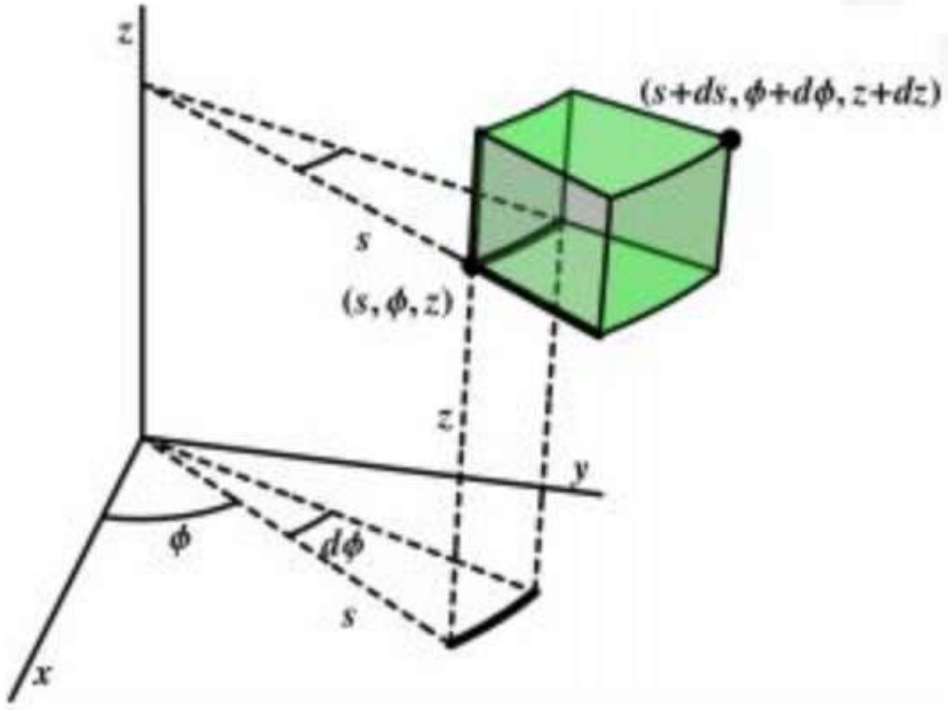
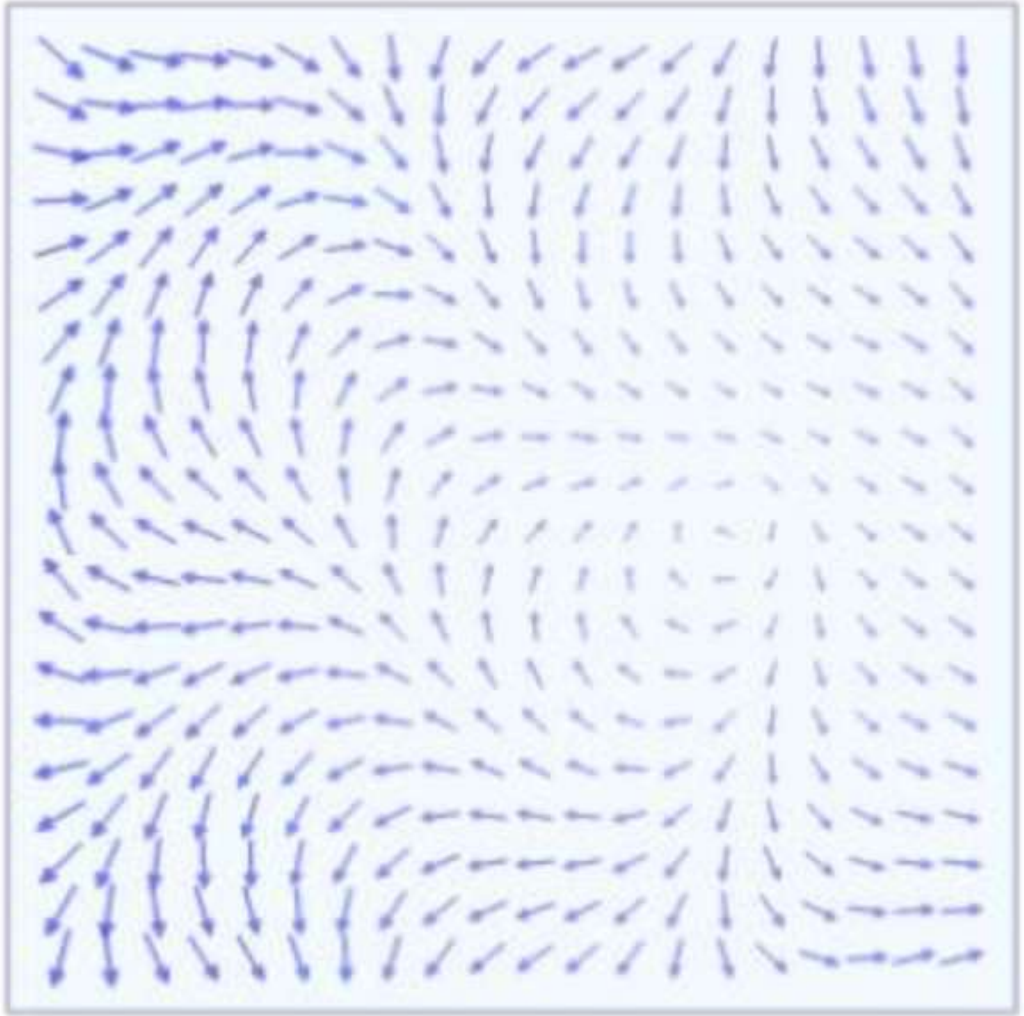
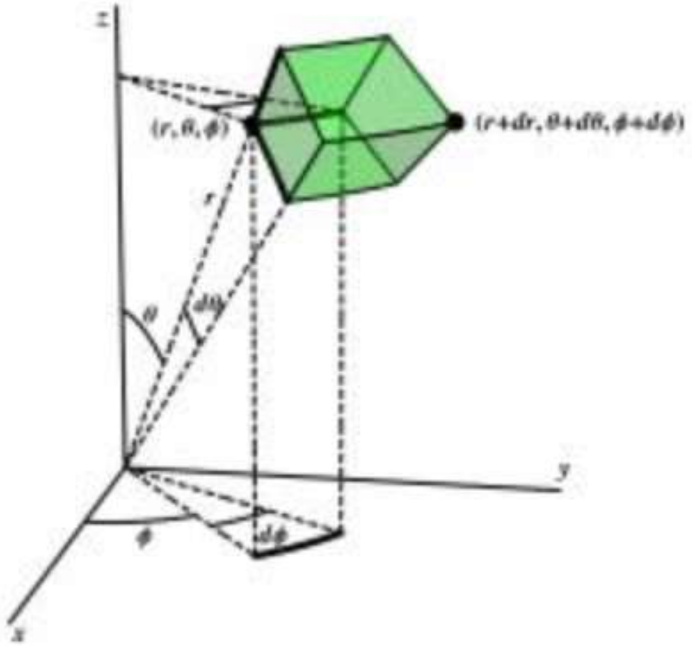
Coordinate systems

Adda247

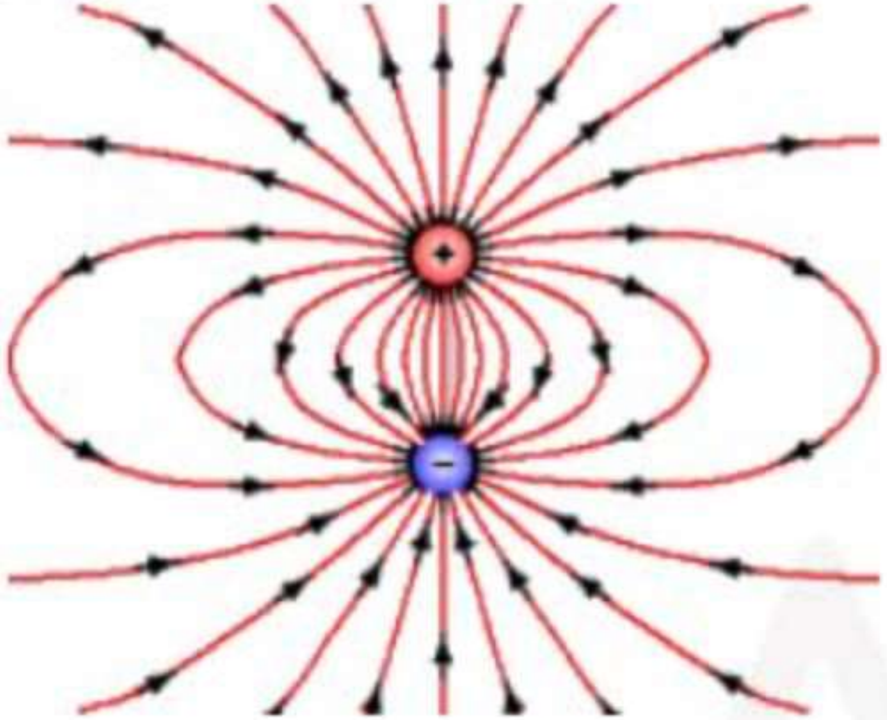
Introduction to Vector Integrals

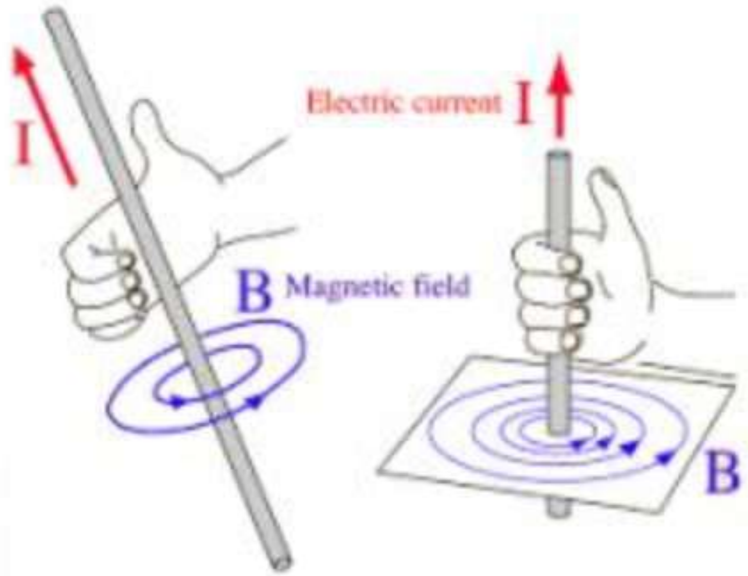


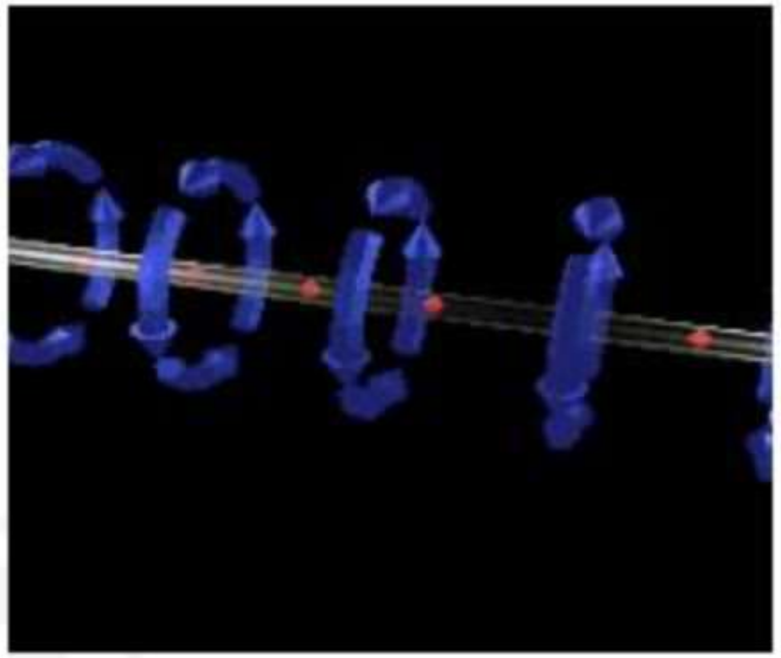


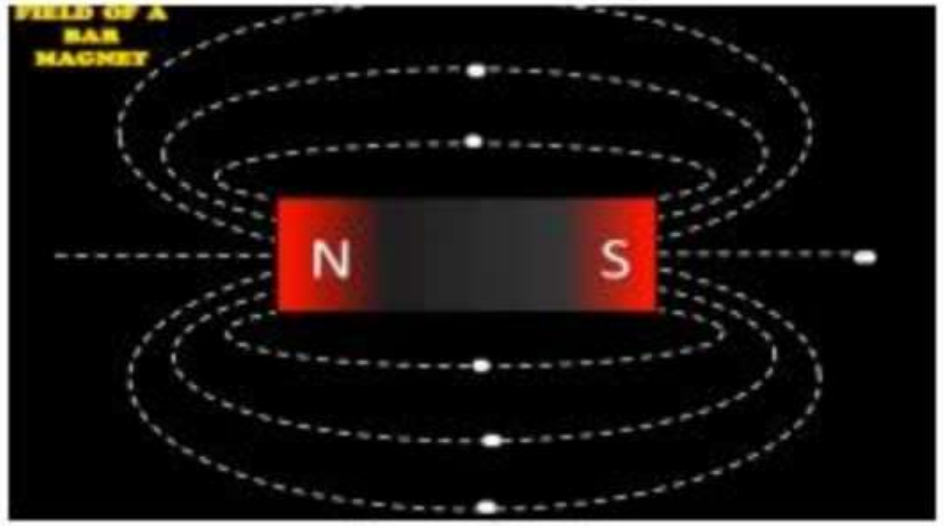


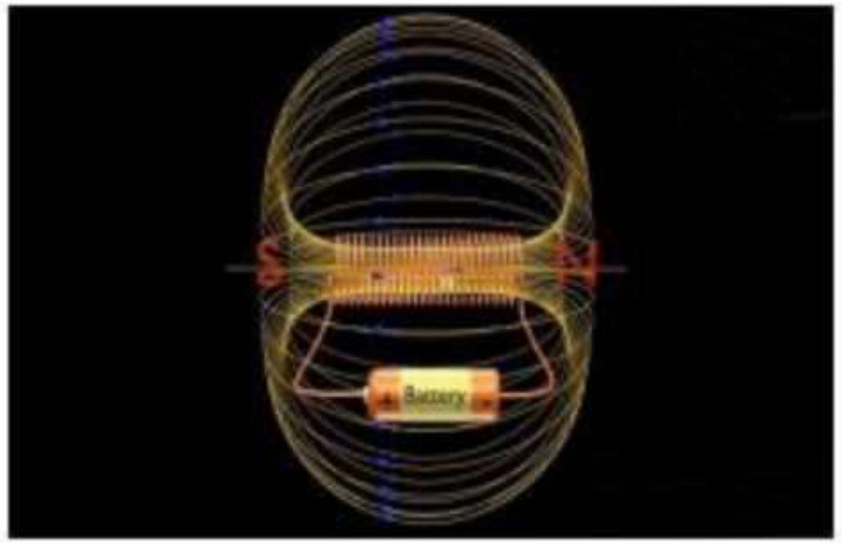
Recap

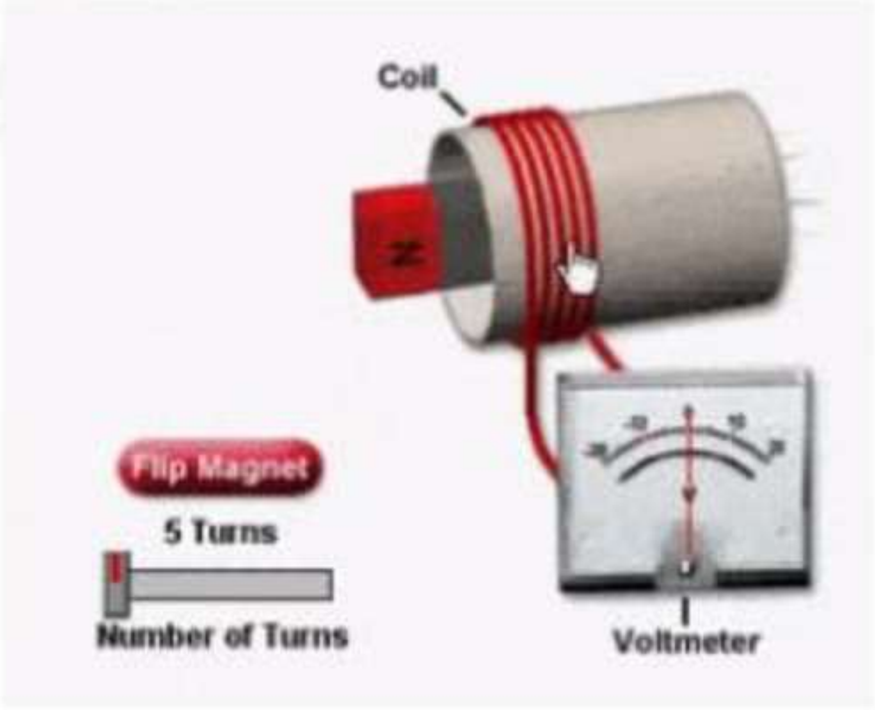



I
Electric current I
B Magnetic field
B

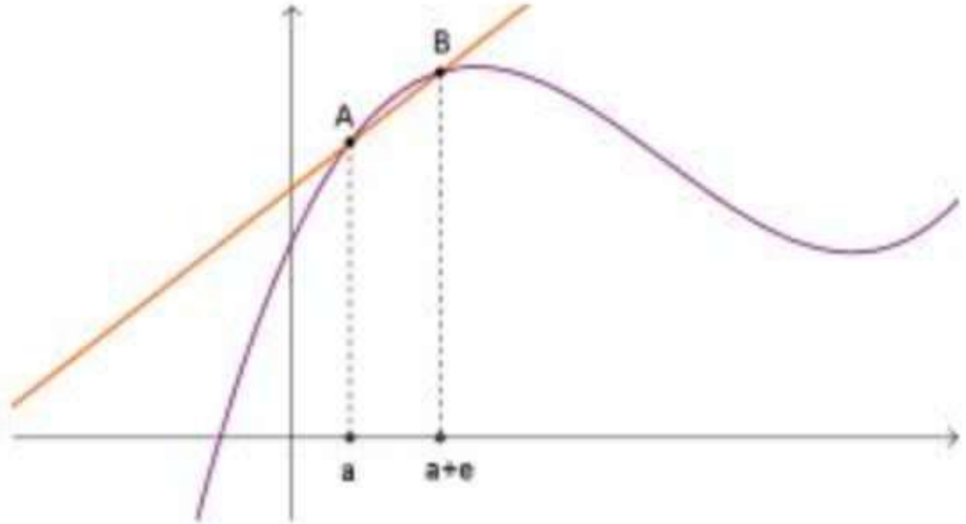



FIELD OF A
BAR
MAGNET
N
S




Coil
Flip Magnet
5 Turns
Number of Turns
Voltmeter

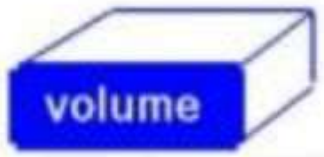
Recap



A scalar quantity has only magnitude.
A vector quantity has both magnitude and direction.

