Questions on vector calculus

Recap

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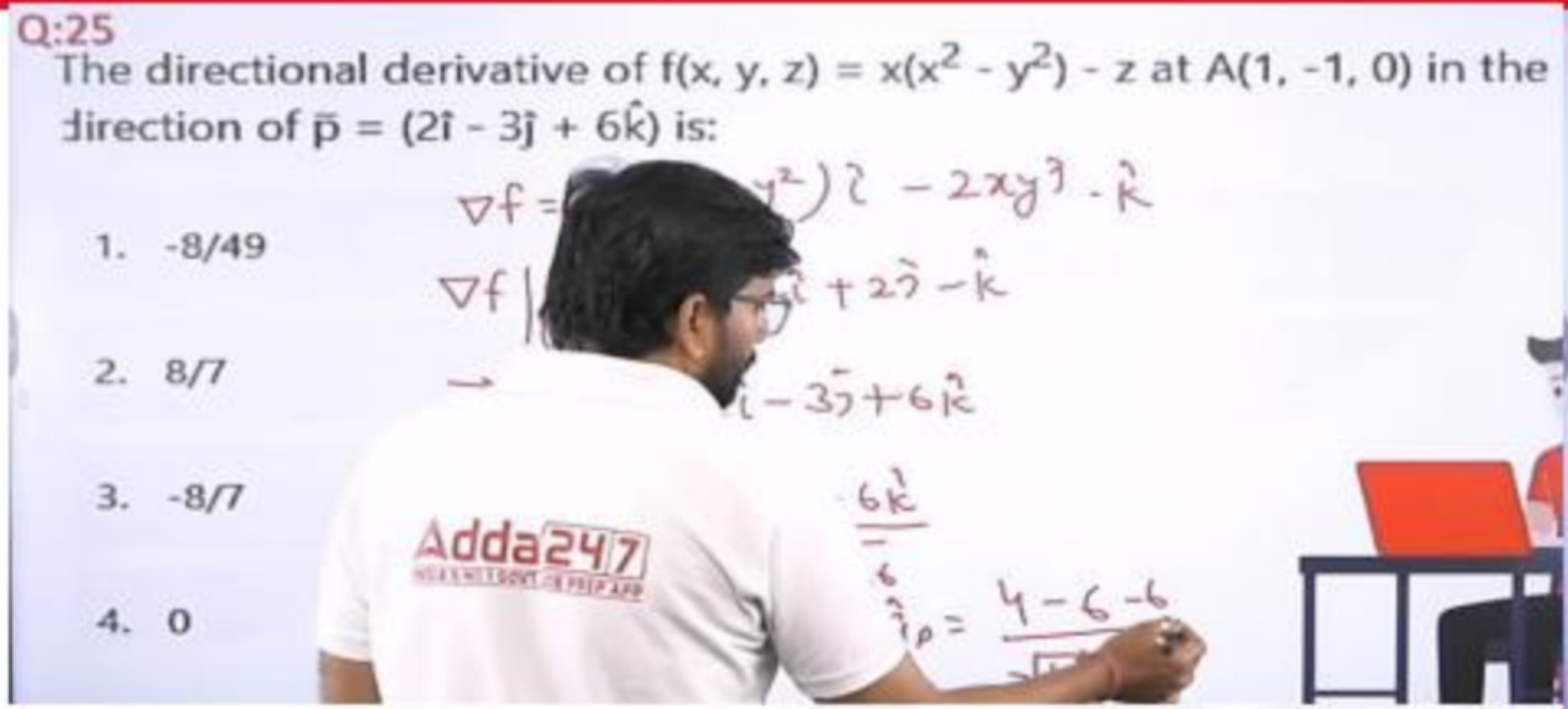
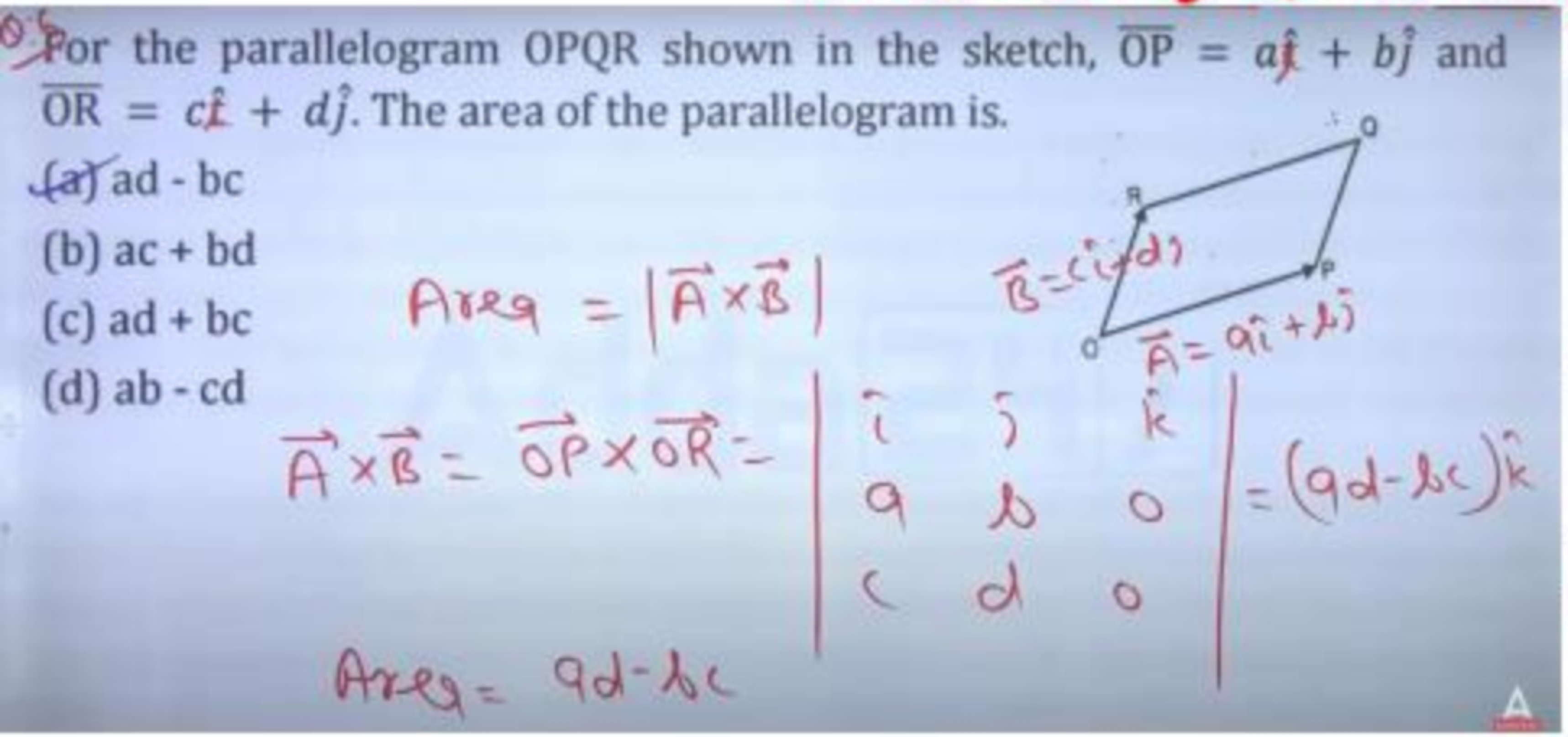
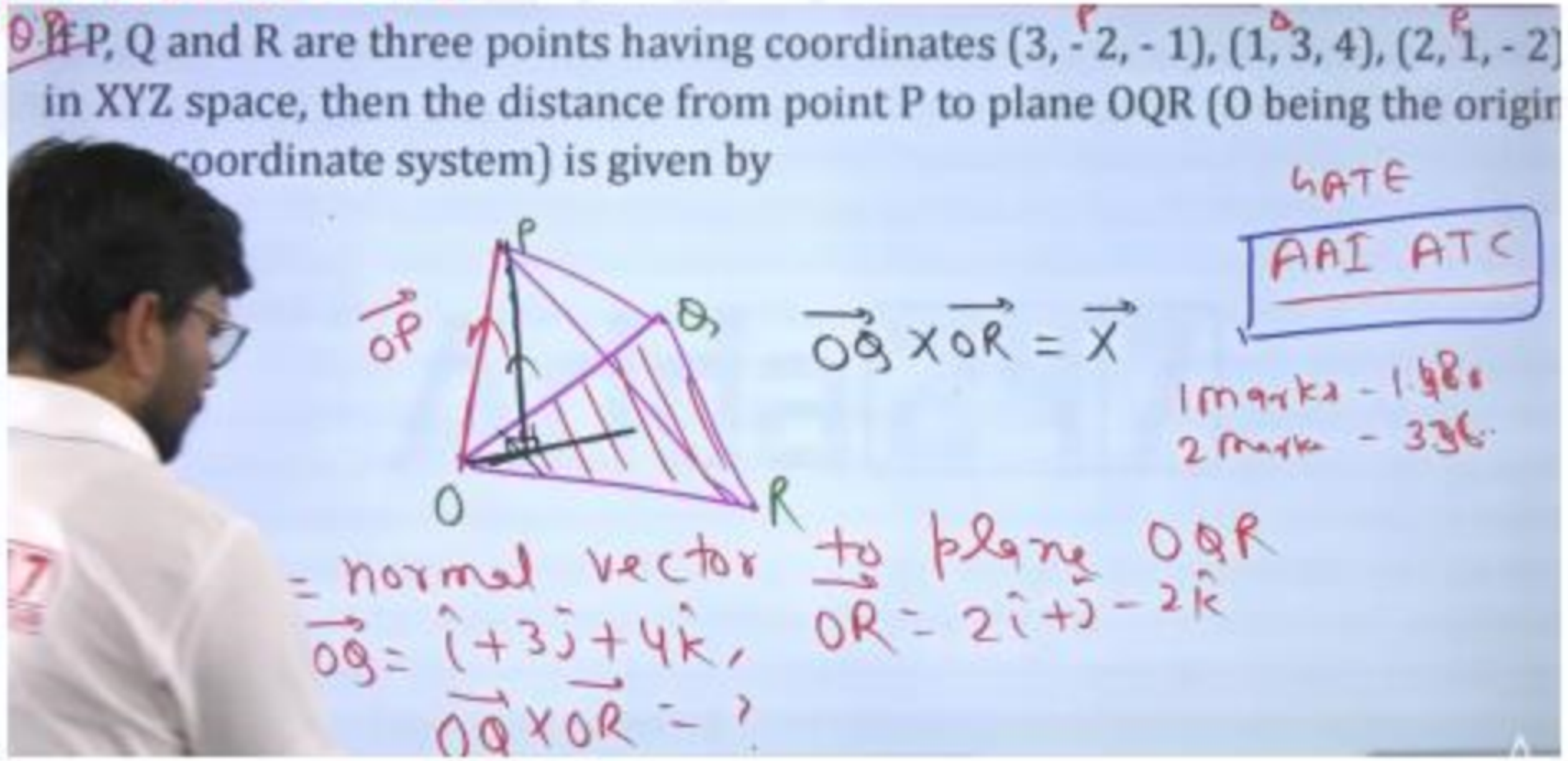


