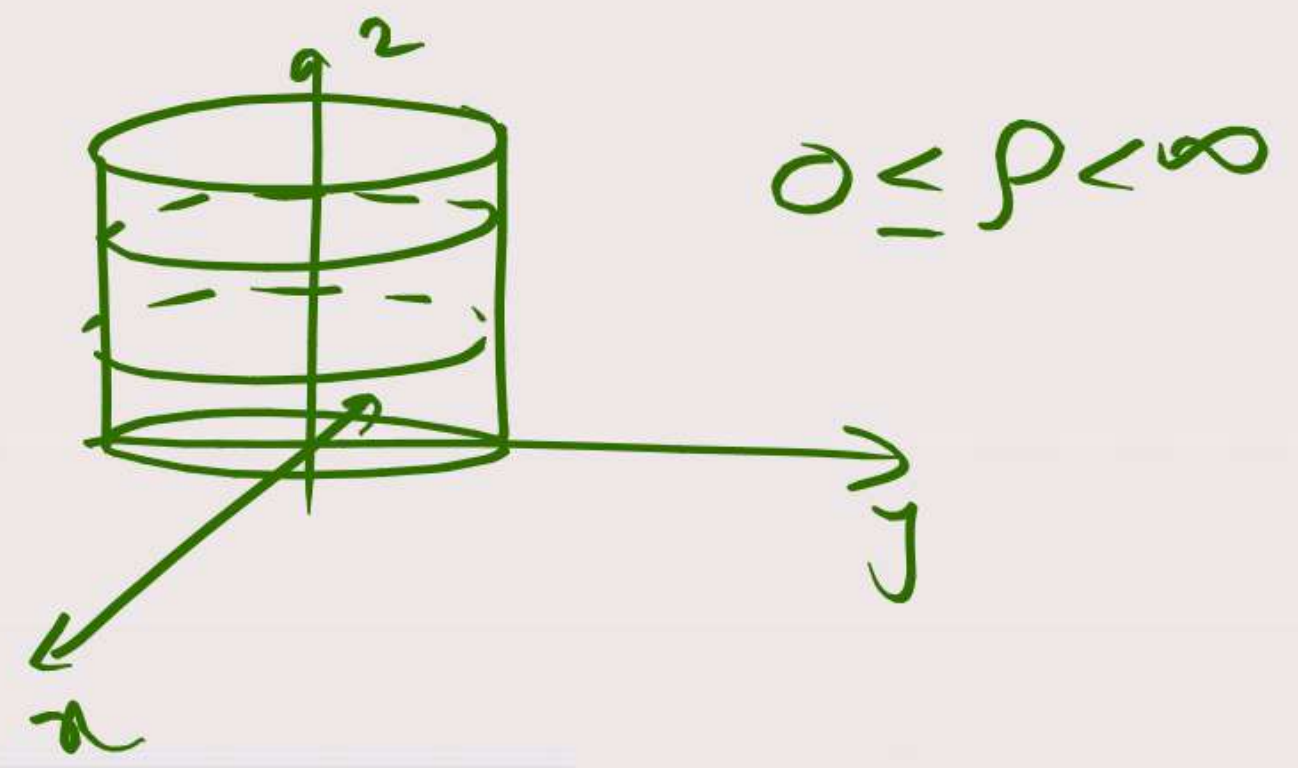
Cylindrical Coordinate systems $P(\rho, \phi, z)$