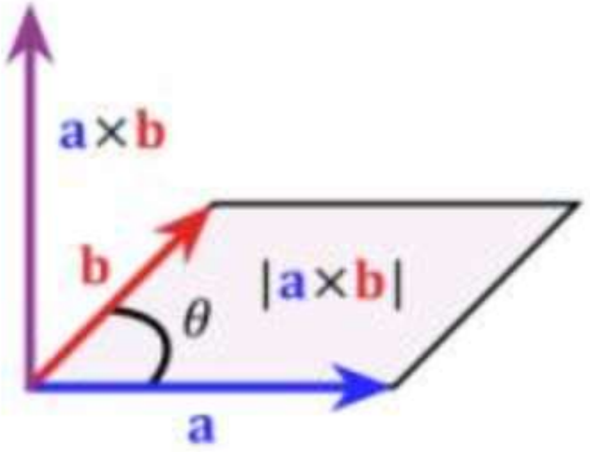
Scalar Quantities

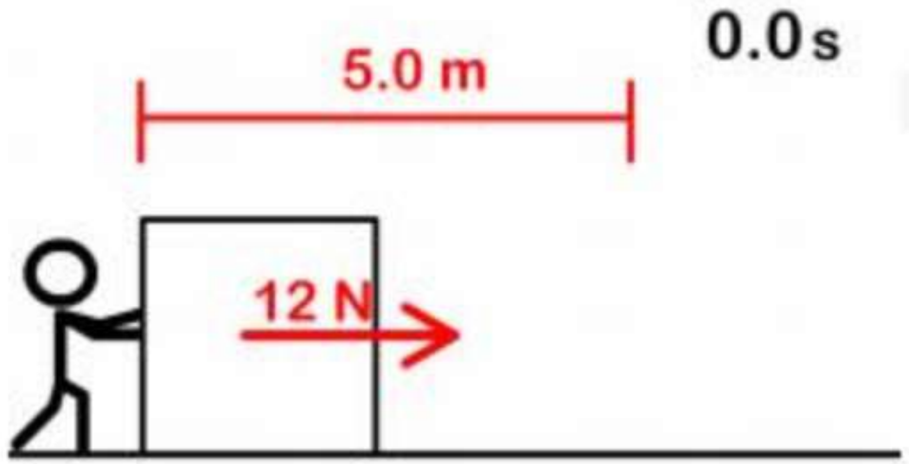
length, area, volume
speed
mass, density
pressure
temperature
energy, entropy
work, power

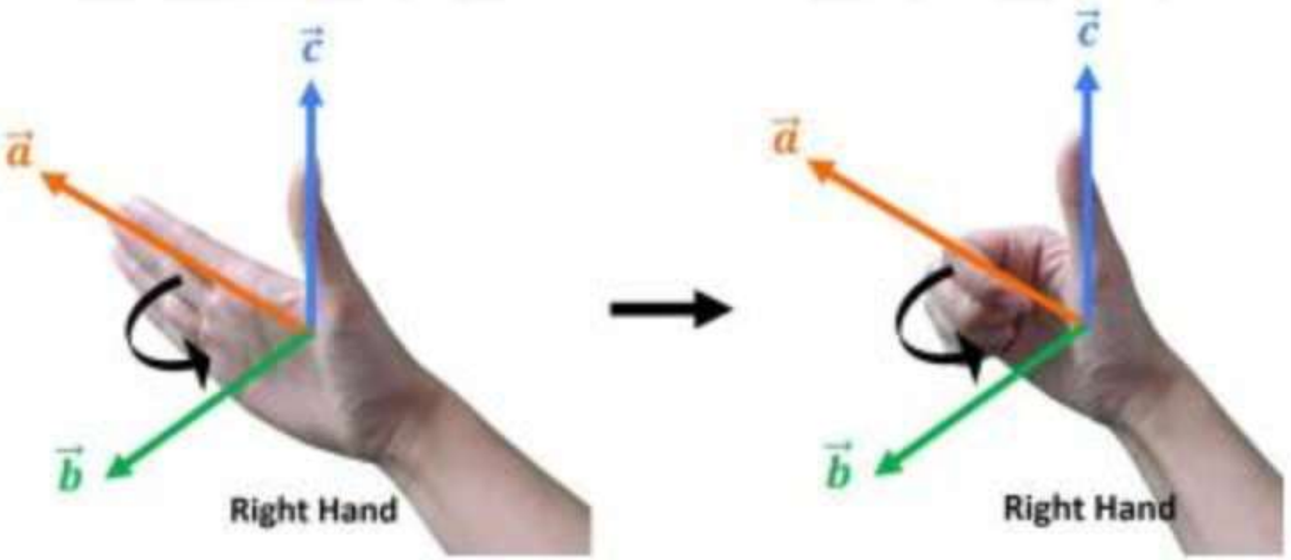


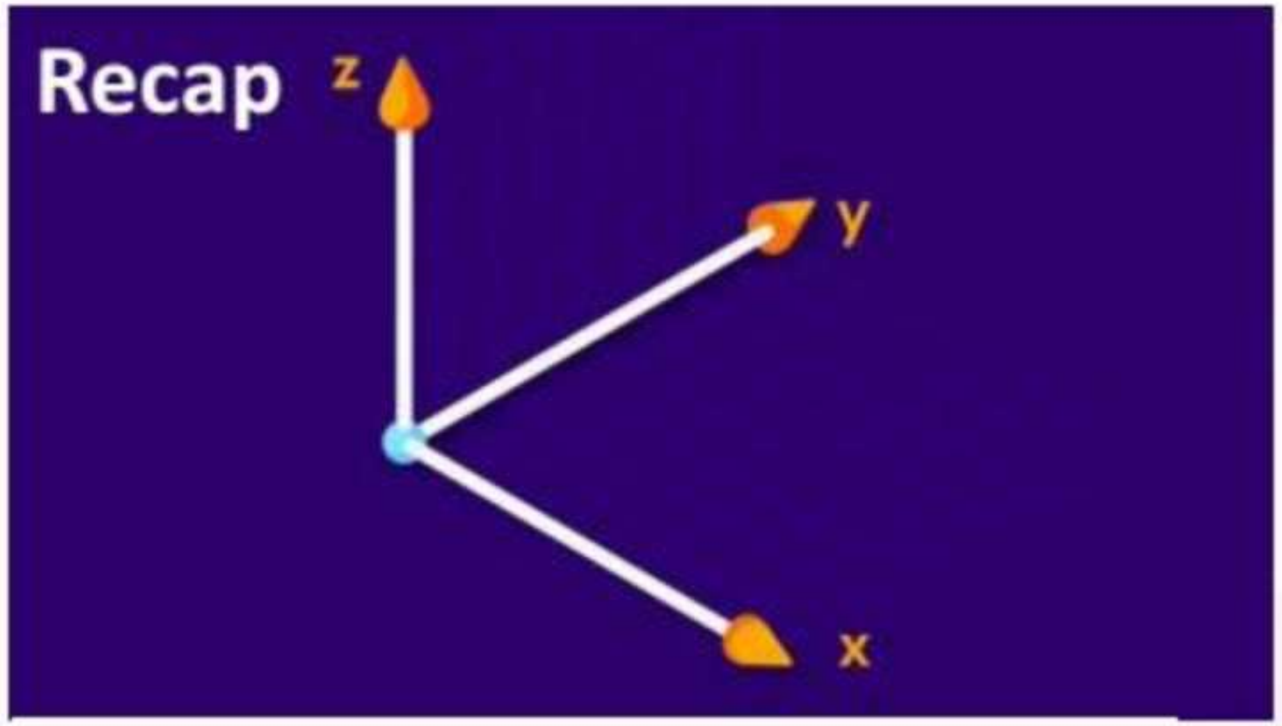
Vector Quantities

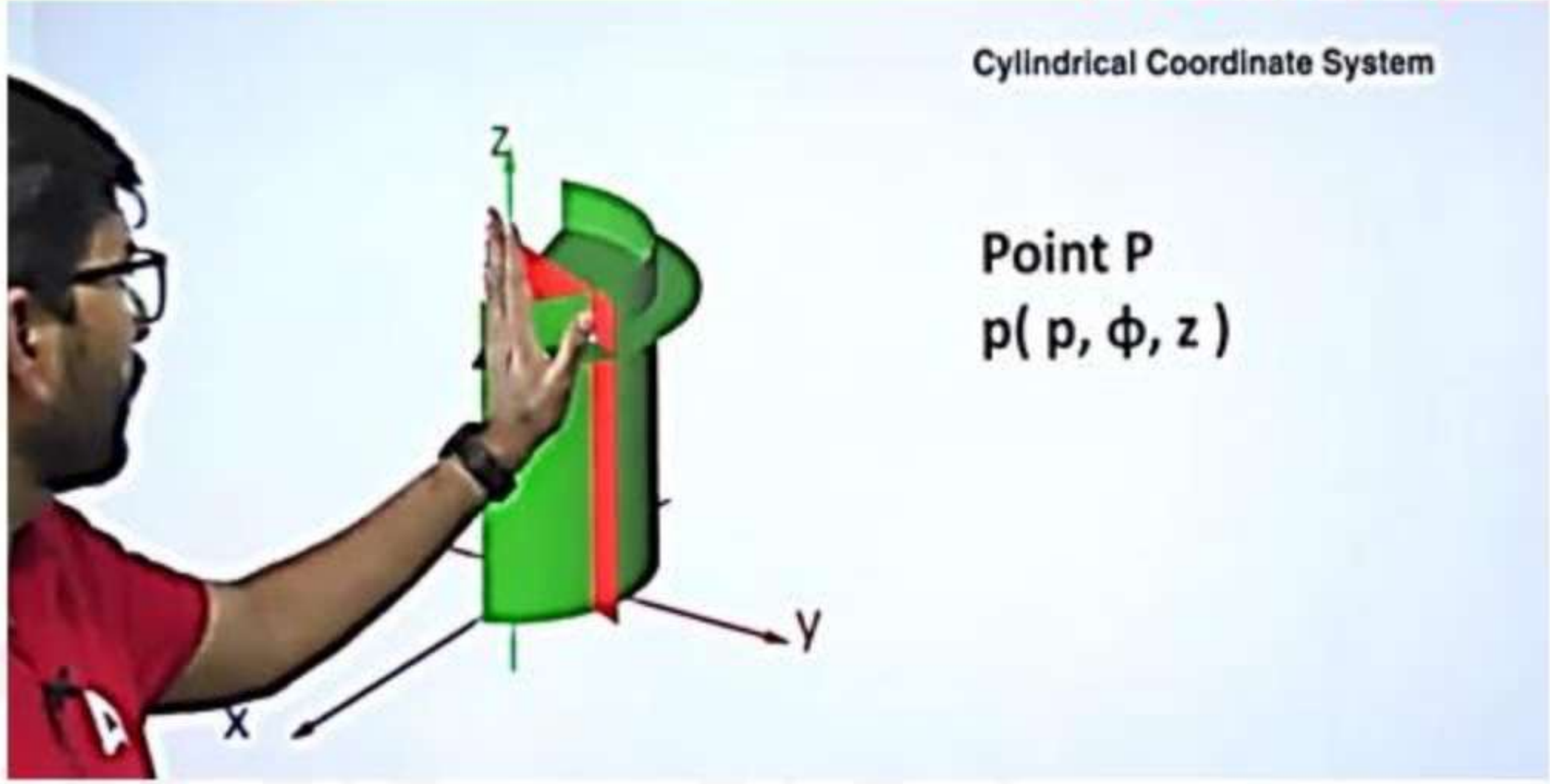
displacement
velocity
acceleration
momentum
force
lift, drag, thrust
weight

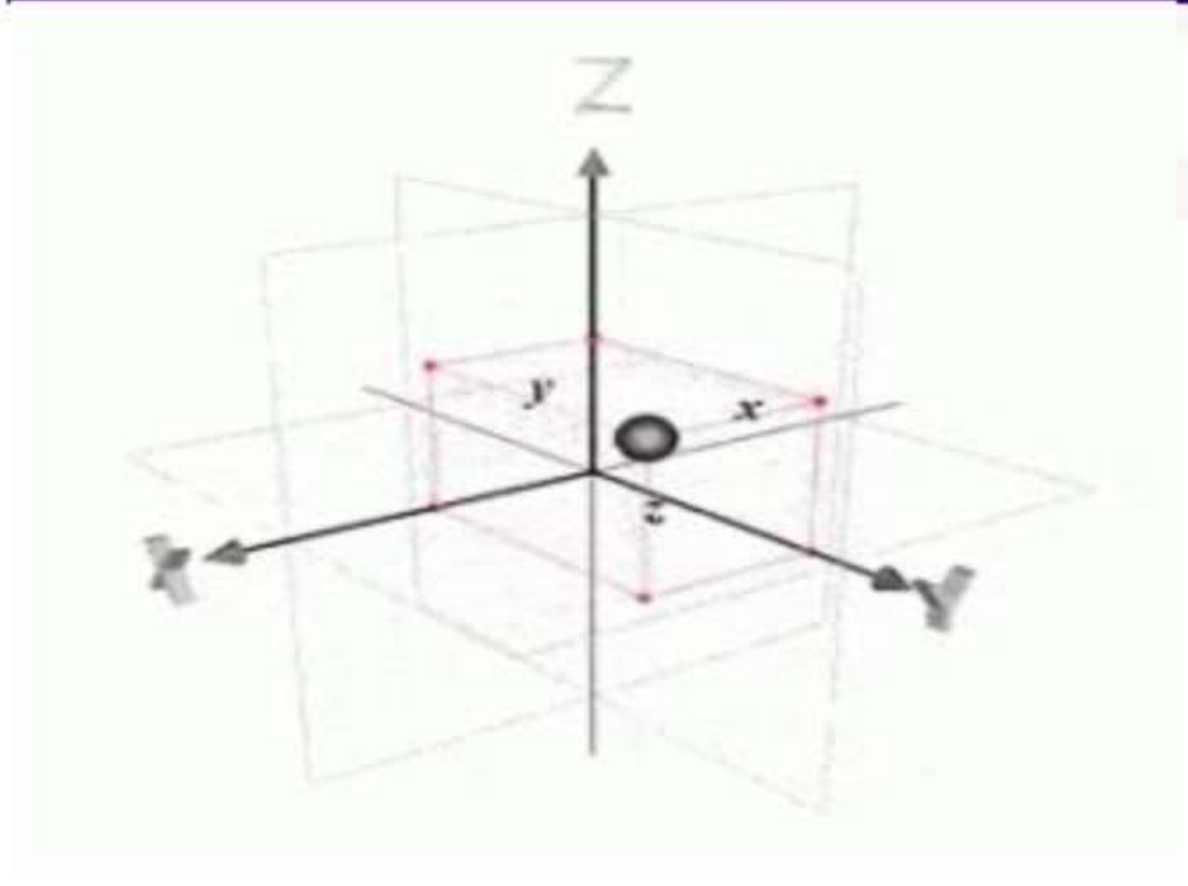


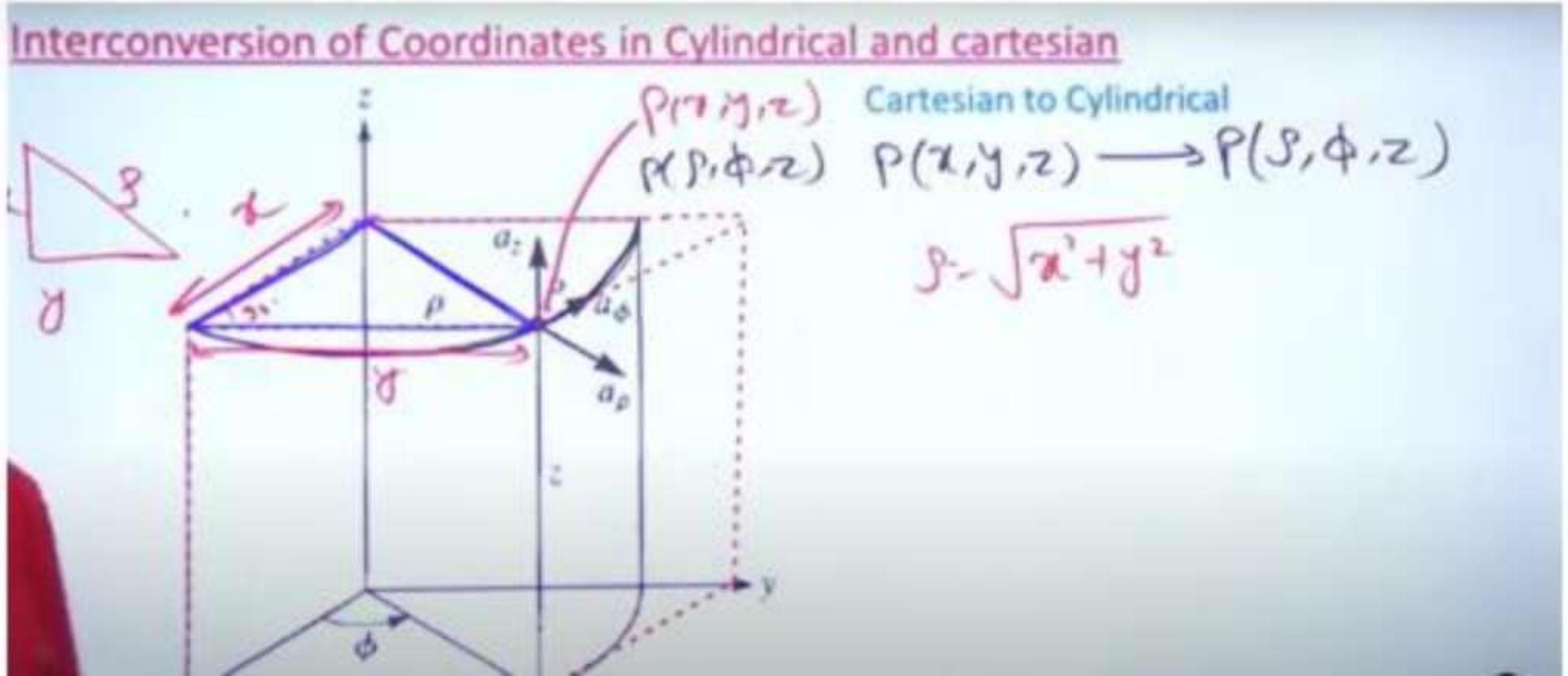





Recap
z
y
x


Cylindrical Coordinate System
Z
Point P
p(p, ϕ, z)
y
X


Z
y
x
z

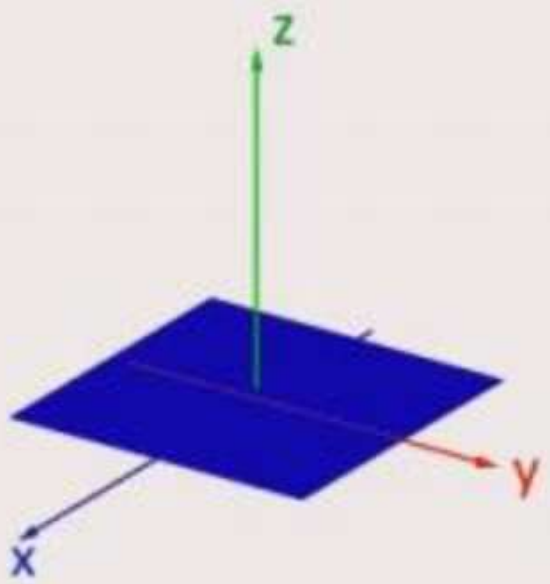

Interconversion of Coordinates in Cylindrical and cartesian
Cartesian to Cylindrical
P(x,y,z) ⟶ P(ρ,ϕ,z)
ρ = √(x²+y²)