Number of Questions covered-27





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GATE 2024

प्रचण्ड Batch

Electromagnetic Field Theory

QUESTION PRACTICE ON VECTOR CALCULUS

LEC-09

EE &ECE

1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical and Spherical Coordinate systems
7. Vector integrals(Line and closed line)
8. Del Operator ,Gradient and its applications
9. Divergence and Curl

Vector Differentials

Questions on vector calculus

Q: 28 Given a vector field $\vec{F}$, the divergence theorem states that

(a) $\oint_s \vec{F} \cdot d\vec{s} = \int_v (\nabla \cdot \vec{F}) dV$

(b) $\int_s \vec{F} \cdot d\vec{s} = \int_v (\nabla \times \vec{F}) dV$

(c) $\int_s \vec{F} \times d\vec{s} = \int_v \nabla \cdot \vec{F} dV$

(d) $\int_s \vec{F} \times d\vec{s} = \int_v \nabla \times \vec{F} dV$

$\oint \vec{A} \cdot \vec{ds} = \int (\nabla \cdot \vec{A}) dV$

Q: 29 Divergence of the vector field

$\vec{V}(x, y, z) = -(x\cos xy + x)\hat{i} + (y\cos xy)\hat{j} + (\sin z^2 + x^2 + y^2)\hat{k}$ is

