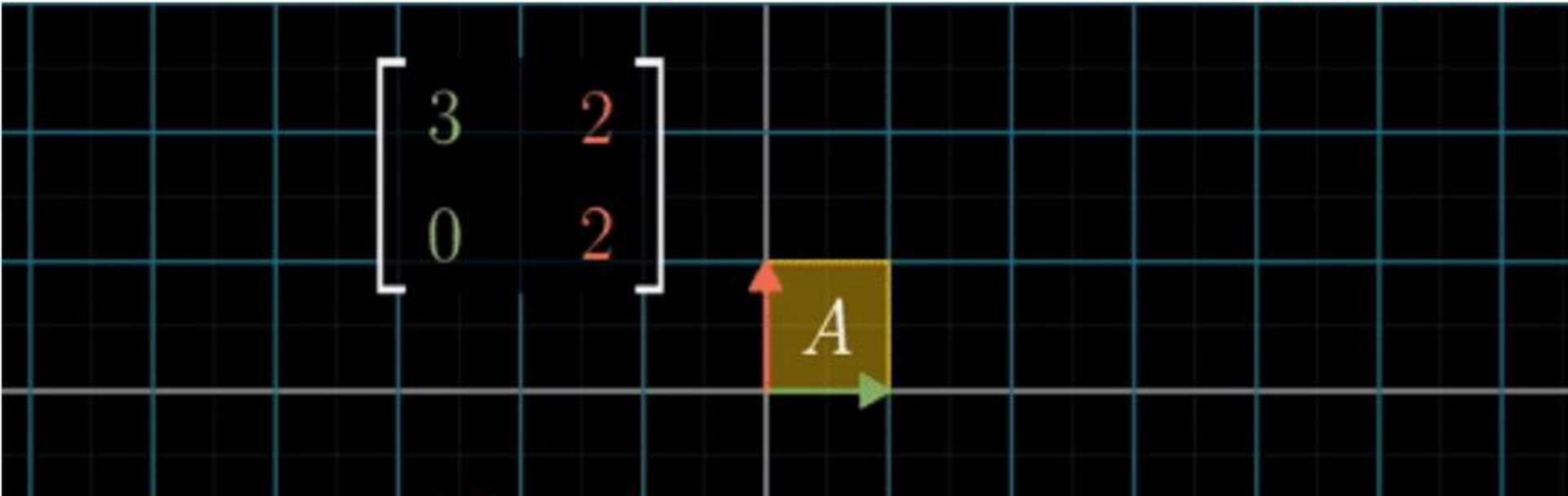
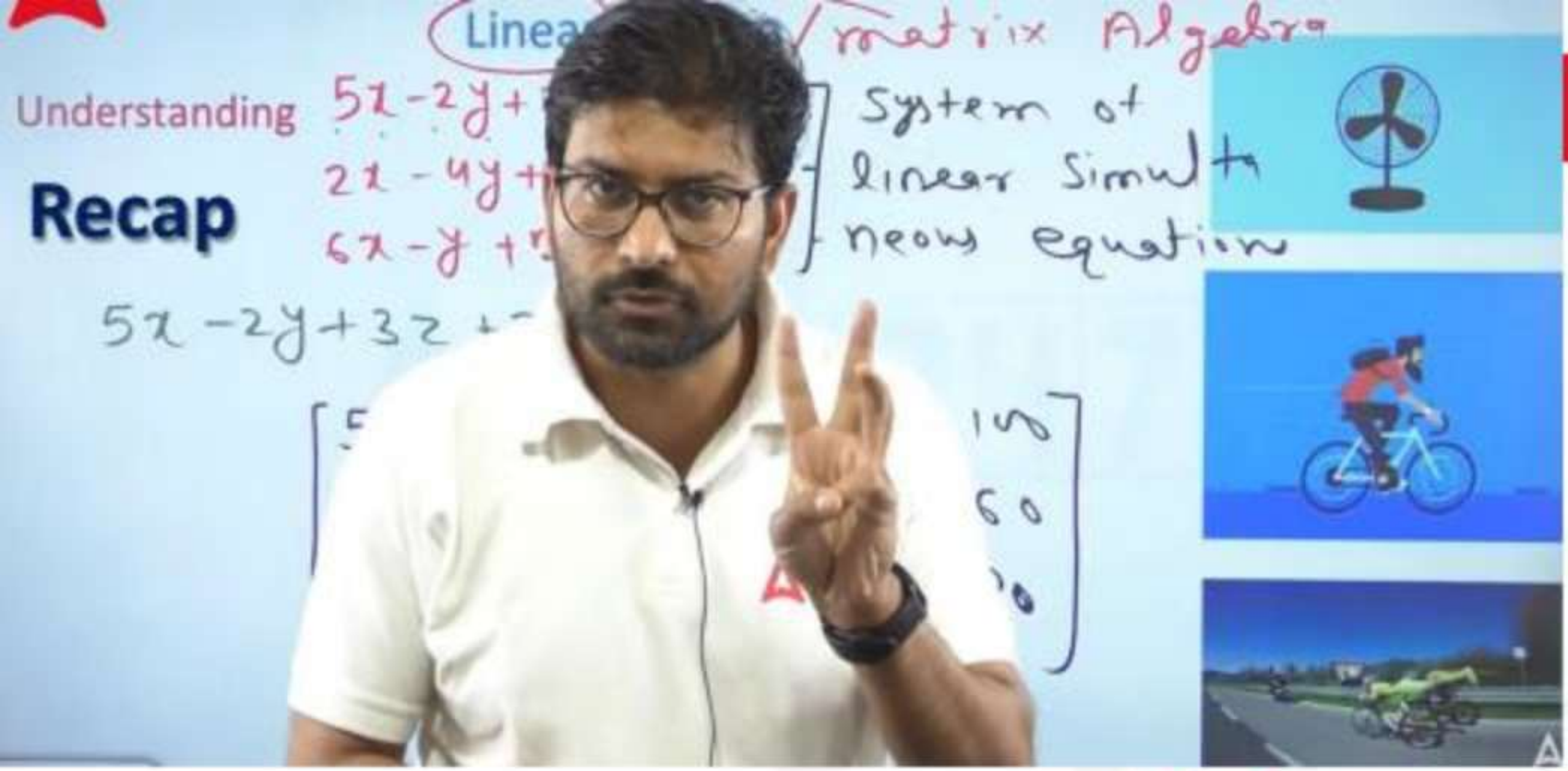
Linear Algebra