Cylindrical Coordinate System

Cylindrical Coordinate System

Constant r surface

$R \geq 0^{o}$

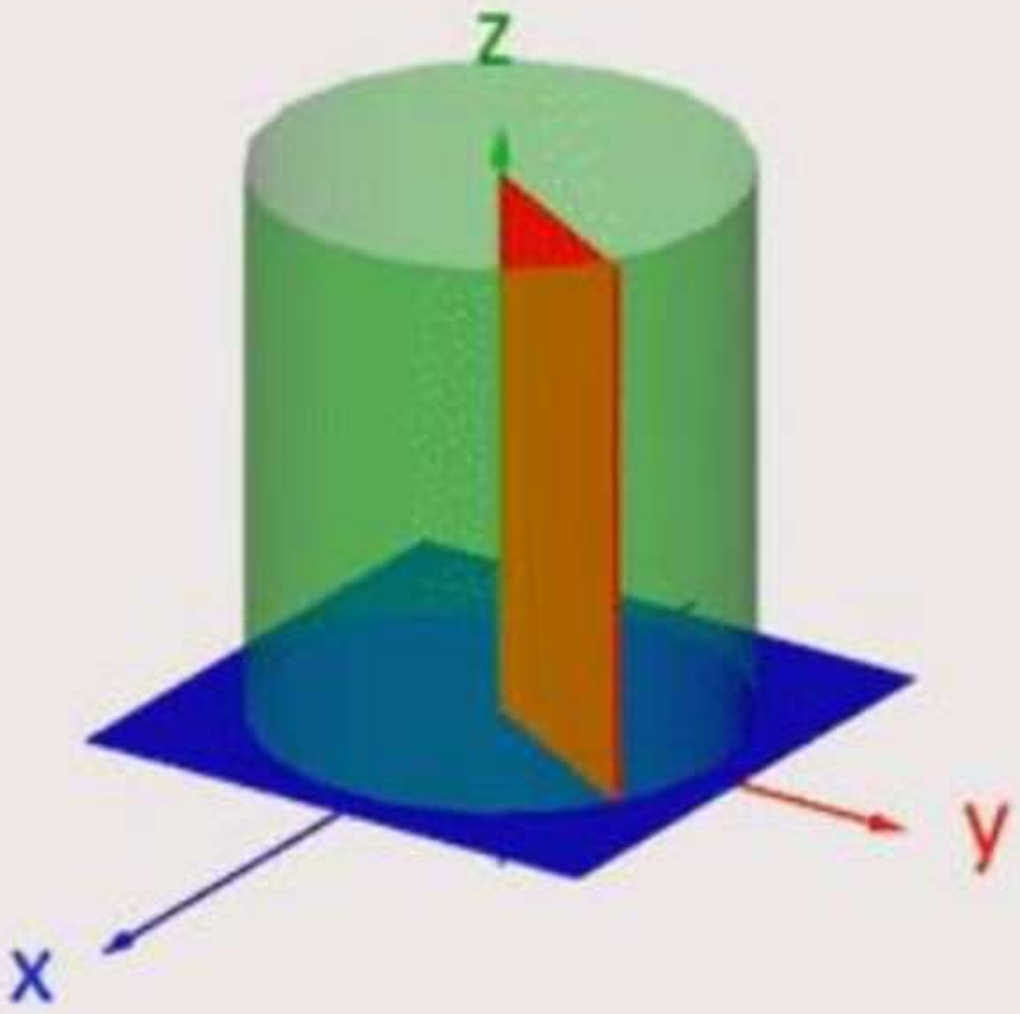


Cylindrical Coordinate System

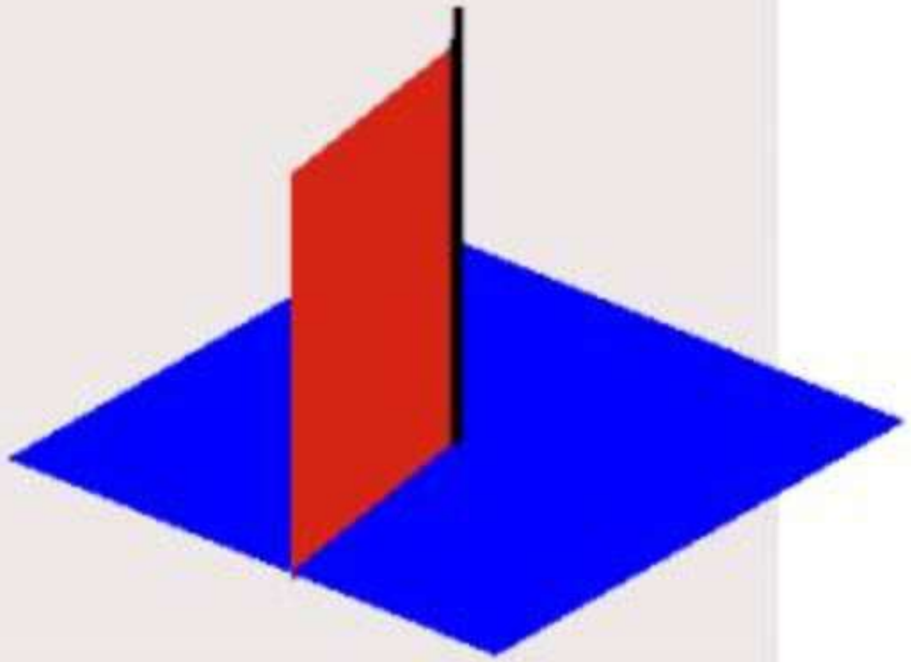
Constant z surface

$-\infty \leq Z \leq +\infty$

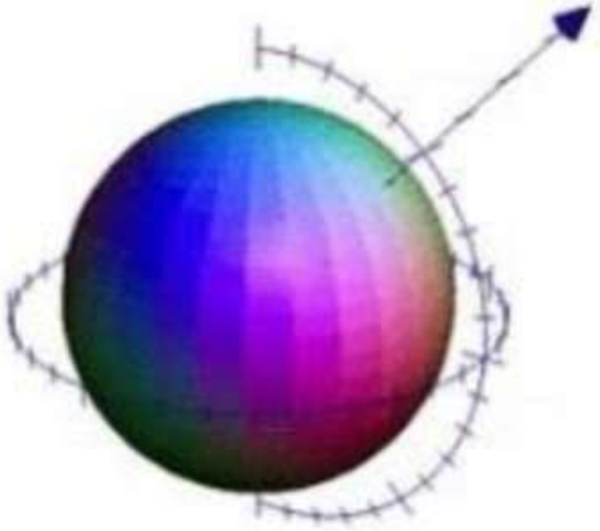
Cylindrical Coordinate System

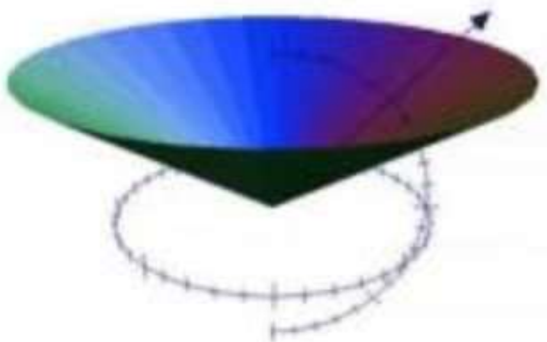


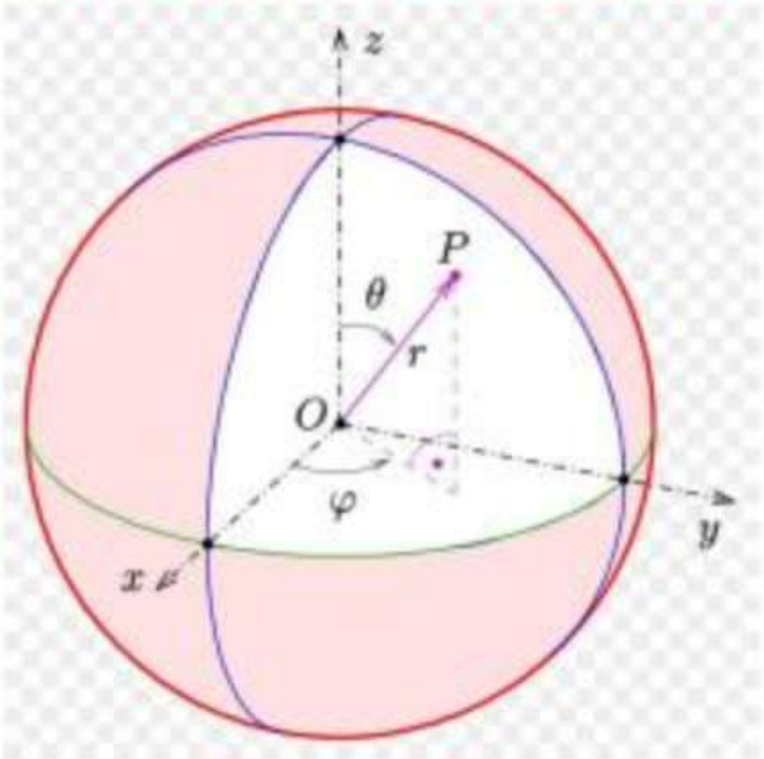
Cylindrical coordinate system planes

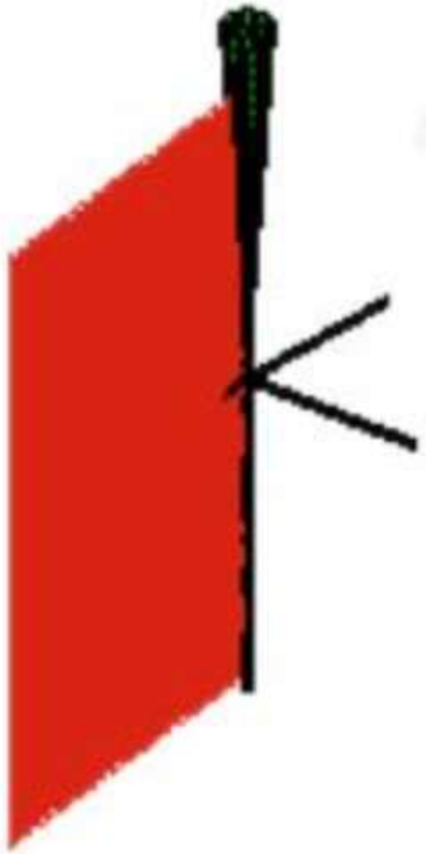


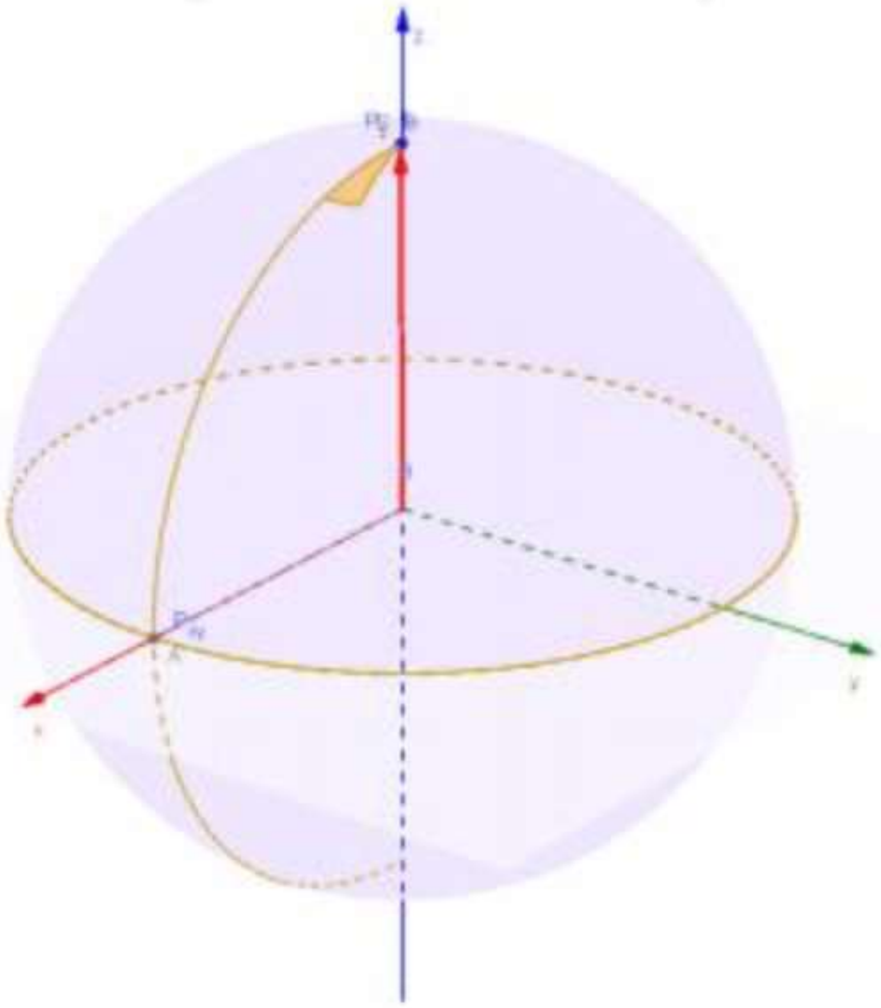
Spherical Coordinates P(r, ϴ, ϕ)











Recap

Q.5 If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $|\vec{a} \times \vec{b}|^2$ will be equal to

(a) $a^2b^2 - (\vec{a} \cdot \vec{b})^2$

(b) $ab - \vec{a} \cdot \vec{b}$

(c) $a^2b^2 + (\vec{a} \cdot \vec{b})^2$

(d) $ab + \vec{a} \cdot \vec{b}$

$\vec{a} \times \vec{b} = ab\sin\theta\, \hat{a}_n$

$|\vec{a} \times \vec{b}| = ab\sin\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2\sin^2\theta$ Ans

$\vec{a} \cdot \vec{b} = ab\cos\theta$ $\sin^2\theta = 1 - \cos^2\theta$

$|\vec{a} \times \vec{b}|^2 = a^2b^2(1 - \cos^2\theta) = a^2b^2 - a^2b^2\cos^2\theta$
$= a^2b^2 - (\vec{a} \cdot \vec{b})^2$

Q.7 The angle between two unit - magnitude coplanar vectors P(0.866, 0.500, 0) and Q(0.259, 0.966, 0) will be

(a) 0°

(b) 30°

(c) 45°

(d) 60°

$\theta = \cos^{-1}$ ... $= \cos^{-1}(0.866 \times 0.259 + \ldots \times 0.966)$

$\cos^{-1}(0.707)$

$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

A(2, −3, 2)

$\vec{OA} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$|\vec{P}| = \sqrt{(0.866)^2 + (0.5)^2}$

$0.707 = \frac{1}{\sqrt{2}}$

Number of Questions covered-9

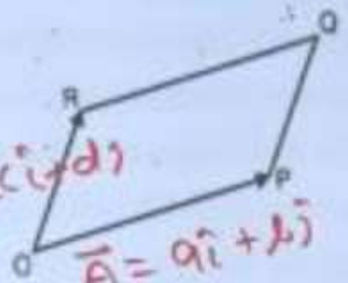
Q.6 For the parallelogram OPQR shown in the sketch, $\overline{OP} = a\hat{i} + b\hat{j}$ and $\overline{OR} = c\hat{i} + d\hat{j}$. The area of the parallelogram is.