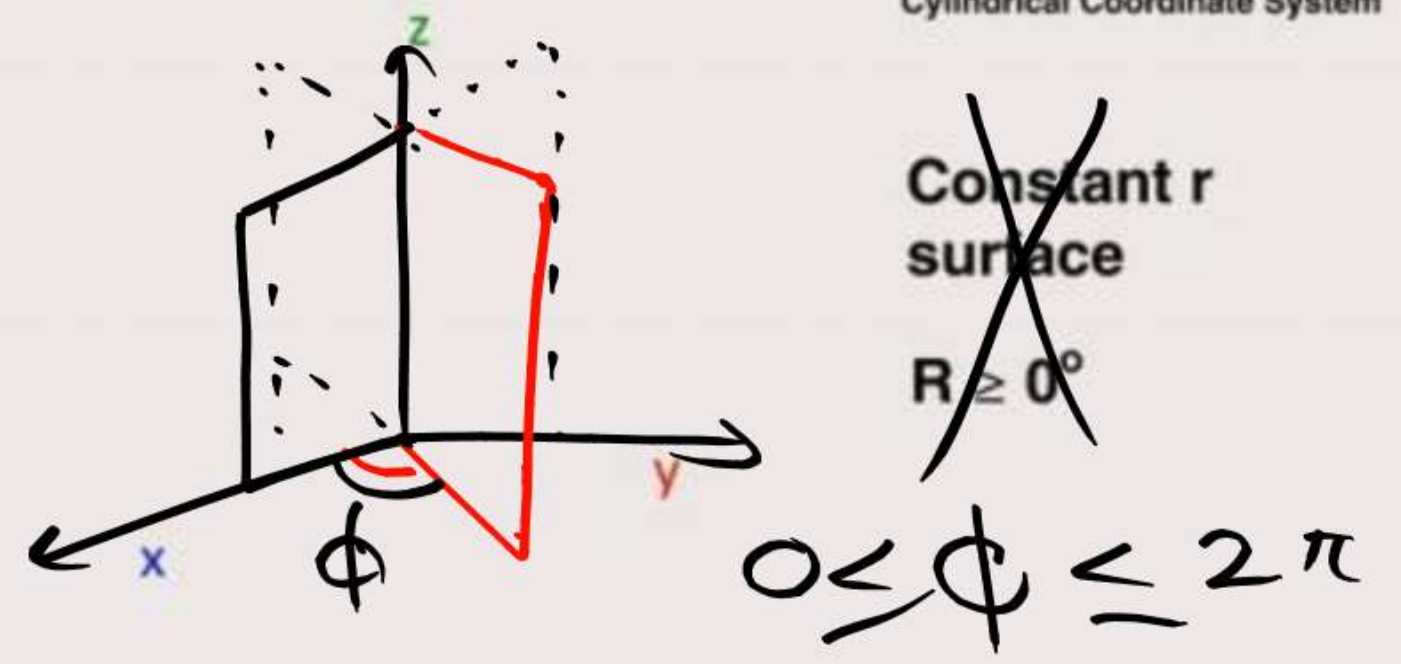
$z = \text{Height}$



Cylindrical Coordinate System

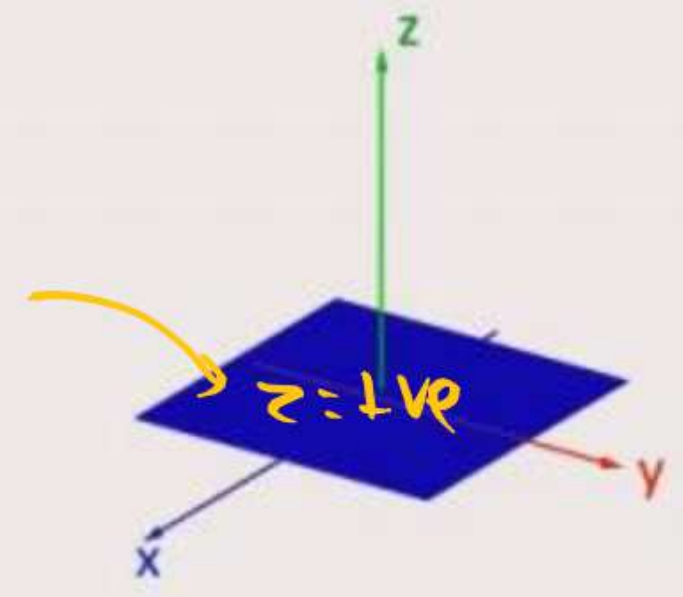