Adda247

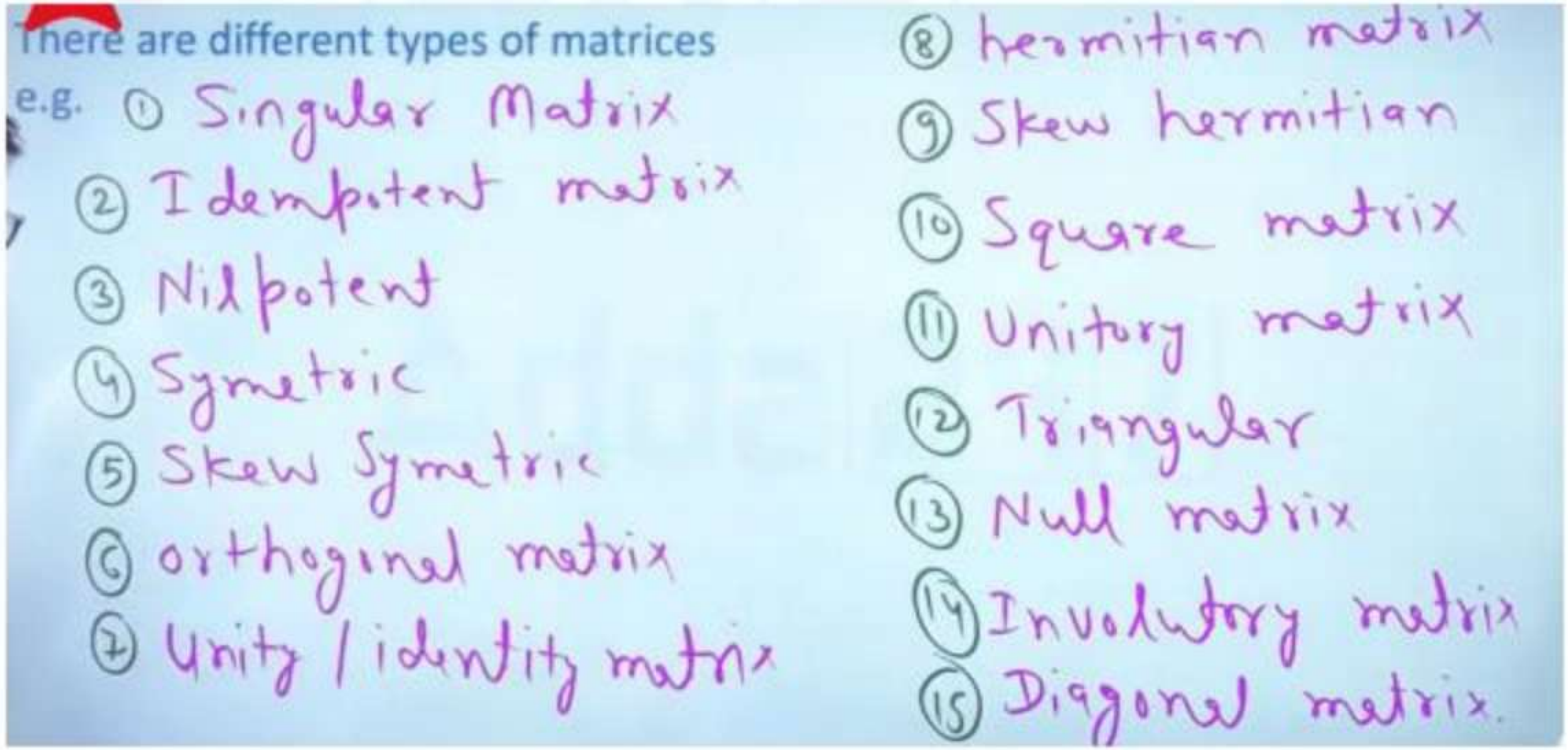


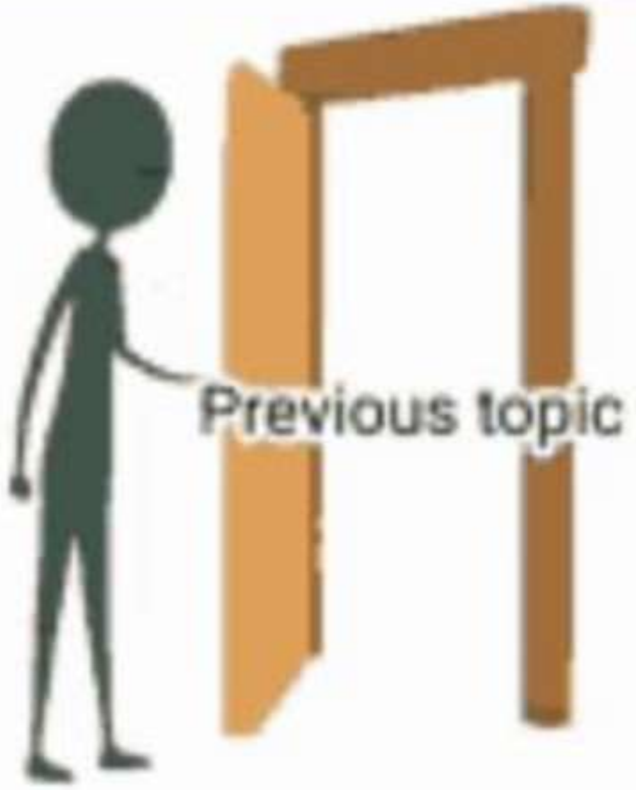
1. *Determinant*
2. *Cofactor*
3. *Minors*
4. *Inverse of matrix*



←Physical Significance of Linear algebra

Different types of matrices →





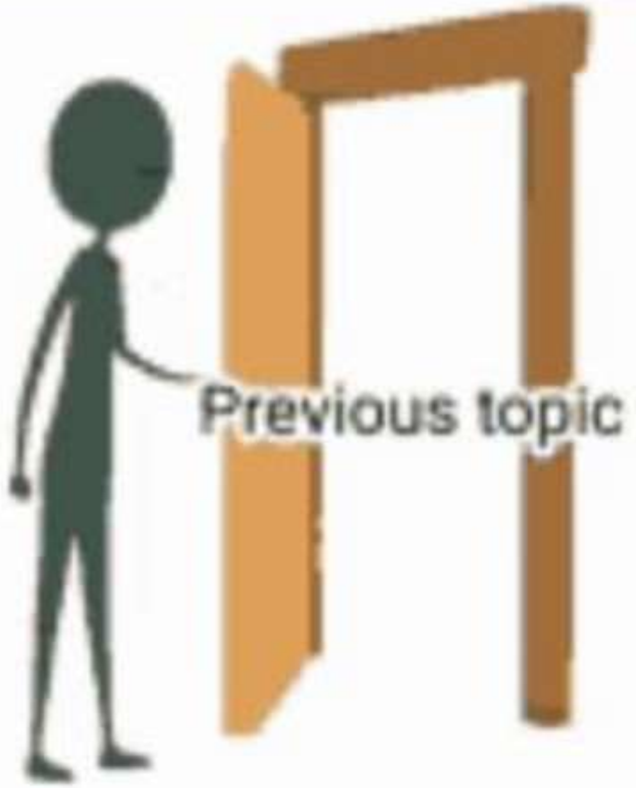
Trace of matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d$$