(a) ad - bc

(b) ac + bd

(c) ad + bc

(d) ab - cd

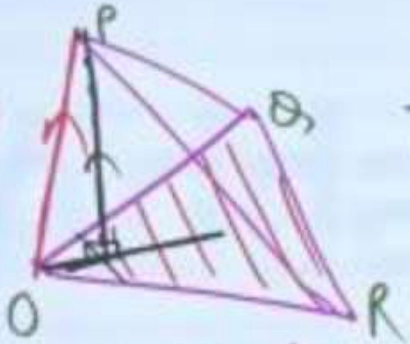


Area $= |\vec{A} \times \vec{B}|$

$\vec{A} \times \vec{B} = \vec{OP} \times \vec{OR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ c & d & 0 \end{vmatrix} = (ad - bc)\hat{k}$

Area = ad − bc

Q.8 If P, Q and R are three points having coordinates (3, - 2, - 1), (1, 3, 4), (2, 1, - 2) in XYZ space, then the distance from point P to plane OQR (O being the origin ... coordinate system) is given by



$\vec{OQ} \times \vec{OR} = \vec{X}$

GATE

AAI ATC

1 marks − 1.98
2 marks − 3.96

= normal vector to plane OQR

$\vec{OQ} = \hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{OR} = 2\hat{i} + \hat{j} - 2\hat{k}$

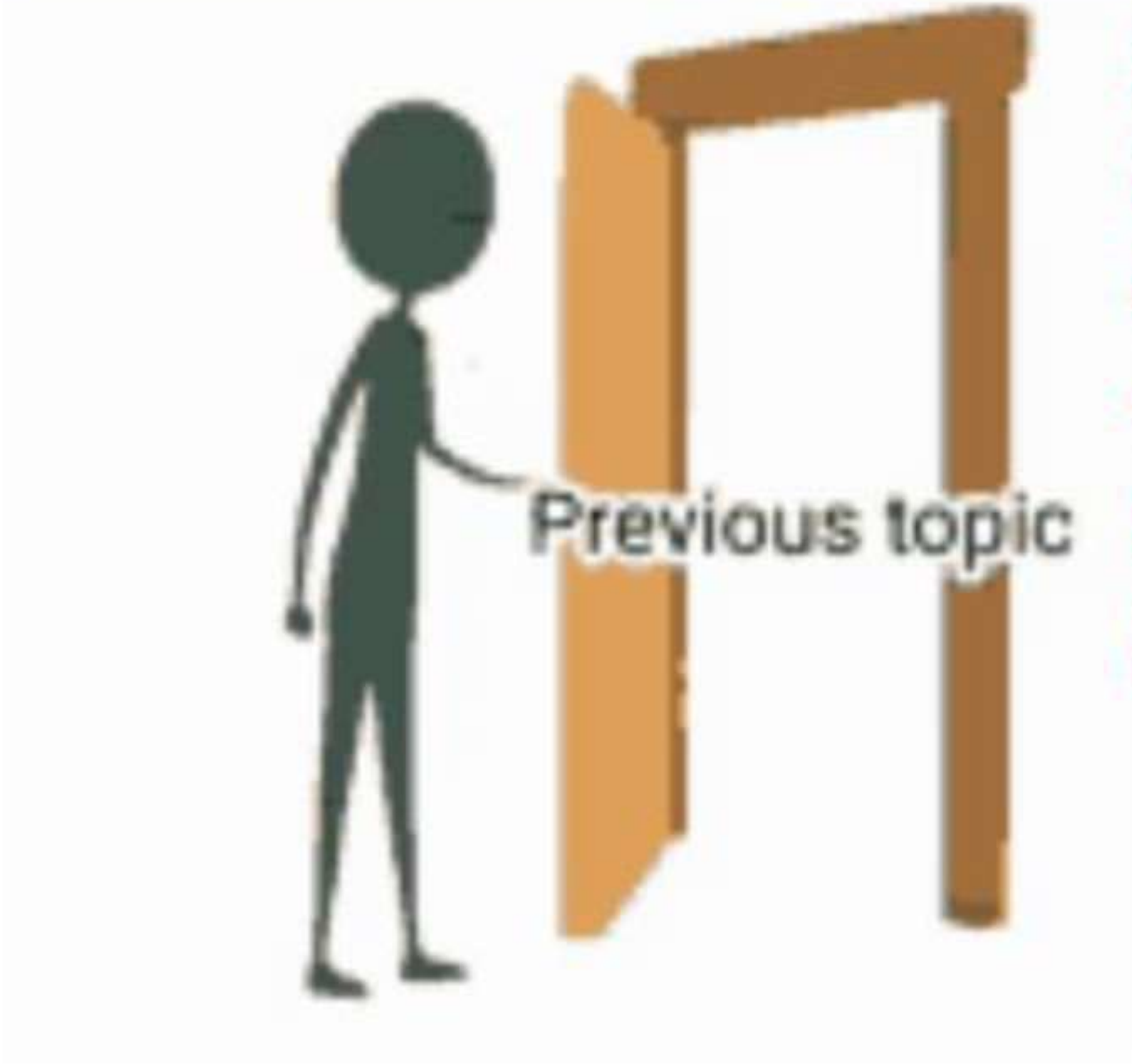
$\vec{OQ} \times \vec{OR} = ?$

WELCOME

TO Adda247

"If You are **here**, You are one step closer to your **GOAL**."

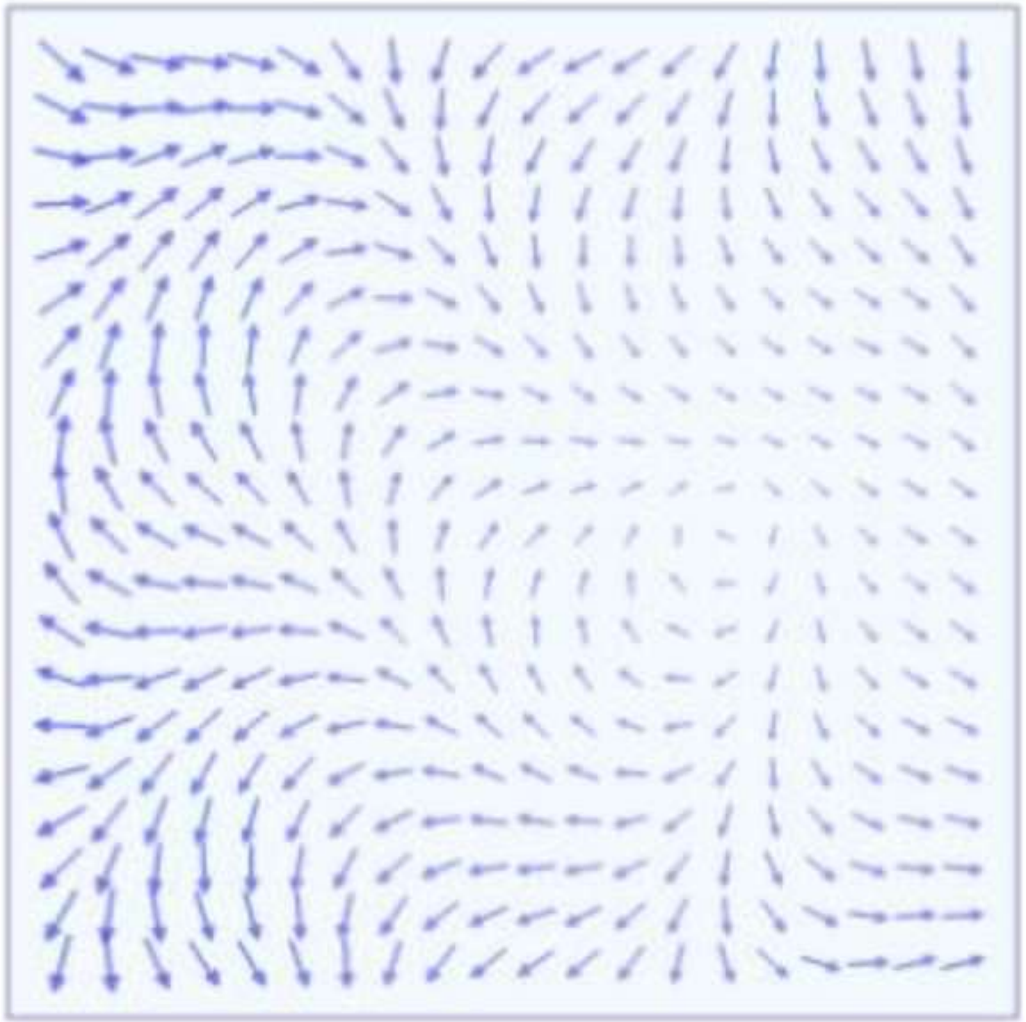

GATE 2024
GATE
प्रचण्ड Batch
Electromagnetic Field Theory
UNDERSTANDING OF VECTOR INTEGRALS
LEC-05
EE &ECE

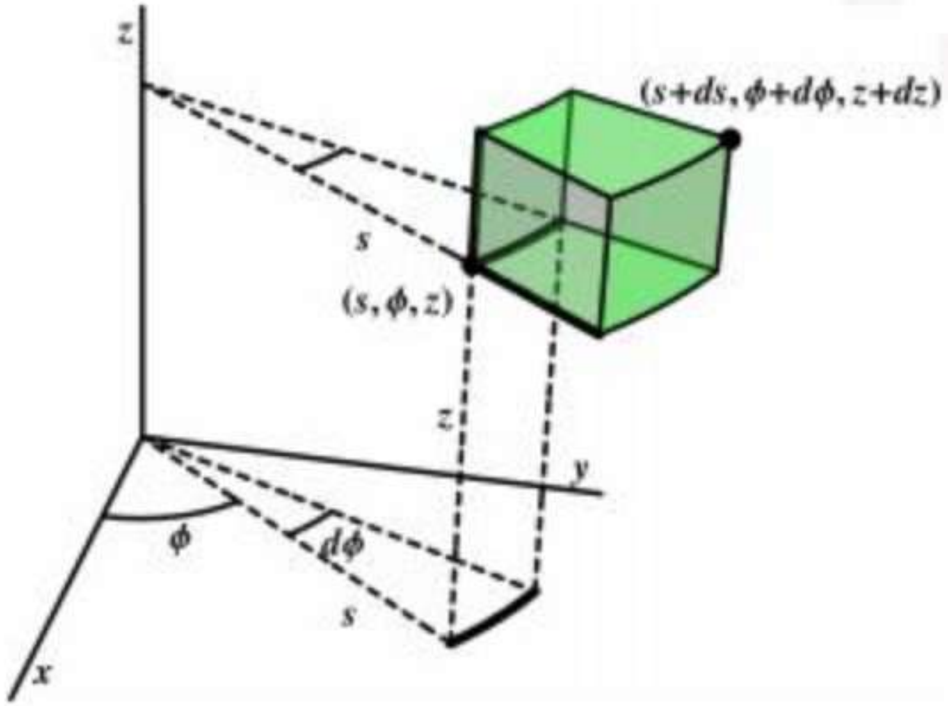


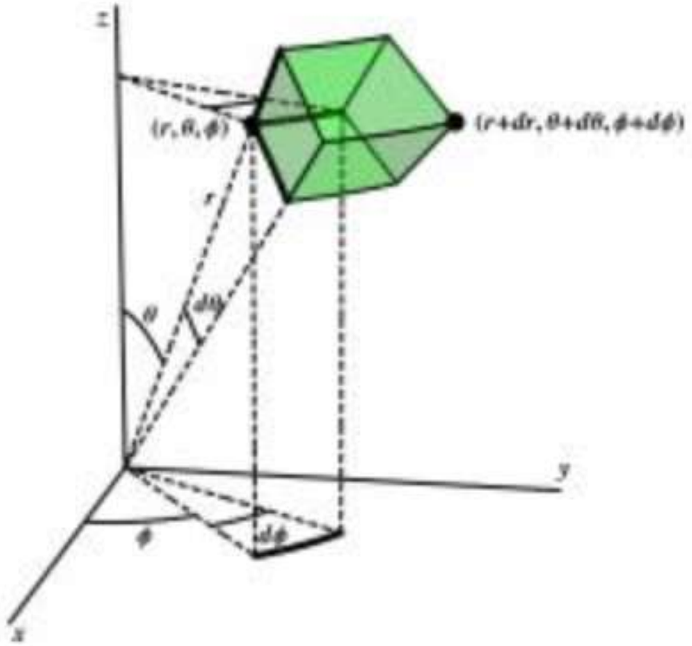
1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical and Spherical Coordinate systems

Coordinate systems

Introduction to Vector Integrals







Vector Integrals Introduction

$$\int_{-2}^{+1} f(x)\,dx$$

$dx \rightarrow$ small chang 'x'

$\searrow$ differential element

$$\int f(x,y)\,dx$$

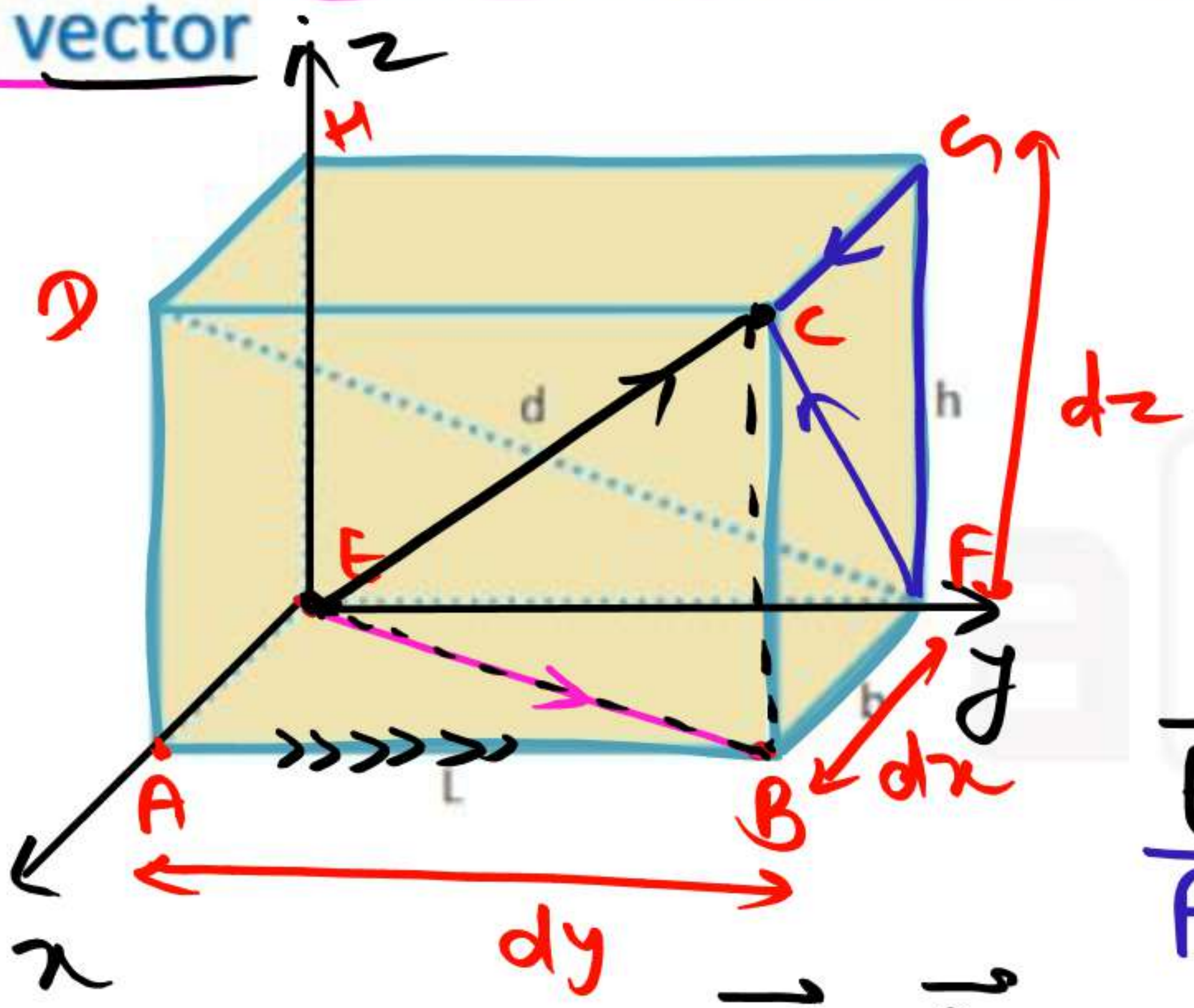
$$\int f(x,y)\,dx\,dy$$

$$\int \left[f_1(x,y)\,dx + f_2(x,y)\,dy\right]$$

$$\int_{-2}^{1} f(x,y)\,dy$$

Small Elements in cartesian Coordinate systems

Small Length vector



$\vec{AB} = dy\,\hat{a}_y$

$\vec{BA} = -dy\,\hat{a}_y$

$\vec{GC} = dx\,\hat{a}_x = \vec{HD} = \vec{FB} = \vec{EA}$

$\vec{BC} = dz\,\hat{a}_z = \vec{AD} = \vec{FG} = \vec{EH}$

$\vec{EB} = \vec{EA} + \vec{AB} = dx\,\hat{a}_x + dy\,\hat{a}_y$

$\vec{FC} = dz\,\hat{a}_z + dx\,\hat{a}_x$

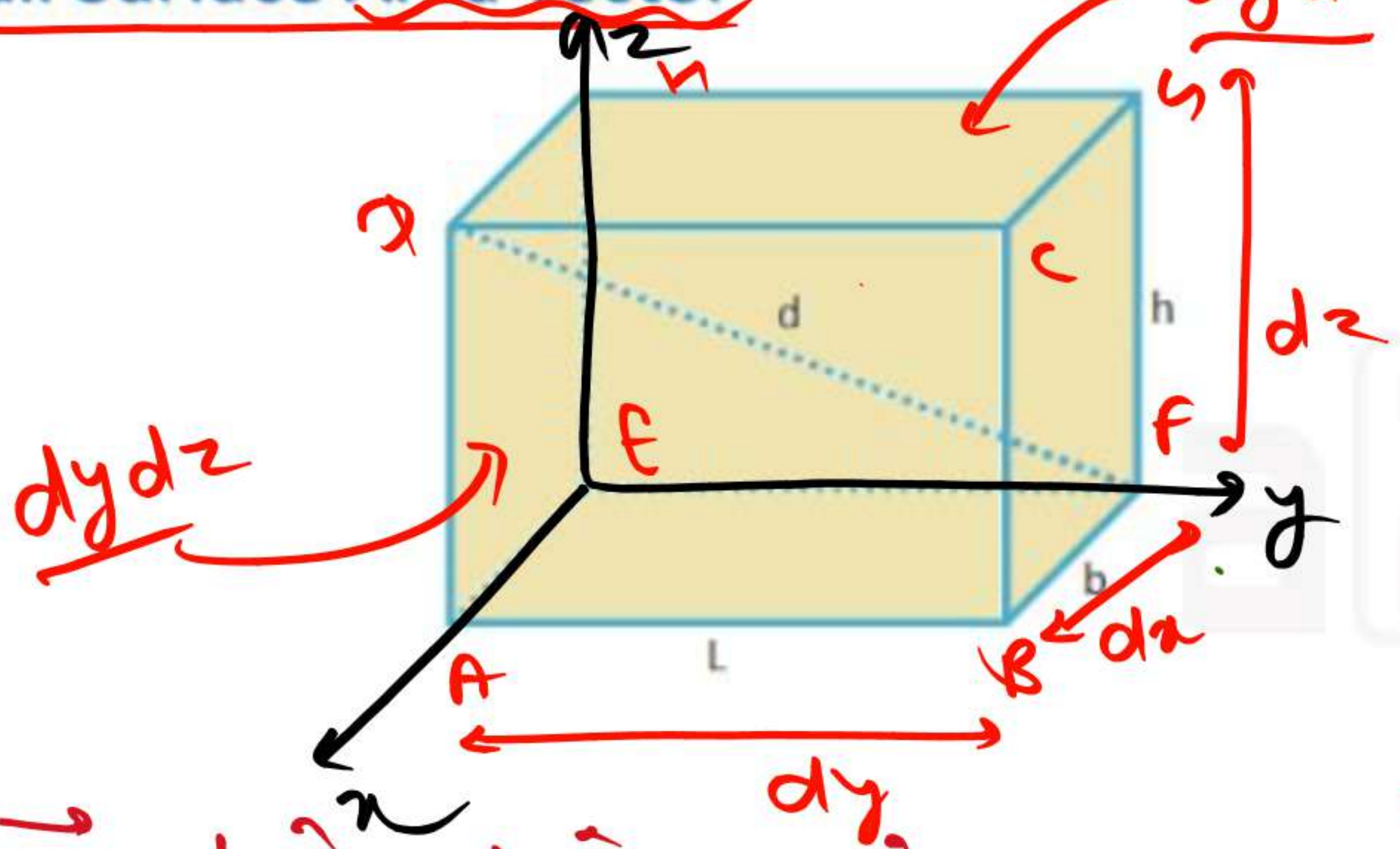
$\vec{FC} = \vec{EB} + \vec{BC} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

in general $\boxed{\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z}$

Small Elements in cartesian Coordinate systems

$\vec{dl}, \vec{ds}, dv$

Small Surface Area vector



* surface vector direction is normally outward from surface.

