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Math is a journey, not a destination.

GATE 2024

GATE

प्रचण्ड Batch

Lec-04

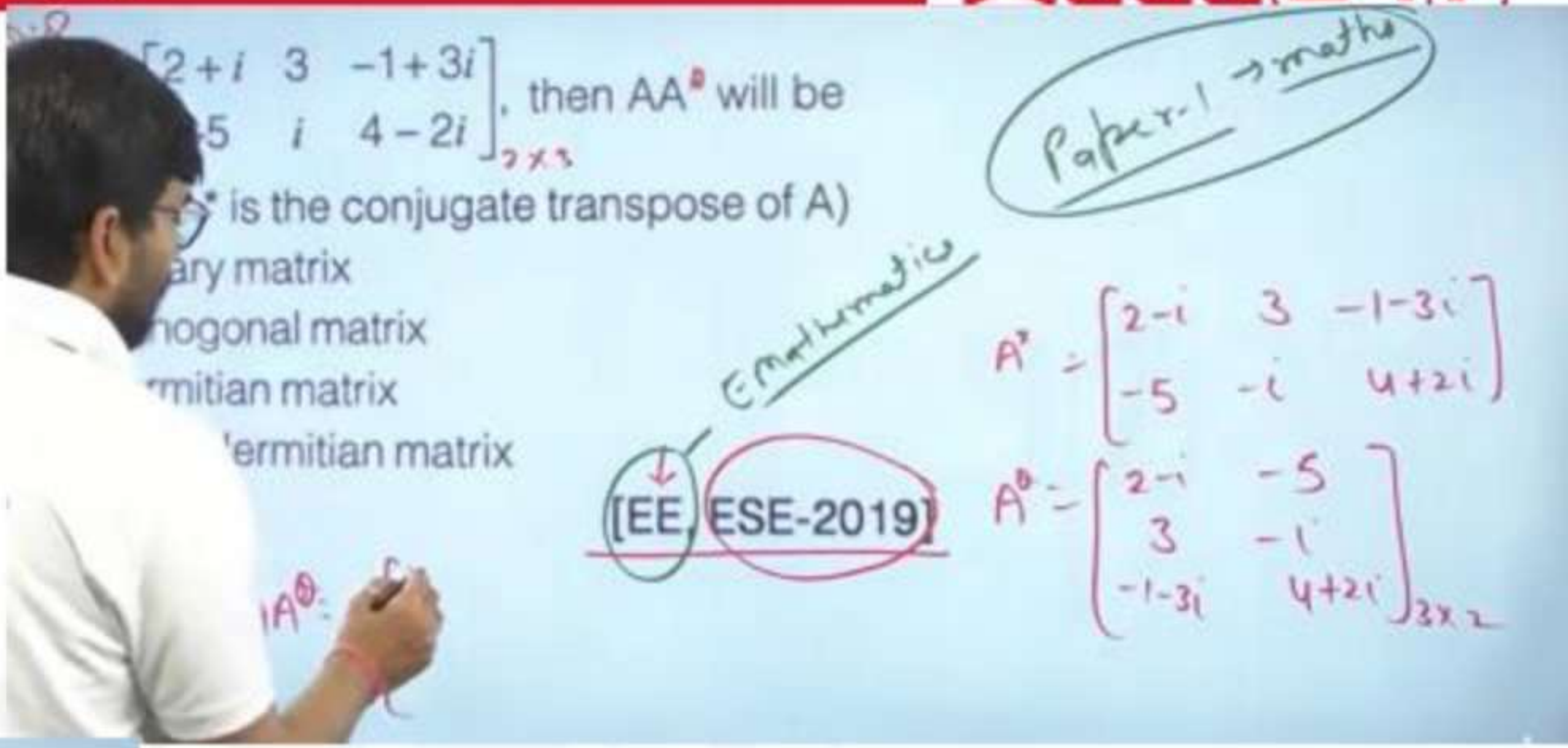
Engineering Mathematics

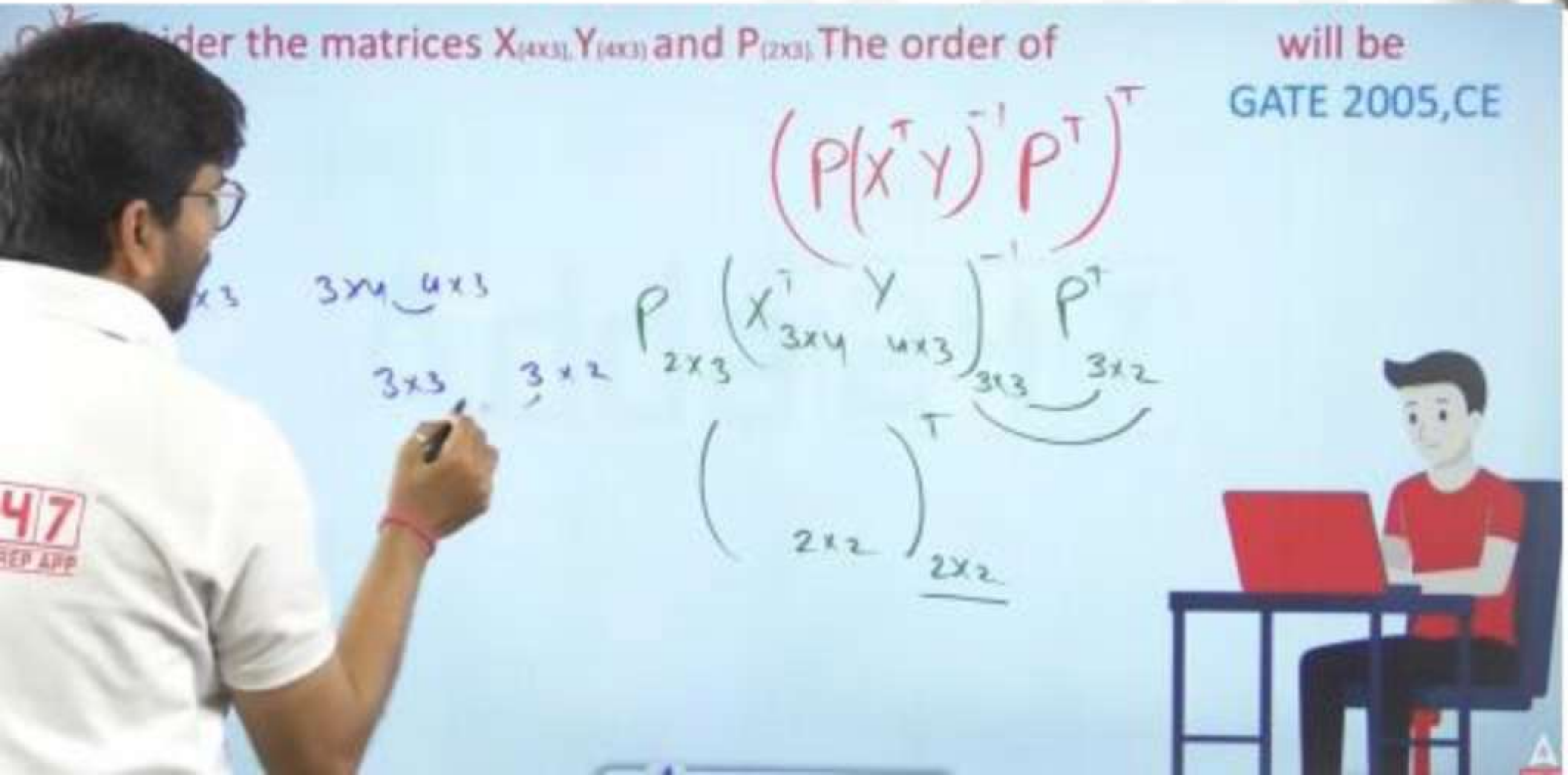
LINEAR ALGEBRA

Question practice on basics of matrices

Recap

ESE Question Practice →





← GATE Question Practice

Number of questions covered-12

Properties of Inverse of Matrices

$$A \longrightarrow A^{-1} = \frac{\operatorname{adj} A}{|A|}$$

① $(A^{-1})^{-1} = A$

② $(A \cdot B)^{-1} = B^{-1} A^{-1} \Rightarrow (ABC)^{-1} = C^{-1} B^{-1} A^{-1}$

③ For singular matrix as $|A| = 0$ so its inverse matrix is not defined i.e. it is non-invertible