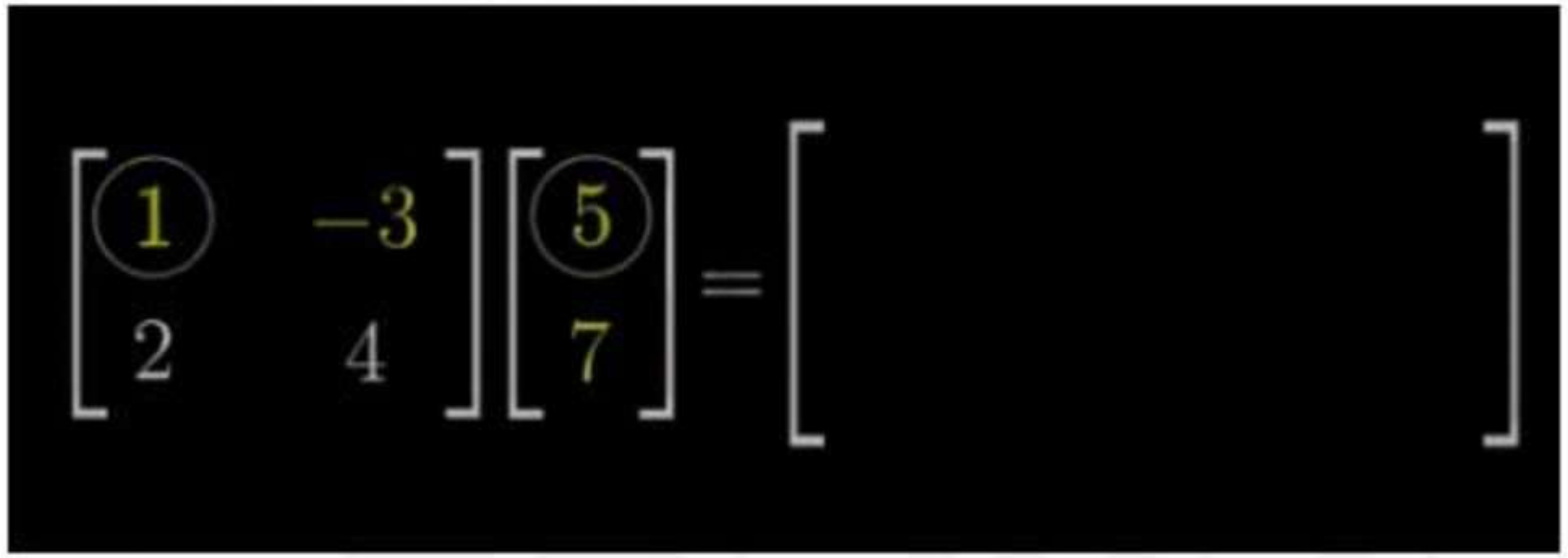
* It is defined for square matrix

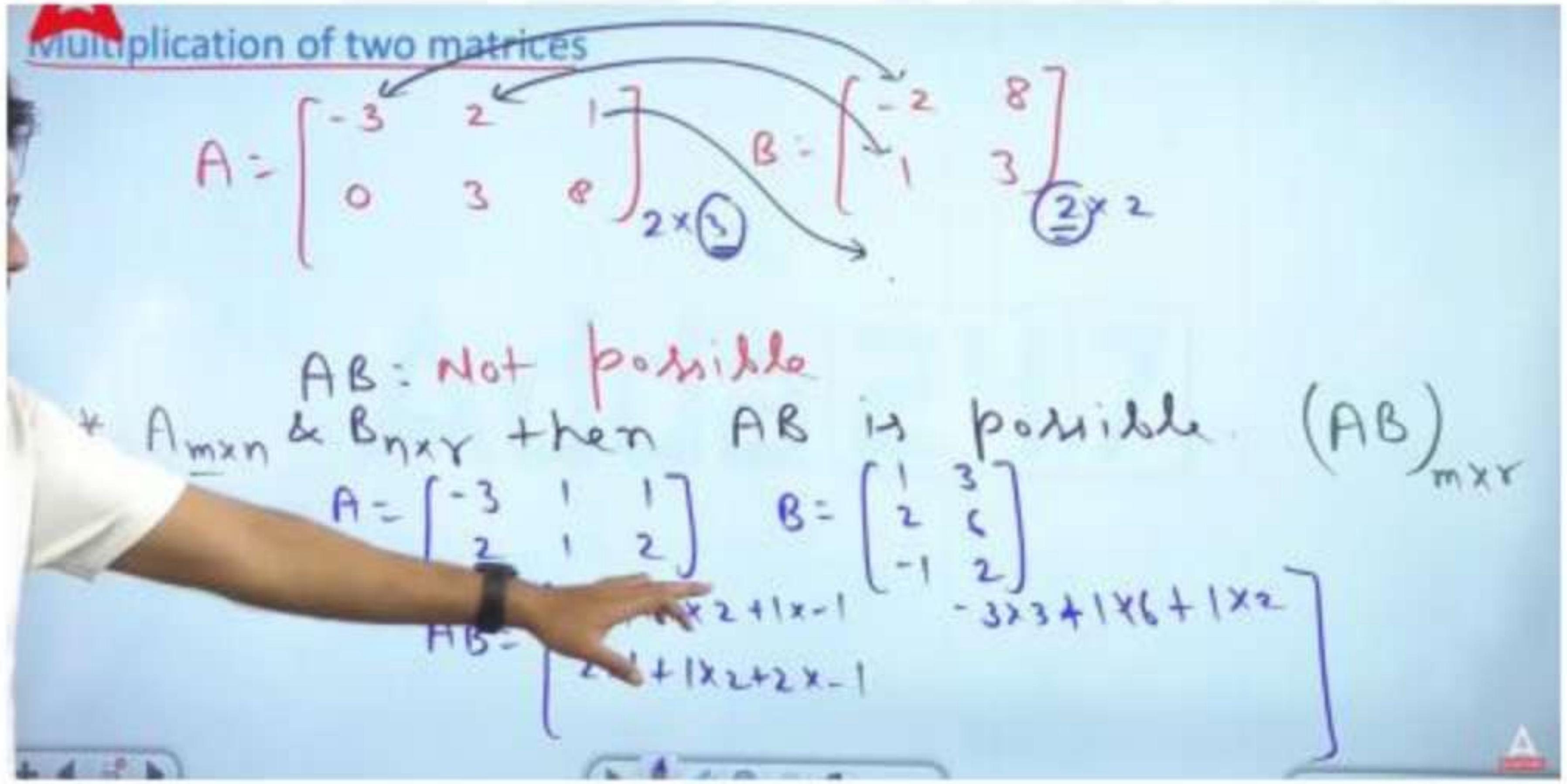
$$A = \begin{bmatrix} -5 & 2 & 1 \\ 0 & 3 & 4 \\ 2 & -1 & 8 \end{bmatrix}$$