$\vec{ds} = dy\,dz\,\hat{a}_x$ (ABCD front)

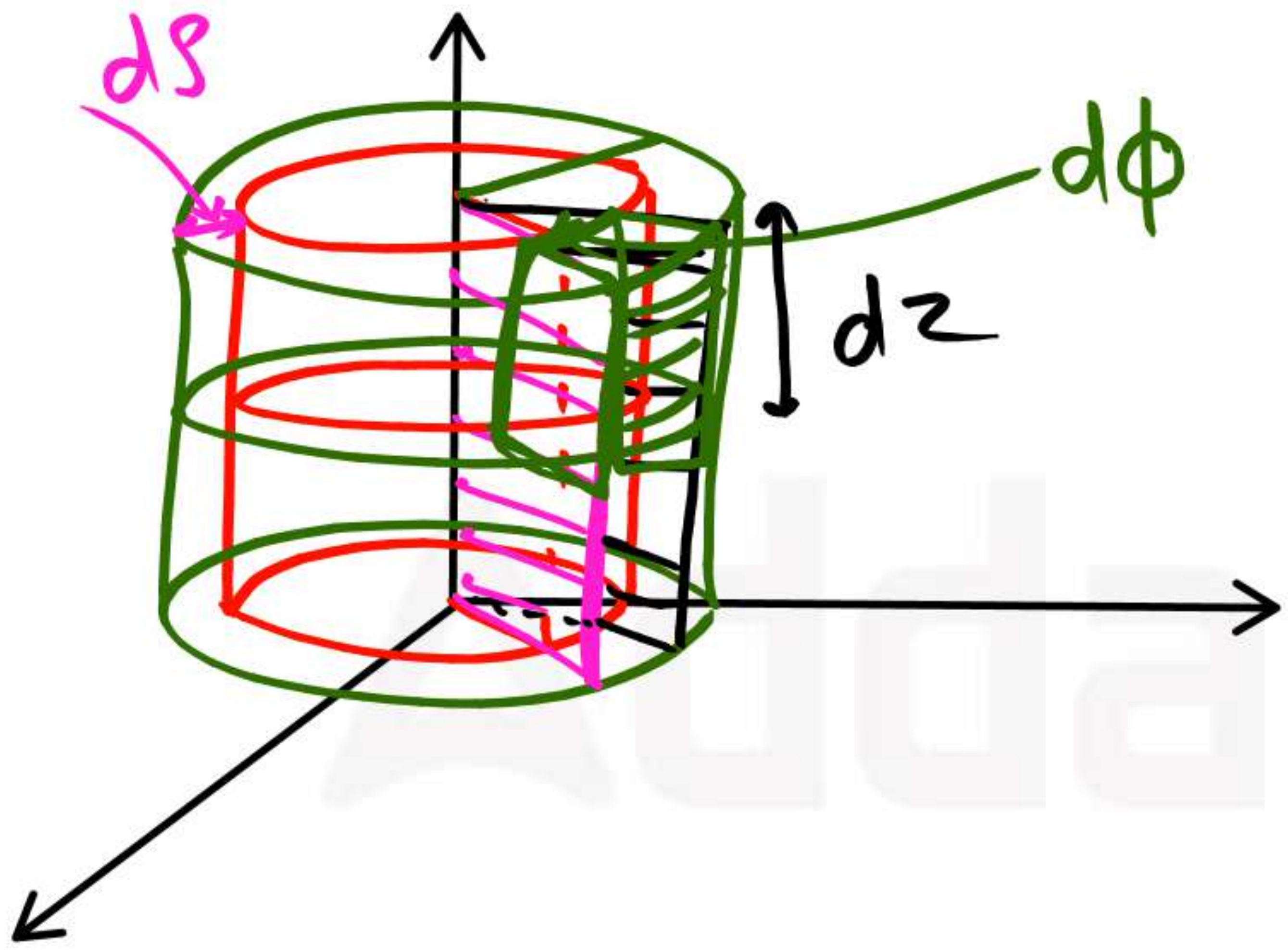
$\vec{ds} = dx\,dy\,\hat{a}_z$ (CDGH top)

$\vec{ds} = dx\,dz\,\hat{a}_y$ (BCFG right)

$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

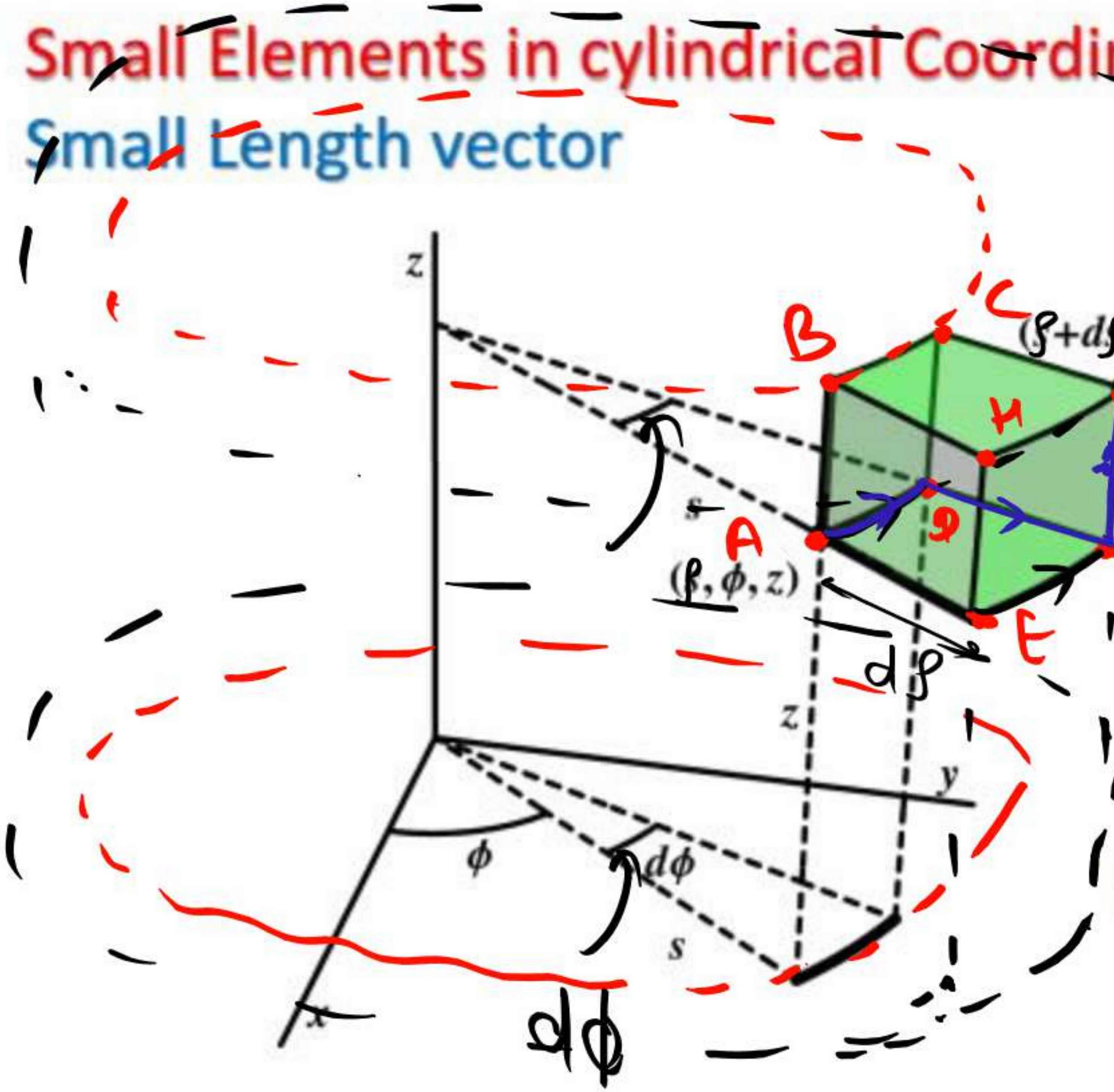
Small volume $= dx\,dy\,dz$ ← scalar.

Small Elements in cylindrical Coordinate systems



Small Elements in cylindrical Coordinate systems

Small Length vector



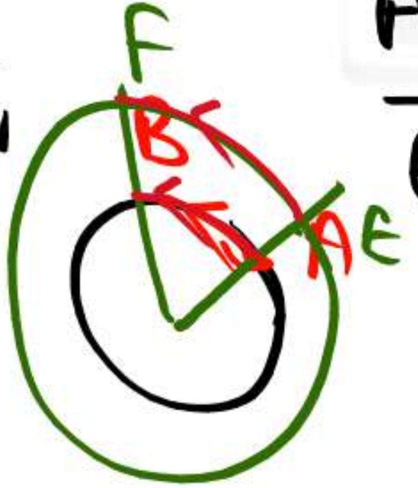
$\vec{dl} = ??$

$\vec{AE} = d\rho\,\hat{a}_\rho$

$\vec{AB} = dz\,\hat{a}_z$

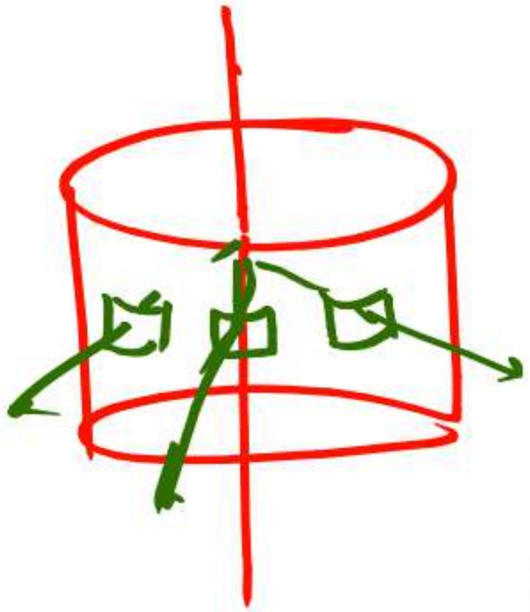
$\vec{AD} = \rho\, d\phi\,\hat{a}_\phi$ ✗

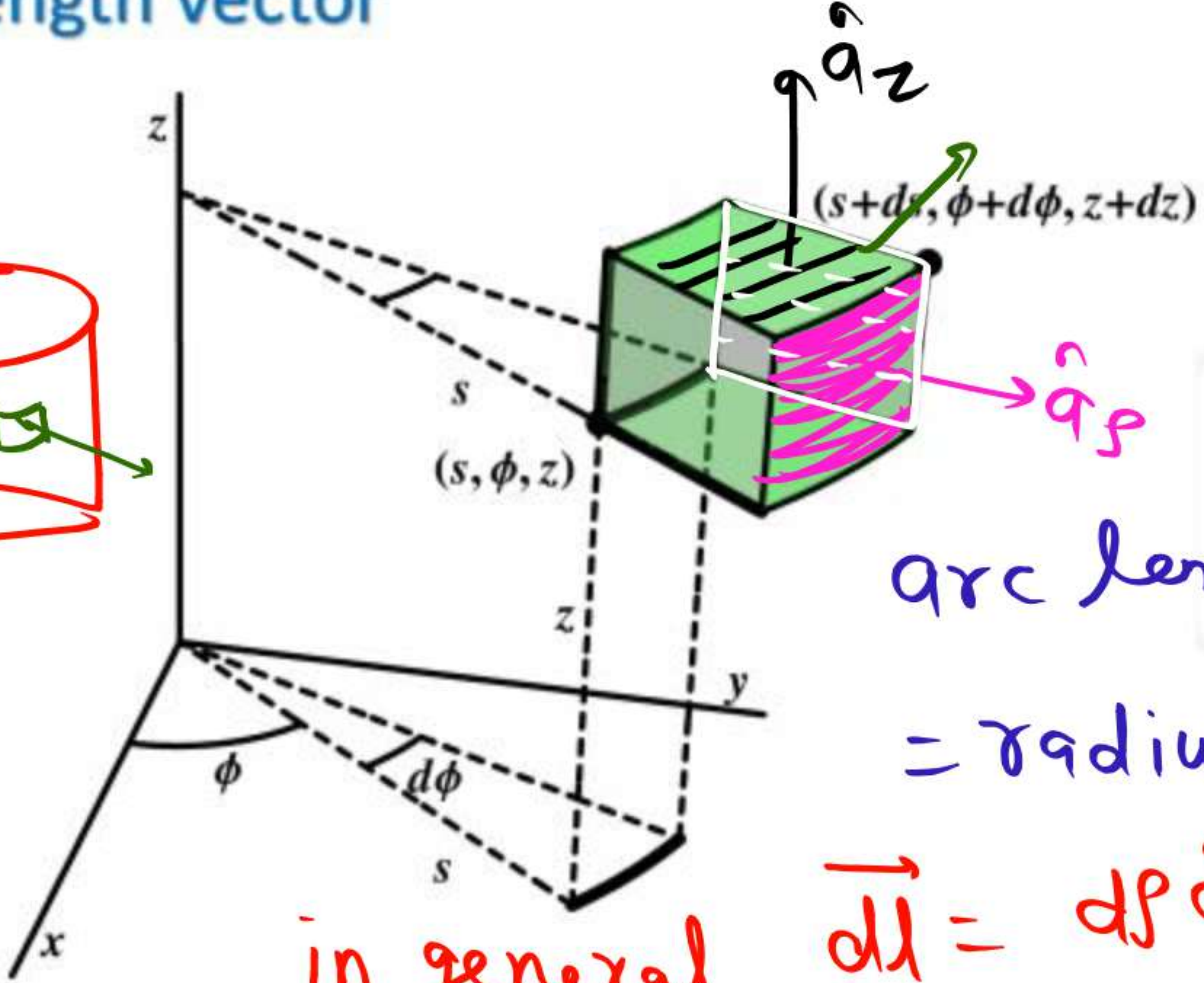
$\vec{EF} = (\rho + d\rho)\, d\phi\,\hat{a}_\phi$

