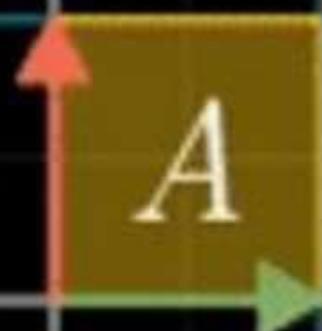


$$\begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$$



- 1. Determinant**
- 2. Cofactor**
- 3. Minors**
- 4. Inverse of matrix**

← Physical Significance of Linear algebra

Understanding Linear Algebra

System of linear Simultaneous equations

Recap

$5x - 2y + 3z + \dots$

$2x - 4y + \dots$

$6x - y + \dots$

$\begin{bmatrix} 5 \\ -2 \\ 3 \\ \dots \end{bmatrix}$

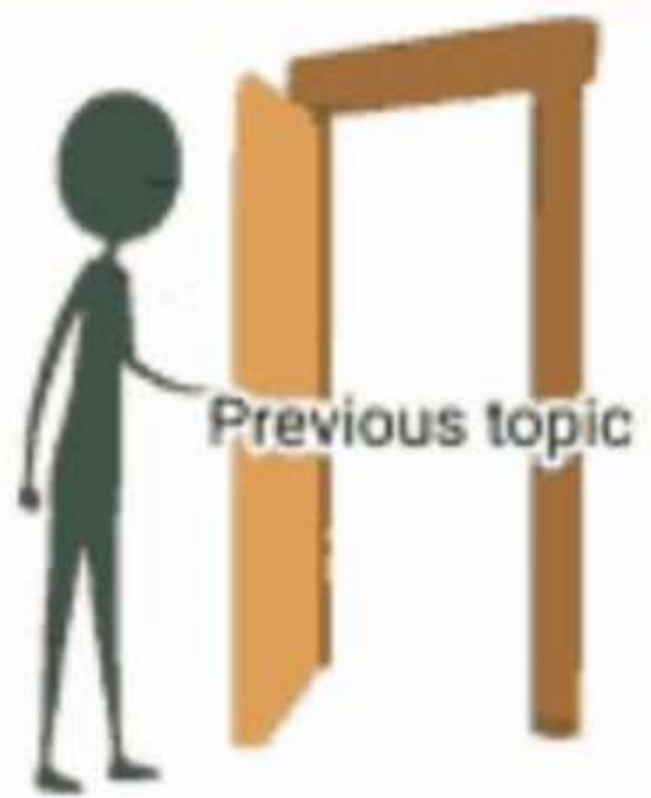
$\begin{bmatrix} 15 \\ 0 \\ 0 \\ \dots \end{bmatrix}$

Different types of matrices →

There are different types of matrices

- e.g.
- ① Singular Matrix
 - ② Idempotent matrix
 - ③ Nilpotent
 - ④ Symmetric
 - ⑤ Skew Symmetric
 - ⑥ orthogonal matrix
 - ⑦ Unity / identity matrix

- ⑧ hermitian matrix
- ⑨ Skew hermitian
- ⑩ Square matrix
- ⑪ Unitary matrix
- ⑫ Triangular
- ⑬ Null matrix
- ⑭ Involutory matrix
- ⑮ Diagonal matrix.



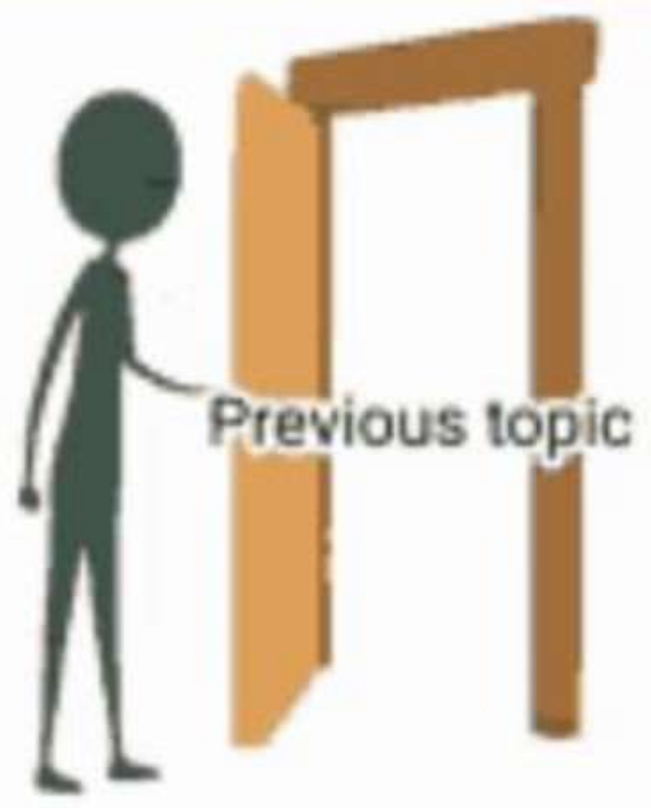
Trace of matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d$$