Q: 13

If $\Delta = \begin{vmatrix} p & p^2 & (p^3-1) \\ q & q^2 & (q^3-1) \\ r & r^2 & (r^3-1) \end{vmatrix} = 0$, in which p, q, r are different. The value of pqr is

(a) 3

(b) 1

(c) 2.5

(d) 3.5

[ESE-2021]

$$\begin{vmatrix} p & p^2 & p^3-1 \\ q & q^2 & q^3-1 \\ r & r^2 & r^3-1 \end{vmatrix} = 0$$

$$p(q^2r^3-q^2-r^2q^3+r^2)-p^2(qr^3-q-rq^3+r)$$

$$+(p^3-1)(qr^2-rq^2)=0$$

$$pq^2r^3-pq^2-pr^2q^3+pr^2-p^2qr^3+p^2q+p^2rq^3-p^2r$$

$$+p^3qr^2-p^3rq^2-qr^2+rq^2=0$$

$$pqr(qr^2-rq^2-pr^2+pq^2+p^2r-p^2q)$$

$$-(\_\ \_\ \_\ \_)=0$$

$$(pqr-1)(\_\ \_\ \_\ \_)=0$$

$$pqr=1$$

Q:14 Multiplication of real valued square matrices of same dimension is

(a) not always possible to compute

(b) commutative

(c) always positive definite

(d) associative

[GATE-2020 ME SET-I]

$A_{3\times3}$ $B_{3\times3}$

$AB \neq BA$

$ABC = A(BC) = (AB)C$

Q.15

Consider a 2 × 2 matrix $M = [v_1\ \ v_2]$ where, v_1 and v_2 are the column vectors. Suppose $M^{-1} = \begin{bmatrix} u_1^T \\ u_2^T \end{bmatrix}$, where u_1^T and u_2^T are the row vectors. Consider the following statements :

Statement I : $u_1^T\ v_1 = 1$ and $u_2^T\ v_2 = 1$

Statement II : $u_1^T v_2 = 0$ and $u_2^T v_1 = 0$

(a) Statement 1 is true and statement 2 is false

(b) Both the statements are false

(c) Statement 2 is true and statement 1 false

(d) Both the statements are true.

[GATE-2019 (EE) 2 Marks]

$v_1 = \begin{bmatrix} a \\ b \end{bmatrix}$ $\quad v_2 = \begin{bmatrix} c \\ d \end{bmatrix}$

$m^{-1} = \dfrac{adj\, m}{|m|} = \dfrac{1}{(ad-bc)}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

$u_1^T = \begin{bmatrix} \dfrac{d}{ad-bc} & -\dfrac{c}{ad-bc} \end{bmatrix}$

$u_2^T = \begin{bmatrix} \dfrac{-b}{ad-bc} & \dfrac{a}{ad-bc} \end{bmatrix}$

$u_1^T v_1 = \begin{bmatrix} \dfrac{d}{\Delta} & -\dfrac{c}{\Delta} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \dfrac{ad}{\Delta} - \dfrac{bc}{\Delta} = 1$

Q:16

The inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ is

$|A| = 20 - 45 + 20$
$= -5$

$$\begin{array}{c|ccccc} 2 & 3 & 4 & 2 & 3 \\ \hline 4 & 3 & 1 & 4 & 3 \\ 1 & 2 & 4 & 1 & 2 \\ 2 & 3 & 4 & 2 & 3 \\ 4 & 3 & 1 & 4 & 3 \end{array}$$

(a) $\begin{bmatrix} 2 & -\frac{4}{5} & -\frac{9}{5} \\ -3 & \frac{4}{5} & \frac{14}{5} \\ 1 & -\frac{1}{5} & -\frac{6}{5} \end{bmatrix}$

✓ (b) $\begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & -\frac{14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}$

(c) $\begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

(d) $\begin{bmatrix} -10 & 4 & 9 \\ 15 & -4 & -14 \\ -5 & 1 & 6 \end{bmatrix}$

$Cof(A) = \begin{bmatrix} 10 & -15 & 5 \\ -4 & 4 & -1 \\ -9 & 14 & -6 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 10 & -4 & -9 \\ -15 & 4 & 14 \\ 5 & -1 & -6 \end{bmatrix}$

[GATE-2019 (CE) SHIFT-I ; 1 Mark]

Q:17

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A^{-1})$ is 0.25 ___ (correct to two decimal places)

$|A| = 4$

$A^{-1} \rightarrow \frac{1}{|A|} = \frac{1}{4} = 0.25$

[GATE-2018 (ME-Afternoon Session)]

$A \longrightarrow |A|$

$A^{-1} \longrightarrow \frac{1}{|A|}$

* for triangular matrix determinant multiplication of diagonal elements.

