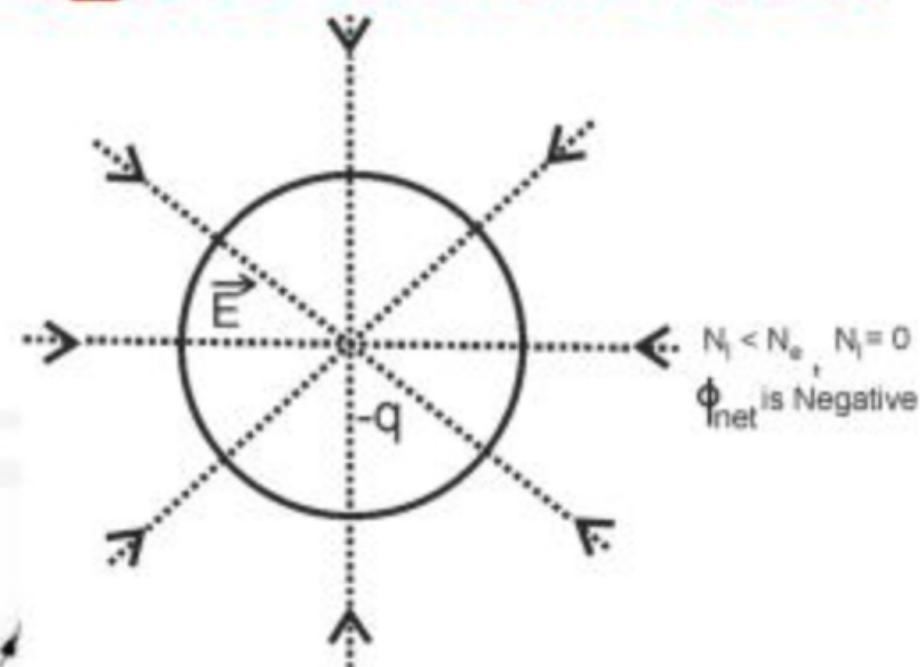




today's
topics

Electric Field Calculations through Coulomb's law

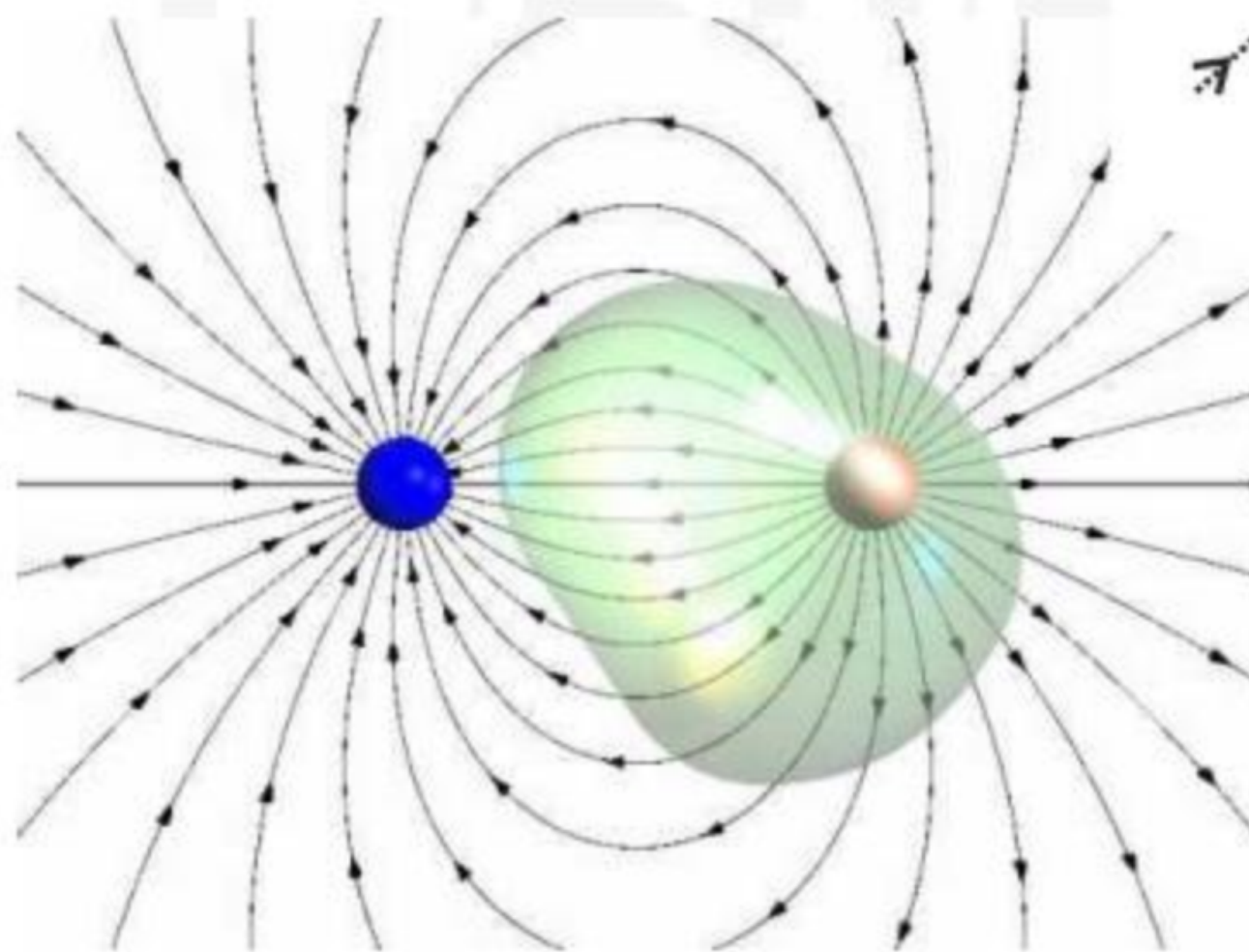
Gauss Law



Electrostatic attraction



Electrostatic repulsion



Q:25 The directional derivative of $f(x, y, z) = x(x^2 - y^2) - z$ at $A(1, -1, 0)$ in the direction of $\vec{p} = (2\hat{i} - 3\hat{j} + 6\hat{k})$ is:

- 8/49
- 8/7
- 8/7
- 0

Handwritten notes:
 $\nabla f = (2xy^2)\hat{i} - 2xy^2\hat{j} - \hat{k}$
 $\nabla f|_A = (2(-1)^2)\hat{i} - 2(1)(-1)^2\hat{j} - \hat{k} = 2\hat{i} - 2\hat{j} - \hat{k}$
 $\vec{p} = 2\hat{i} - 3\hat{j} + 6\hat{k}$
 $|\vec{p}| = \sqrt{4+9+36} = \sqrt{49} = 7$
 $\frac{\vec{p}}{|\vec{p}|} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{7}$

Q:21 Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \frac{y\hat{i} - x\hat{j}}{x^2 + y^2}$

- Circular path $x^2 + y^2 = 1$ described clockwise.
- The square formed by the lines $x = \pm 1, y = \pm 1$, counter clockwise.

Handwritten notes:
 (i) $\oint_C \frac{y dx - x dy}{x^2 + y^2}$
 $x = \cos \theta, y = \sin \theta$
 $dx = -\sin \theta d\theta, dy = \cos \theta d\theta$