* It is defined for square matrix

$$A = \begin{bmatrix} -5 & 2 & 1 \\ 0 & 3 & 4 \\ 2 & -1 & 0 \end{bmatrix}$$

$$\text{trace} = 6$$



Matrix Multiplications

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Multiplication of two matrices

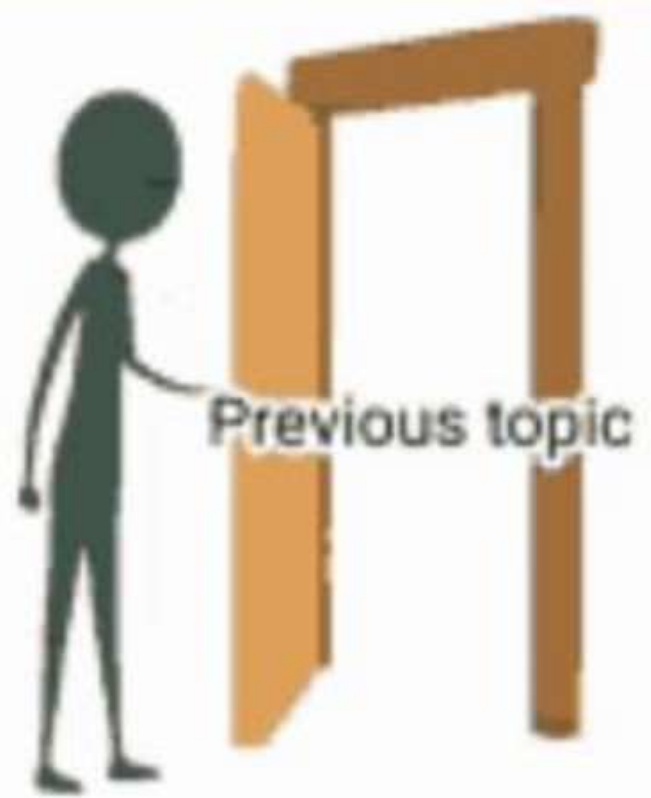
$A = \begin{bmatrix} -3 & 2 & 1 \\ 0 & 3 & 8 \end{bmatrix}_{2 \times 3}$
 $B = \begin{bmatrix} -2 & 8 \\ 1 & 3 \end{bmatrix}_{2 \times 2}$

AB : Not possible

\bullet $A_{m \times n}$ & $B_{n \times r}$ then AB is possible. $(AB)_{m \times r}$

$A = \begin{bmatrix} -3 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$
 $B = \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ -1 & 2 \end{bmatrix}$

$AB = \begin{bmatrix} -3 \times 1 + 1 \times 2 + 1 \times -1 & -3 \times 3 + 1 \times 6 + 1 \times 2 \\ 2 \times 1 + 1 \times 2 + 2 \times -1 & \dots \end{bmatrix}$



Matrix Multiplications

$$\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

Matrix Multiplications Concept question →

Q: There are three matrices $P_{4 \times 2}, Q_{2 \times 4}$ and $R_{4 \times 1}$. The minimum of multiplications required to compute PQR is 16

GATE 2013, CE

Sol: $PQR = (PQ)R = \dots$

$P_{4 \times 2} \cdot Q_{2 \times 4} = (PQ)_{4 \times 4}$
 number of multiplications = $2 \times 4 \times 4 = 32$