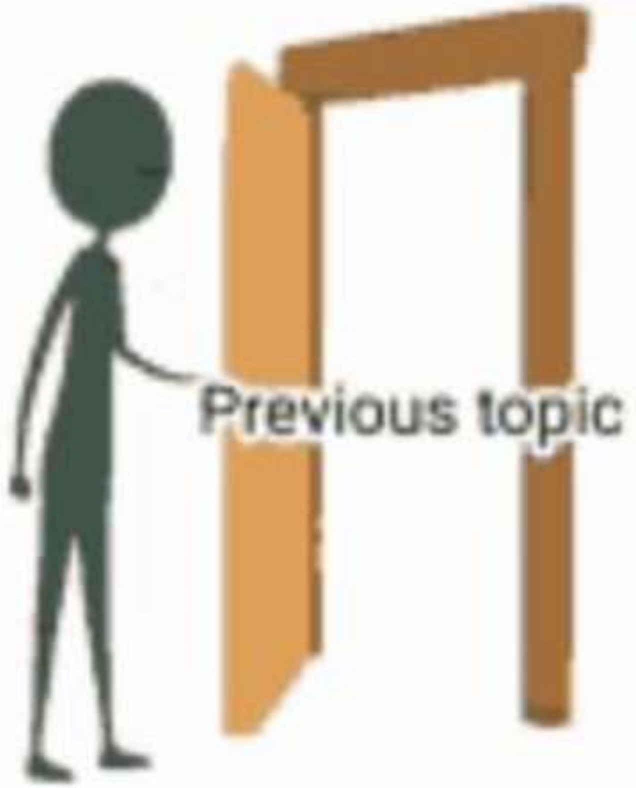
trace = 6



Matrix Multiplications





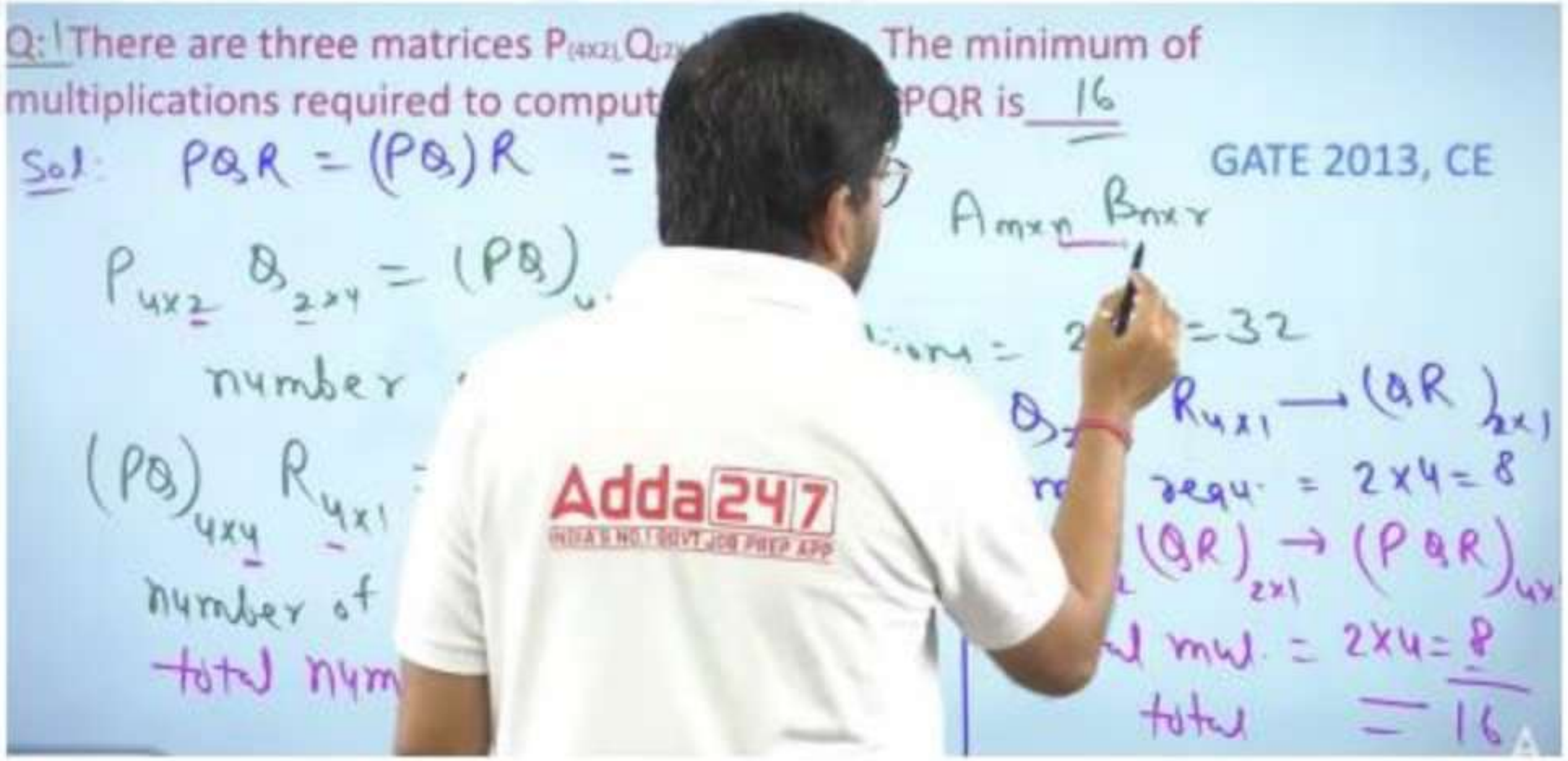


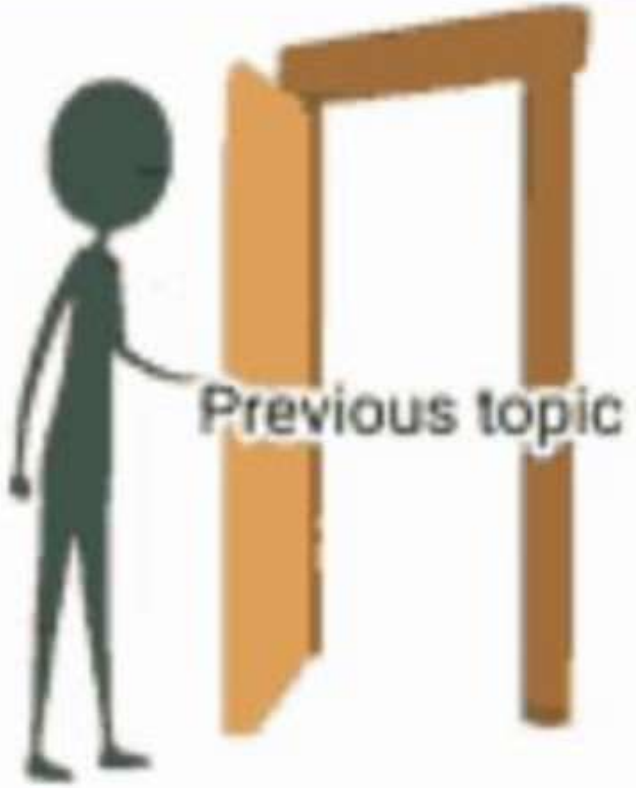
Matrix Multiplications

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} \; \end{bmatrix}$$

