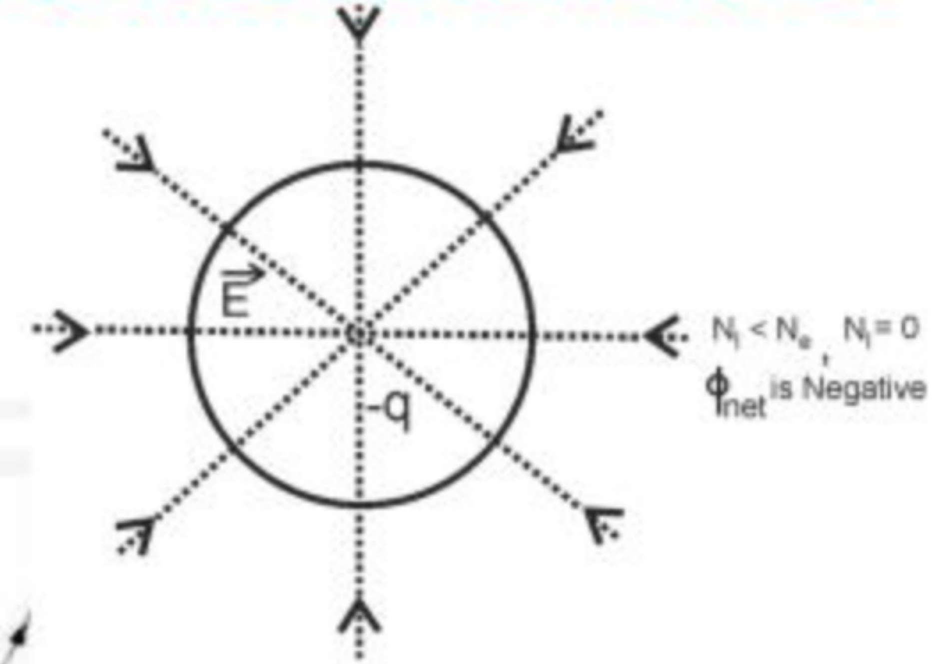
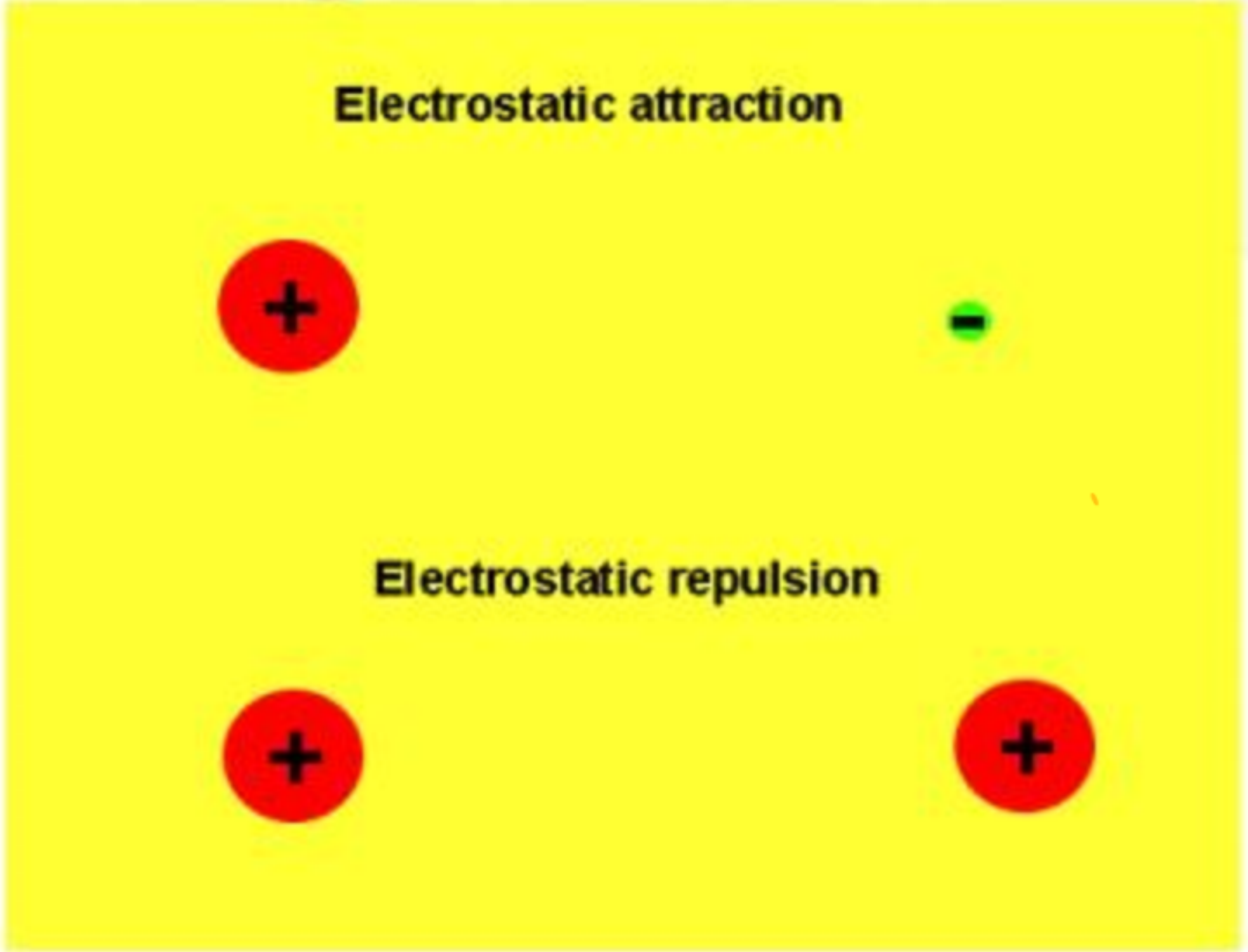
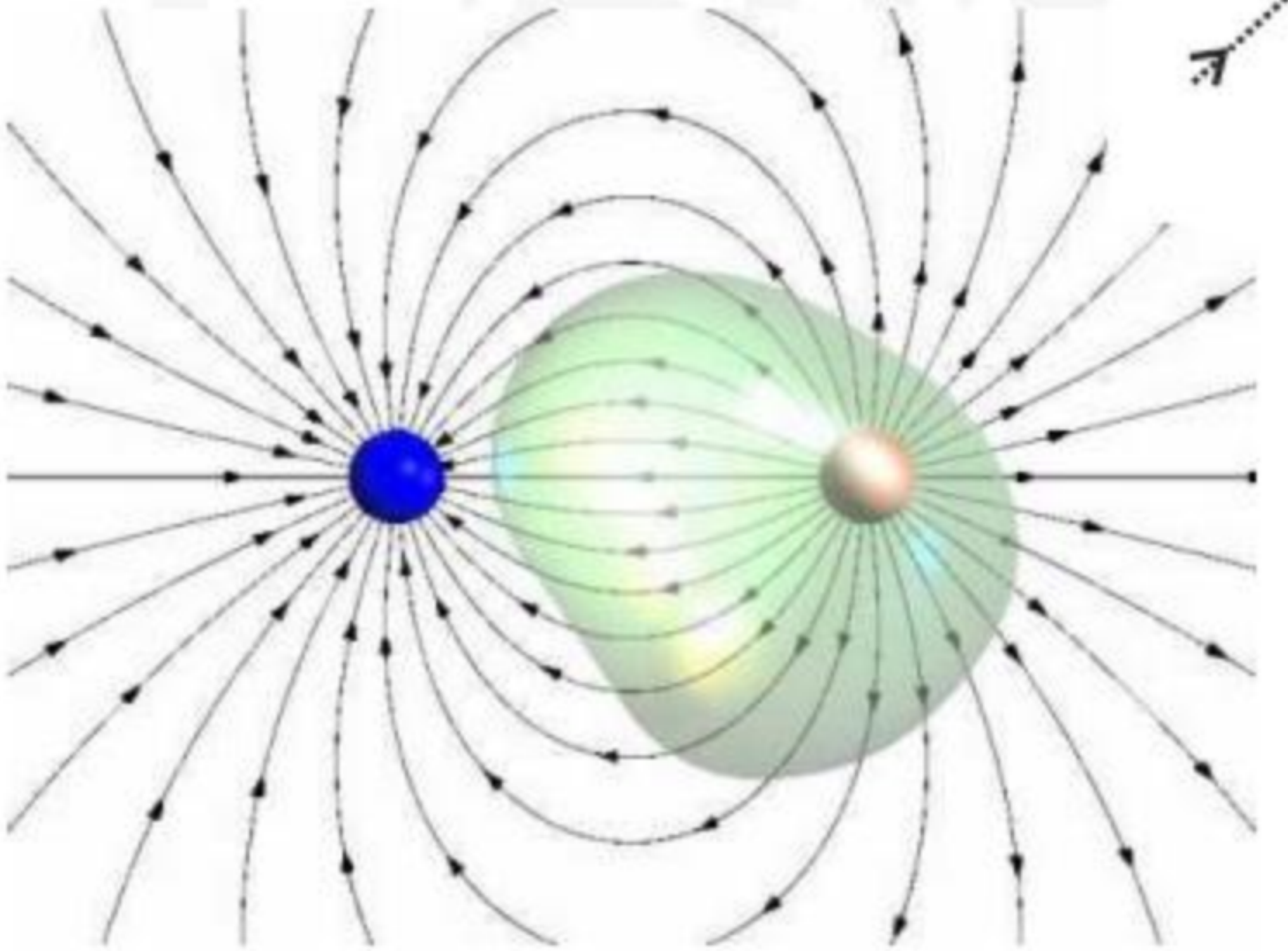
Electric Field Calculations through Coulomb's law