Q.18 Which one of the following matrices is singular?

(a) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ 6-5 = 1

(b) $\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ 6-4 = 2

(c) $\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ 12-12=0

(d) $\begin{bmatrix} 4 & 3 \\ 6 & 2 \end{bmatrix}$ 8-18 = -10

[GATE-2018 (CE-Morning Session)]

Q.19 For the given orthogonal matrix Q.

$$Q = \begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$$

The inverse is

X (a) $\begin{bmatrix} 3/7 & 2/7 & 6/7 \\ -6/7 & 3/7 & 2/7 \\ 2/7 & 6/7 & -3/7 \end{bmatrix}$

X (b) $\begin{bmatrix} -3/7 & -2/7 & -6/7 \\ 6/7 & -3/7 & -2/7 \\ -2/7 & -6/7 & 3/7 \end{bmatrix}$

✓ (c) $\begin{bmatrix} 3/7 & -6/7 & 2/7 \\ 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \end{bmatrix}$

X (d) $\begin{bmatrix} -3/7 & -6/7 & -2/7 \\ -2/7 & -3/7 & -6/7 \\ -6/7 & -2/7 & 3/7 \end{bmatrix}$

[GATE-2018 (CE-Morning Session)]

$$Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$$

For any square matrix

$AA^{-1} = I$ — ①

For orthogonal matrix $AA^T = I$ — ②

$\Rightarrow A^T = A^{-1}$

* For orthogonal matrix $\boxed{A^{-1} = A^T}$

Q:20

If $A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 8 & 4 \end{bmatrix}$. AB^T is equal to

(a) $\begin{bmatrix} 38 & 28 \\ 32 & 56 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 40 \\ 42 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 43 & 27 \\ 34 & 50 \end{bmatrix}$

(d) $\begin{bmatrix} 38 & 32 \\ 28 & 56 \end{bmatrix}$

[GATE-2017 CE Session-II]

$A = \begin{bmatrix} 1 & 5 \\ 6 & 2 \end{bmatrix}$ $B^T = \begin{bmatrix} 3 & 8 \\ 7 & 4 \end{bmatrix}$

$AB^T = \begin{bmatrix} 38 & 26 \\ & \end{bmatrix}$

21. The matrix P is the inverse of a matrix Q. If I denotes the identity matrix, which one of the following options is correct?

(a) PQ = I but QP ≠ I

(b) QP = I but PQ ≠ I

(c) PQ = I and QP = I

(d) PQ – QP = I

[GATE-2017 CE Session-I]

$P = Q^{-1}$

$PQ = QP = I$

$AA^{-1} = A^{-1}A = I$

Q:22

The matrix $A = \begin{bmatrix} a & 0 & 3 & 7 \\ 2 & 5 & 1 & 3 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & b \end{bmatrix}$ has det(A) = 100

and trace (A) = 14. The value of |a–b| is ______.

[GATE-2016-EC-Set-2; 2 Marks]

Trace = ?? = addition of diagonal elements

$a+7+b=14$

$\Rightarrow a+b=7$

$|A|=100$

$10ab=100 \Rightarrow ab=10$

$a=5, 2$

$b=2, 5$

$a-b=3, -3$

$|a-b|=3$

Q.23

Let $M^4 = I$, (where I denotes the identity matrix) and $M \neq I$ and $M^2 \neq I$ and $M^3 \neq I$. Then, for any natural number k, M^{-1} equals:

(a) M^{4k+1} (b) M^{4k+2}

(c) M^{4k+3} (d) M^{4k}

[GATE-2016-EC-SET-1; 1 Mark]