$(PQ)_{4 \times 4} \cdot R_{4 \times 1}$
 number of multiplications = $4 \times 4 \times 1 = 16$

total number of multiplications = $32 + 16 = 48$

$P_{4 \times 2} \cdot (QR)_{2 \times 1}$
 number of multiplications = $2 \times 4 \times 1 = 8$

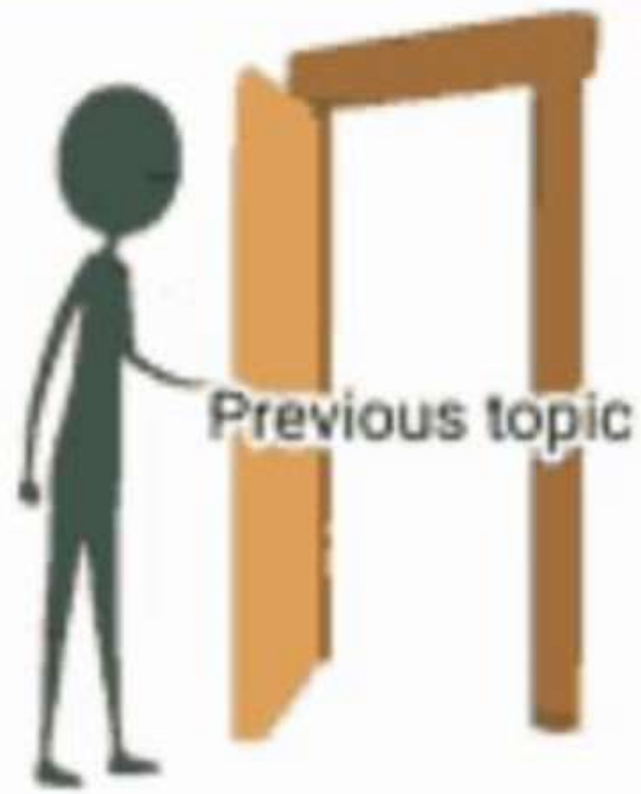
$(PQ)_{4 \times 4} \cdot R_{4 \times 1}$
 number of multiplications = $4 \times 4 \times 1 = 16$

total = $8 + 16 = 24$

$(PQ)_{4 \times 4} \cdot R_{4 \times 1}$
 number of multiplications = $4 \times 4 \times 1 = 16$

total = $32 + 16 = 48$

total = 16



Transpose of matrix

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$M \quad M^T$$
$$\begin{bmatrix} \square & \square & \square \\ \square & \square & \square \\ \square & \square & \square \end{bmatrix} \quad \begin{bmatrix} \square & \square \\ \square & \square \\ \square & \square \end{bmatrix}$$

Recap

ESE Question Practice →


$\begin{bmatrix} 2+i & 3 & -1+3i \\ -5 & i & 4-2i \end{bmatrix}_{2 \times 3}$, then AA^{θ} will be
 (where θ is the conjugate transpose of A)
 Hermitian matrix
 Orthogonal matrix
 Symmetric matrix
 Normal matrix

Handwritten notes:
 Paper-1 → maths
 E-mathematics
 [EE ESE-2019]
 $A^{\theta} = \begin{bmatrix} 2-i & 3 & -1-3i \\ -5 & -i & 4+2i \end{bmatrix}$
 $A^{\theta} = \begin{bmatrix} 2-i & -5 \\ 3 & -i \\ -1-3i & 4+2i \end{bmatrix}_{3 \times 2}$

Consider the matrices $X_{(4 \times 3)}$, $Y_{(4 \times 3)}$ and $P_{(2 \times 3)}$. The order of $(P(X^T Y)^{-1} P^T)^T$ will be
 GATE 2005, CE