Cylindrical Coordinate System



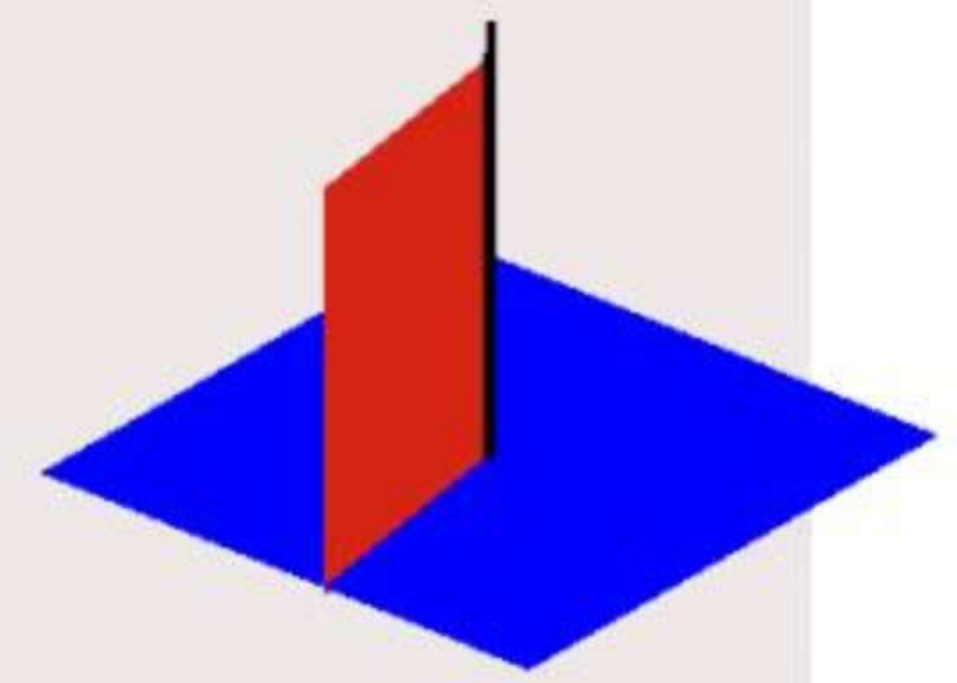
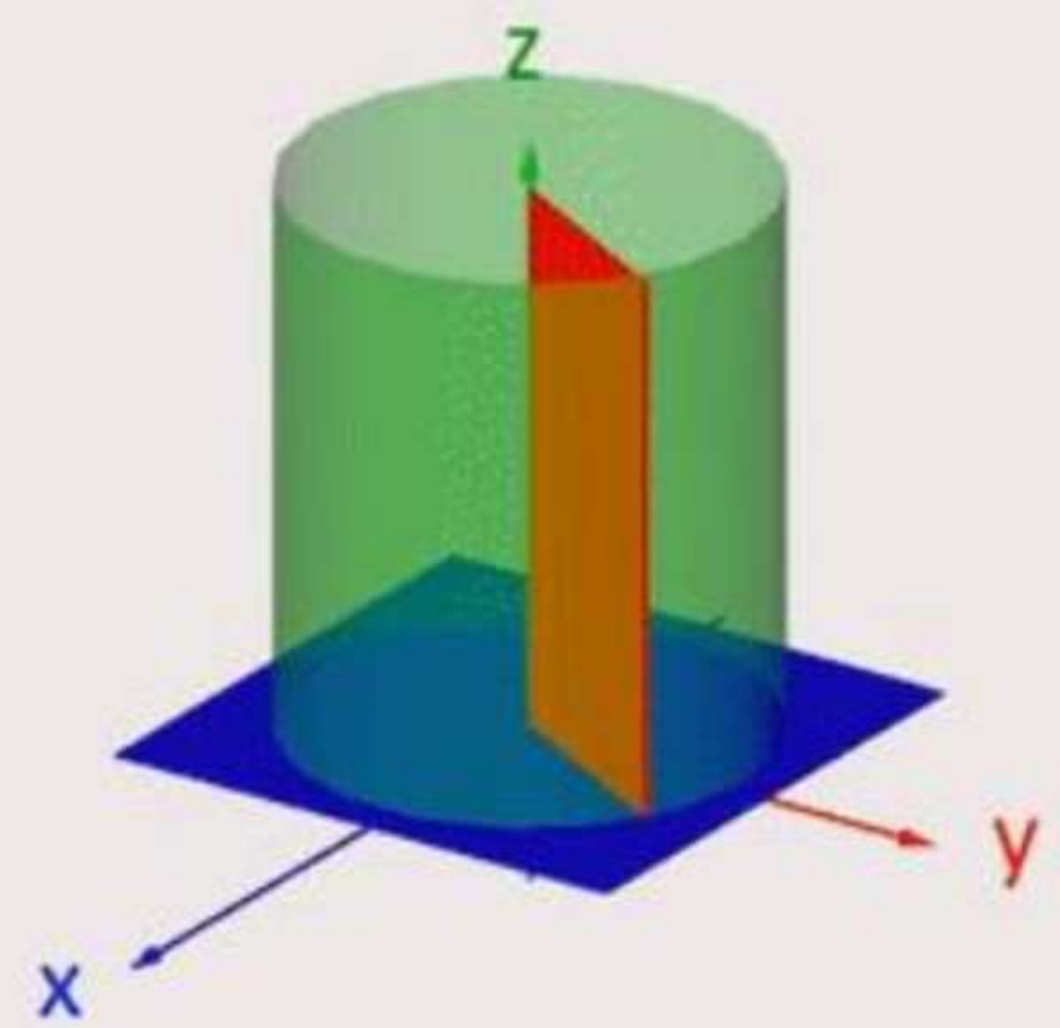
Cylindrical Coordinate System