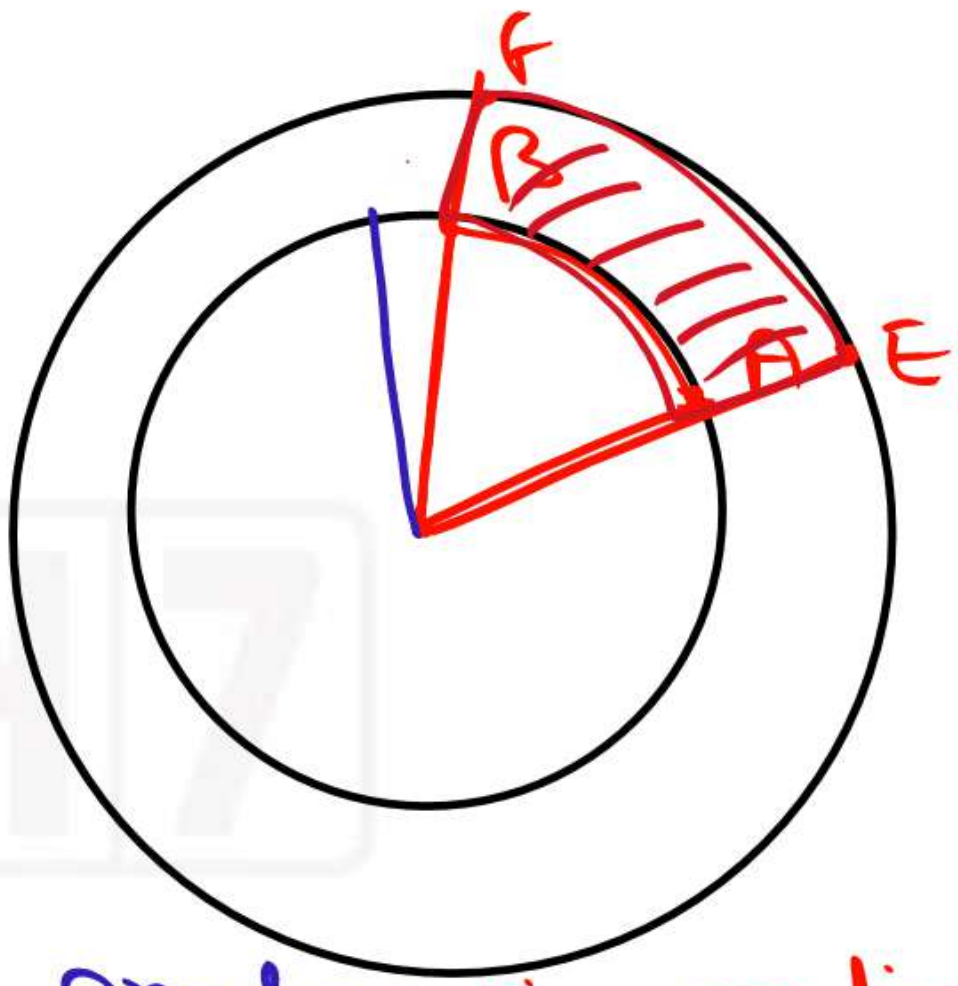


Small Elements in cylindrical Coordinate systems

Small Length vector





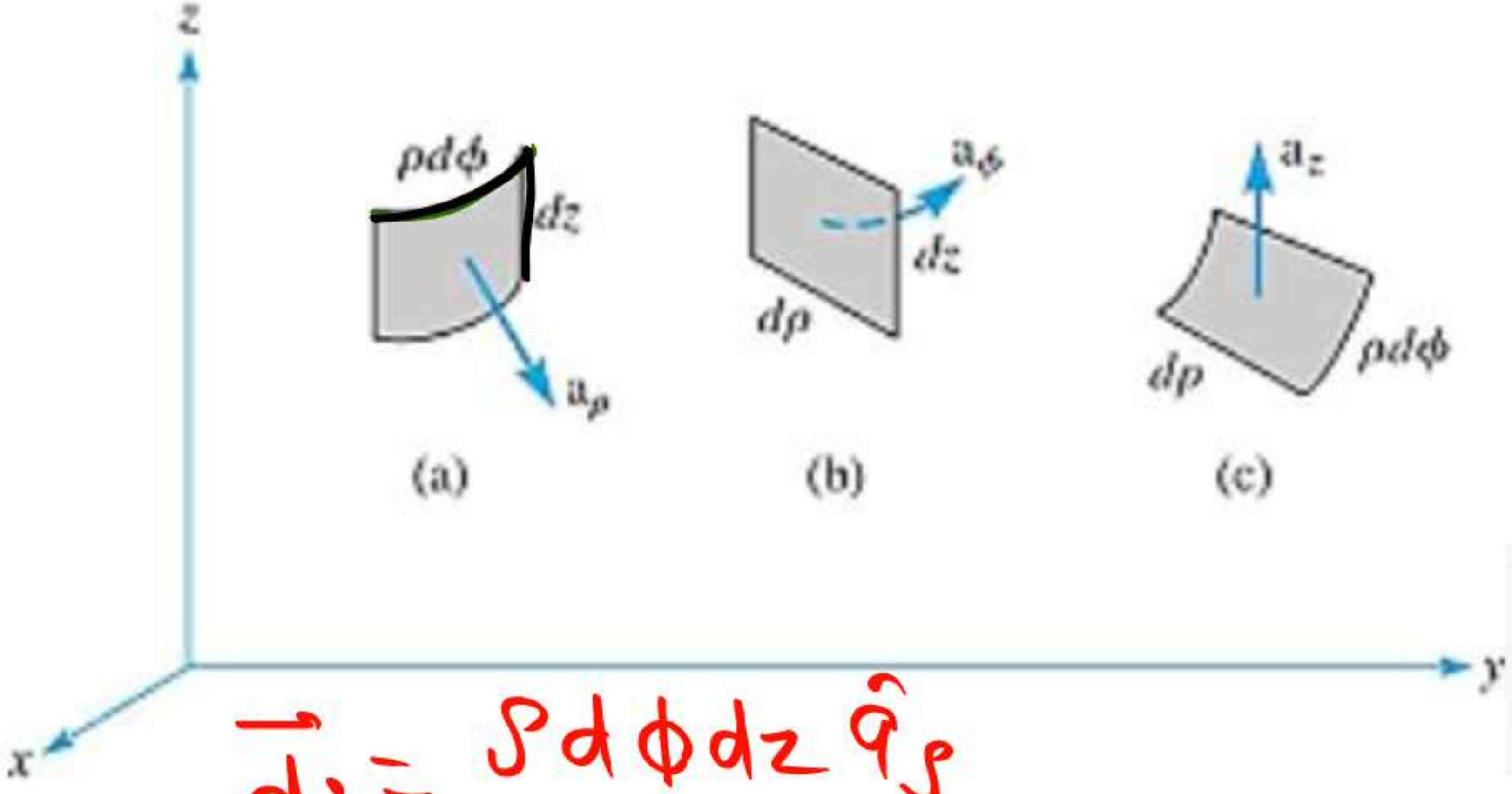


arc length

= radius × arc angle ← (in radians)

in general $\vec{dl} = d\rho\,\hat{a}_\rho + \rho\, d\phi\,\hat{a}_\phi + dz\,\hat{a}_z$

Small Surface Area vector

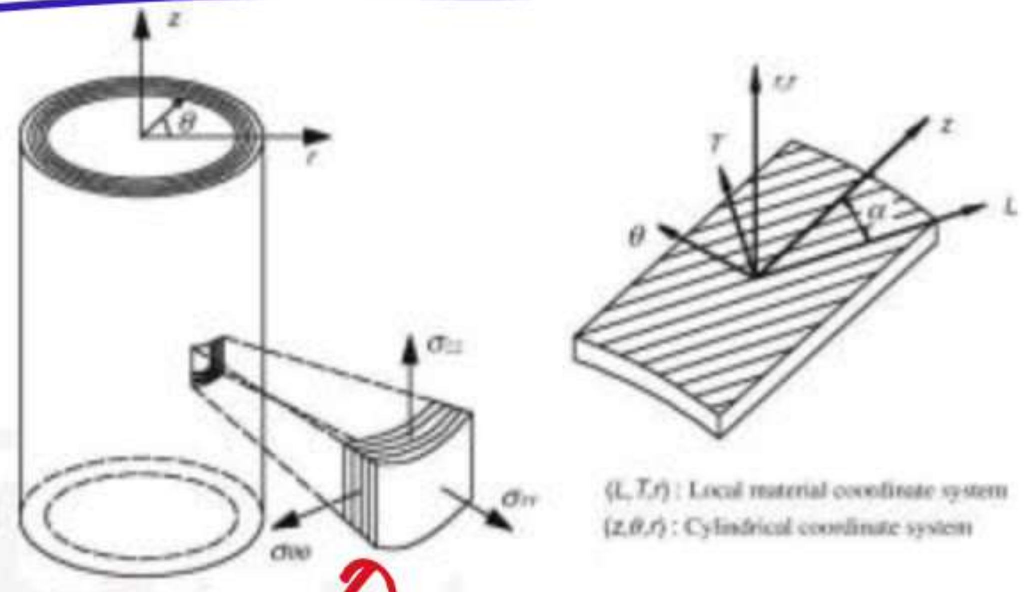


$$\vec{ds} = \rho\, d\phi\, dz\, \hat{a}_\rho$$

$$d\rho\, dz\, \hat{a}_\phi$$

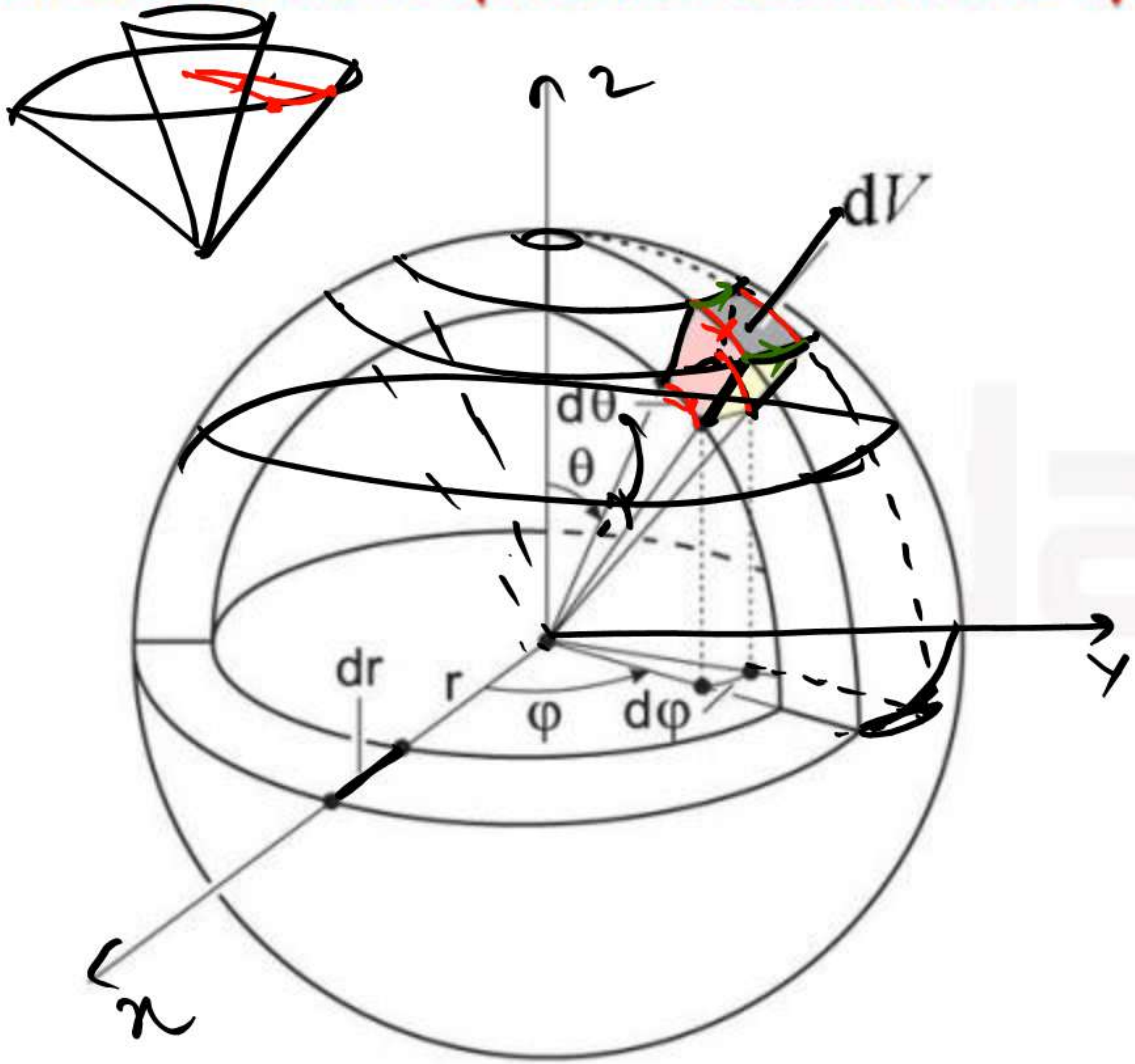
$$\rho\, d\rho\, d\phi\, \hat{a}_z$$

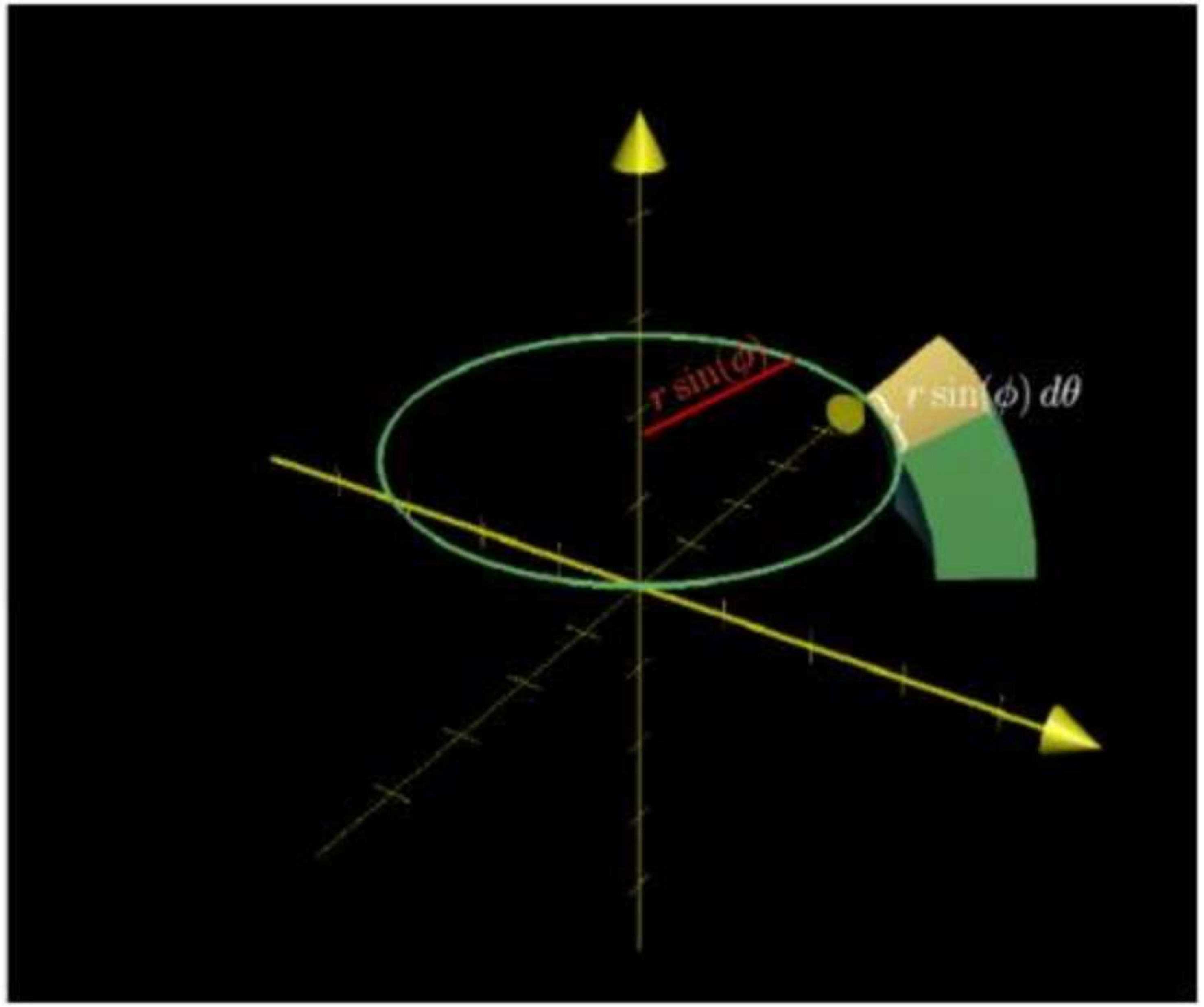
Small volume



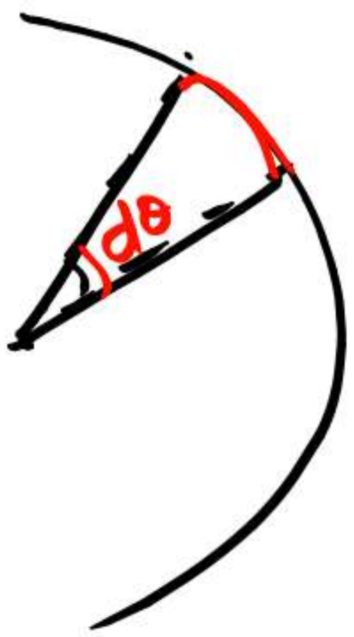
$$dV = \rho\, d\rho\, d\phi\, dz$$

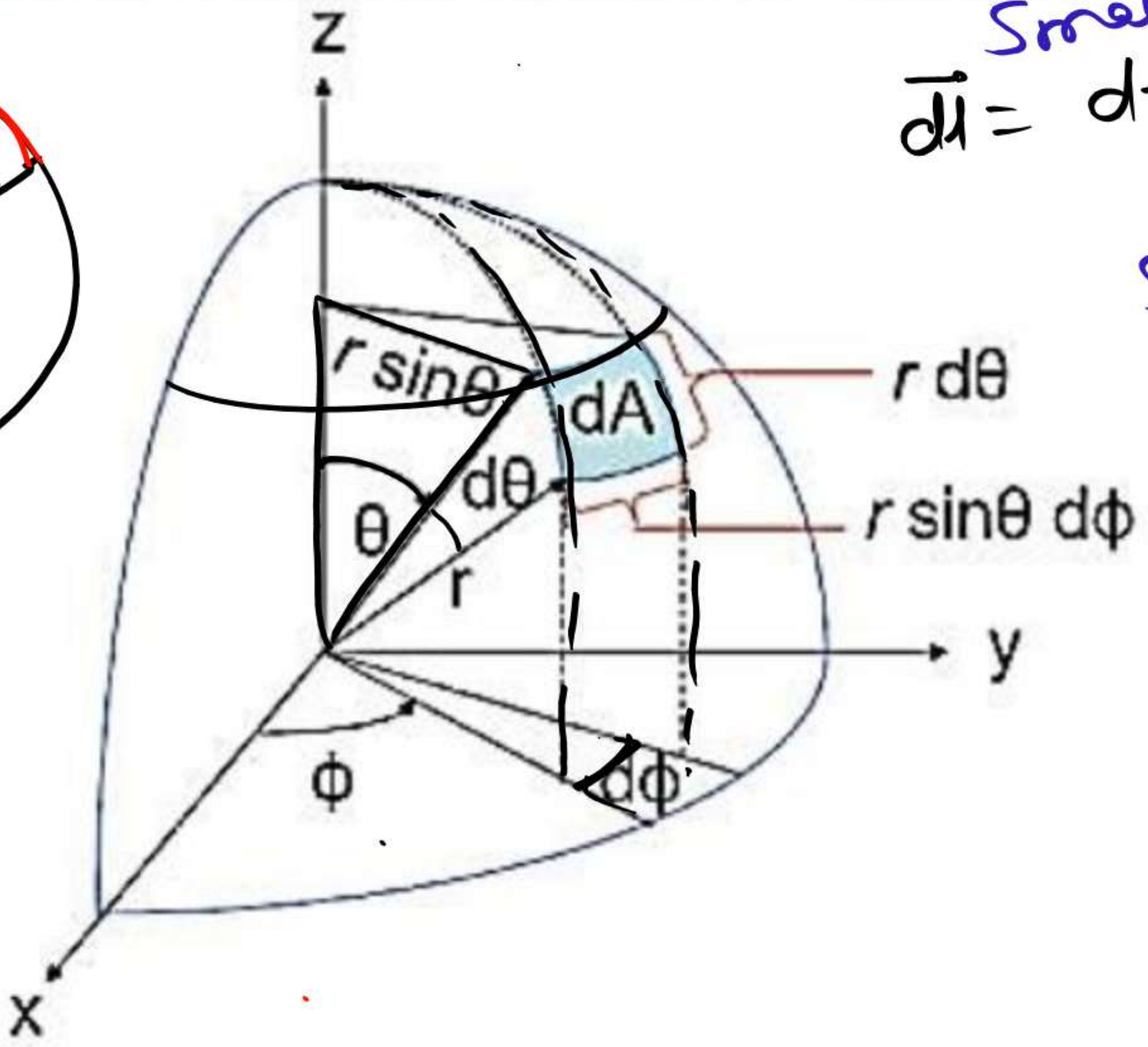
Small Elements in spherical Coordinate systems