 $\int_0^{2\pi} \frac{\sin \theta (-\sin \theta d\theta) - \cos \theta (\cos \theta d\theta)}{1} = \int_0^{2\pi} -\sin^2 \theta - \cos^2 \theta d\theta = \int_0^{2\pi} -1 d\theta = -2\pi$

Number of Questions covered-54

Q:54 Which one of the following describes the relationship among the three vectors, $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} + 3\hat{j} + \hat{k}$ and $5\hat{i} + 6\hat{j} + 4\hat{k}$

- The vectors are mutually perpendicular
- The vectors are linearly independent
- The vectors are linearly independent
- The vectors are unit vectors

Handwritten notes:
 $\vec{A} \cdot \vec{B} = 2 + 3 + 4 = 9$

Q:54 P, Q and R are three points having coordinates $(3, -2, -1)$, $(1, 3, 4)$, $(2, 1, -2)$ in XYZ space, then the distance from point P to plane OQR (O being the origin in coordinate system) is given by

Handwritten notes:
 $\vec{OQ} = \hat{i} + 3\hat{j} + 4\hat{k}$
 $\vec{OR} = 2\hat{i} + \hat{j} - 2\hat{k}$
 $\vec{OQ} \times \vec{OR} = \vec{X}$
 $\vec{X} = \text{normal vector to plane OQR}$
 $\vec{OP} = 3\hat{i} - 2\hat{j} - \hat{k}$
 $\text{Distance} = \frac{|\vec{OP} \cdot \vec{X}|}{|\vec{X}|}$

MARKS
 1 mark - 1.48
 2 mark - 3.36

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closer to your
GOAL.”