Constant z surface
 $-\infty \leq z \leq +\infty$

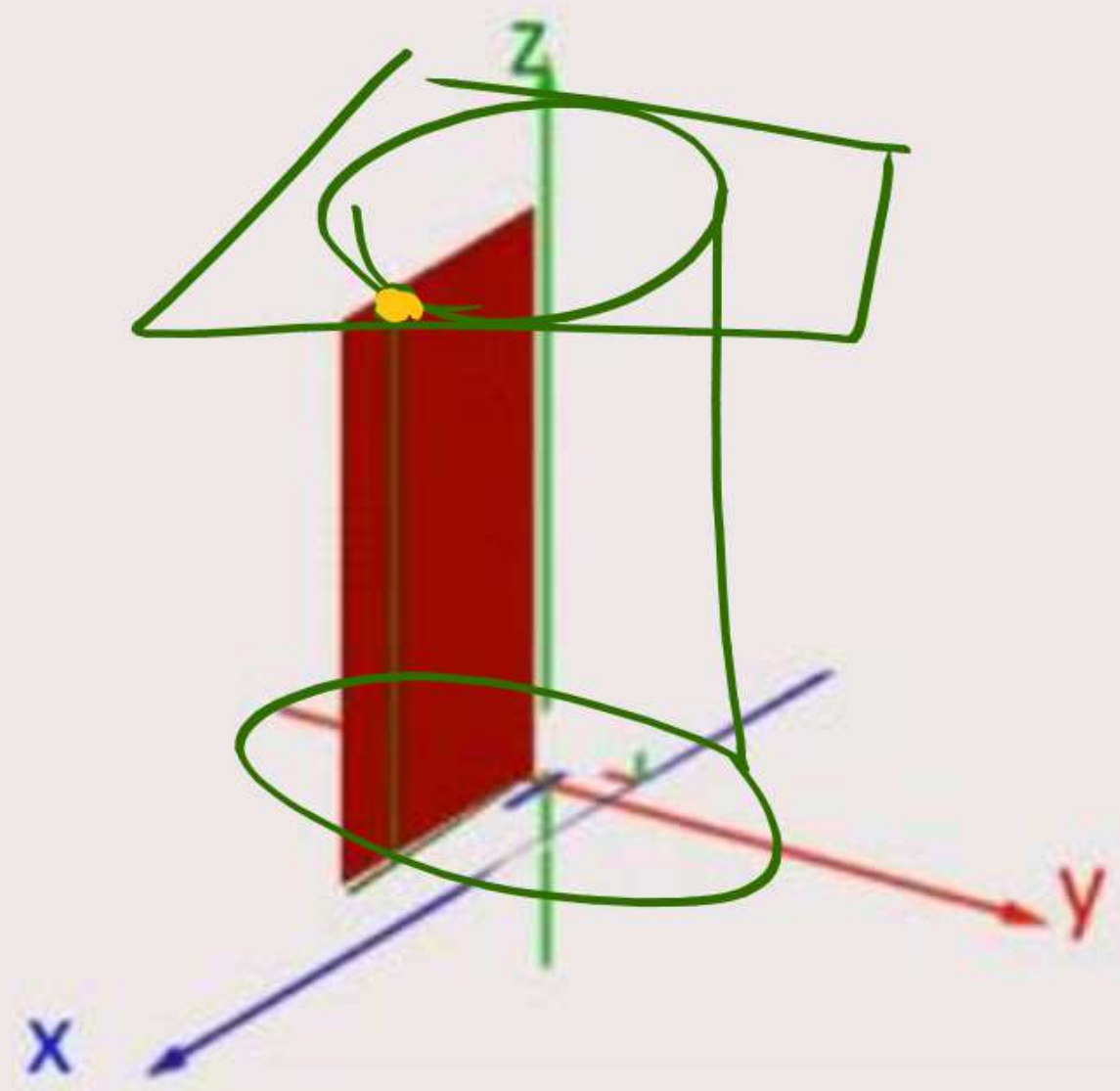


Cylindrical Coordinate System

Cylindrical coordinate system planes



Cylindrical Coordinate System



Point P

$$P(\rho, \phi, z)$$

Cylindrical Coordinate Points in 3-D