Gauss Law

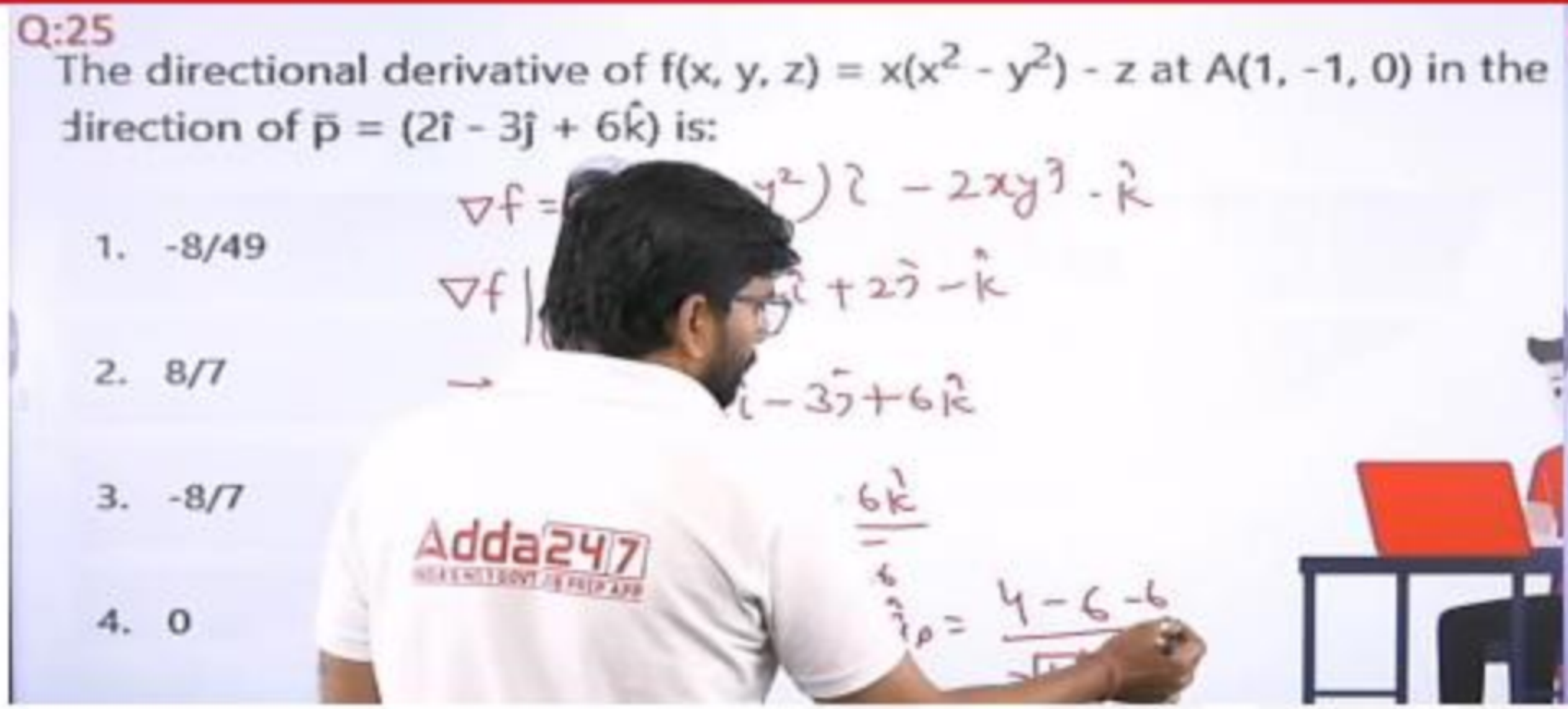




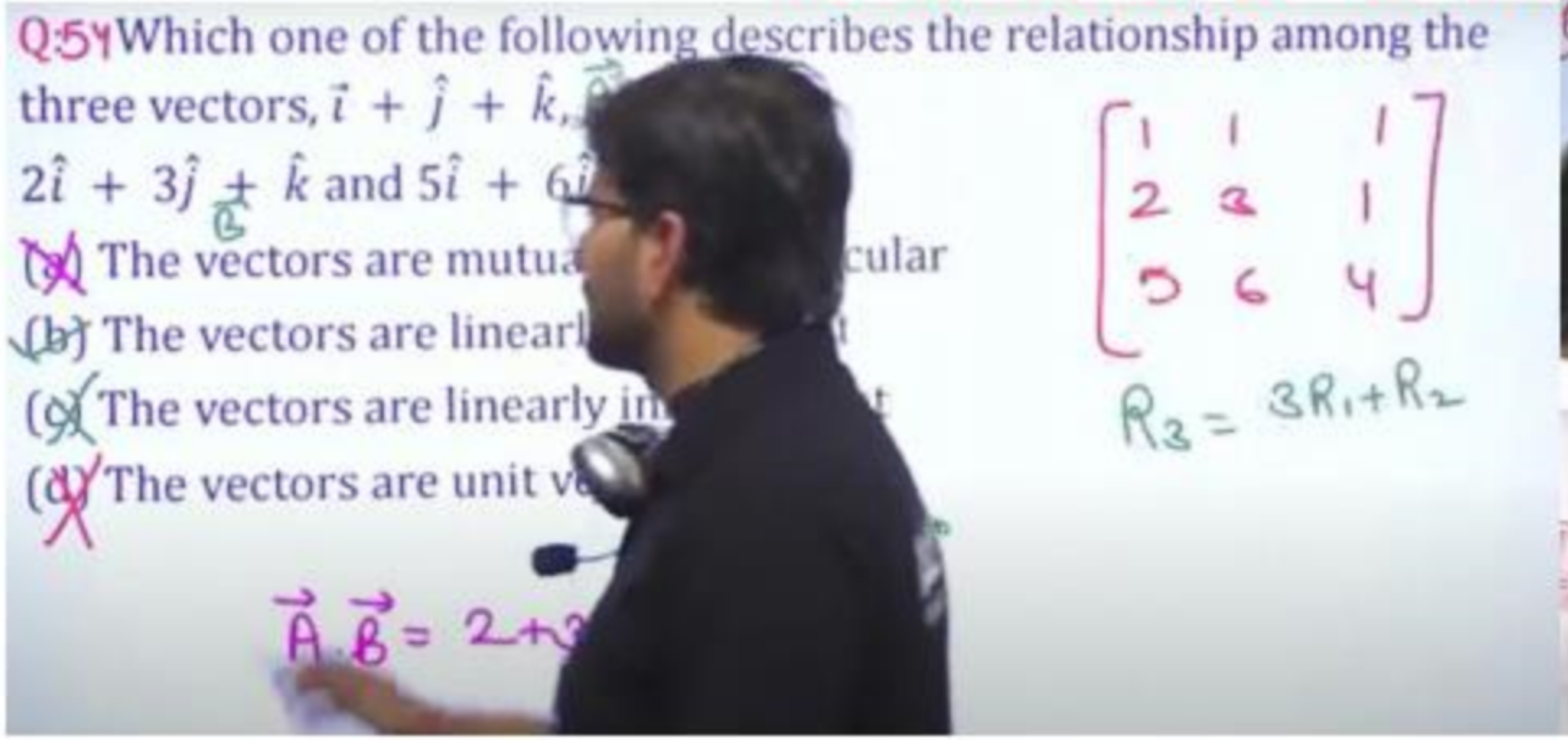


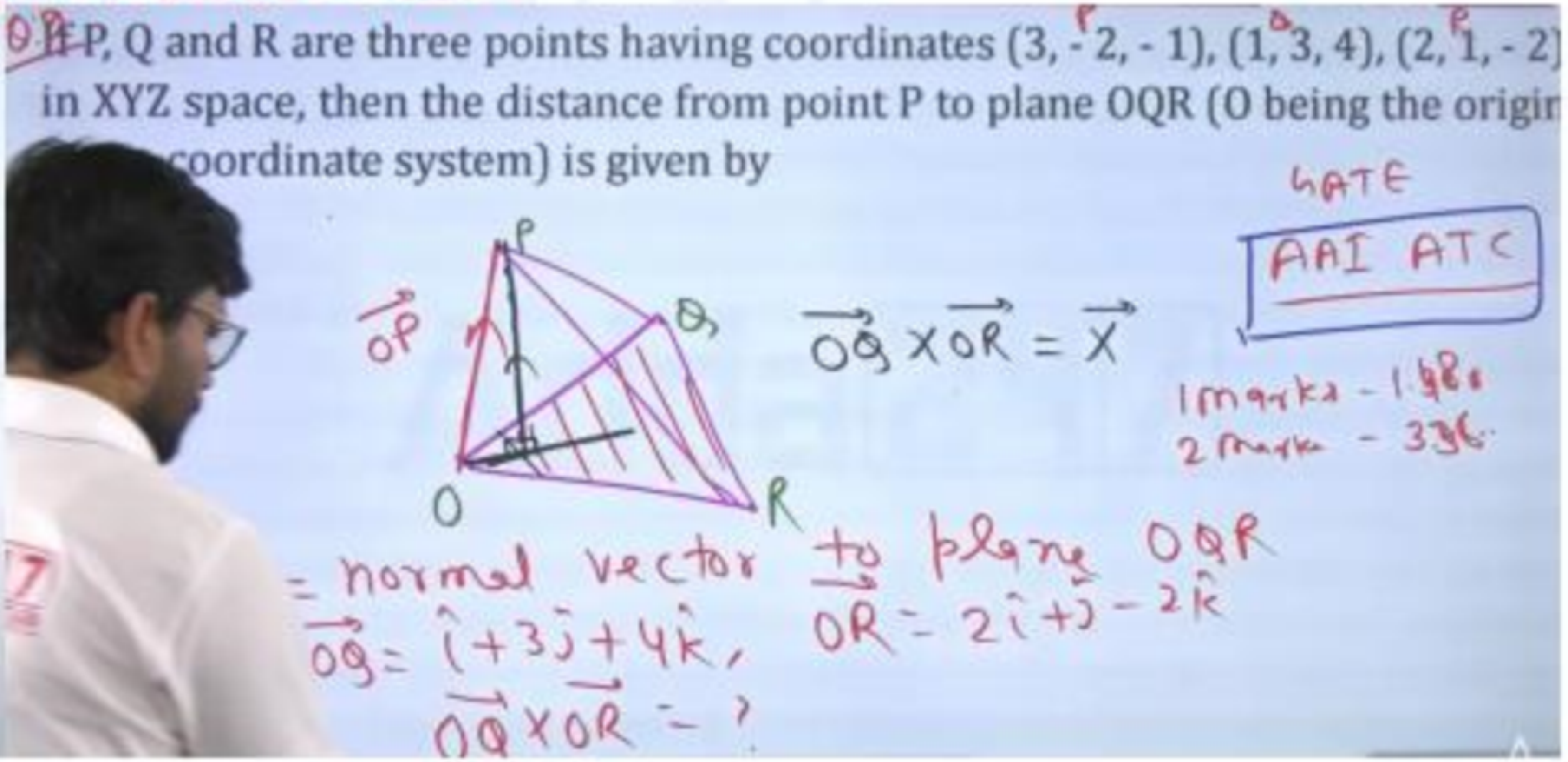
Recap

Dedicated batches available on **ADDA247** App, Use