$m^4 = I$

$m^{-1} \cdot m \cdot m^3 = m^{-1}$

$m^3 = m^{-1}$

$AI = A$

$m^{-1} = m^3 \text{ or } m^7 \text{ or } m^{11}$

$m^{-1} = m$

$m^4 = I$

$m = m^5$

$m^2 = m^6$

$m^{-1} = m^3 = m^7$

$m^4 = m^8$

$m^5 = m^9$

$m^6 = m^{10}$

$m^7 = m^{11}$

Q:24

A real square matrix A is called skew-symmetric if

(a) $A^T = A$ (b) $A^T = A^{-1}$

(c) $A^T = -A$ (d) $A^T = A + A^{-1}$

[GATE-2016-ME-SET-1; 1 Mark]

$A^T = A \rightarrow$ Symetric

$A^T = -A \rightarrow$ Skew-symmetric

25

Perform the following operations in the matrix

$$\begin{bmatrix} 3 & 4 & 45 \\ 7 & 9 & 105 \\ 13 & 2 & 195 \end{bmatrix} A \rightarrow$$

(i) Add the third row to the second row

(ii) Subtract the third column from the first column.

The determinant of the resultant matrix is ______.

[GATE-2015 (CS-Set 2)]

$R_2 \leftarrow R_2 + R_3$

linear transformations

$$\begin{bmatrix} 3 & 4 & 45 \\ 20 & 11 & 300 \\ 13 & 2 & 197 \end{bmatrix}$$

$$\begin{bmatrix} -42 & 4 & 45 \\ -280 & 11 & 340 \\ -184 & 2 & 197 \end{bmatrix}$$

$|A| = -2340$

Q:26

For $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, the determinant of

$A^T A^{-1}$ is

(a) $\sec^2 x$ (b) $\cos 4x$

(c) 1 (d) 0

[2015-EC-SET-3; 1 Mark]

$$A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\sec^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\det(A^T A^{-1}) = |A^T| \cdot |A^{-1}|$$

$$= \cancel{|A|} \cdot \frac{1}{\cancel{|A|}} = 1$$

$A \rightarrow |A|$

$A^T \rightarrow |A^T| = |A|$

$A^{-1} \rightarrow |A^{-1}| = \frac{1}{|A|}$

Q:27

For a given matrix $P = \begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ $i = \sqrt{-1}$,

the inverse of matrix P is

(a) $\frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$ (b) $\frac{1}{25}\begin{bmatrix} i & 4-3i \\ 4+3i & -i \end{bmatrix}$

(c) $\frac{1}{24}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$ (d) $\frac{1}{25}\begin{bmatrix} 4+3i & -i \\ i & 4-3i \end{bmatrix}$

[GATE-2015-ME-Set-3 ; 2 Marks]

$P^{-1} = \frac{1}{24}\begin{bmatrix} 4-3i & i \\ -i & 4+3i \end{bmatrix}$

Q.28

If the matrix A is such that $A = \begin{bmatrix} 2 \\ -4 \\ 7 \end{bmatrix} \begin{bmatrix} 1 & 9 & 5 \end{bmatrix}$,

then the determinant of A is equal to ______.

[GATE-2014 (CS-Set 2)]

$$A = \begin{bmatrix} 2 & 18 & 10 \\ -4 & -36 & -20 \\ 7 & 63 & 35 \end{bmatrix}$$

$$|A| = 0$$

Q:29

The determinant of matrix A is 5 and the determinant of matrix B is 40. The determinant of matrix AB is______.

[2014-EE-SET-2; 1 Mark]

$|A| = 5$

$|B| = 40$

$|AB| = |A||B| = 5 \times 40 = \underline{200}$

Q: 30

The determinant of matrix $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 2 & 3 & 0 & 1 \\ 3 & 0 & 1 & 2 \end{bmatrix}_{4\times4}$ is $\rightarrow$ $\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 0 \\ 0 & 3 & -6 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

$- \quad + \quad - \quad +$

[GATE-2014-CE-SET-II ; 2 Marks]

$+2(6+6) = 24$

Q:3) Which of the following equations is a correct identify for arbitrary 3 × 3 real matrices P, Q and R?