$$P_{2 \times 3} (X_{3 \times 4}^T Y_{4 \times 3})^{-1} P_{3 \times 2}^T$$

$$\left((P(X^T Y)^{-1} P^T)^T \right)_{2 \times 2}$$



← GATE Question Practice

Number of questions covered-12



today's
topics

1. Determinant
2. Cofactor
3. Minors
4. Adjoint Matrix
5. Inverse of matrix

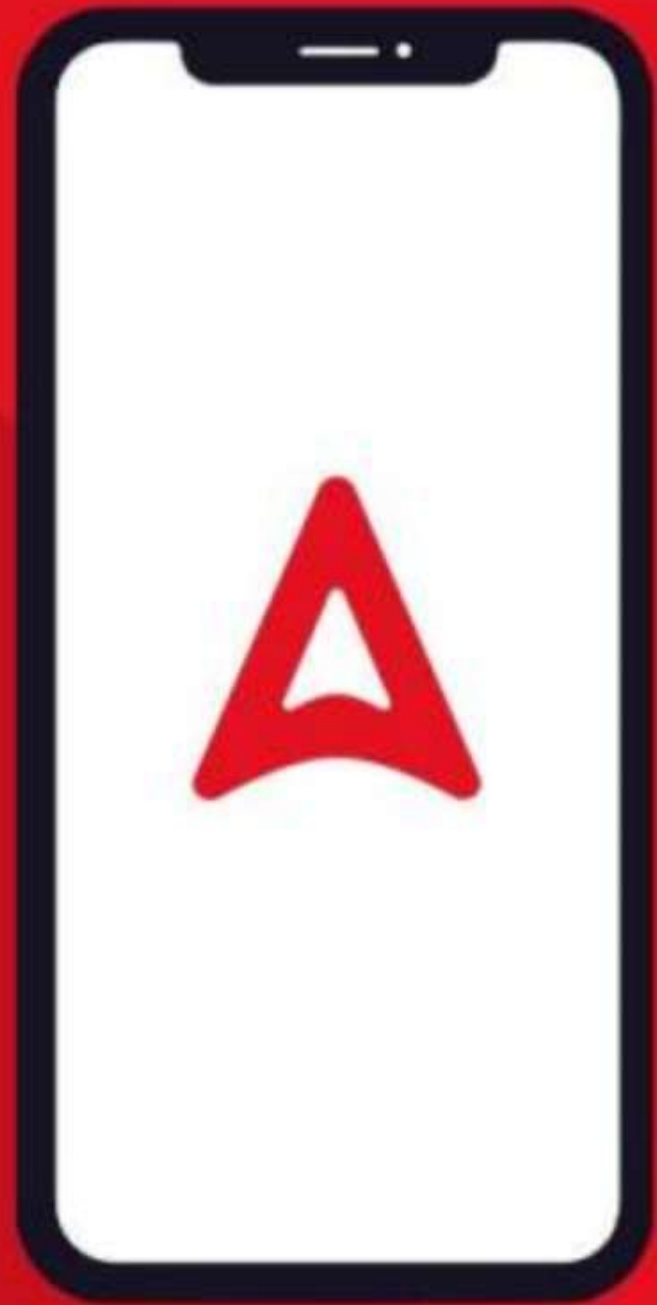
$$\begin{bmatrix} 3 & 2 \\ 0 & 2 \end{bmatrix}$$