Matrix Multiplications

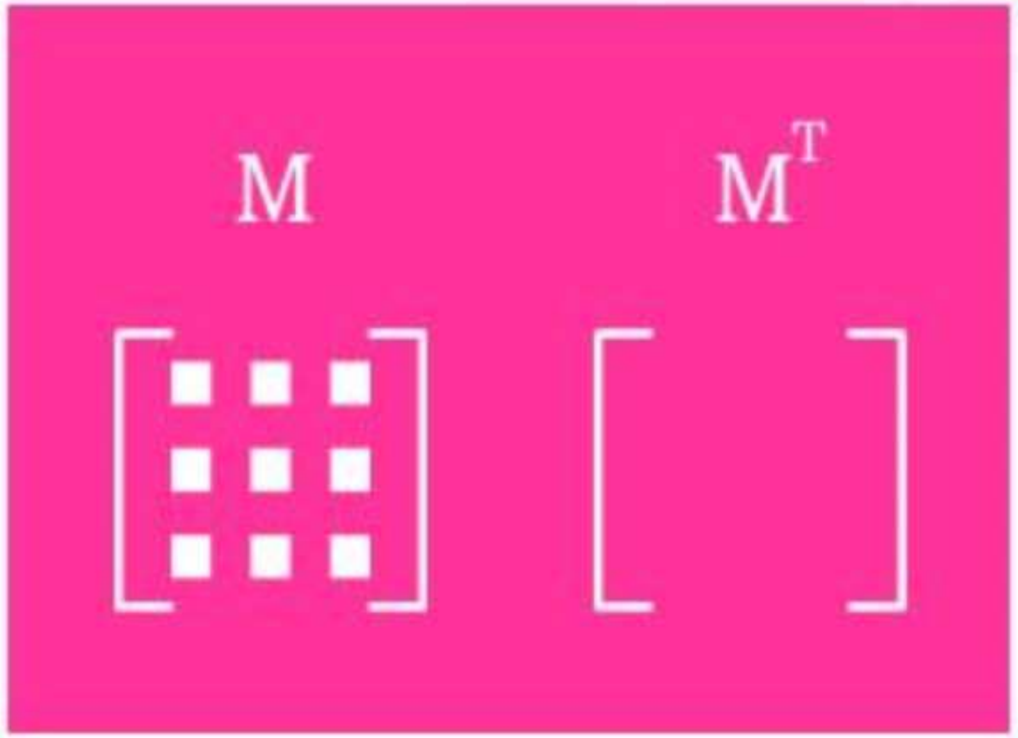
Concept question →





Transpose of matrix

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

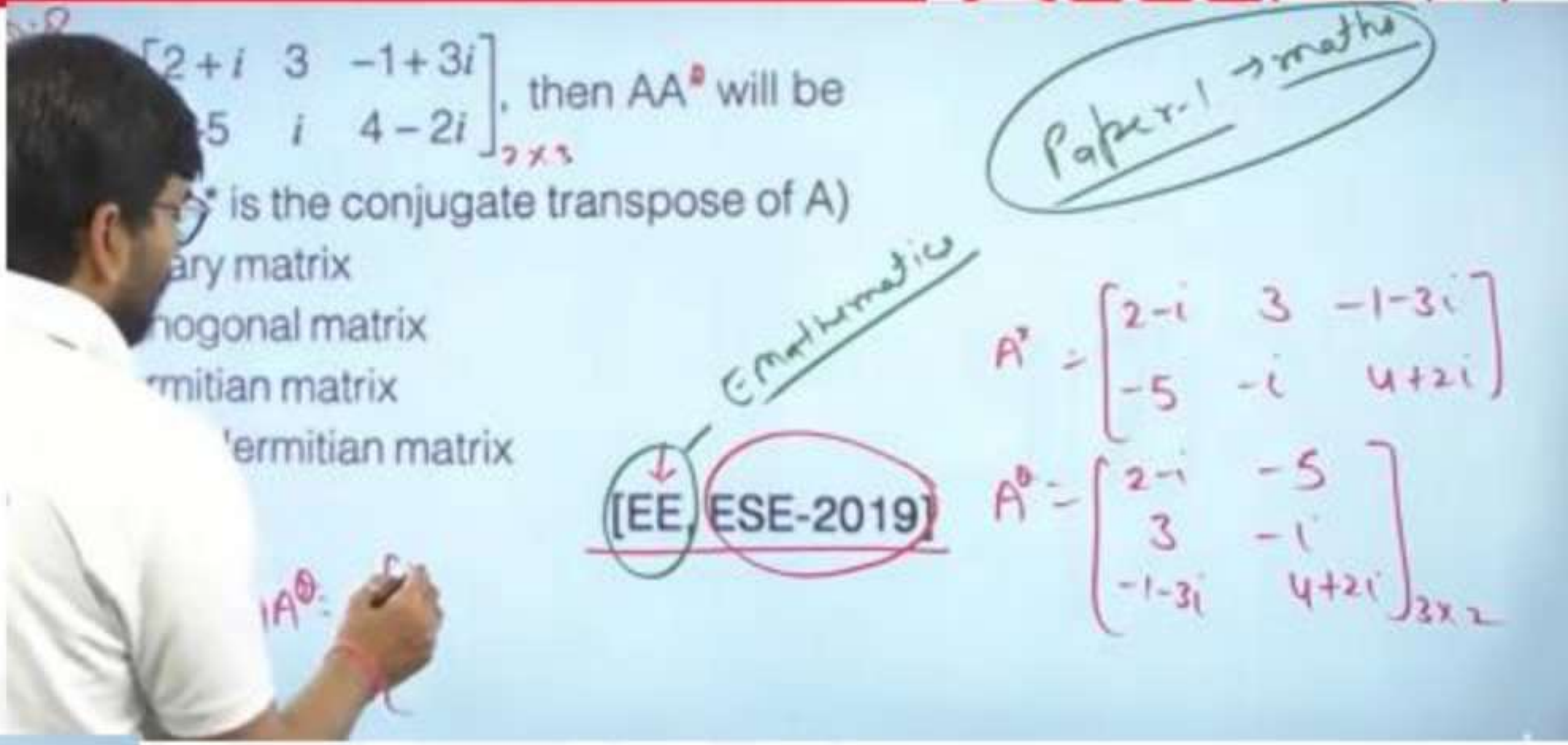


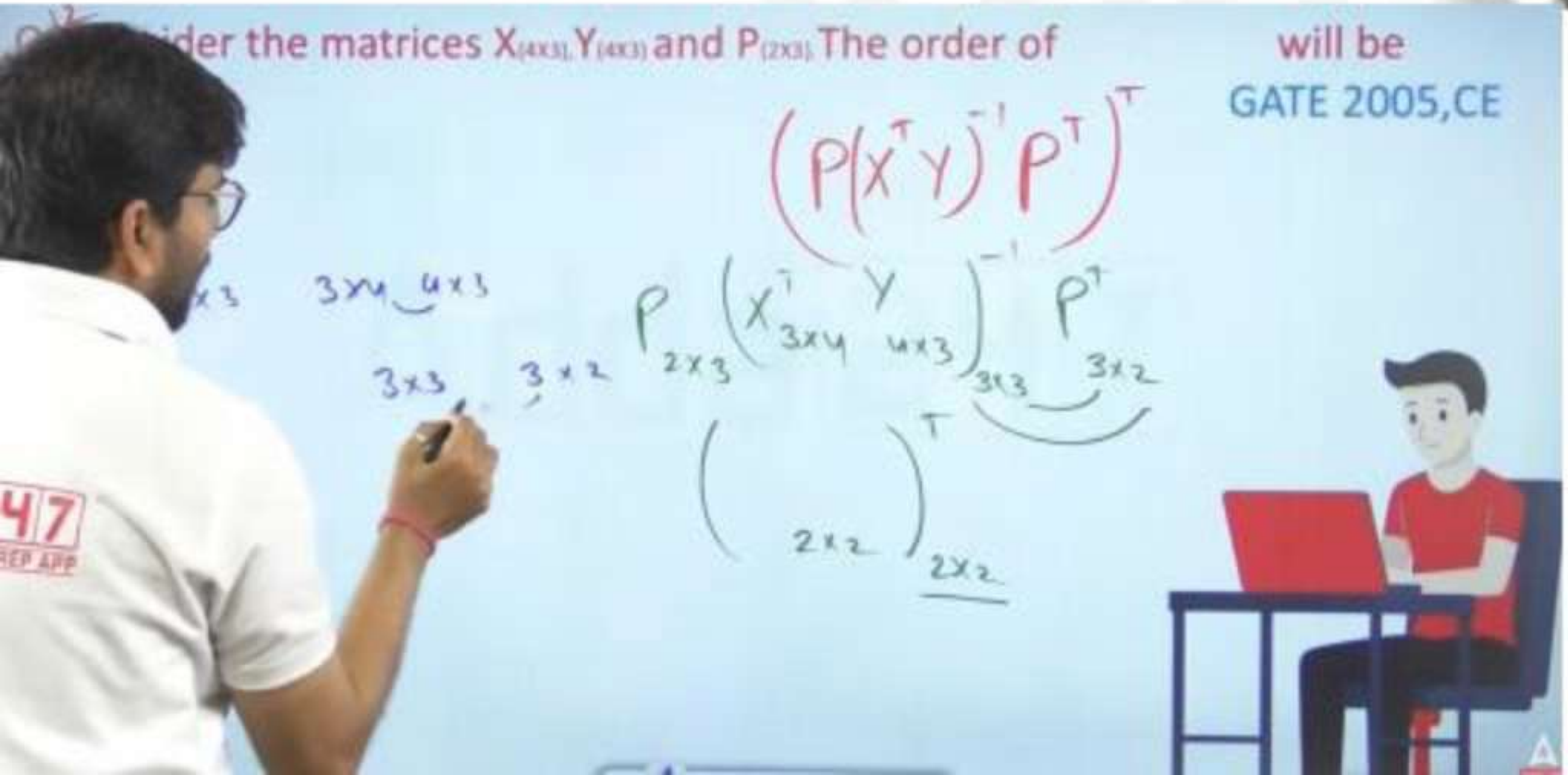
A