Small Elements in spherical Coordinate systems





Small length vector

$$\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

Small surface area vector.

$$\vec{ds} = r^2\sin\theta\,d\theta\,d\phi\,\hat{a}_r$$

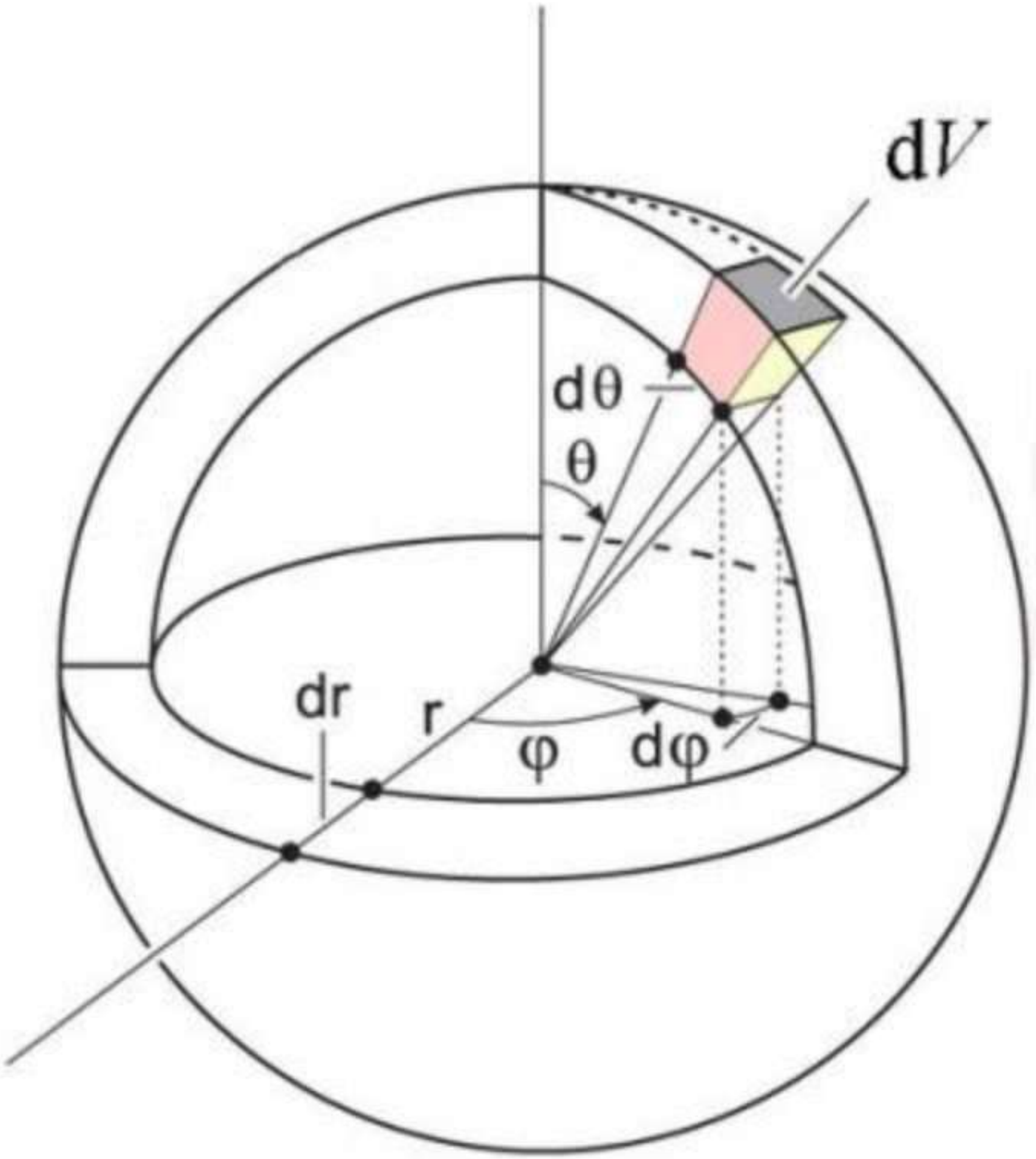
or

$$r\,dr\sin\theta\,d\phi\,\hat{a}_\theta$$

or

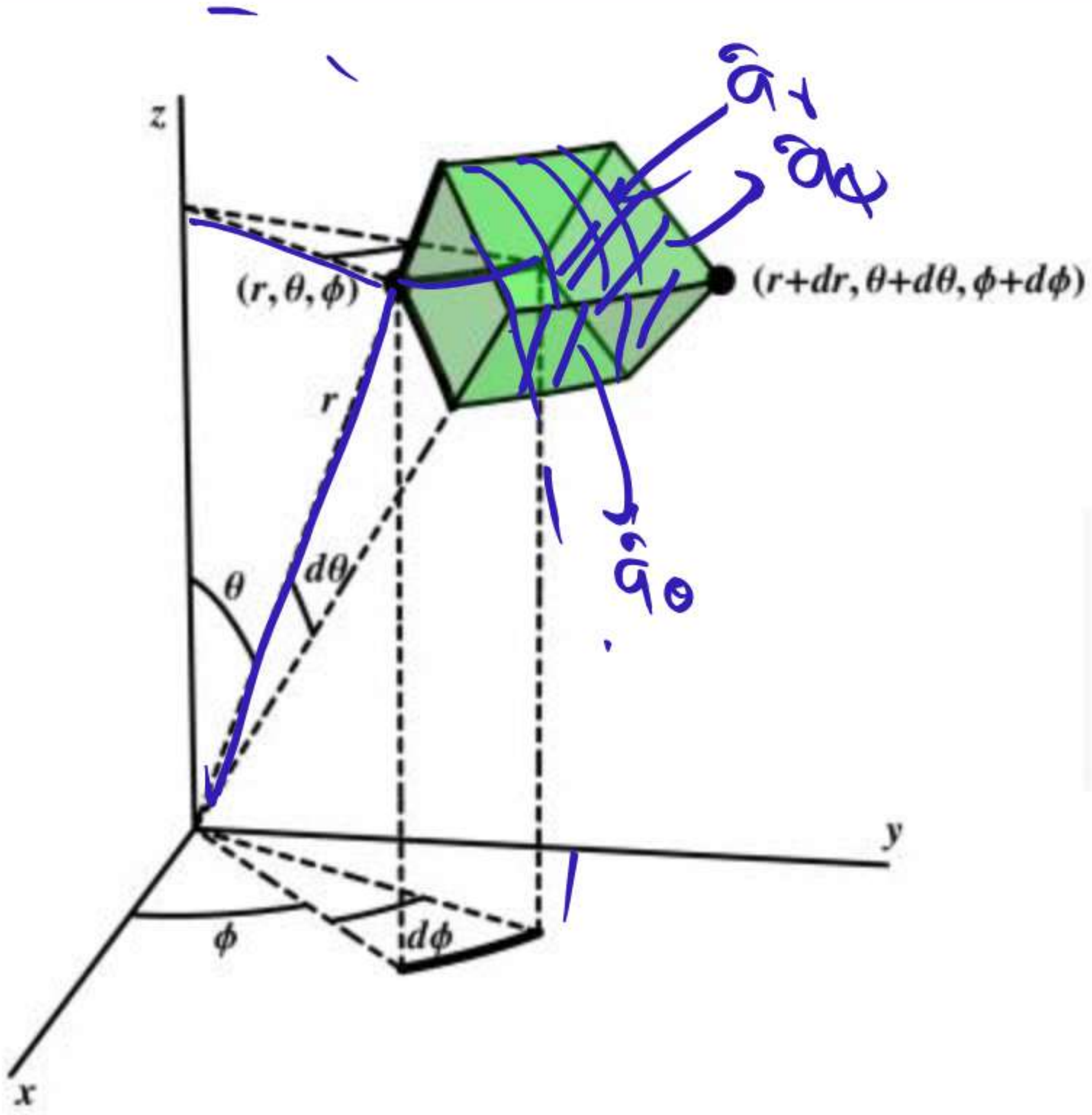
$$r\,dr\,d\theta\,\hat{a}_\phi$$

Small Elements in spherical Coordinate systems



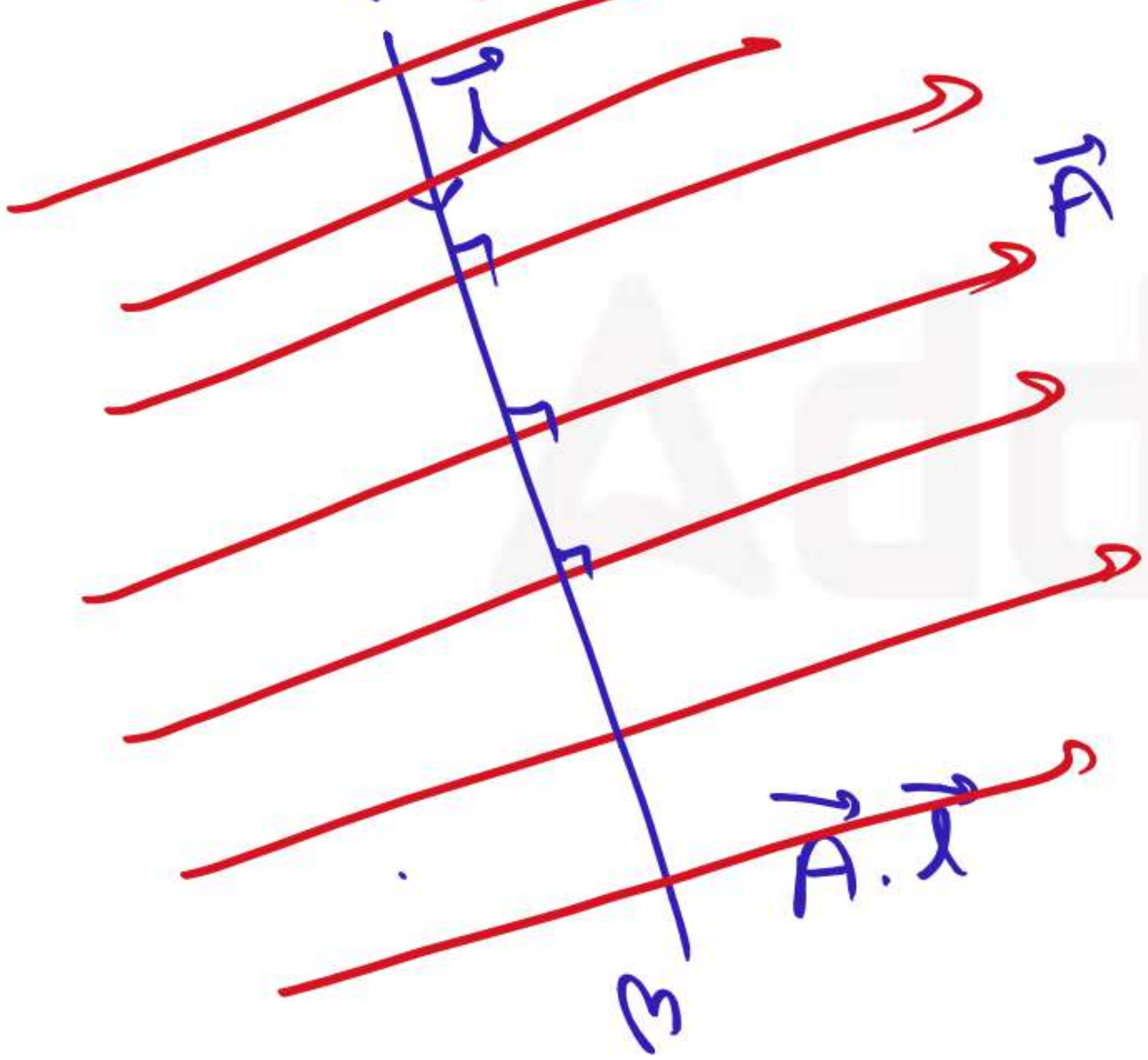
Small volume

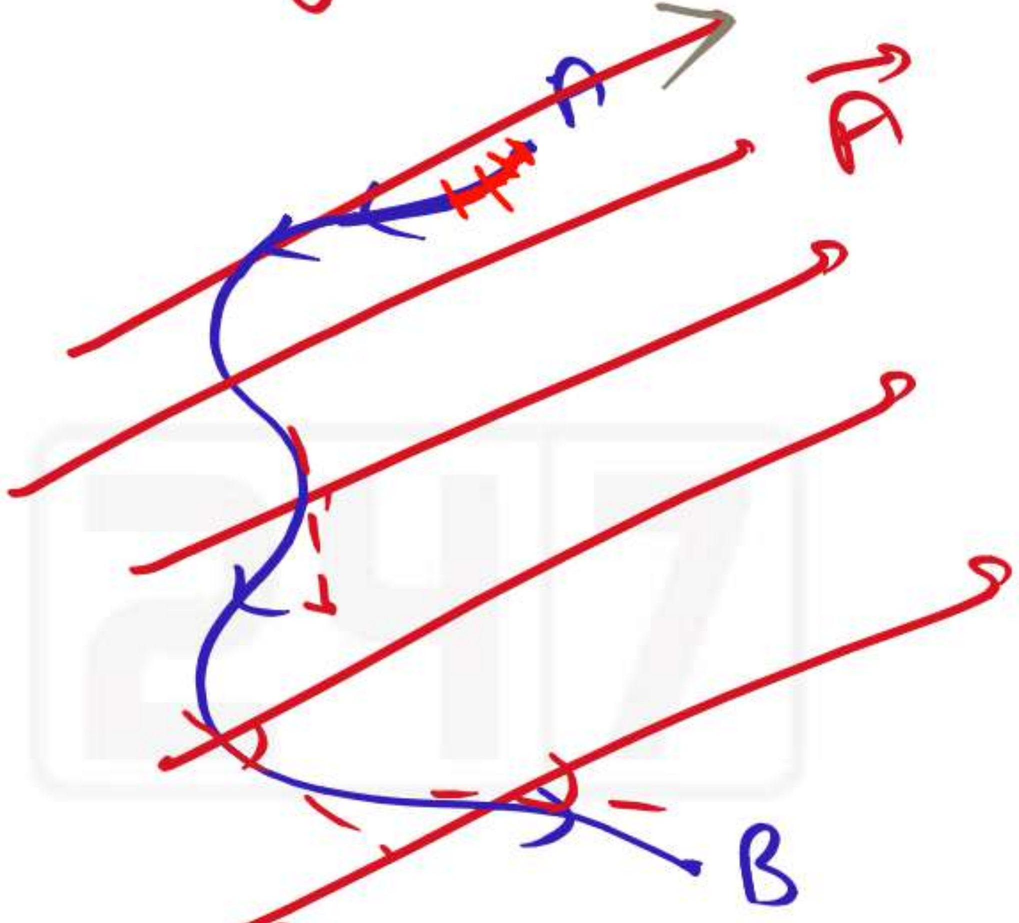
$dv = r^2 \sin\theta \, dr \, d\theta \, d\phi$


z
(r, θ, φ)
(r+dr, θ+dθ, φ+dφ)
r
θ
dθ
y
φ
dφ
x

Line Integral

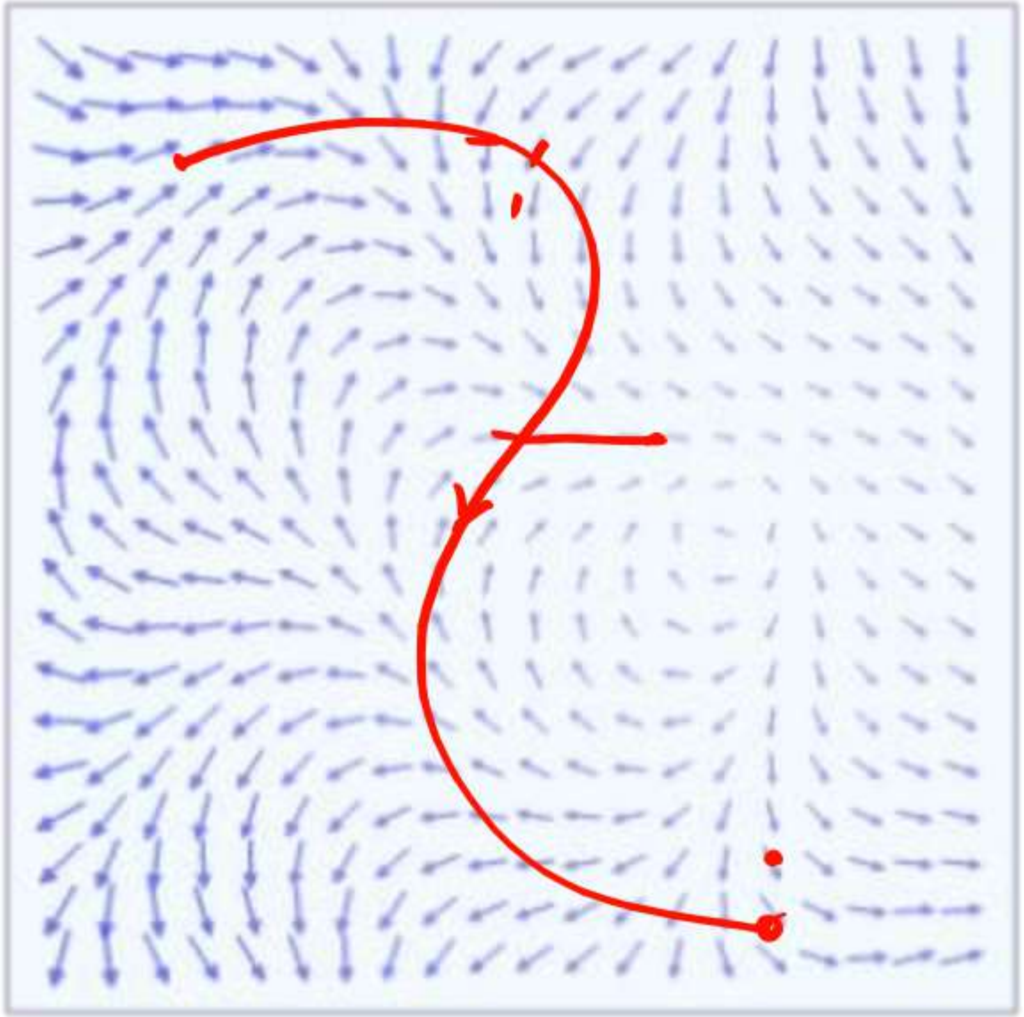
$$\int_A^B \vec{A} \cdot \vec{dl} \rightarrow \text{line integral.}$$