(a) $P(Q + R) = PQ + RP$

(b) $(P - Q)^2 = P^2 - 2PQ + Q^2$

(c) $\det(P + Q) = \det P + \det Q$

(d) $(P + Q)^2 = P^2 + PQ + QP + Q^2$

[GATE-2014; 1 Mark]

$(P-Q).(P-Q)$

$= P^2 - PQ - QP + Q^2$

Q.22 Given that the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 0 \\ 2 & 6 & 4 \\ -1 & 0 & 2 \end{bmatrix}$$ A

3x3

is –12, the determinant of the matrix

$$\begin{bmatrix} 2 & 6 & 0 \\ 4 & 12 & 8 \\ -2 & 0 & 4 \end{bmatrix}$$ is

$B = 2A$

$A \rightarrow |A|$

$B = 2A = |2A| = 2|A|$ X

$= (2)^3 |A|$

$= 8 \times -12 = -96$

(a) –96 (b) –24

(c) 24 (d) 96

[GATE-2014-ME-Set-1; 1 Mark]

Q:33 Which one of the following does NOT equal

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} ?$$

$C_2 \leftarrow C_2 + C_1$
$C_3 \leftarrow C_3 + C_2$

$$\begin{vmatrix} 1 & x^2+x & x+1 \\ 1 & y^2+y & y+1 \\ 1 & z^2+z & z+1 \end{vmatrix}$$

$C_2 \leftarrow C_2 + C_1$
$C_3 \leftarrow C_3 + C_1$

(a) $\begin{vmatrix} 1 & x(x+1) & x+1 \\ 1 & y(y+1) & y+1 \\ 1 & z(z+1) & z+1 \end{vmatrix}$ (b) $\begin{vmatrix} 1 & x+1 & x^2+1 \\ 1 & y+1 & y^2+1 \\ 1 & z+1 & z^2+1 \end{vmatrix}$

$R_1 \leftarrow R_1 - R_2$
$R_2 \leftarrow R_2 - R_3$

(c) $\begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$ (d) $\begin{vmatrix} 2 & x+y & x^2+y^2 \\ 2 & y+z & y^2+z^2 \\ 1 & z & z^2 \end{vmatrix}$

$R_1 \leftarrow R_1 + R_2$
$R_2 \leftarrow R_2 + R_3$

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→ 9 PM → Comm for ECE.

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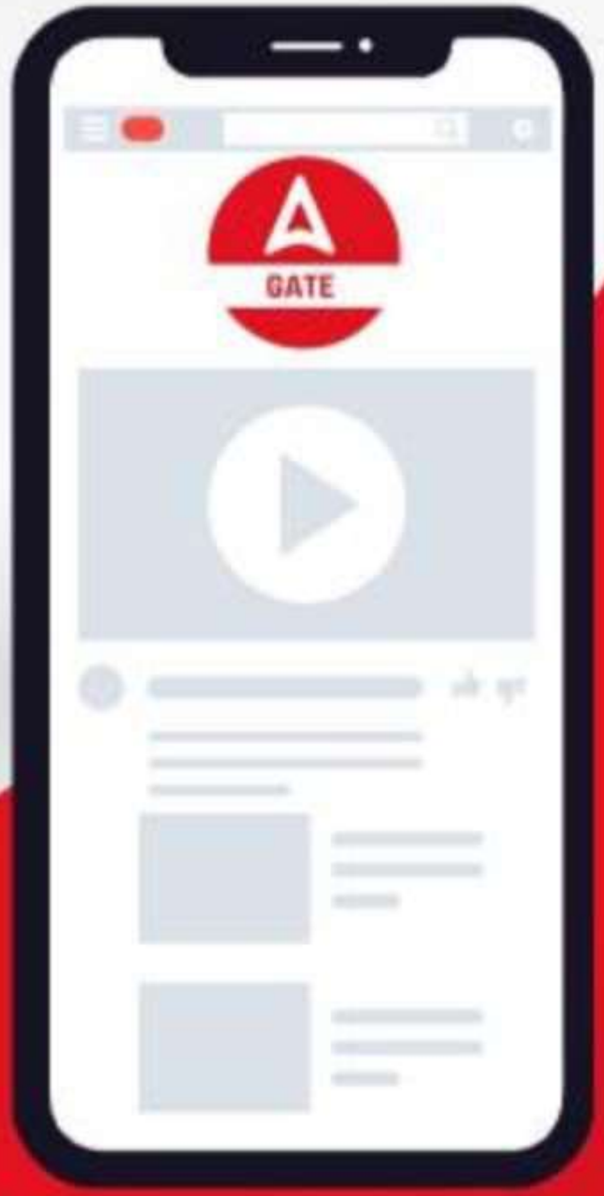
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