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Recap

ESE Question Practice →

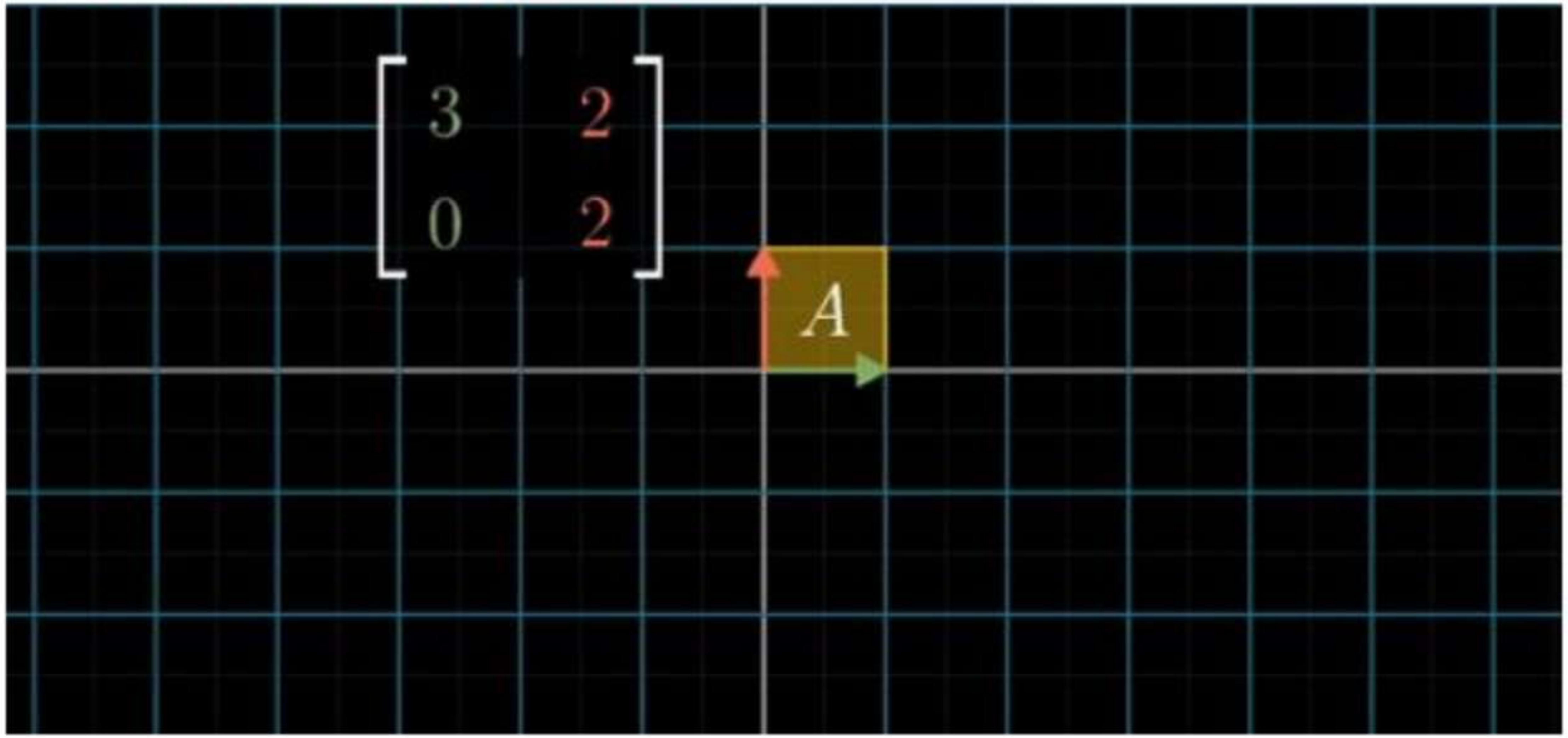




← GATE Question Practice

Number of questions covered-12

1. **Determinant**
2. **Cofactor**
3. **Minors**
4. **Adjoint Matrix**
5. **Inverse of matrix**