(a) $2z\cos z^2$

(b) $\sin xy + 2z\cos z^2$

(c) $x\sin xy - \cos z$

(d) none of these

$\nabla \cdot \vec{V} = -(1 + \cancel{\cos xy} - \cancel{xy\sin xy})$

$+ \{\cancel{\cos xy} - \cancel{xy\sin xy}\}$

$+ 2z\cos z^2.$

$\nabla \cdot \vec{V} = 1 + 2z\cos z^2$

Q:30 The direction of vector A is radially outward from the origin, with $|A| = kr^n$ where $r^2 = x^2 + y^2 + z^2$ and k is a constant. The value of n for which $\nabla . A = 0$ is

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(a) - 2

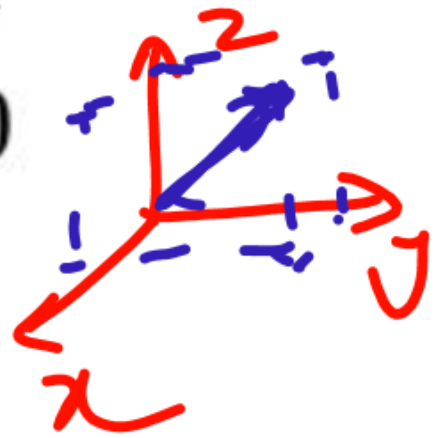
(b) 2

(c) 1

(d) 0

$$|A| = kr^n$$

$$r^2 = x^2 + y^2 + z^2$$



$$\vec{A} = \underbrace{kr^n}_{A_r}\hat{a}_r + 0\,\hat{a}_\theta + 0\,\hat{a}_\phi$$

$$\nabla . \vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \underbrace{\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta A_\theta\right)}_{0} + \underbrace{\frac{1}{r\sin\theta}\frac{\partial (A_\phi)}{\partial\phi}}_{0}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^{n+2}\right) = \frac{k}{r^2}(n+2)r^{n+1} = 0$$

$$\Rightarrow n+2=0$$

$$n=-2$$

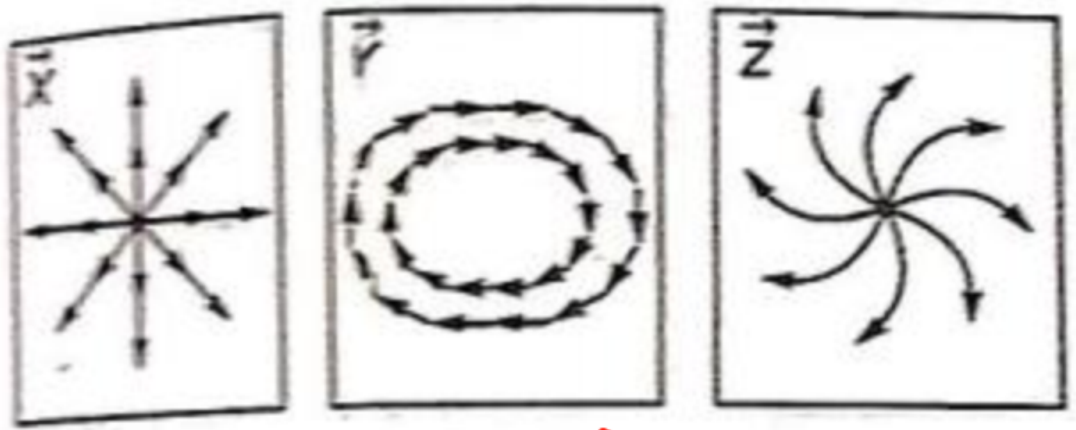
Q:31 Match List - I with List - II the following:

List - I	List - II
A. Stoke's Theorem	1 $\oiint D \cdot ds = Q$
B. Gauss's Theorem	2 $\oint f(z)dz = 0$
C. Divergence Theorem	3 $\iiint (\nabla \cdot A)dv = \oiint A \cdot ds$
D. Cauchy's Integral Theorem	4 $\iint (\nabla \times A) \cdot ds = \oint A \cdot dl$

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	4	1	3	2
(c)	4	3	1	2
(d)	3	4	2	1

Q:32 The figures show diagrammatic representations of vector fields $\vec{X}_1\ \vec{Y}'$ and $\vec{Z}$, respectively. Which one of the following choices is true?