GATE 2024



प्रचण्ड Batch

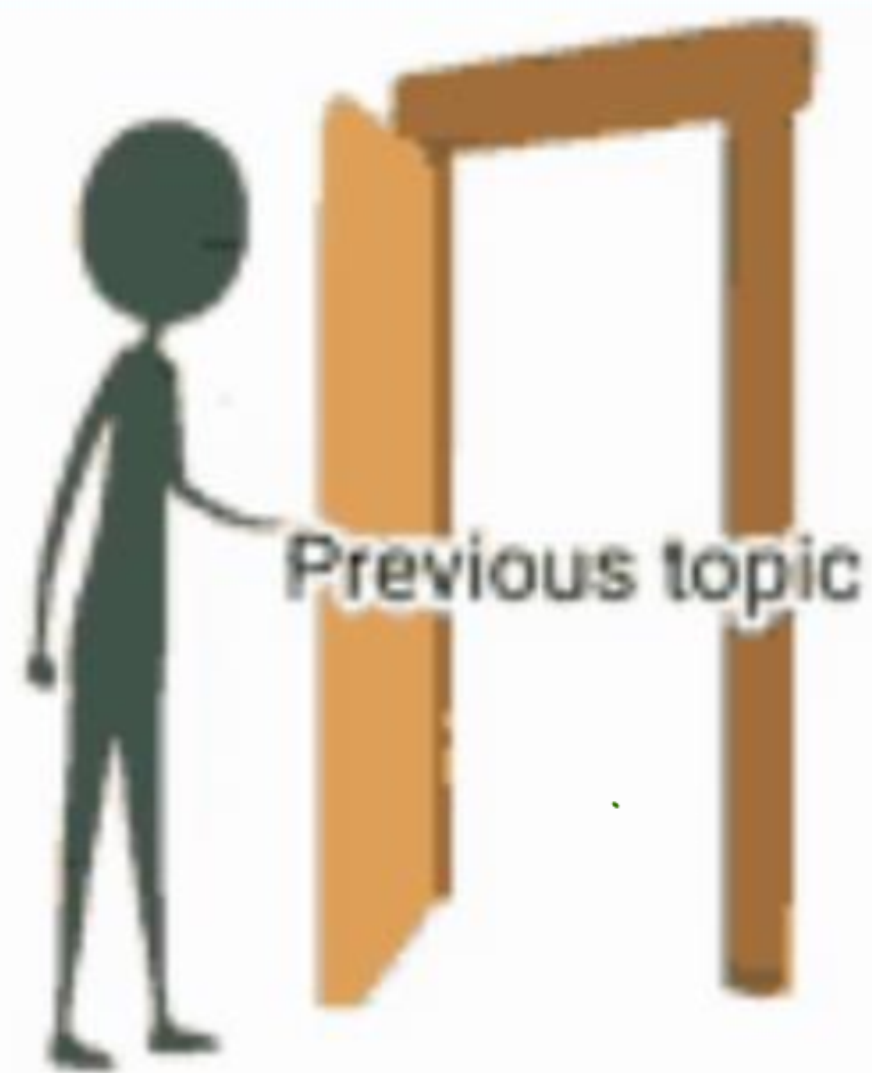
Electromagnetic Field Theory

**ELECTRIC FIELD CALCULATION
THROUGH COULOMB'S LAW AND GAUSS LAW**

LEC-10

EE & ECE





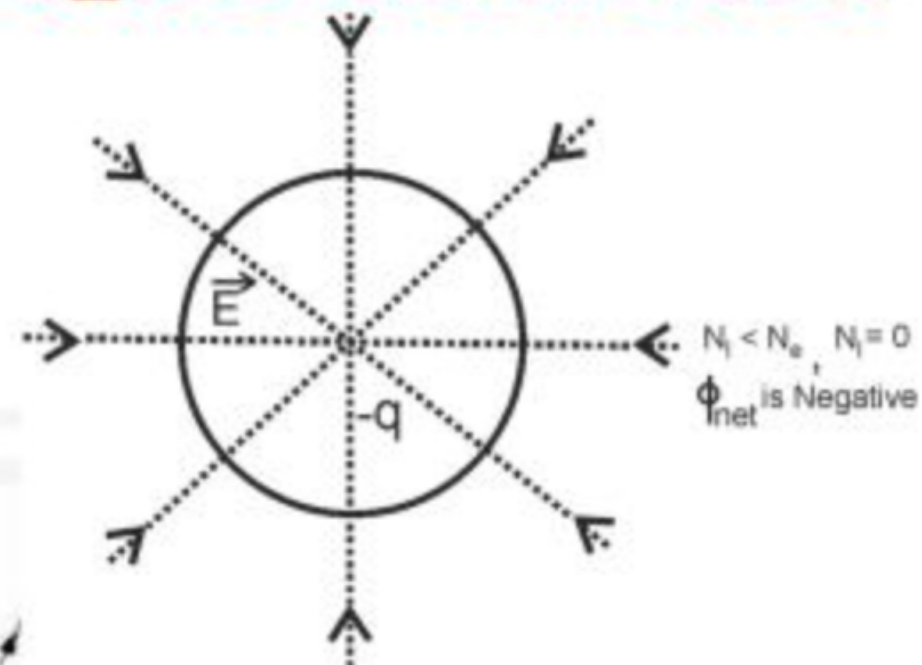
- 1. Basic introduction of Fields**
- 2. Vectors, Scalars and Tensors**
- 3. Position vector and vector between points**
- 4. Magnitude and direction of vector**
- 5. Dot and cross products and its applications**
- 6. Cartesian and Cylindrical and Spherical Coordinate systems**
- 7. Vector integrals(Line and closed line)**
- 8. Del Operator ,Gradient and its applications**
- 9. Divergence and Curl**
- 10. Question practice on Vector calculus**