offer code **Y657**



Number of Questions covered-54





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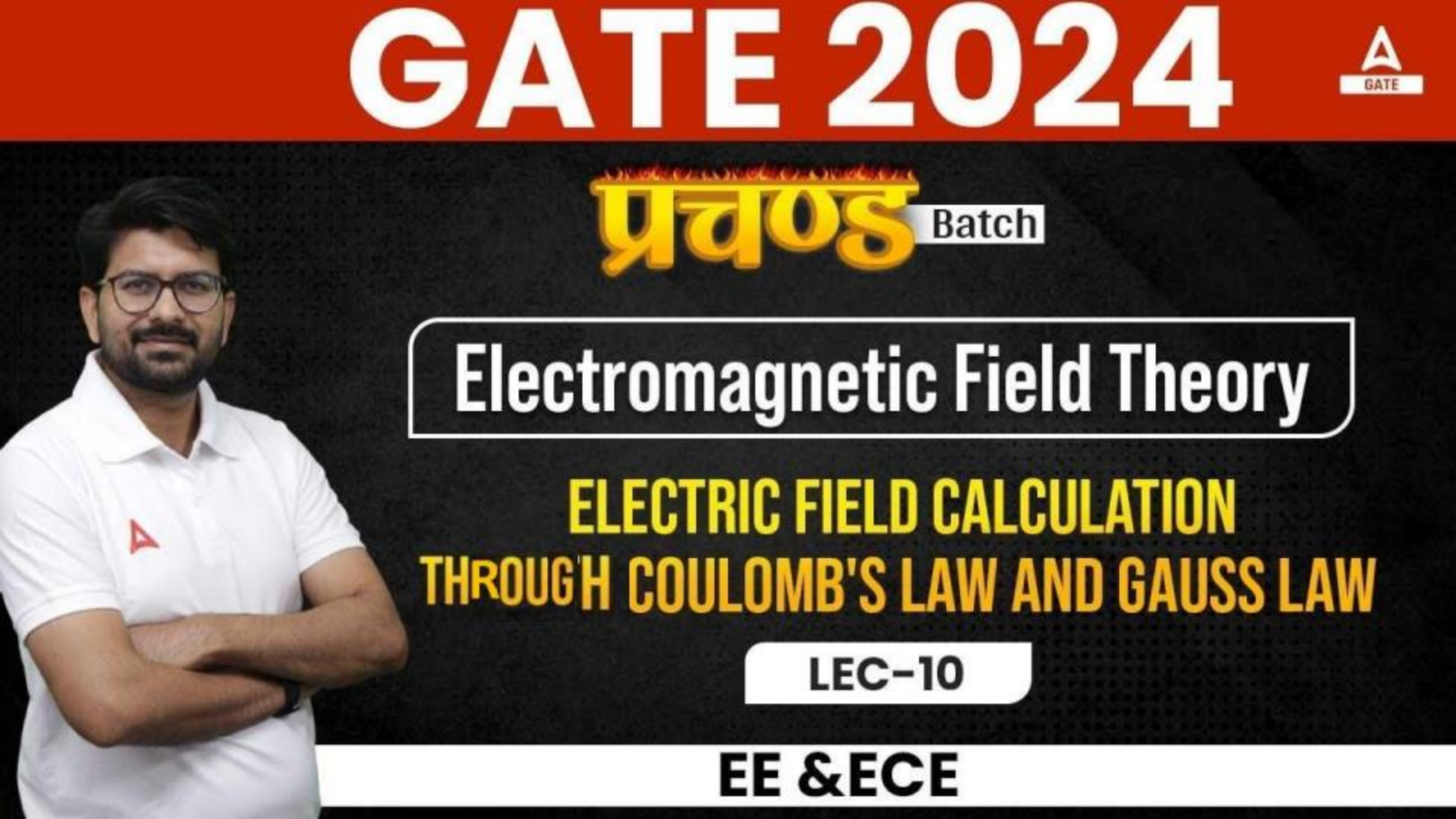
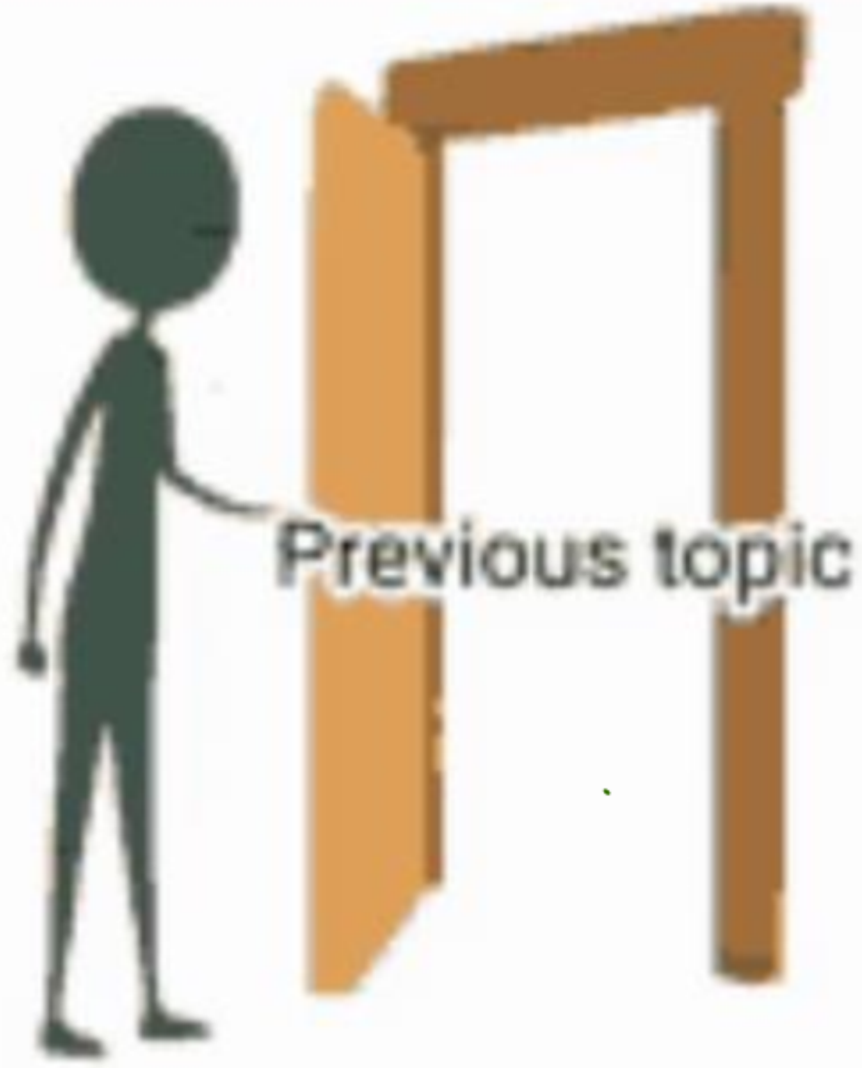