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GATE 2024



प्रचण्ड Batch

Engineering Mathematics

LINEAR ALGEBRA

**(DETERMINANT, MINORS, COFACTORS
AND INVERSE OF MATRICES)**



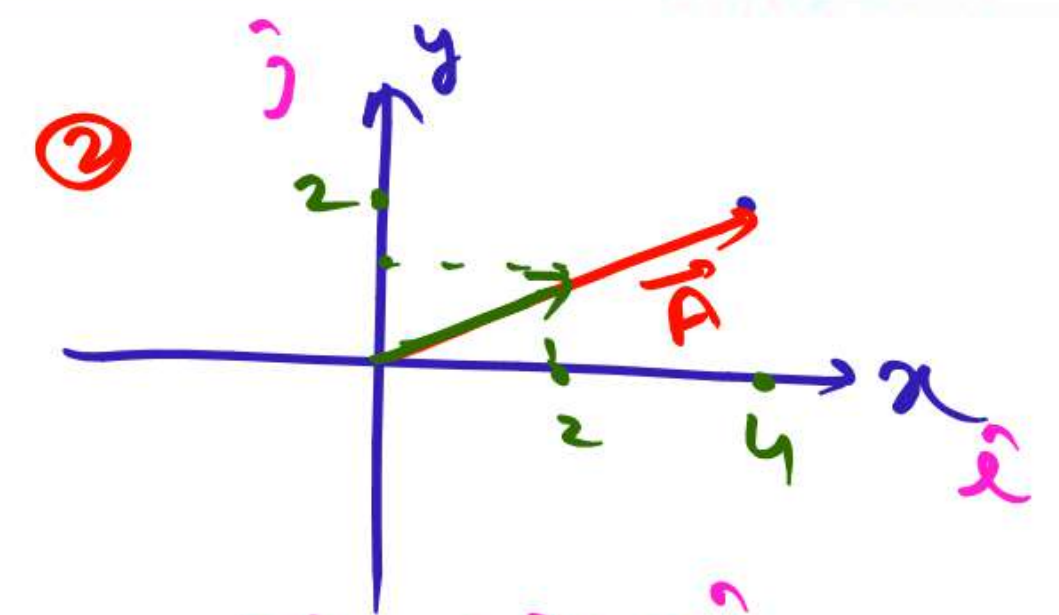
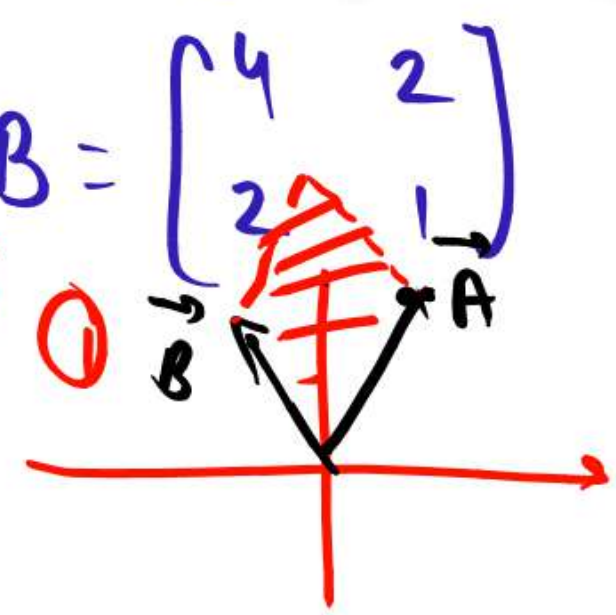
Determinant of matrix

$$\det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2 \times 2} = ad - bc$$

① $A = \begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}$ $|A| = 6 + 4 = 10$

① $B = \begin{bmatrix} 4 & 2 \\ 4 & -4 \end{bmatrix}$ $|B| = 4 - 4 = 0$

$\vec{A} = 2\hat{i} + 4\hat{j}$
 $\vec{B} = -\hat{i} + 3\hat{j}$



$\vec{A} = 4\hat{i} + 2\hat{j}$
 $\vec{B} = 2\hat{i} + \hat{j}$



$$\det(A) = \det \begin{bmatrix} +a & -b & +c \\ -d & +e & -f \\ +g & h & i \end{bmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

or

$$-d \begin{vmatrix} b & c \\ h & i \end{vmatrix} + e \begin{vmatrix} a & c \\ g & i \end{vmatrix} - f \begin{vmatrix} a & b \\ g & h \end{vmatrix}$$

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 9 & 3 & 4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 1 & 6 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$