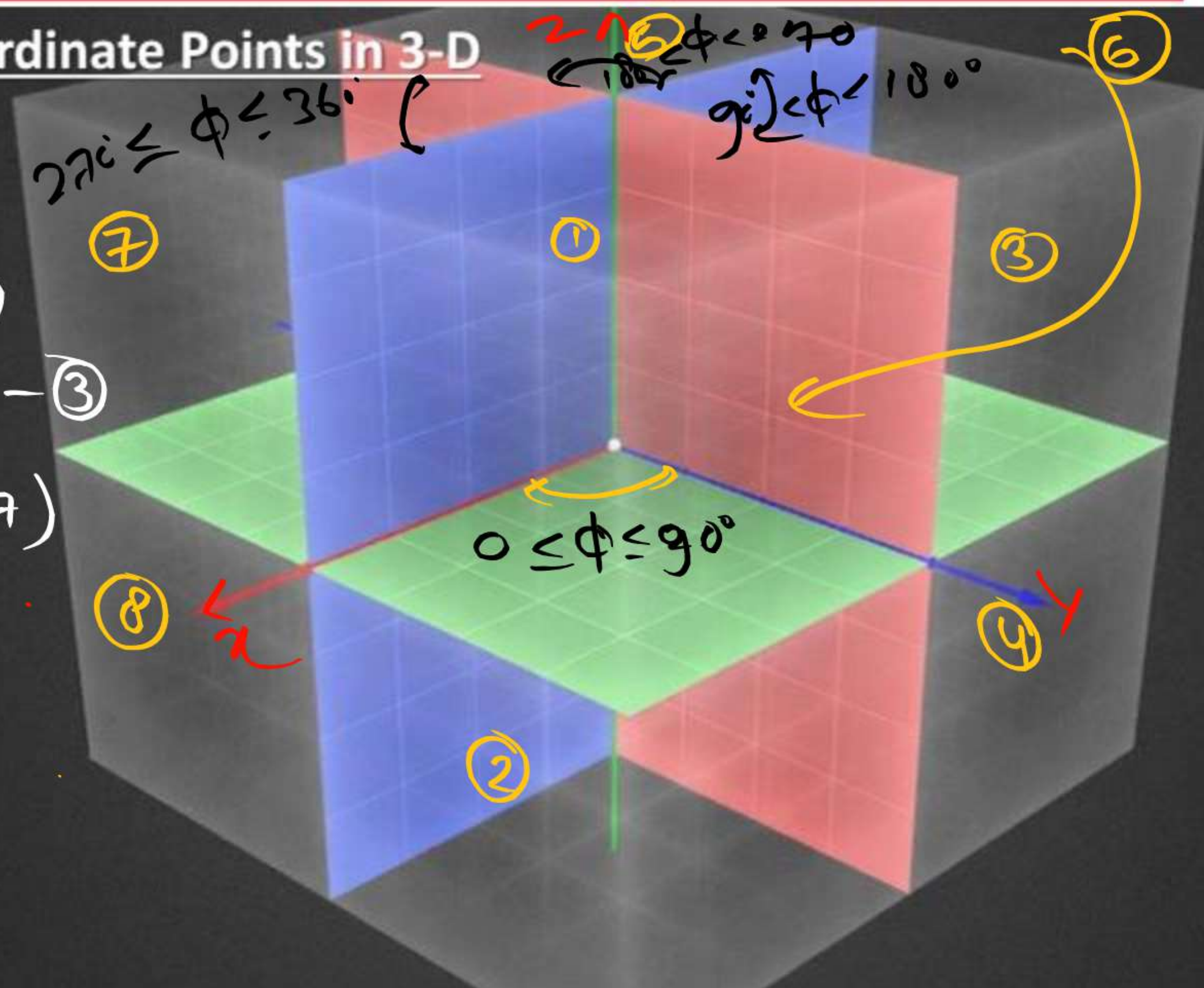
$A(3, 70^\circ, -2)$

→ ②

$B(2, 143^\circ, 1)$ — ③

$C(2, 315^\circ, -7)$

→ ⑧



$270^\circ \leq \phi \leq 360^\circ$

⑦

$180^\circ \leq \phi < 270^\circ$
 $90^\circ \leq \phi < 180^\circ$

①

③

⑥

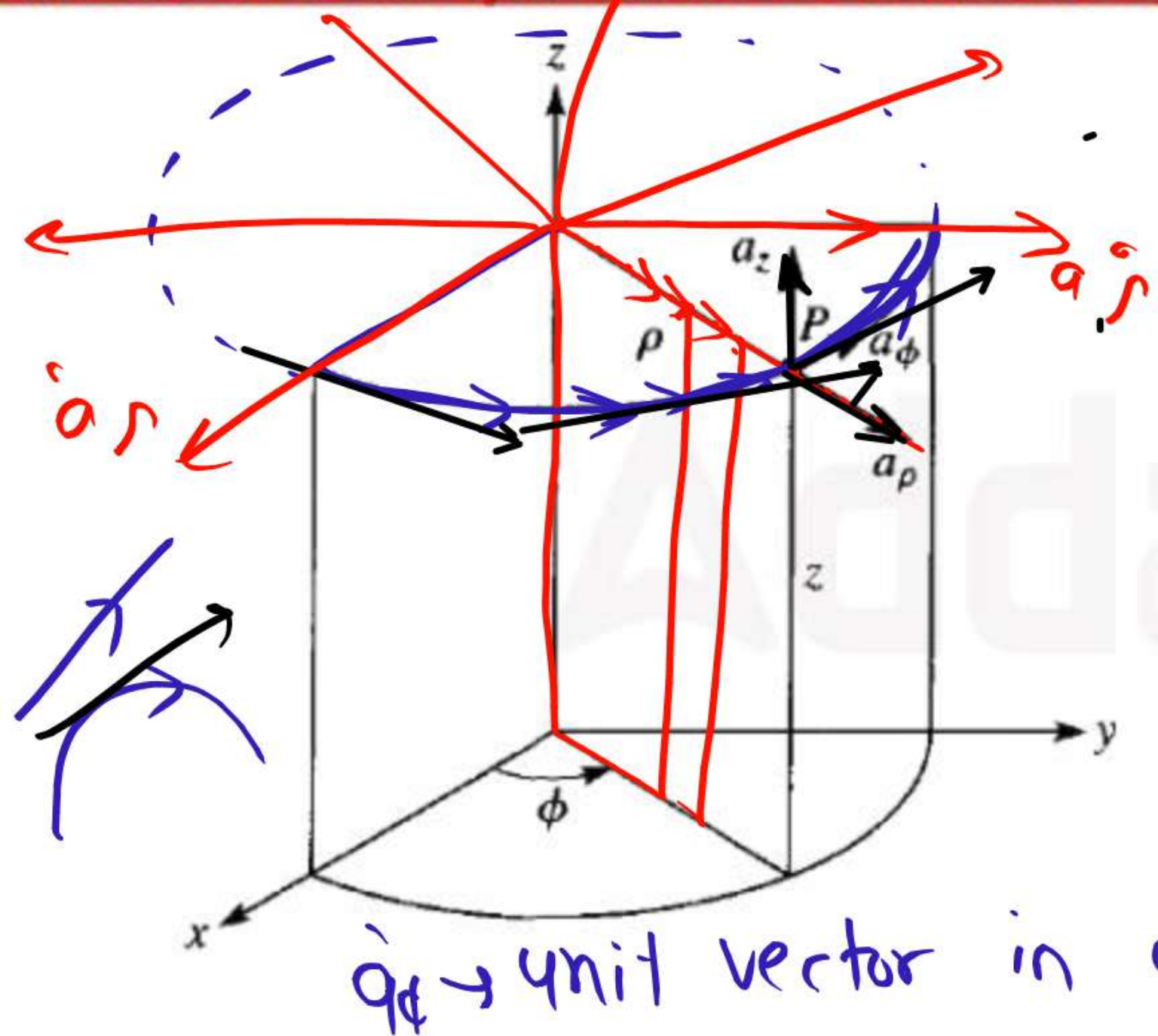
$0 \leq \phi \leq 90^\circ$

⑧

②

④

Unit Vectors in cylindrical Coordinate systems



\hat{a}_ρ = unit vector in the direction in which ρ increases & ϕ & z fixed.

\hat{a}_ϕ → unit vector in radially outward direction from z -axis.

\hat{a}_ϕ → unit vector in circular rotation around z -axis.