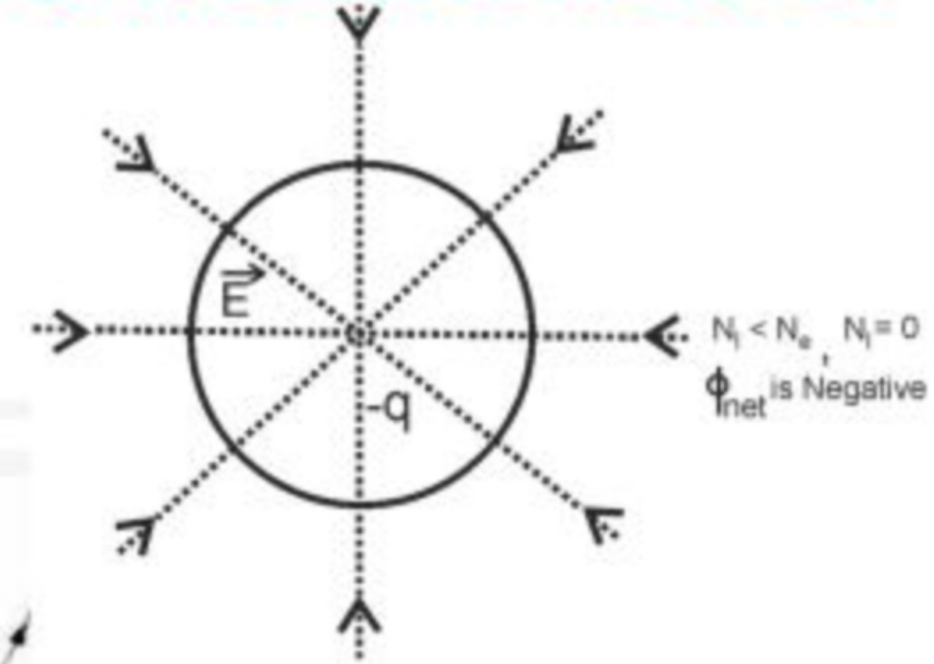

GATE 2024
GATE
प्रचण्ड Batch
Electromagnetic Field Theory
ELECTRIC FIELD CALCULATION
THROUG'H COULOMB'S LAW AND GAUSS LAW
LEC-10
EE &ECE