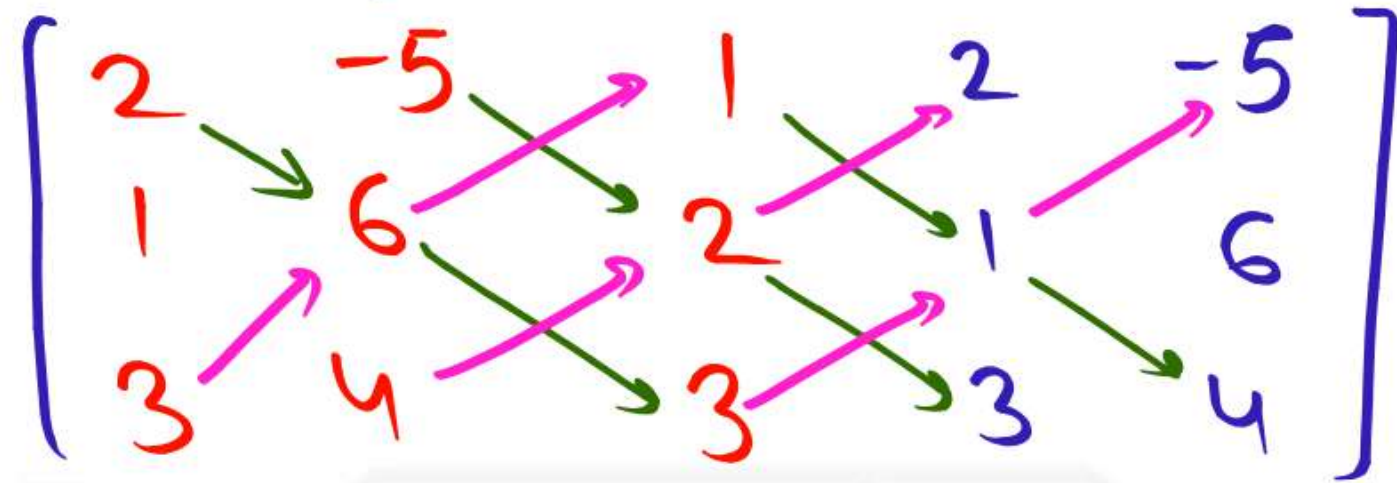
$$\Rightarrow |A| = +2(15) = 30$$

$$\Rightarrow |A| = 2(10) + 5(-3) + 1(-14) = 20 - 15 - 14 = -9$$

Trick to find determinant of 3X3 matrix

$4pper = 18 + 16 - 15 = 19$

$$A = \begin{bmatrix} 2 & -5 & 1 \\ 1 & 6 & 2 \\ 3 & 4 & 3 \end{bmatrix}$$



$lower = 36 - 30 + 4 = 10$

$|A| = lower - 4pper$
 $= 10 - 19 = -9$

* determinant is defined only for a square matrix.

$$A = \begin{bmatrix} 2 & -1 & -1 & 3 \\ 0 & 2 & 4 & 1 \\ -1 & -2 & 3 & 2 \\ 1 & 1 & 3 & -1 \end{bmatrix}$$

$$\rightarrow B = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ o & p & q & m \end{bmatrix}$$

$$|A| = 2 \begin{vmatrix} 2 & 4 & 1 \\ -2 & 3 & 2 \\ 1 & 3 & -1 \end{vmatrix} - 1 \begin{vmatrix} 0 & 4 & 1 \\ -1 & 3 & 2 \\ 1 & 3 & -1 \end{vmatrix} + 1 \begin{vmatrix} 0 & 2 & 1 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{vmatrix} - 3 \begin{vmatrix} 0 & 2 & 4 \\ -1 & -2 & 3 \\ 1 & 1 & 3 \end{vmatrix}$$

Properties of Determinant of MatrixImp ~~A~~

$$A \rightarrow |A|$$

1. \tilde{B} = Linear transformed matrix of A

$$|B| = |A|$$

* with linear transformations in $R_2 \leftarrow R_2 - 2R_1$
a matrix determinant does not change.

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 1 & 2 & 1 \\ 3 & 4 & 2 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

Properties of Determinant of Matrix

$$A \rightarrow |A|$$

② * If all elements of a row or column of a matrix are zero, then determinant of that matrix will be zero.

$$A = \begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix} \quad |A| = 0$$

$$B = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$$C = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 0 & 4 \\ 9 & 0 & 3 \end{bmatrix}$$
$$|C| = 0$$

Properties of Determinant of Matrix

$$A \rightarrow |A|$$

③ If element of one row/column are same or multiple of elements of another row/column then determinant of this matrix will be zero.