today's
topics

Electric Field Calculations through Coulomb's law

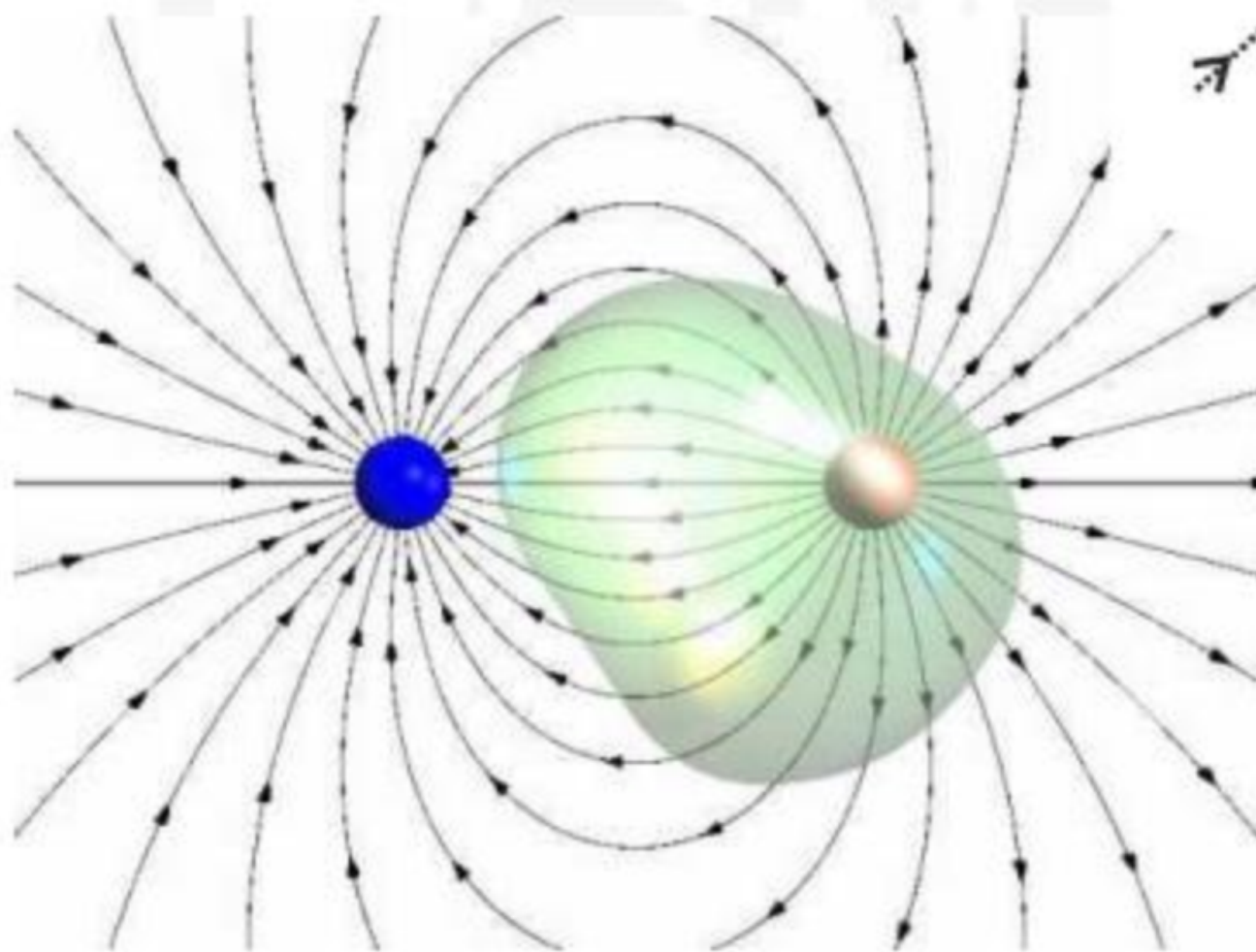
Gauss Law



Electrostatic attraction



Electrostatic repulsion

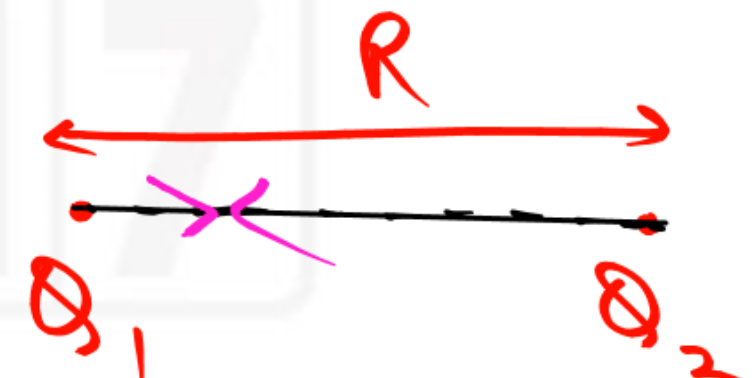


Coulomb's Law:- Electric Force on a charge Q_1 due to another charge Q_2 is..

- (i) proportional to product of both the charges
- (ii) inversely proportional to square of distance between them
- (iii) in the direction of line joining these two charges

$F \propto Q_1 Q_2$
 $F \propto \frac{1}{R^2}$
 $F \propto \frac{Q_1 Q_2}{R^2} \hat{r}_{21}$

$\Rightarrow \vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{r}_{21}$ ← force on Q_1
 $\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{r}_{12}$ ← force on Q_2



$$\hat{q}_{R12} = -\hat{q}_{R21}$$

$$\vec{F}_{12} = -\vec{F}_{21}$$

Q_1 due to Q_2

$$\vec{F}_e = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{q}_R$$

Here $\hat{q}_R \rightarrow$ unit vector in the direction from generating charge to the charge on action