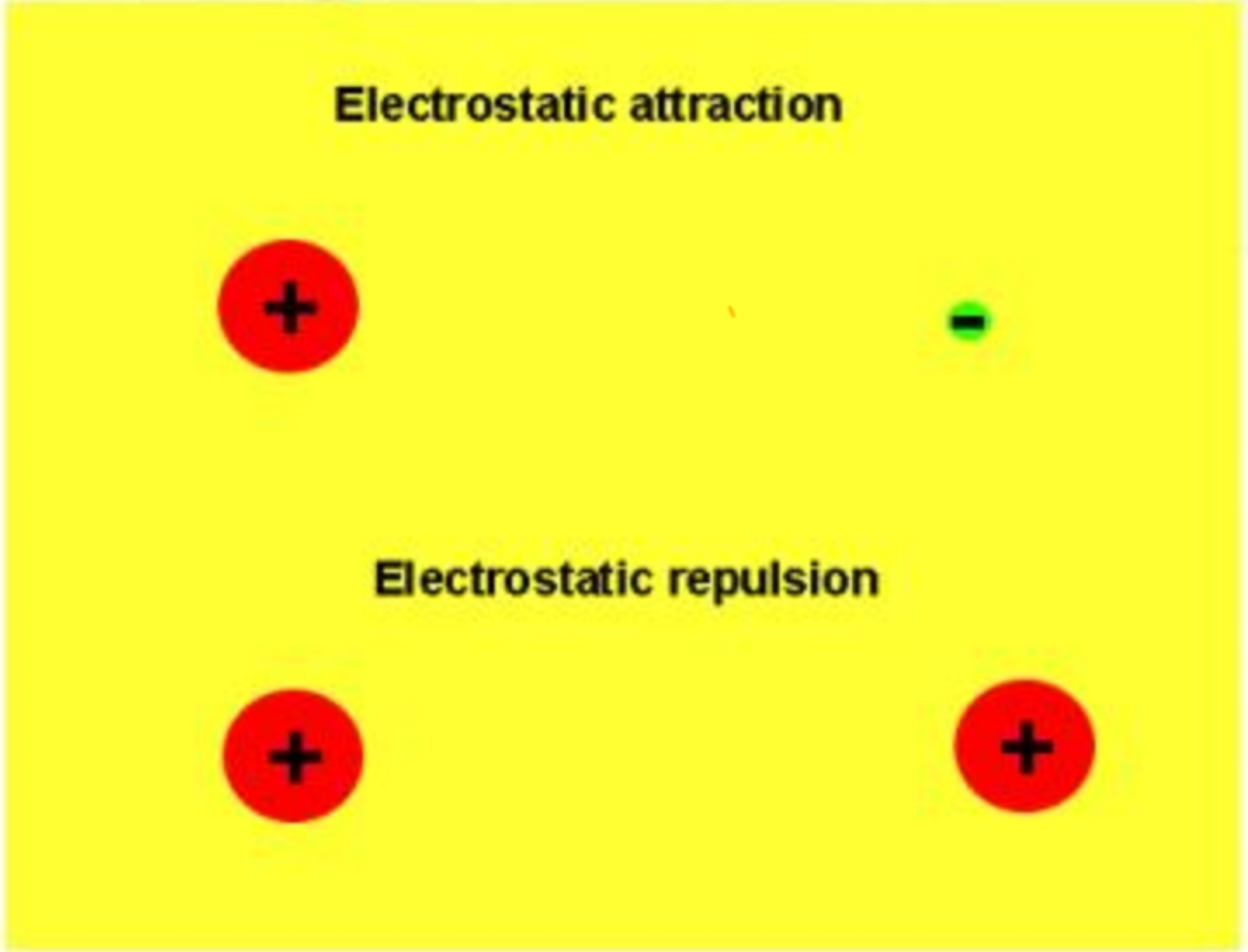


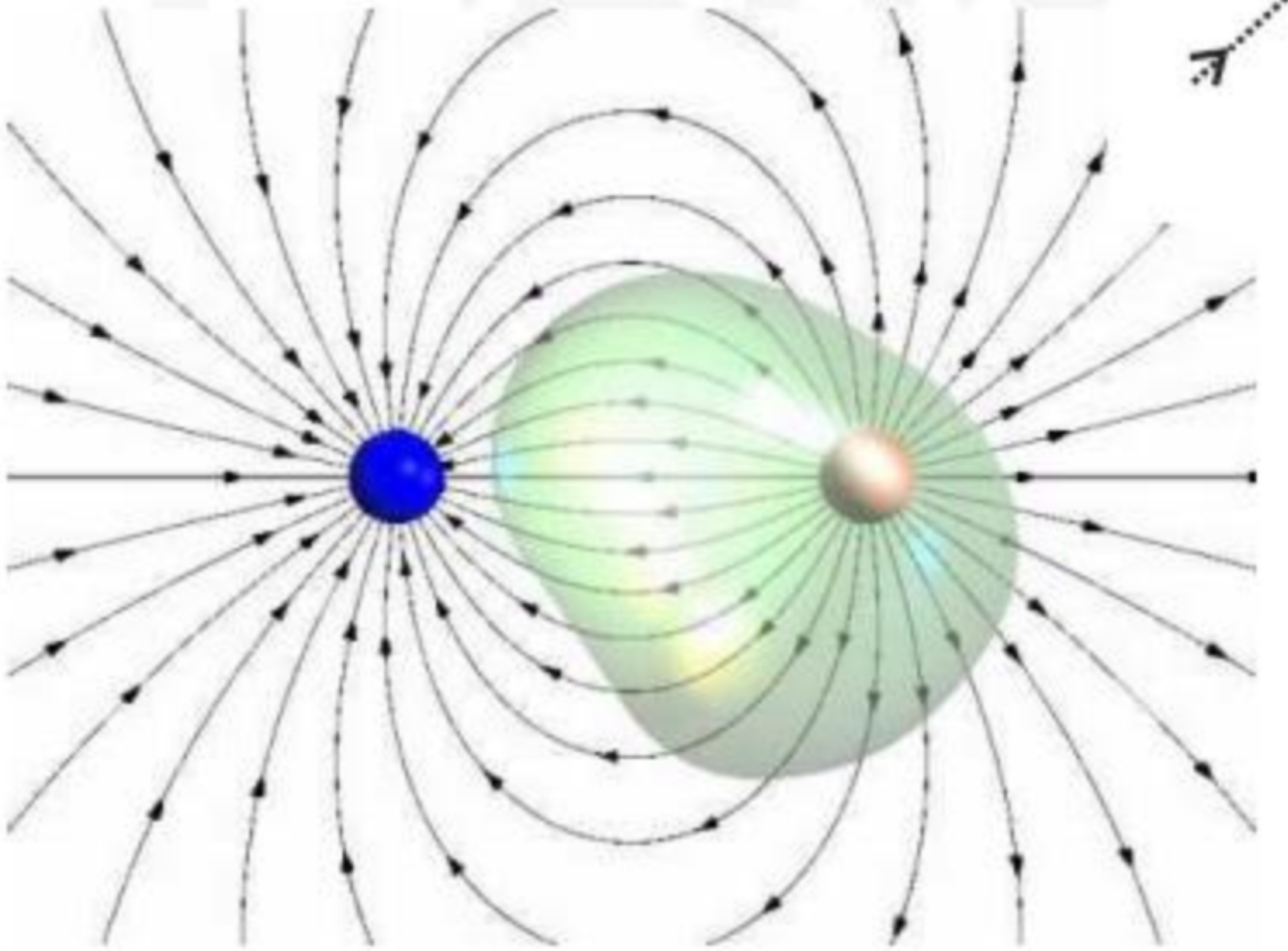
1. Basic introduction of Fields
2. Vectors, Scalars and Tensors
3. Position vector and vector between points
4. Magnitude and direction of vector
5. Dot and cross products and its applications
6. Cartesian and Cylindrical and Spherical Coordinate systems
7. Vector integrals(Line and closed line)
8. Del Operator ,Gradient and its applications
9. Divergence and Curl
10. Question practice on Vector calculus

Electric Field Calculations through Coulomb's law

Gauss Law





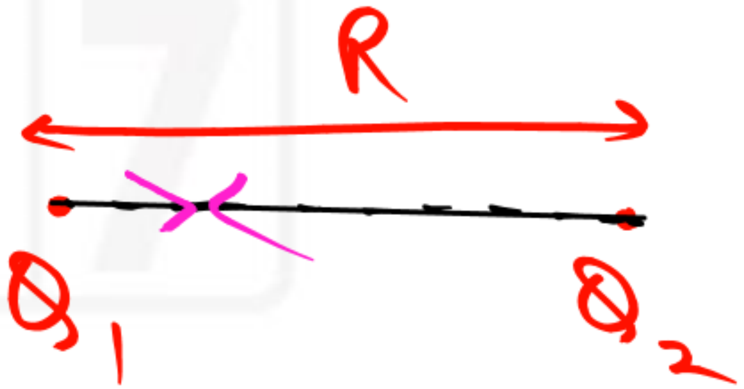


Coulomb's Law:- Electric Force on a charge Q1 due to another charge Q2 is..