Interconversion of Coordinates in Cylindrical and cartesian

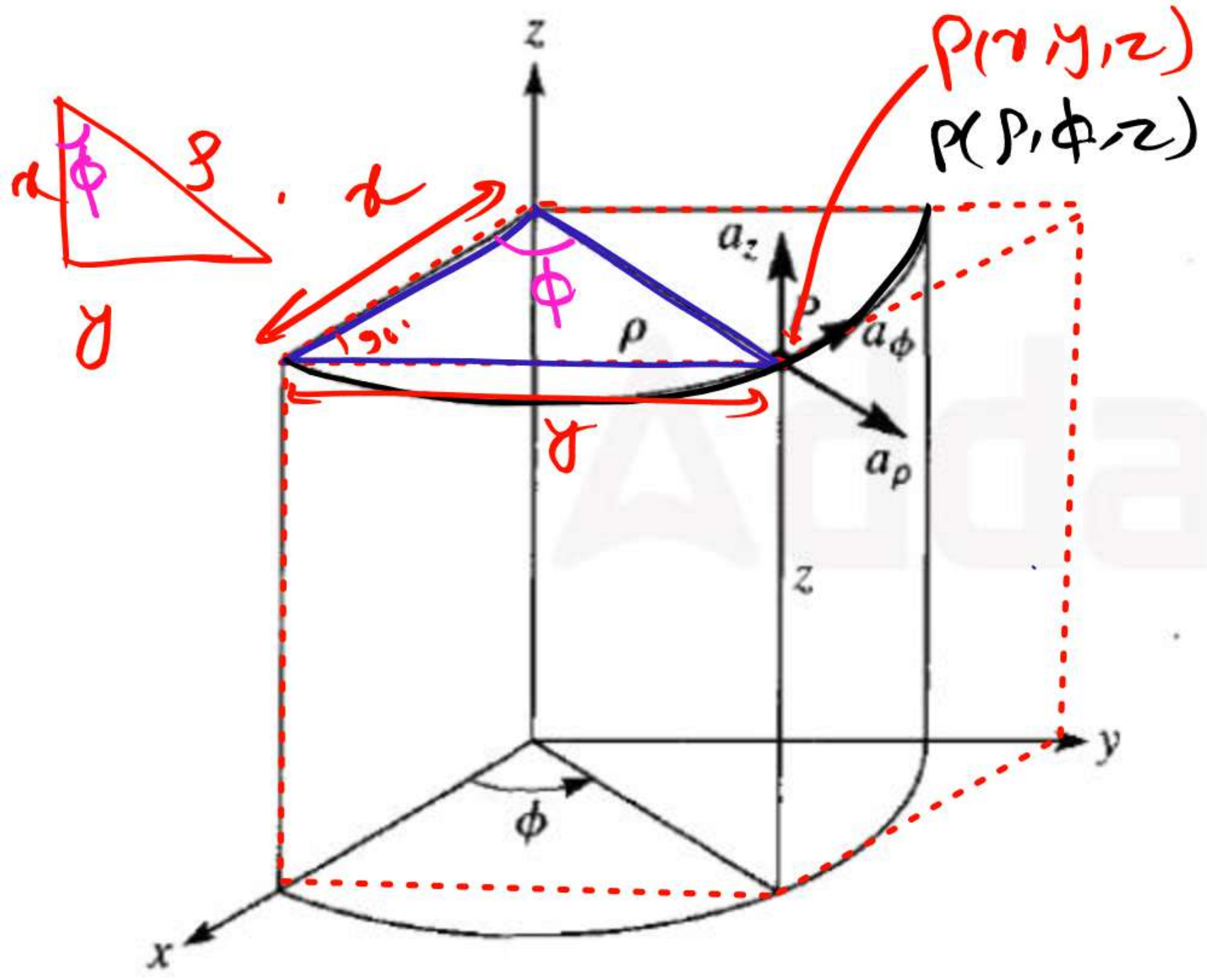
Cartesian to Cylindrical

$P(x, y, z) \longrightarrow P(\rho, \phi, z)$

$\rho = \sqrt{x^2 + y^2}$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$z = z$