$$A = \begin{bmatrix} 5 & 2 \\ 10 & 4 \end{bmatrix} \Rightarrow |A| = 0$$

Properties of Determinant of Matrix

$$A \rightarrow |A|$$

④ When two rows/columns are interchanged then determinant of new transformed matrix 'B' has determinant given by

$$\text{⑤ } \begin{array}{l} A \xrightarrow{n \times n} |A| \\ KA \xrightarrow{} |KA| = K^n |A| \end{array} \quad |B| = -|A|$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$B = KA = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \Rightarrow |B| = k^2 ad - k^2 bc = k^2(ad - bc)$$

$$|A| = ad - bc$$

Properties of Determinant of Matrix

$A \rightarrow |A|$

$$A = \begin{matrix} 3 \times 3 \\ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \end{matrix}$$

$$|A| = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - \dots$$

$$B = kA = \begin{bmatrix} ka & kb & kc \\ kd & ke & kf \\ kg & kh & ki \end{bmatrix}$$

$$|kA| = ka \begin{vmatrix} ke & kf \\ kh & ki \end{vmatrix} - \dots$$

$$|B| = k^3 \left[a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - \dots \right] = k^3 |A|$$

Properties of Determinant of Matrix

$$A \rightarrow |A|$$

$$\textcircled{6} \quad A \rightarrow |A|$$

$$B = A^T \rightarrow |A^T| = |A|$$

$$\textcircled{7} \quad A \rightarrow |A|$$

$$B \rightarrow |B|$$

$$C = AB \rightarrow |C| = |A| \cdot |B|$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

$$\textcircled{8}$$

$$A \rightarrow |A|$$

$$A^2 = A \cdot A = |A|^2$$

$$A^m \rightarrow |A^m| = (|A|)^m$$

Properties of Determinant of Matrix

$$\textcircled{10} I_{n \times n} \rightarrow |I| = 1$$

$$A \rightarrow |A|$$

$$\textcircled{9} A^{-1} \rightarrow (|A|)^{-1} = \frac{1}{|A|}$$

$$AA^0 = I$$

$$\textcircled{11} \text{ for orthogonal matrix } AA^T = I$$

$$\det(AA^T) = \det(I)$$

$$|A| |A^T| = 1$$

$$(|A|)^2 = 1$$

* determinant of orthogonal matrix has magnitude unity always. $|A| = \pm 1$

Properties of Determinant of Matrix

$$A \rightarrow |A|$$

$$\textcircled{12} \quad \begin{aligned} A &\rightarrow |A| \\ B &\rightarrow |B| \end{aligned}$$

$$A+B \rightarrow |A+B| \neq |A| + |B|$$

$$\textcircled{13} \quad A_{n \times n} \rightarrow |A|$$

$B =$ multiple by ' k ' to any one row/column of ' A '

$$B \rightarrow |B| = k |A|$$

if A has $|A|=0$
then matrix A is
called singular matrix.

$$A = \begin{vmatrix} 5 & 2 & 8 \\ 3 & 4 & 16 \\ 2 & 1 & 4 \end{vmatrix} = 4 \begin{vmatrix} 5 & 2 & 2 \\ 3 & 4 & 4 \\ 2 & 1 & 1 \end{vmatrix} = 0$$

Minors \rightarrow With removing one or more row or column or both to get a square matrix then this resulting square matrix is called one minor of original matrix.

$$A = \begin{bmatrix} 5 & -2 & 8 & 6 \\ 2 & 1 & 3 & 4 \\ 2 & -5 & 6 & 1 \end{bmatrix}_{3 \times 4}$$

$$\begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 5 & -2 & 8 \\ 2 & 1 & 3 \\ 2 & -5 & 6 \end{bmatrix}_{3 \times 3}$$

$$= {}^4 C_2 \times {}^3 C_1$$

$$= \frac{{}^4 P_2}{{}^2 P_2} \times \frac{{}^3 P_1}{{}^2 P_1}$$

$$= \frac{4 \times 3}{2} \times 3$$

$$= 18$$

Cofactor

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}_{2 \times 2}$$

$$\text{Cof}(5) = + (4) = 4$$

$$\text{Cof}(2) = - (-1) = 1$$

$$\text{Cof}(-1) = - 2$$

$$\text{Cof}(4) = 5$$

$$\text{Cof}(A) = \begin{bmatrix} 4 & 1 \\ -2 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 3 & -2 & 1 \end{bmatrix}_{3 \times 3}$$

$$\text{Cof}(A) = \begin{bmatrix} 10 & 11 & -8 \\ -1 & -1 & 1 \\ -6 & -7 & 5 \end{bmatrix}_{3 \times 3}$$

Inverse of matrix

Inverse of a matrix 'A' is matrix 'B' such

that $AB = BA = I$.

$$A_{2 \times 3} B_{3 \times 2} \quad B_{3 \times 2} A_{2 \times 3}$$

$$A_{3 \times 3} B_{3 \times 3}$$

$$B_{3 \times 3} A_{3 \times 3}$$

$(AB)_{2 \times 2} \neq BA_{3 \times 3}$

\Rightarrow Matrix 'A' for which inverse of matrix is to be defined should be square matrix only

\Rightarrow Inverse matrix is also square matrix of same order.