(i) proportional to product of both the charges

(ii) inversely proportional to square of distance between them

(iii) in the direction of line joining these two charges



$$F \propto Q_1 Q_2$$

$$F \propto \frac{1}{R^2}$$

$$\vec{F}_e \propto \frac{Q_1 Q_2}{R^2}\hat{a}_{R_{21}} \Rightarrow \vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{21}} \leftarrow \text{force on } Q_1$$

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}} \leftarrow \text{force on } Q_2$$

$$\hat{a}_{R_{12}} = -\hat{a}_{R_{21}}$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Q_1 due to Q_2

$$\vec{F}_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

Here $\hat{a}_R \rightarrow$ unit vector in the direction from generating charge to the charge on action

Here $\varepsilon_0 \rightarrow$ electric permitivity of free space

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

$$= \frac{10^{-9}}{36\pi} \text{ F/m}$$

$C \leftarrow$ farrad

$$C = \frac{\varepsilon A}{d}$$

Coulomb's Law:- Electric Force on a charge Q1 due to another charge Q2 is..

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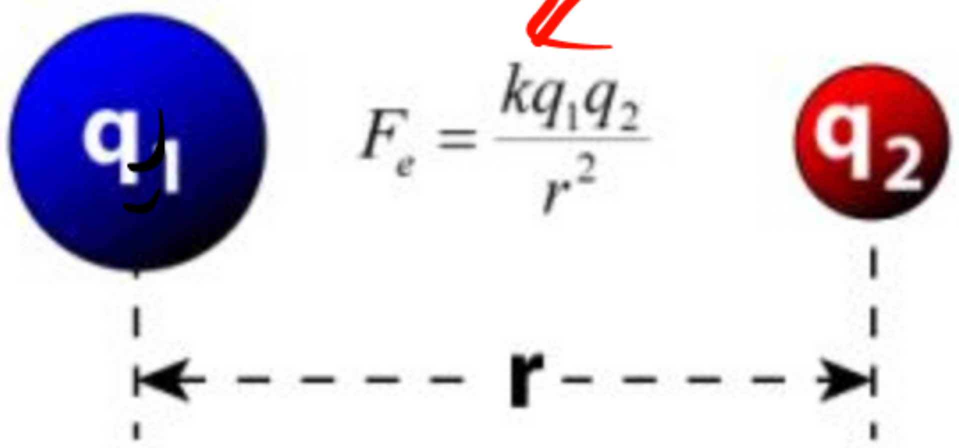
(ii) inversely proportional to square of distance between them