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GATE 2024

GATE

प्रचण्ड Batch

Engineering Mathematics

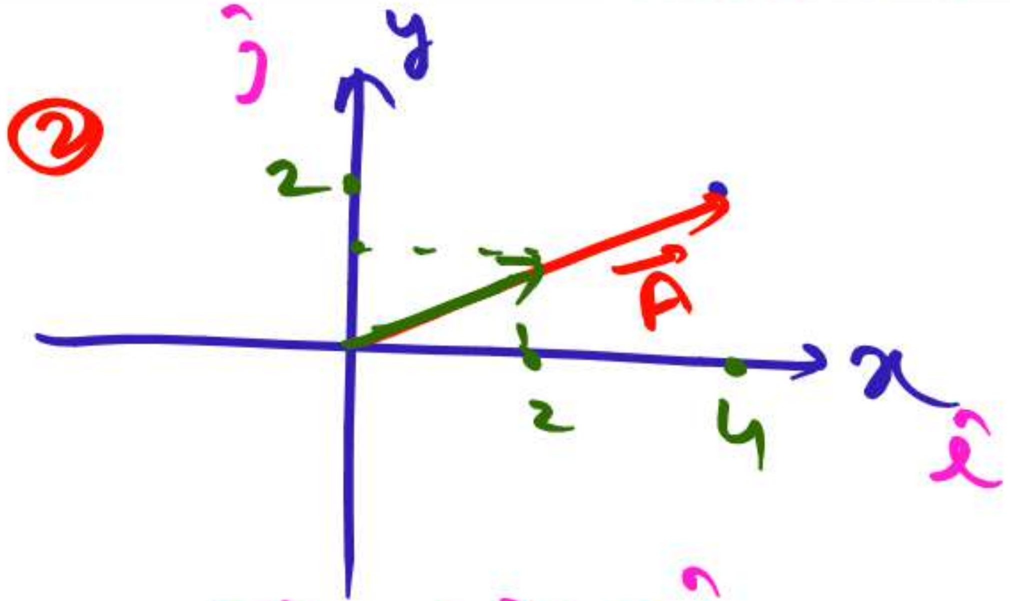
LINEAR ALGEBRA

DTERMINANT,MINORS,COFACTORS AND INVERSE OF MATRICES)

Determinant of matrix

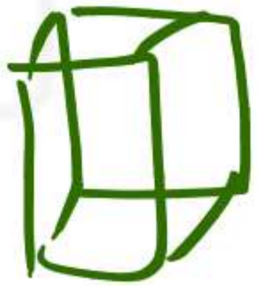
$$\det A = \det\begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2} = ad - bc$$

②



$\vec{A} = 4\hat{i} + 2\hat{j}$

$\vec{B} = 2\hat{i} + \hat{j}$



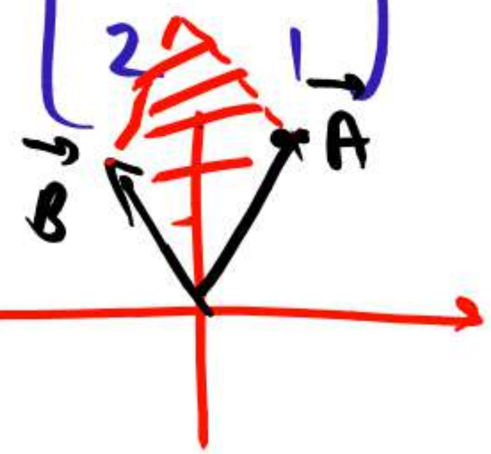
① $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$ $\quad |A| = 6 + 4 = 10$

① $B = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ $\quad |B| = 4 - 4 = 0$

$\vec{A} = 2\hat{i} + 4\hat{j}$

$\vec{B} = -\hat{i} + 3\hat{j}$

①



$$\det(A) = \det\begin{bmatrix} {}^{+}a & {}^{-}b & {}^{+}c \\ {}^{-}d & {}^{+}e & {}^{-}f \\ {}^{+}g & h & i \end{bmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

or

$$-d\begin{vmatrix} b & c \\ h & i \end{vmatrix} + e\begin{vmatrix} a & g \\ c & i \end{vmatrix} - f\begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 0 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix} \Rightarrow |A| = +2(15) = 30$$

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 1 & 6 & 2 \\ 3 & 4 & 3 \end{bmatrix} \Rightarrow |A| = 2(10) + 5(-3) + 1(-14) = 20 - 15 - 14$$

$$= -9$$

Trick to find determinant of 3X3 matrix

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 1 & 6 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$

upper $= 18 + 16 - 15 = 19$

$$\begin{bmatrix} 2 & -5 & 1 & 2 & -5 \\ 1 & 6 & 2 & 1 & 6 \\ 3 & 4 & 3 & 3 & 4 \end{bmatrix}$$

lower $= 36 - 30 + 4 = 10$

$|A| = \text{lower} - \text{upper}$

$= 10 - 19 = -9$

* determinant is defined only for a square matrix.

$$A = \begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & 2 & 4 & 1 \\ 1 & -2 & 3 & 2 \\ 1 & 1 & 3 & -1 \end{bmatrix} \longrightarrow B = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & m \end{bmatrix}$$

$$|A| = 2 \begin{vmatrix} 2 & 4 & 1 \\ -2 & 3 & 2 \\ 1 & 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 4 & 1 \\ 1 & 3 & 2 \\ 1 & 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 2 & 1 \\ 1 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix}$$

$$-3 \begin{vmatrix} 0 & 2 & 4 \\ 1 & -2 & 3 \\ 1 & 1 & 3 \end{vmatrix}$$

Properties of Determinant of Matrix

Imp

$A \rightarrow |A|$

1. B = Linear transformed matrix of A

$|B| = |A|$

$$\begin{array}{cccc} & C_1 & C_2 & C_3 & \\ A = & \left[\begin{array}{ccc} 2 & -1 & 4 \\ 1 & 2 & 1 \\ 3 & 4 & 2 \end{array}\right] & & & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array}$$

$R_2 \leftarrow R_2 - 2R_1$

* with linear transformations in a matrix determinant does not changes.

Properties of Determinant of Matrix

$A \rightarrow |A|$

② * If all elements of a row or column of a matrix are zero, then determinant of that matrix will be zero.

$A = \begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix} \quad |A| = 0$

$C = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 0 & 4 \\ 4 & 0 & 3 \end{bmatrix}$

$B = \begin{pmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 0 & 0 & 0 \end{pmatrix} = 0$

$|C| = 0$

Properties of Determinant of Matrix

$A \rightarrow |A|$

③ If element of one row/column are same or multiple of elements of another row/column then determinant of this matrix will be zero.

$$A = \begin{bmatrix} 5 & 2 \\ 10 & 4 \end{bmatrix} \Rightarrow |A| = 0$$

Properties of Determinant of Matrix

$A \rightarrow |A|$

(4) when two rows/columns are interechanged then determinant of new transformed matrix 'B' has determinant given by

$|B| = -|A|$

(5) $A_{n\times n} \longrightarrow |A|$

$kA \longrightarrow |kA| = k^n|A|$

$A_{2\times 2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad |A| = ad - bc$

$B = kA = \begin{pmatrix} ka & kb \\ kc & kd \end{pmatrix} \Rightarrow |B| = k^2ad - k^2bc = k^2(ad - bc)$

Properties of Determinant of Matrix

A → |A|

$$A_{3\times3} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \qquad |A| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} \cdots$$

$$B = KA = \begin{bmatrix} Ka & Kb & Kc \\ Kd & Ke & Kf \\ Kg & Kh & Ki \end{bmatrix}$$

$$|KA| = Ka \begin{vmatrix} Ke & Kf \\ Kh & Ki \end{vmatrix}$$

$$|B| = K^3 \left[a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - - - \right] = K^3 |A|$$

Properties of Determinant of Matrix

$A \rightarrow |A|$

(6) $A \longrightarrow |A|$

$B = A^T \longrightarrow |A^T| = |A|$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

(7) $A \longrightarrow |A|$

$B \longrightarrow |B|$

$C = A.B \longrightarrow |C| = |A|.|B|$

(8) $A \longrightarrow |A|$

$A^2 = A.A = |A|^2$

$\vdots$

$A^m \longrightarrow |A^m| = (|A|)^m$

Properties of Determinant of Matrix

(10) $I_{n\times n} \longrightarrow |I| = 1$

$A \rightarrow |A|$

(9) $A^{-1} \longrightarrow (|A|)^{-1} = \frac{1}{|A|}$

$AA^{\theta} = I$

(11) For orthogonal matrix $AA^{T} = I$

$$\det(AA^{T}) = \det(I)$$

$$|A|\,|A^{T}| = 1$$

$$(|A|)^{2} = 1$$

$$|A| = \pm 1$$

* determininant of orthogonal matrix has magnitude unity always.