Interconversion of Coordinates in Cylindrical and cartesian

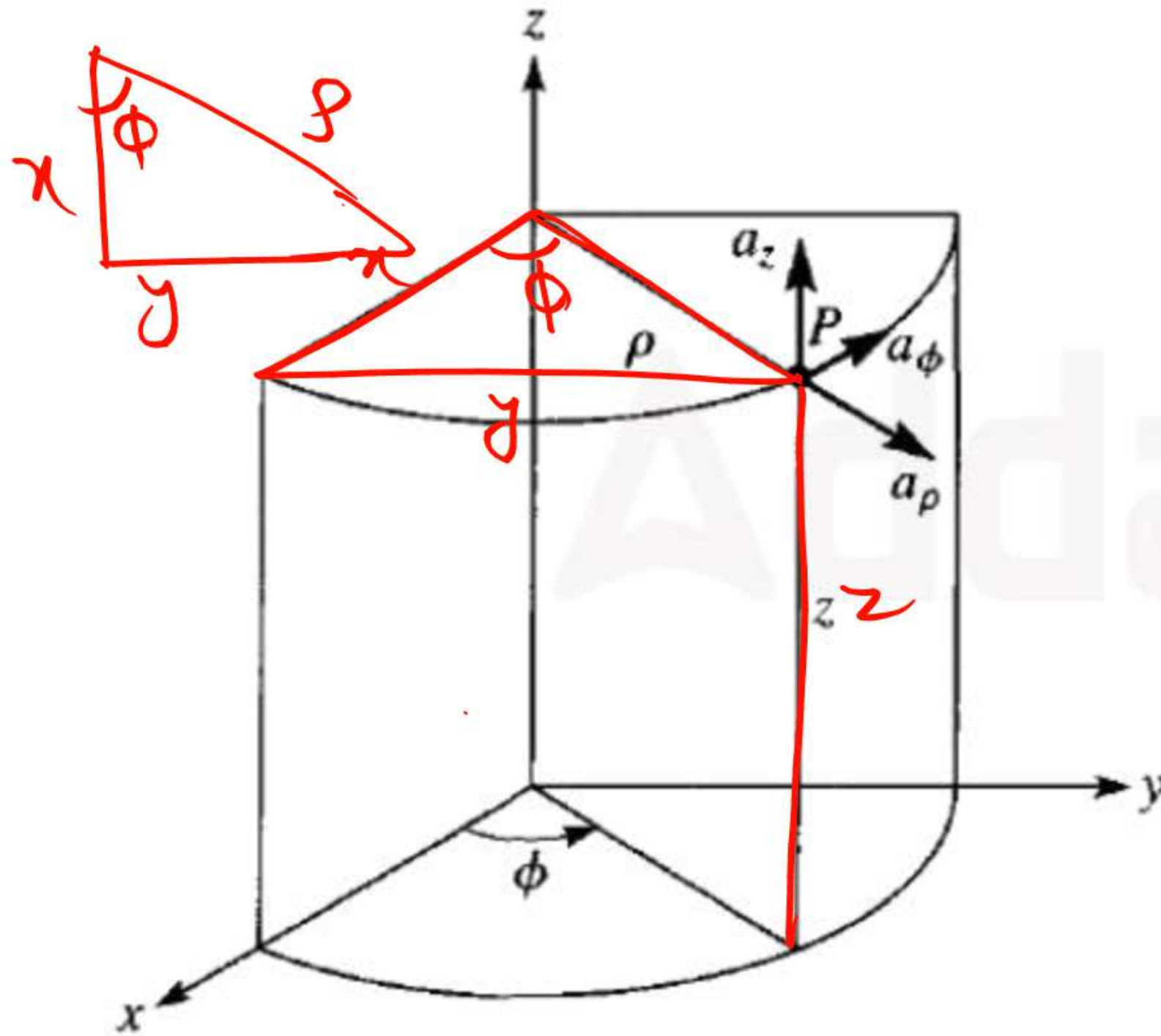
Cylindrical to Cartesian

$$P(\rho, \phi, z) \longrightarrow P(x, y, z)$$

$$x = \rho \cos \phi$$

$$y = \rho \sin \phi$$

$$z = z$$



GATE 2024



Th/Fri/Sat
9pm

प्रचण्ड Batch

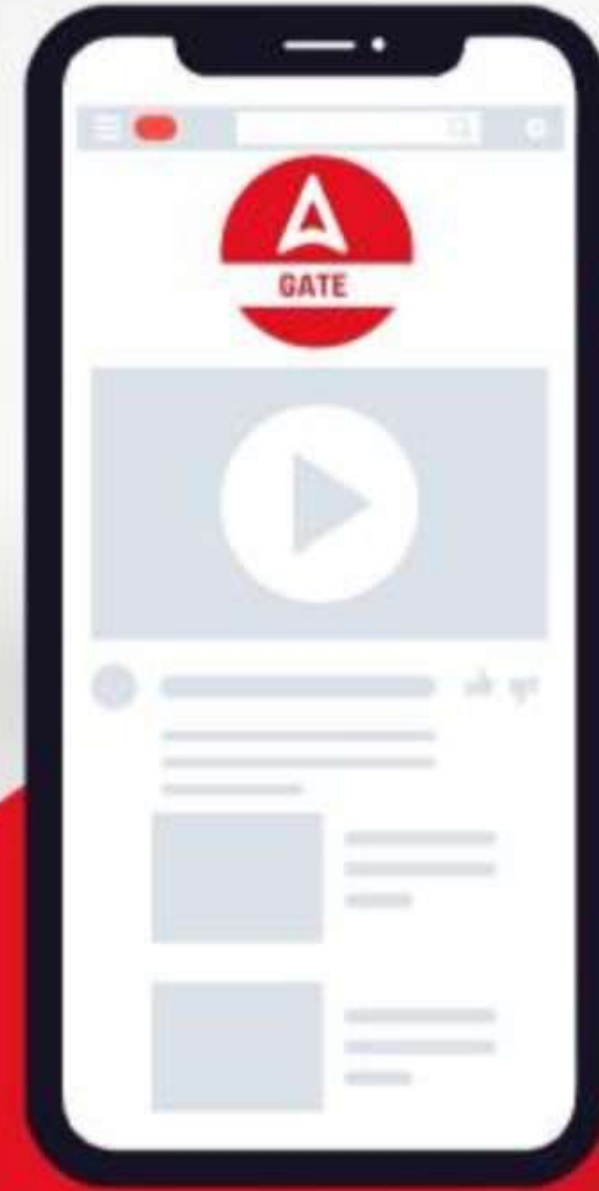
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