(iii) in the direction of line joining these two charges

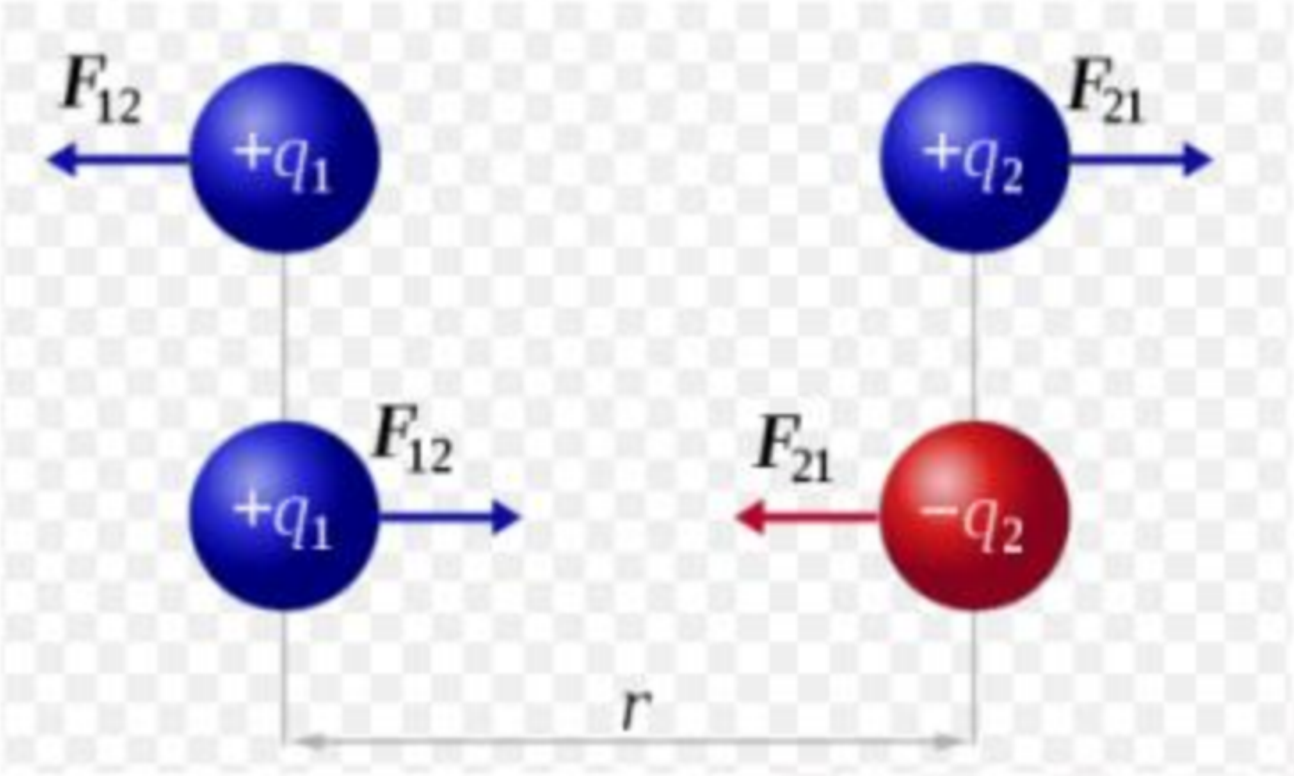
$K = \frac{1}{4\pi\varepsilon_0}$



$F_e = \frac{kq_1q_2}{r^2}$

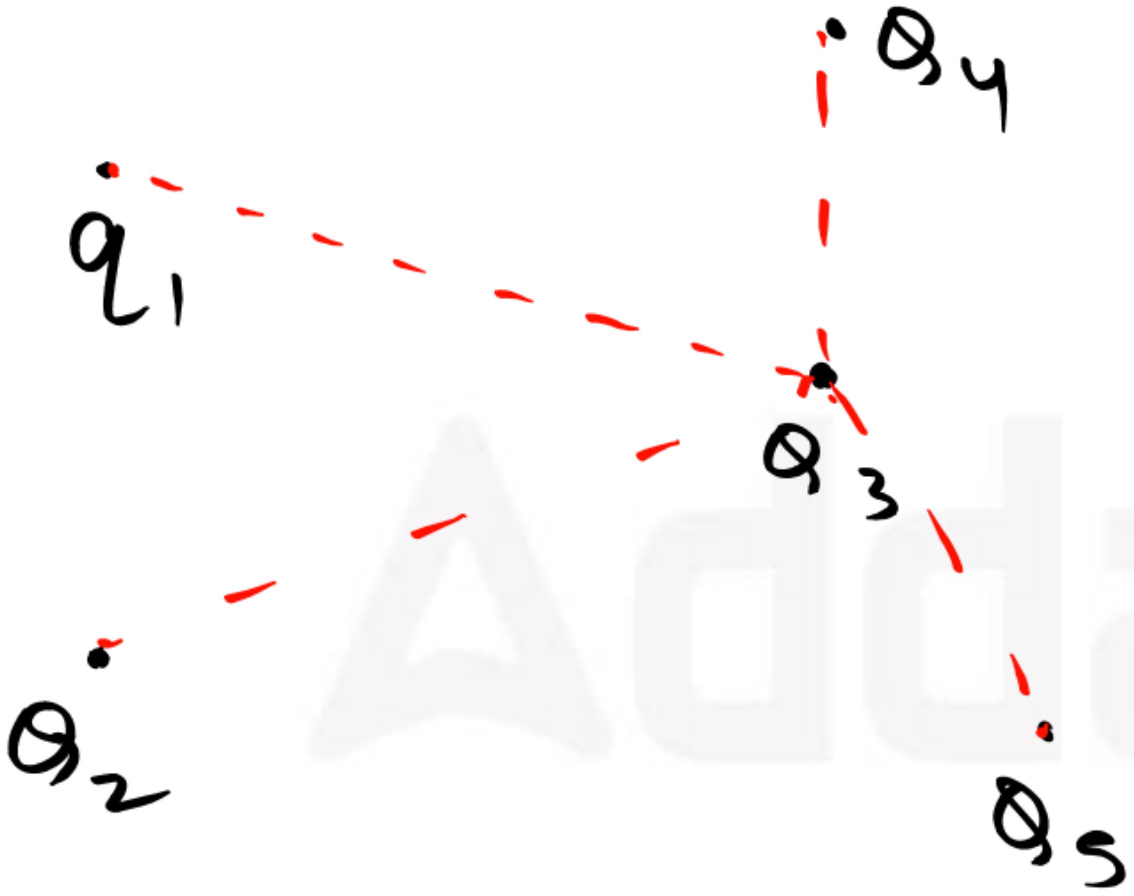
$$\vec{F_e} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

$$\vec{F_e} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\frac{\vec{R}}{R} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^3}\vec{R}$$



$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Electric Force due to multiple Charges

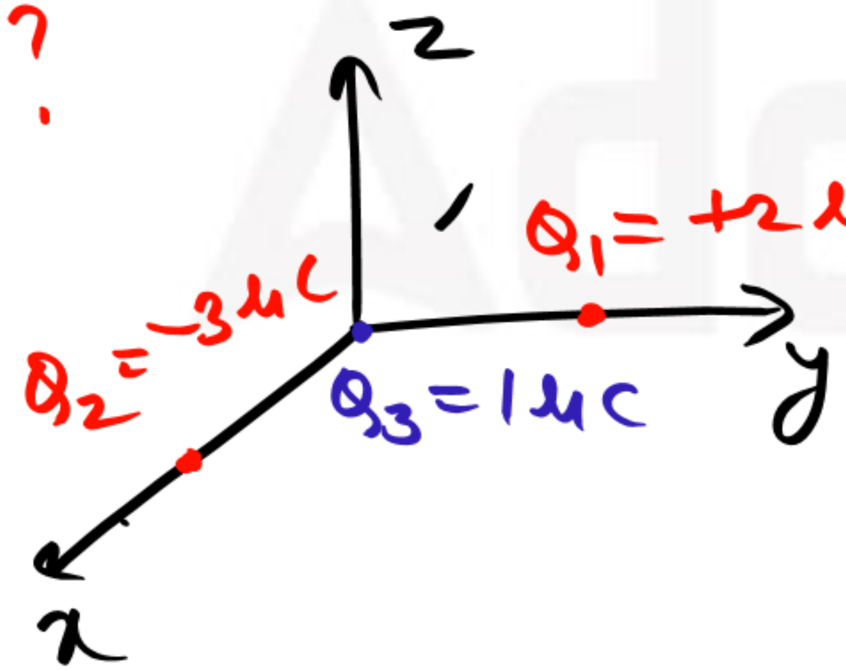


Force on Q_3

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} + \vec{F}_{43} + \vec{F}_{53}$$

Q:55 If three charges of $+2\mu C, -3\mu C$ & $1\mu C$ are located at $(0,1,0)$, $(1,0,0)$ & $(0,0,0)$ respectively then find electric force on the charge at origin?

Sol:



$$\vec{F_3} = \vec{F_{13}} + \vec{F_{23}}$$

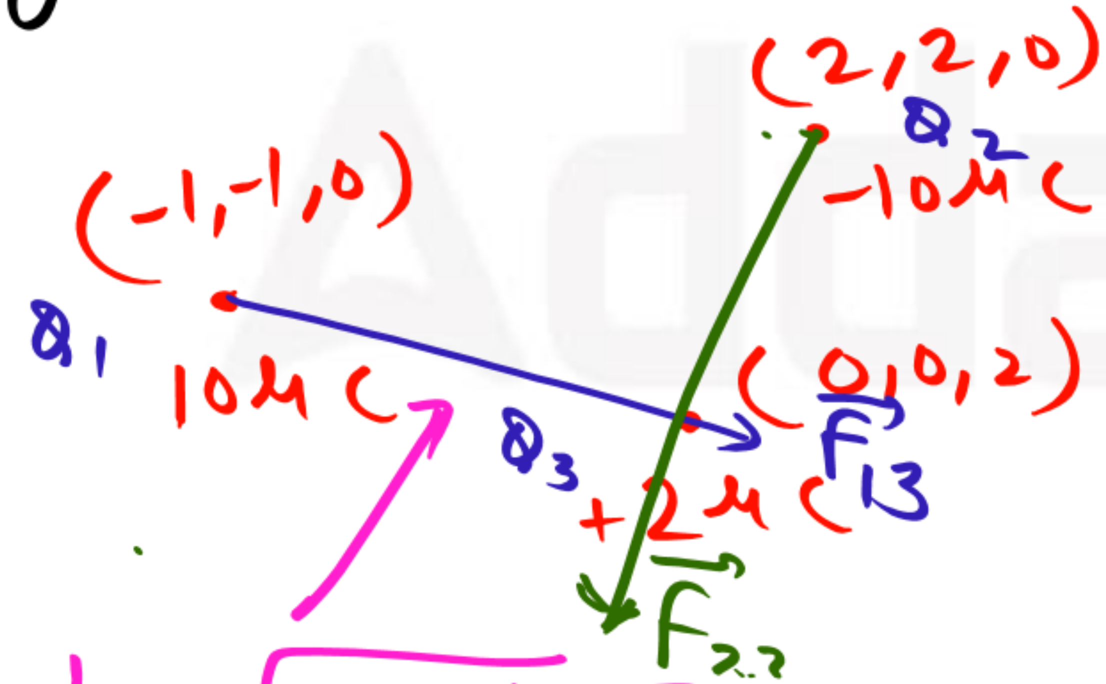
$$= \frac{-2\times10^{-12}}{4\pi\times\frac{10^{-9}}{36\pi}(1)^2}\hat{j} + \frac{3\times10^{-12}}{4\pi\times\frac{10^{-9}}{36\pi}(1)^2}\hat{i}$$

$$\vec{F} = (-18\hat{j} + 27\hat{i})\times10^{-3} \text{ Newton}$$

$$\vec{F} = (27\hat{i} - 18\hat{j}) \text{ mili Newton}$$

Q:56 If two charge of $10\mu C$ & $-10\mu C$ are located at $(-1,-1,0)$ & $(2,2,0)$ then find force on a charge of $+2\mu C$ placed at $(0,0,2)$.