'B' is termed as A^{-1} .

$$AA^{-1} = A^{-1}A = I$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$\text{adj}(A) = [\text{cof}(A)]^T$$

* if A is singular matrix

e.g. $A = \begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix} \Rightarrow |A| = 0$

$$A^{-1} = \frac{1}{0} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}^T = \frac{1}{0} \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$$

then A is called non-invertible matrix

$$= \begin{pmatrix} \infty & -\infty \\ -\infty & \infty \end{pmatrix}$$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$P(A) = \begin{bmatrix} a & -c \\ -b & d \end{bmatrix} \Rightarrow \text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 2 \\ -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{22} \begin{bmatrix} 4 & -2 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{4}{22} & -\frac{2}{22} \\ \frac{1}{22} & \frac{5}{22} \end{bmatrix}$$

Short trick to find cofactor matrix of 3x3 matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 3 & -2 & 1 \end{bmatrix}$$

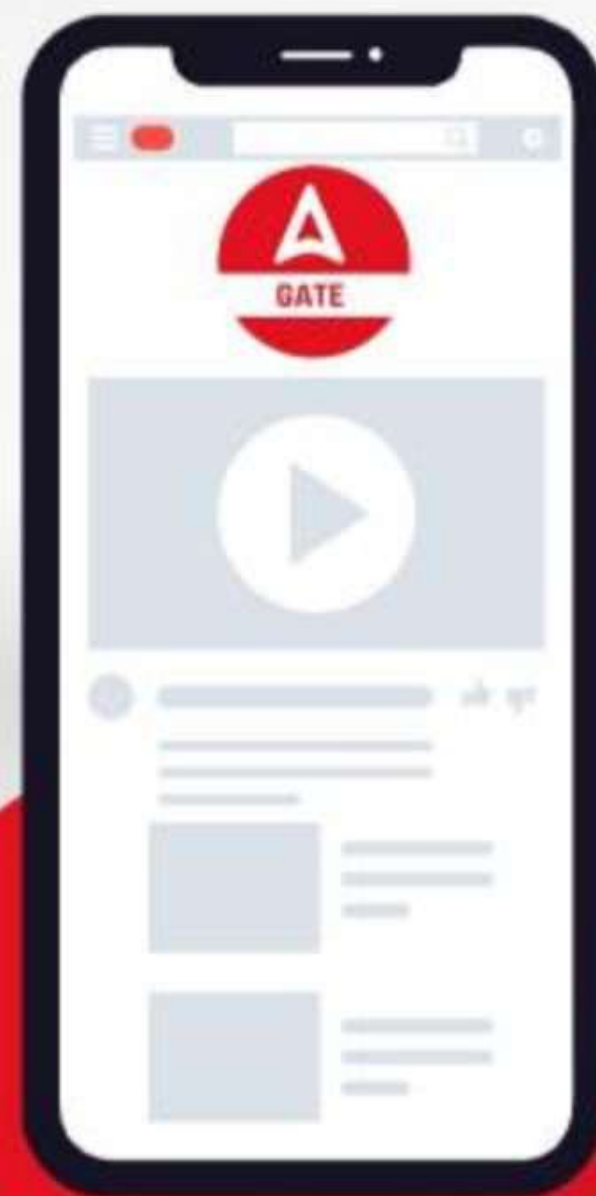
$$\text{Cof}(A) = \begin{bmatrix} 10 & 11 & -8 \\ -1 & -1 & 1 \\ -6 & -7 & 5 \end{bmatrix}$$

2	-1	1	2	-1
1	2	4	1	2
3	-2	1	3	-2
2	-1	1	2	-1
1	2	4	1	2



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