(a) $\nabla \cdot \vec{X} = 0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z} = 0$

(b) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y} = 0, \nabla \times \vec{Z} \neq 0$

(c) $\nabla \cdot \vec{X} \neq 0, \nabla \times \vec{Y} \neq 0, \nabla \times \vec{Z} \neq 0$

(d) $\nabla \cdot \vec{X} = 0, \nabla \times \vec{Y} = 0, \nabla \times \vec{Z} = 0$

$\nabla \times \vec{X} = 0$ $\nabla \times \vec{Y} \neq 0,$ $\nabla \times \vec{Z} \neq 0$

$\nabla \cdot \vec{X} \neq 0$ $\nabla \cdot \vec{Y} = 0$ $\nabla \cdot \vec{Z} \neq 0$

Q:33 The directional derivative of $f(x,y,z) = 2x^2 + 3y^2 + z^2$ at the point P(2, 1, 3) in the direction of the vector $\vec{a}$ = i - 2k is

(a) - 2.785

(b) - 2.145

(c) - 1.789

(d) 1.000

$\nabla f \cdot \hat{a}$

$\nabla f = 4x\hat{i} + 6y\hat{j} + 2z\hat{k}$

$\nabla f\Big|_{(2,1,3)} = 8\hat{i} + 6\hat{j} + 6\hat{k}$

$\hat{a} = \dfrac{\hat{i} - 2\hat{k}}{\sqrt{5}}$

$d.d. = \nabla f \cdot \hat{a} = \dfrac{8}{\sqrt{5}} - \dfrac{12}{\sqrt{5}} = \dfrac{-4}{\sqrt{5}}$

Q:34 For a scalar function $f(x, y, z) = x^2 + 3y^2 + 2z^2$, the gradient at the point P(1, 2, - 1) is

(a) $2\vec{i} + 6\vec{j} + 4\vec{k}$

(b) $2\vec{i} + 12\vec{j} - 4\vec{k}$

(c) $2\vec{i} + 12\vec{j} + 4\vec{k}$

(d) $\sqrt{56}$

$\nabla f = 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$

$\nabla f\big|_{(1,2,-1)} = 2\hat{i} + 12\hat{j} - 4\hat{k}$

Q: 35 For a scalar function $f(x,y,z) = x^2 + 3y^2 + 2z^2$, the directional derivative at the point P(1, 2, - 1) in the direction of a vector $\vec{i} - \vec{j} + 2\vec{k}$ is

(a) -18

(b) $-3\sqrt{6}$

(c) $3\sqrt{6}$

(d) 18

$\nabla f = 2x\hat{i} + 6y\hat{j} + 4z\hat{k}$

$\nabla f \big|_{(1,2,-1)} = 2\hat{i} + 12\hat{j} - 4\hat{k}$

$\nabla f \cdot \hat{a}_A = (2\hat{i} + 12\hat{j} - 4\hat{k}) \cdot \left(\frac{\hat{i} - \hat{j} + 2\hat{k}}{\sqrt{6}}\right)$

$= \frac{2 - 12 - 8}{\sqrt{6}} = \frac{-18}{\sqrt{6}}$

$= \frac{-3 \times 6}{\sqrt{6}} = -3\sqrt{6}$

Q: 36 The directional derivative of the field $u(x, y, z) = x^2 - 3yz$ in the direction of the vector $(\bar{i} + \bar{j} - 2\bar{k})$ at point (2, - 1, 4) is ______.

Q: 37 Equation of the line normal to function $f(x) = (x - 8)^{2/3} + 1$ at $P(0, 5)$ is