Sol:



$d_{13} = \sqrt{1+1+4} = \sqrt{6}$

$\hat{a}_{R_{13}} = \dfrac{\vec{R}_{13}}{R_{13}} = \dfrac{\hat{i}+\hat{j}+2\hat{k}}{\sqrt{6}}$

$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$

$$= \frac{20\times10^{-12}}{4\pi\times\frac{10^{-9}}{36\pi}\times 6}\frac{(\hat{i}+\hat{j}+2\hat{k})}{\sqrt{6}} - \frac{20\times10^{-12}}{4\pi\times\frac{10^{-9}}{36\pi}\times 12}\cdot\frac{(-2\hat{i}-2\hat{j}+2\hat{k})}{\sqrt{12}}$$

$$30\times10^{-3}\frac{(\hat{i}+\hat{j}+2\hat{k})}{\sqrt{6}} + 15\times10^{-3}\frac{(2\hat{i}+2\hat{j}-2\hat{k})}{\sqrt{12}}$$

$$=\left(30\left(\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{12}}\right)\hat{i}+30\left(\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{12}}\right)\hat{j}+30\left(\frac{2}{\sqrt{6}}-\frac{1}{\sqrt{12}}\right)\hat{j}\right)mH$$

Electric Field Intensity

$$\vec{F}_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

electric field intensity due to a charge Q at a point in space is electric force on a test charge of +1 coulomb's at that point.

Q_1 Q_2

3 μC 20 μ

6 μC

Q R +1C

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R = \frac{\vec{F}}{q}$$

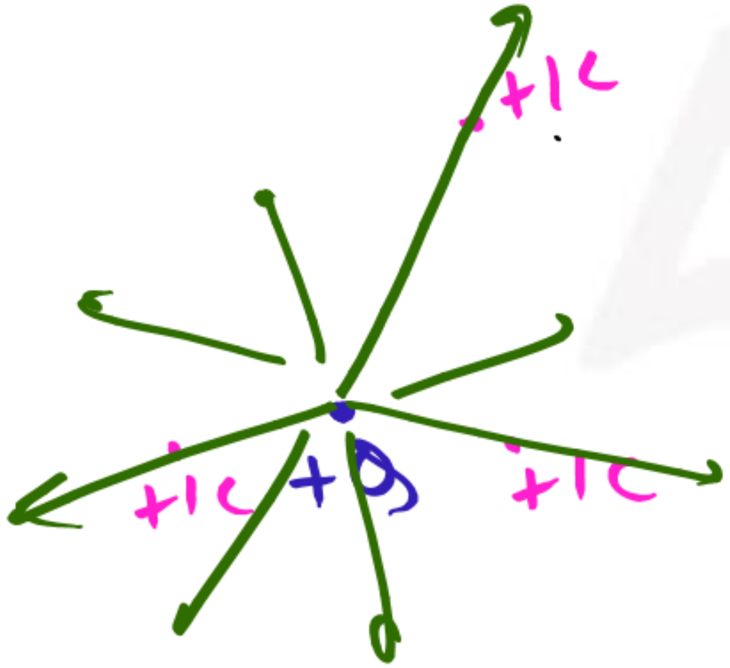
electric field intensity is force per unit charge.

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\Rightarrow \boxed{\vec{F} = q\vec{E}}$$

$\vec{F_e} \rightarrow$ Newton

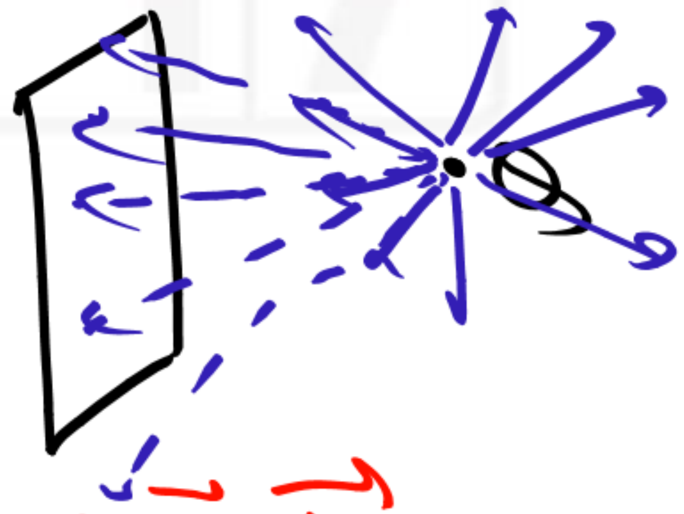
$\vec{E} = \frac{\vec{F_e}}{q} \rightarrow$ Newton/c



Electric Flux and Electric flux density

$$\text{electric flux} = \int \vec{E} \cdot \vec{ds}$$

$$= \int \frac{Q}{4\pi\varepsilon_0 R^2} \hat{a}_R \cdot \vec{ds}$$



Note:→ electric flux is never given by $\psi = \int \vec{E} \cdot \vec{ds}$

Electric Flux and Electric flux density

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$

electric field intensity is material dependent quantity.

$$\vec{D} = \varepsilon_0\vec{E} = \frac{Q}{4\pi R^2}\hat{a}_R \Rightarrow \vec{D} \text{ is material independent.}$$

So electric flux is given by $\psi = \int \vec{D}\cdot\vec{ds}$

$\vec{D}\cdot\vec{ds} \rightarrow$ amount of electric flux from small surface

$\vec{D} \rightarrow$ electric flux per unit area (electric flux density)

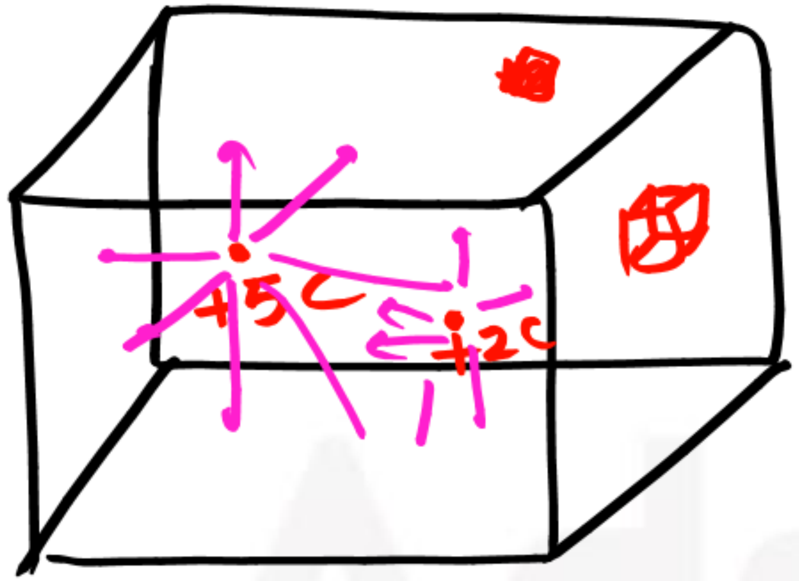
$$\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R$$

$$\psi = \int \vec{D} \cdot \vec{ds}$$

flux

flux per unit area.

Gauss Law :→



$\oint \vec{D} \cdot \vec{ds}$

It states that net outward flux from a volume is equal to total enclosed charge within the volume.

$$\Psi_{net} = Q_{enc}$$

$$\oint \vec{D}\cdot\vec{ds} = \int \rho_v \, dv$$

$$\oint \vec{D}\cdot\vec{ds} = Q_{enc}$$

Ψ_n = flux = Coulo.

$D \rightarrow$ flux per unit area

$D \rightarrow C/m^2$

Applying divergence theorem

$$\int (\nabla\cdot\vec{D})\, dv = \int \rho_v \, dv \leftarrow \text{integral form}$$

$$\Rightarrow \nabla\cdot\vec{D} = \rho_v \leftarrow \text{maxwell's first equation in point form or differential form}$$

THANKS FOR

Watching

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