Here $\epsilon_0 \rightarrow$ electric permittivity of free space

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ F/m}$$

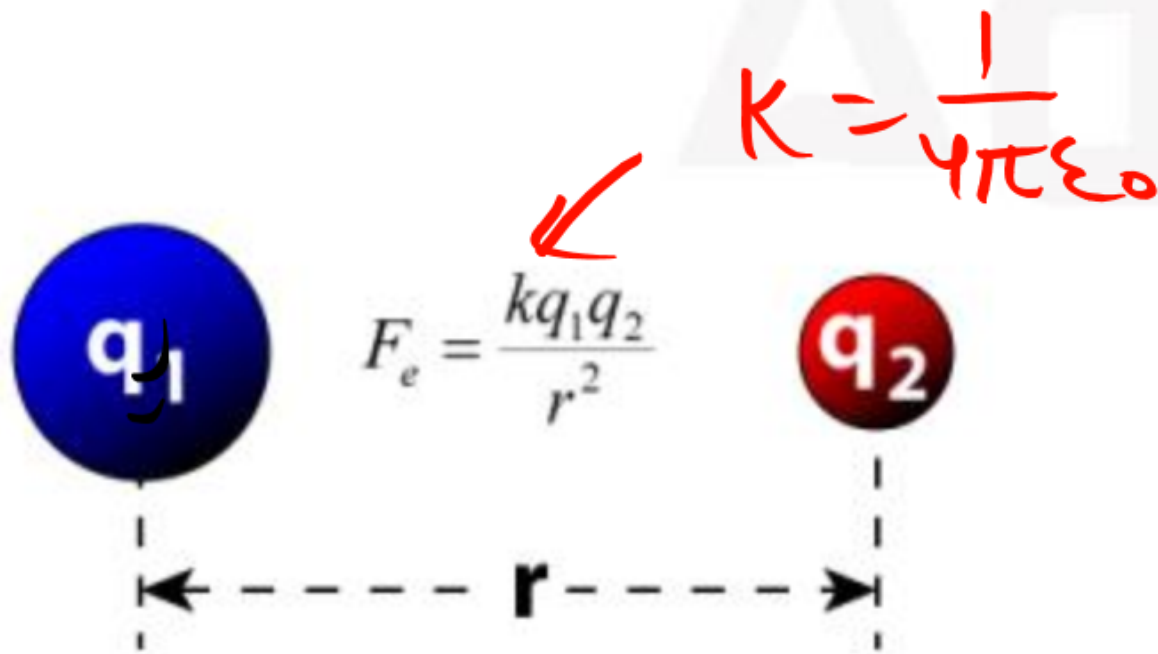
$$= \frac{10^{-9}}{36\pi} \text{ F/m}$$

$C \leftarrow$ Farad

$$C = \frac{QA}{d}$$