(a) y = 3x - 5

(b) y = 3x + 5

(c) 3y = x + 15

(d) 3y = x - 15

Sol: 36

$\nabla u = 2x\hat{i} - 3z\hat{j} - 3y\hat{k}$

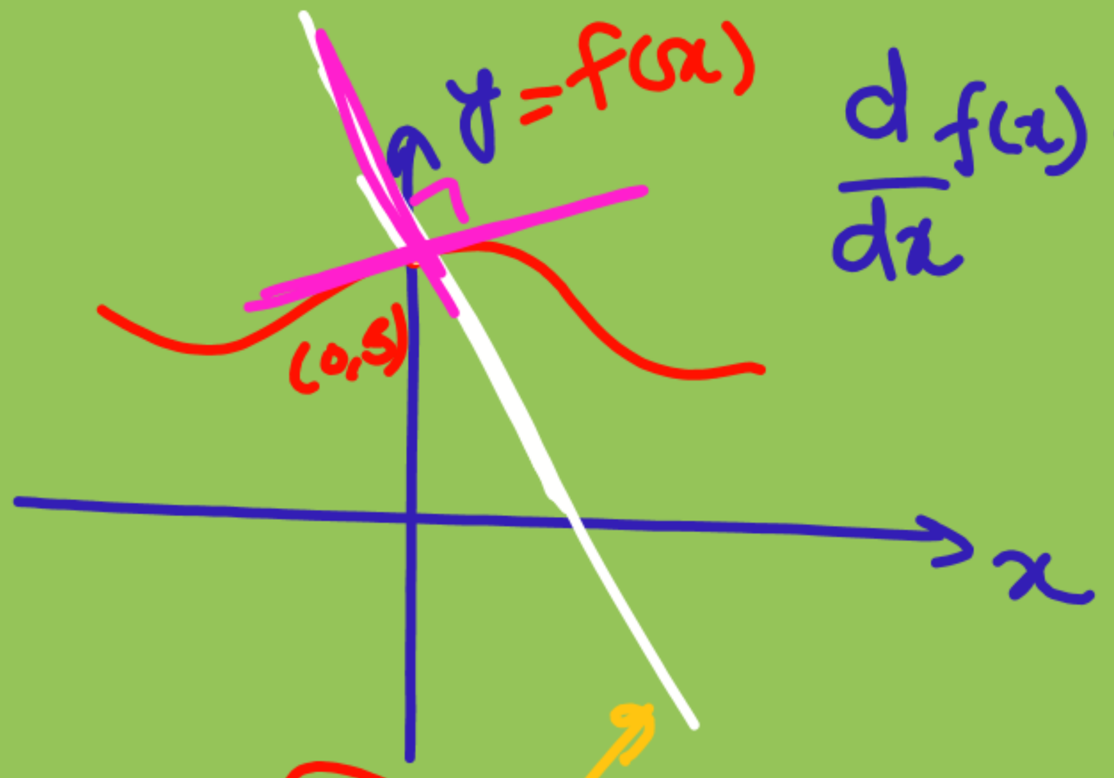
$\nabla u|_{(2,-1,4)} = 4\hat{i} - 12\hat{j} + 3\hat{k}$

$\nabla u \cdot \hat{a}_A = (4\hat{i} - 12\hat{j} + 3\hat{k})\left(\frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}\right)$

$= \frac{4 - 12 - 6}{\sqrt{6}} = \frac{-14}{\sqrt{6}} = -5.715$

Sol: 3-

$f(x) = (x-8)^{\frac{2}{3}} + 1 \qquad P(0,5)$



$\frac{d}{dx} f(x) = \frac{2}{3}(x-8)^{-\frac{1}{3}}$

$= \frac{2}{3}(-8)^{-\frac{1}{3}}$

$= \frac{2}{3(-8)^{\frac{1}{3}}}$

$= -\frac{1}{3}$

$y = (x-8)^{\frac{2}{3}} + 1$

at $x = 0$

$y = (0-8)^{\frac{2}{3}} + 1$

$= \left((-8)^{\frac{1}{3}}\right)^2 + 1$

$y = 4 + 1 = 5$

$y = mx + c$

$y = (x-8)^{\frac{2}{3}} + 1$

$y - (x-8)^{\frac{2}{3}} - 1 = 0 \Rightarrow f(x,y,z) = 0$

$$\nabla f = -\frac{2}{3}(x-8)^{-\frac{1}{3}}\hat{i} + \hat{j}$$

$$\nabla f\Big|_{(0,5)} = \frac{-2}{3}(-8)^{\frac{1}{3}}\hat{i} + \hat{j}$$

$$m_2 = \frac{-1}{-\frac{1}{3}} = 3$$

$$m_1 m_2 = -1$$

$$m_2 = \frac{-1}{m}$$

$$y = mx + c$$

$$y = 3x + 5$$

Q:38 The divergence of the vector field $(x-y)\hat{i} + (y-x)\hat{j} + (x + y + z)\hat{k}$ is

$\nabla \cdot \vec{A} = 1 + 1 + 1 = 3$

(a) 0

(b) 1

(c) 2

(d) 3

Q:39 The directional derivative of the scalar function $f(x, y, z) = x^2 + 2y^2 + z$ at the point P = (1, 1, 2) in the direction of the vector $\vec{a} = 3\hat{i} - 4\hat{j}$ is.