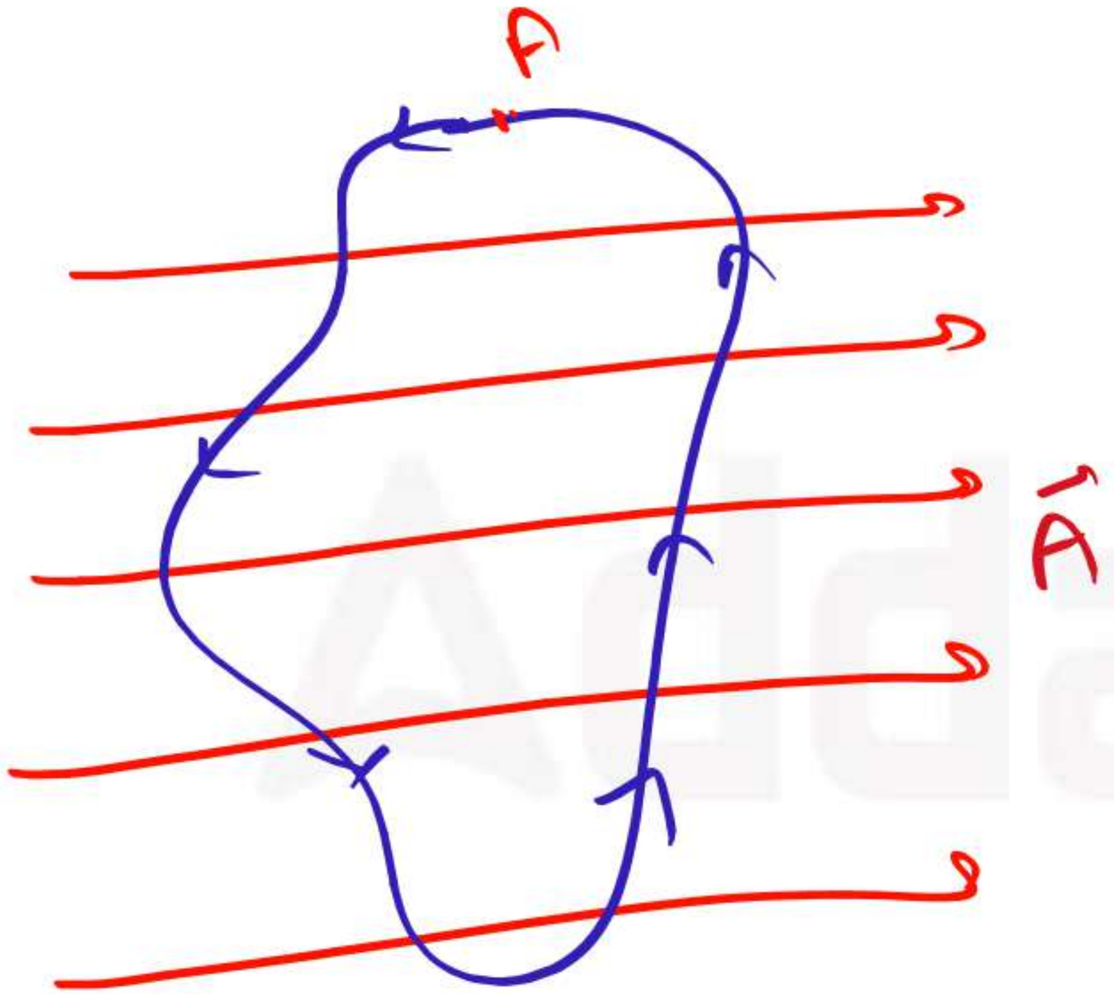


$$\int_A^B \vec{A} \cdot \vec{dl}$$

Physical Significance of Line Integral

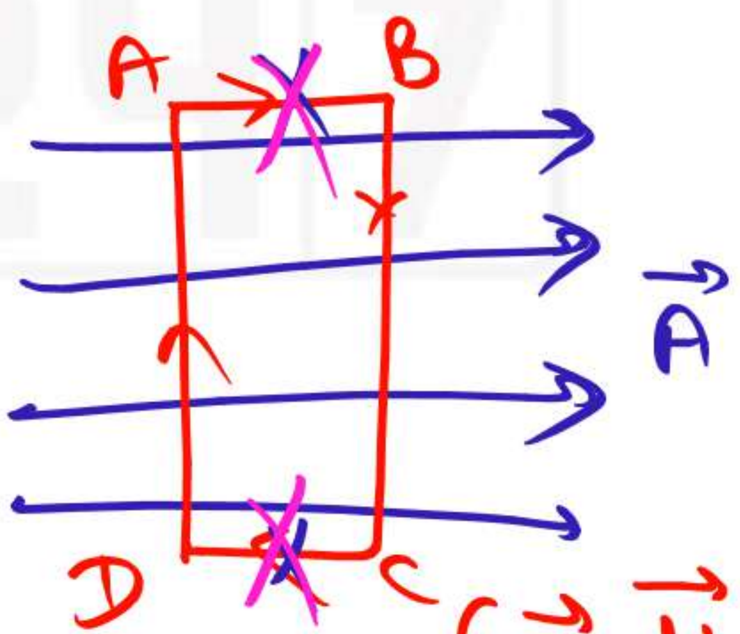


Closed Line Integral



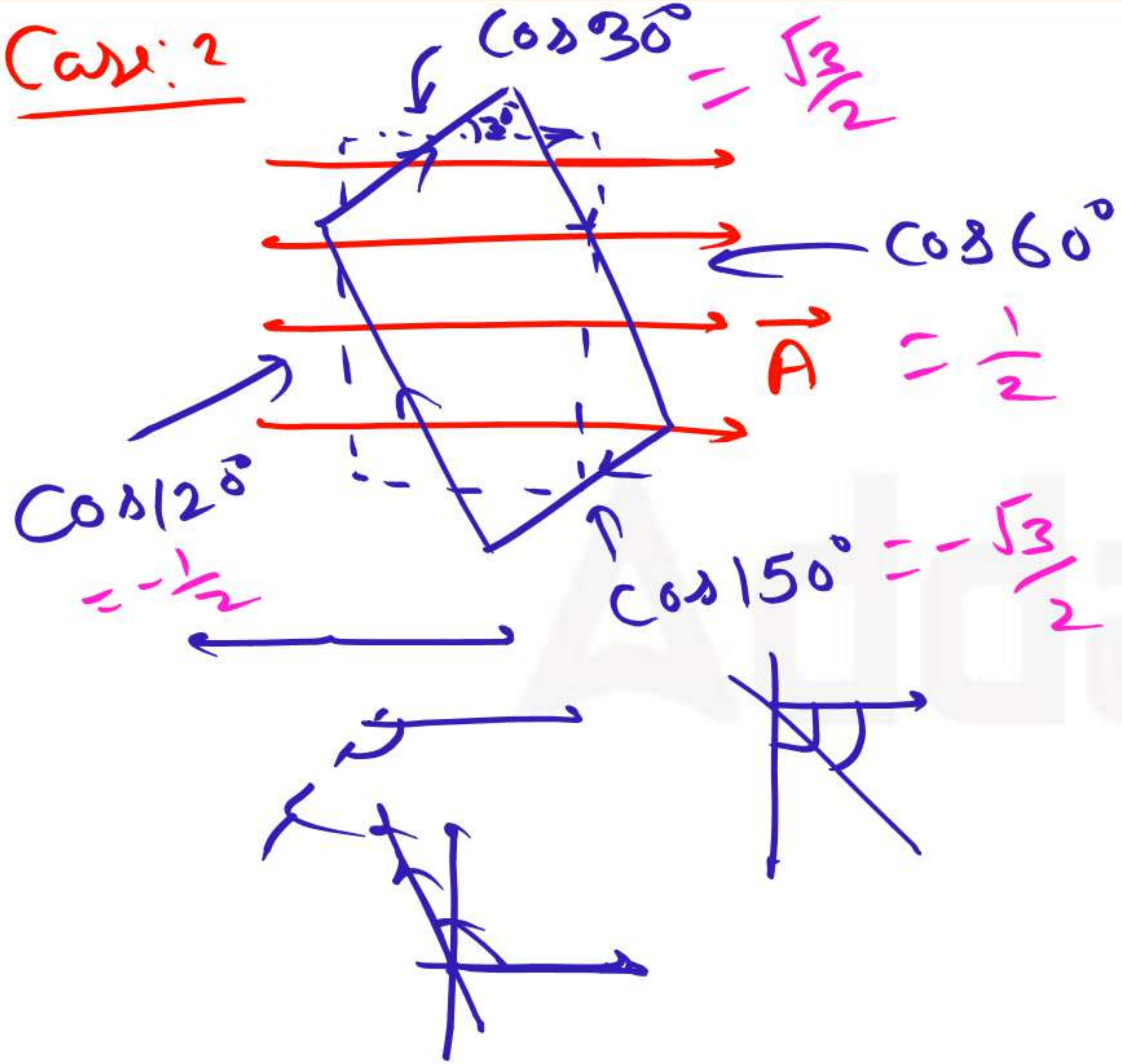
$\oint \vec{A}\cdot\vec{dl}$

Case : I



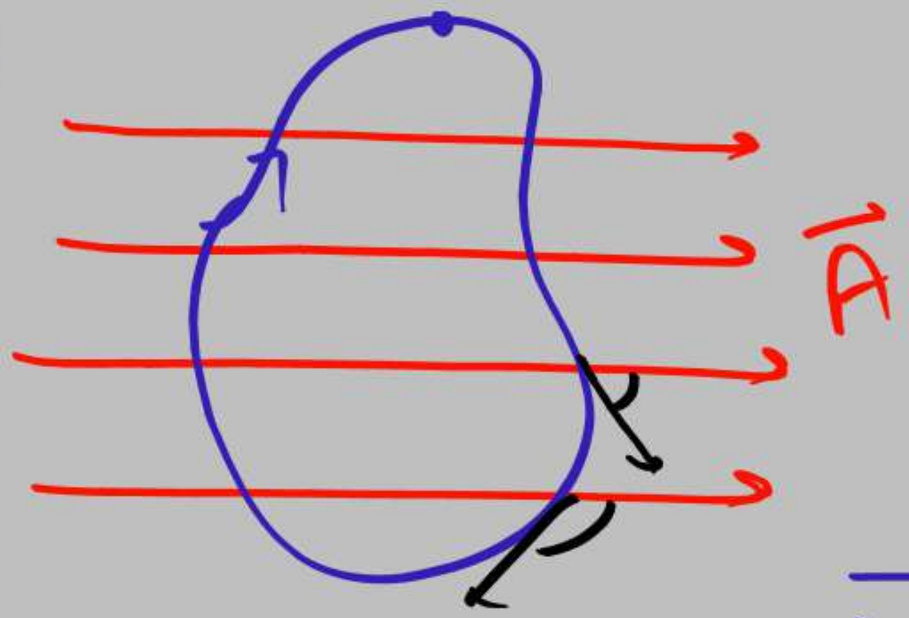
$\oint \vec{A}\cdot\vec{dl} = 0$

Case: 2

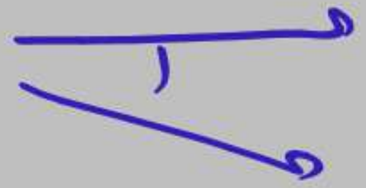


$$\oint \vec{A} \cdot \vec{dl} = 0$$

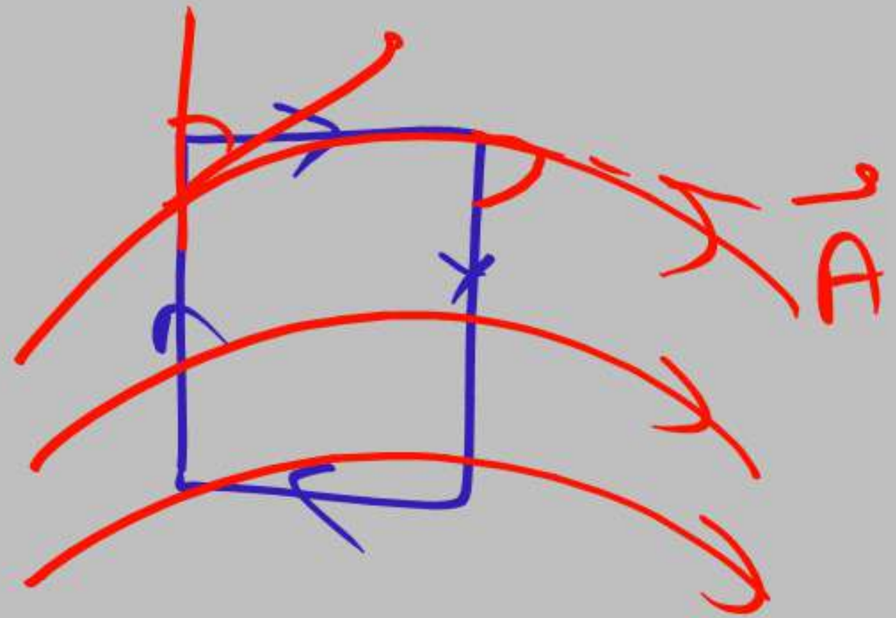
Case: 3



$$\oint \vec{A} \cdot \vec{dl} = 0$$

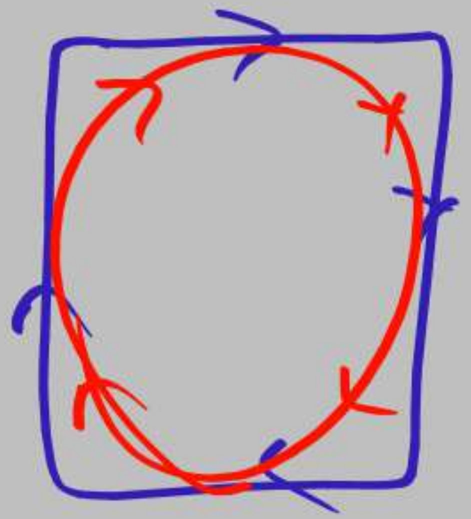


Case: 4



$$\oint \vec{A} \cdot \vec{dl} \neq 0 = X$$

Case: 5



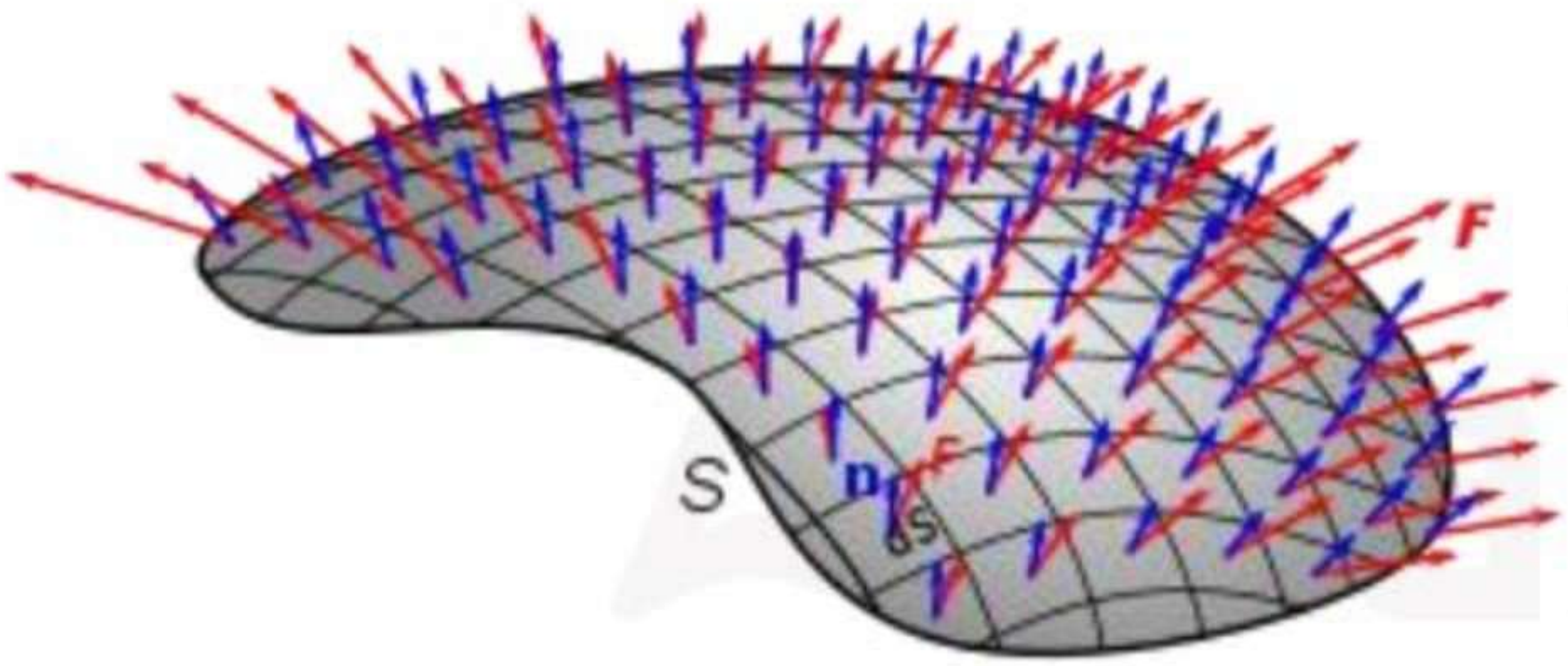
$$\oint \vec{A} \cdot \vec{dl} \neq 0 = Y$$

$$X < Y$$

It is evident that closed line integral of a vector is the measure of how much a vector is circulative around that closed line.

Closed line integral ⟶ circulation.

Volume Integral



Volume Integral

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Engineering Mathematics

LINEAR ALGEBRA

Question practice on basics of matrices


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