Properties of Determinant of Matrix

$A \rightarrow |A|$

(12) $A \longrightarrow |A|$

$B \longrightarrow |B|$

$A+B \longrightarrow |A+B| \neq |A| + |B|$

if A has $|A| = 0$ then matrix A is Called singular matrix.

(13) $A_{n \times n} \longrightarrow |A|$

B = multiple by 'K' to any one row/column of 'A'

$B \rightarrow |B| = K|A|$

$$A = \begin{vmatrix} 5 & 2 & 8 \\ 3 & 4 & 16 \\ 2 & 1 & 4 \end{vmatrix} = 4\begin{vmatrix} 5 & 2 & 2 \\ 3 & 4 & 4 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

Minors :→ With removing one or more row or column or both to get a square matrix then this resulting square matrix is called one minor of original matrix.

$$A = \begin{bmatrix} 5 & -2 & 8 & 6 \\ 2 & 1 & 3 & 4 \\ 2 & -5 & 6 & 1 \end{bmatrix}_{3\times 4}$$

$$\begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}_{2\times 2}$$

$$\begin{bmatrix} 5 & -2 & 8 \\ 2 & 1 & 3 \\ 2 & -5 & 6 \end{bmatrix}_{3\times 3}$$

$$= {}^4C_2 \times {}^3C_1$$

$$= \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \frac{\lfloor 3}{\lfloor 2 \lfloor 1}$$

$$\frac{4\times 3}{2} \times 3$$

$$18$$

<u>Cofactor</u>

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}_{2\times 2}$$

$\text{cof}(5) = +(4) = 4$

$\text{cof}(2) = -(-1) = 1$

$\text{cof}(-1) = -2$

$\text{cof}(4) = 5$

$$\text{Cof}(A) = \begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 3 & -2 & 1 \end{bmatrix}_{3\times 3}$$

$$Cof(A) = \begin{bmatrix} 10 & 11 & -8 \\ -1 & -1 & 1 \\ -6 & -7 & 5 \end{bmatrix}_{3\times 3}$$

Inverse of matrix

Inverse of a matrix 'A' is matrix 'B' such that $AB = BA = I$.

$A_{2\times 3} B_{3\times 2}$ $B_{3\times 2} A_{2\times 3}$ | $A_{3\times 3} B_{3\times 3}$ $B_{3\times 3} A_{3\times 3}$

$(AB)_{2\times 2} \neq BA_{3\times 3}$

⇒ Matrix 'A' for which inverse of matrix is to be defined should be square matrix only

⇒ Inverse matrix is also square matrix of same order.

'B' is termed as A^{-1}.

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$adj(A) = [Cof(A)]^T$$

* if A is singular matrix

e.g. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \Rightarrow |A| = 0$

$$A^{-1} = \frac{1}{0}\begin{bmatrix} 3 & -6 \\ -1 & 2 \end{bmatrix}^T = \frac{1}{0}\begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix} = \begin{bmatrix} \infty & -\infty \\ -\infty & \infty \end{bmatrix}$$

then A is called non-invertible matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$C(A) = \begin{bmatrix} a & -c \\ -b & a \end{bmatrix} \Rightarrow adj\,A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}$$

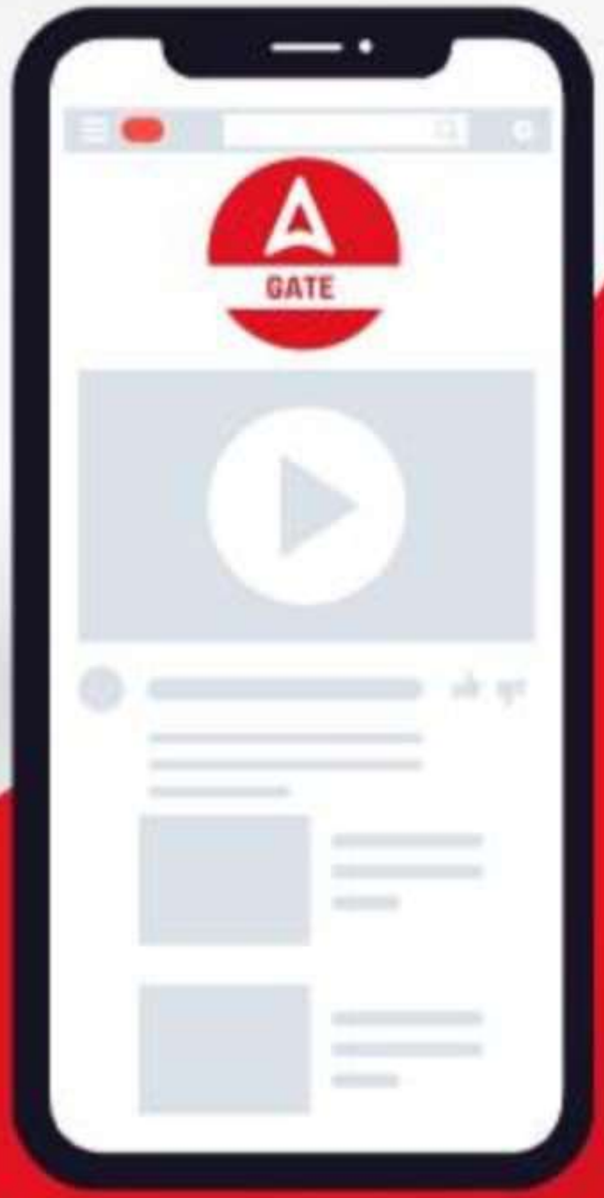
$$A^{-1} = \frac{1}{22}\begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{22} & -\frac{2}{22} \\ \frac{1}{22} & \frac{5}{22} \end{bmatrix}$$

Short trick to find cofactor matrix of 3x3 matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 3 & -2 & 1 \end{bmatrix}$$

$$\begin{array}{c|cccc} 2 & -1 & 1 & 2 & -1 \\ \hline 1 & 2 & 4 & 1 & 2 \\ 3 & -2 & 1 & 3 & -2 \\ 2 & -1 & 1 & 2 & -1 \\ 1 & 2 & 4 & 1 & 2 \end{array}$$

$$\mathrm{Cof}(A) = \begin{bmatrix} 10 & 11 & -8 \\ -1 & -1 & 1 \\ -6 & -7 & 5 \end{bmatrix}$$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