(a) - 4

(b) - 2

(c) - 1

(d) 1

$\nabla f = 2x\hat{i} + 4y\hat{j} + \hat{k}$

$\nabla f \big|_{(1,1,2)} = 2\hat{i} + 4\hat{j} + \hat{k}$

$\nabla f \cdot \hat{a}_A = \frac{6 - 16}{\sqrt{25}} = \frac{-10}{5} = -2$

Q: 40 The divergence of the vector field $3xz\hat{i} + 2xy\hat{j} - yz^2\hat{k}$ at a point (1, 1, 1) is equal to

(a) 7

(b) 4

(c) 3

(d) 0

$\nabla \cdot \vec{A} = 3z + 2x - 2yz$

$\nabla \cdot \vec{A}\Big|_{(1,1,1)} = 3 + 2 - 2 = 3$

Q: 41. Velocity vector of a flow field is given as $\vec{V} = 2xy\hat{i} - x^2z\hat{j}$. The vorticity vector at (1, 1, 1) is

(a) $4\hat{i} - \hat{j}$

(b) $4\hat{i} - \hat{k}$

(c) $\hat{i} - 4\hat{j}$

(d) $\hat{i} - 4\hat{k}$

$\overrightarrow{\text{Vorticity}} = \nabla \times \overrightarrow{\text{Velocity}}$

$$\vec{A} = \nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & -x^2z & 0 \end{vmatrix}$$

$$\vec{A} = x^2\hat{i} - (2xz + 2x)\hat{k}$$

$$\vec{A}\Big|_{(1,1,1)} = \hat{i} - 4\hat{k}$$

Q:42 For the spherical surface $x^2 + y^2 + z^2 = 1$, the unit outward normal vector at the point $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ is given by

(a) $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

(b) $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$

(c) $\hat{k}$

(d) $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$

$x^2+y^2+z^2=1$

$f(x,y,z)=0 \Rightarrow x^2+y^2+z^2-1=0$

$\nabla f = 2x\hat{i}+2y\hat{j}+2z\hat{k}$

$\nabla f\Big|_{\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right)} = \sqrt{2}\hat{i}+\sqrt{2}\hat{j} \longrightarrow$ normal Vector

unit normal vector $= \dfrac{\nabla f}{|\nabla f|} = \dfrac{\sqrt{2}\hat{i}+\sqrt{2}\hat{j}}{\sqrt{4}} = \frac{1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{j}$

Q:43 Curl of vector $V(x, y, z) = 2x^2 i + 3z^2 j + y^3 k$ at x = y = z = 1 is

(a) - 3i

(b) 3i

(c) 3i - 4j

(d) 3i - 6k

$$\nabla \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= (3y^2 - 6z)\hat{i} - \hat{j}(0) + \hat{k}(0)$$

$$\nabla \times \vec{V}\Big|_{(1,1,1)} = -3\hat{i}$$

Q:44. Let ϕ be an arbitrary smooth real valued scalar function and V be an arbitrary smooth vectors valued function in a three - dimensional space. Which one of the following is an identity?

(a) $\mathrm{Curl}(\phi\vec{V}) = \nabla(\phi \mathrm{Div}\,\vec{V})$

(b) $\mathrm{Div}\,\vec{V} = 0$

(c) $\mathrm{Div}\,\mathrm{Curl}\,\vec{V} = 0$

(d) $\mathrm{Div}(\phi\vec{V}) = \phi \mathrm{Div}\,\vec{V}$

A $\nabla \times (\phi \vec{V}) = \phi(\nabla \times \vec{V}) + \vec{V} \times \nabla \phi$

$\nabla \cdot (\phi \vec{V}$

Vector identities

gradient $\rightarrow$ Divergence $\rightarrow$ Curl.

① $\nabla.(\nabla V) \rightarrow$ divergence of gradient

② $\nabla \times \nabla V \rightarrow$ Curl of gradient $=0$

③ $\nabla(\nabla.\vec{A}) \rightarrow$ gradient of divergence $=0$ [for linear vector]

④ $\nabla.(\nabla \times \vec{A}) \rightarrow$ Divergence of Curl. $=0$

⑤ $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}$

⑥ $\nabla\cdot(V\vec{A}) = V(\nabla\cdot\vec{A}) + \vec{A}\cdot\nabla V$

Q: 45 For the scalar field $u = \frac{x^2}{2} + \frac{y^2}{3}$, magnitude of the gradient at the point (1, 3) is

(a) $\sqrt{\frac{13}{9}}$

(b) $\sqrt{\frac{9}{2}}$

(c) $\sqrt{5}$

(d) $\frac{9}{2}$

$\nabla u = x\hat{i} + \frac{2y}{3}\hat{j}$

$\nabla u \Big|_{(1,3)} = \hat{i} + 2\hat{j}$

$|\nabla u| = \sqrt{5}$

Q:46 Divergence of the three - dimensional radial vector field $\vec{r}$ is

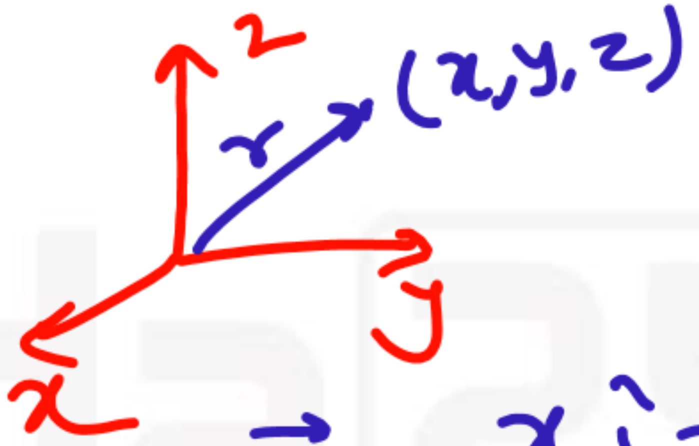
(a) 3

(b) 1/r

(c) $\hat{i} + \hat{j} + \hat{k}$

(d) $3(\hat{i} + \hat{j} + \hat{k})$

$\vec{r}$



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{r} = r\hat{a}_r$

$\nabla \cdot \vec{r} = 1 + 1 + 1 = 3$

Q: 48 The curl of the gradient of the scalar field defined by $V = 2x^2y + 3y^2z + 4z^2x$ is

(a) $4xya_x + 6yza_y + 8zxa_2$

(b) $4a_x + 6a_y + 8a_2$

(c) $(4xy + 4z^2)a_x + (2x^2 + 6yz)a_y + (3y^2 + 8zx)a_2$

(d) 0

$\nabla \times (\nabla V) = 0$ [from vector identity)

Q:49 $\nabla \times \nabla \times P$, where P is a vector is equal to

(a) $P \times \nabla \times P - \nabla^2 P$

(b) $\nabla^2 P + \nabla(\nabla \times P)$

(c) $\nabla^2 P + \nabla \times P$

(d) $\nabla(\nabla \cdot P) - \nabla^2 P$

$\nabla \times \nabla \times \vec{P} = \nabla(\nabla \cdot \vec{P}) - \vec{P}\, \nabla \cdot \nabla$

Q:50 The divergence of the vector field $\vec{A} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ is

(a) 0

(b) 1/3

(c) 1

(d) 3

Q:51 For a vector E, which one of the following statements is NOT TRUE?

(a) If $\nabla \cdot E = 0$, E is called solenoidal. Tr

(b) If $\nabla \times E = 0$, E is called conservative. Tr

(c) It$\nabla \times E = 0$, E is called irrotational. Tr

(d) If $\nabla \cdot E = 0$, E is called irrotational. False

Q: 52. The magnitude of the directional derivative of the function $f(x, y) = x^2 + 3y^2$ in a direction normal to the circle $x^2 + y^2 = 2$, at the point (1, 1), is

(a) $4\sqrt{2}$

(b) $5\sqrt{2}$

(c) $7\sqrt{2}$

(d) $9\sqrt{2}$

Q: 53 Stokes theorem connects

(a) a line integral and a surface integral

(b) a surface integral and a volume integral

(c) a line integral and a volume integral

(d) gradient of a function and its surface integral

Q:54 Which one of the following describes the relationship among the three vectors, $\vec{i} + \hat{j} + \hat{k}$, $\vec{A}$

$2\hat{i} + 3\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 4\hat{k}$?

$\vec{B}$ $\vec{C}$

(a) The vectors are mutually perpendicular

(b) The vectors are linearly dependent

(c) The vectors are linearly independent

(d) The vectors are unit vectors

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 5 & 6 & 4 \end{bmatrix}$$

$R_3 = 3R_1 + R_2$

$\vec{C} = 3\vec{A} + \vec{B}$

$\vec{A} \cdot \vec{B} = 2+3+1 \neq 0$

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