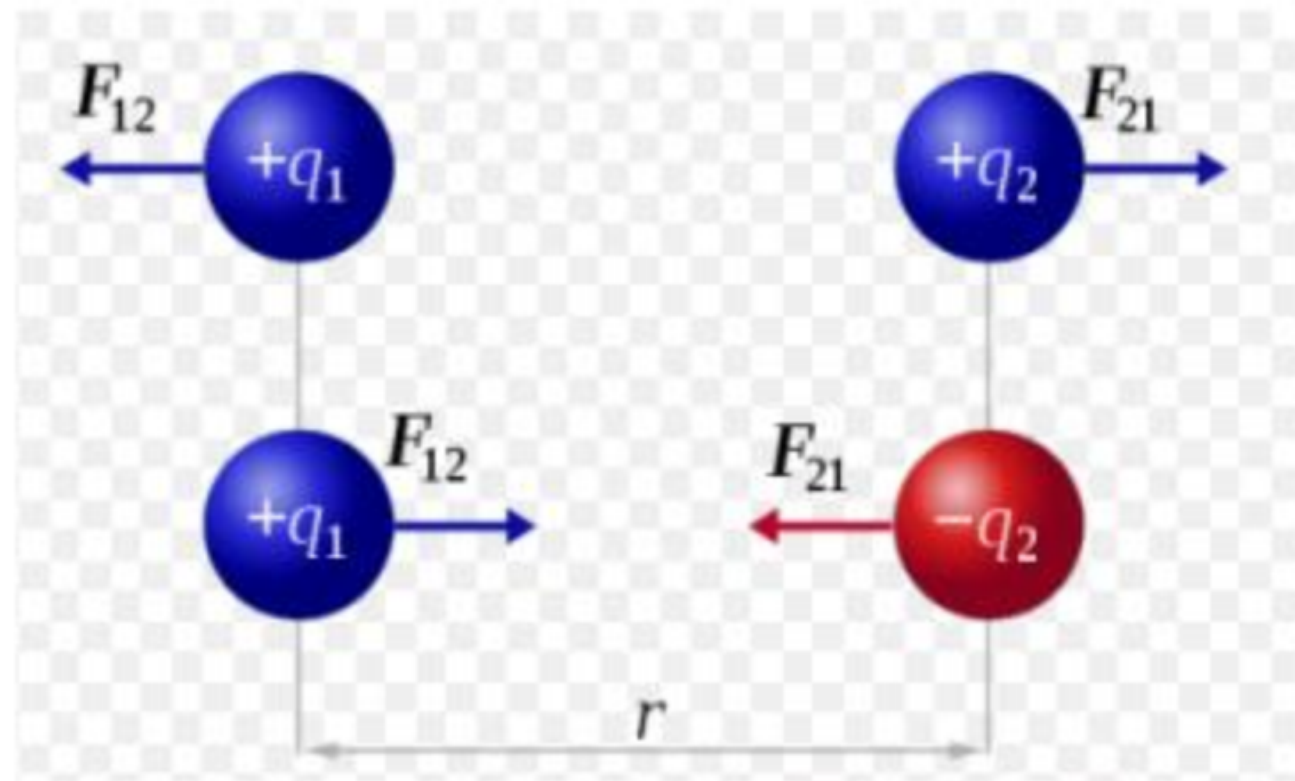
Coulomb's Law:- Electric Force on a charge Q_1 due to another charge Q_2 is..

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$$F_e = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{r}$$

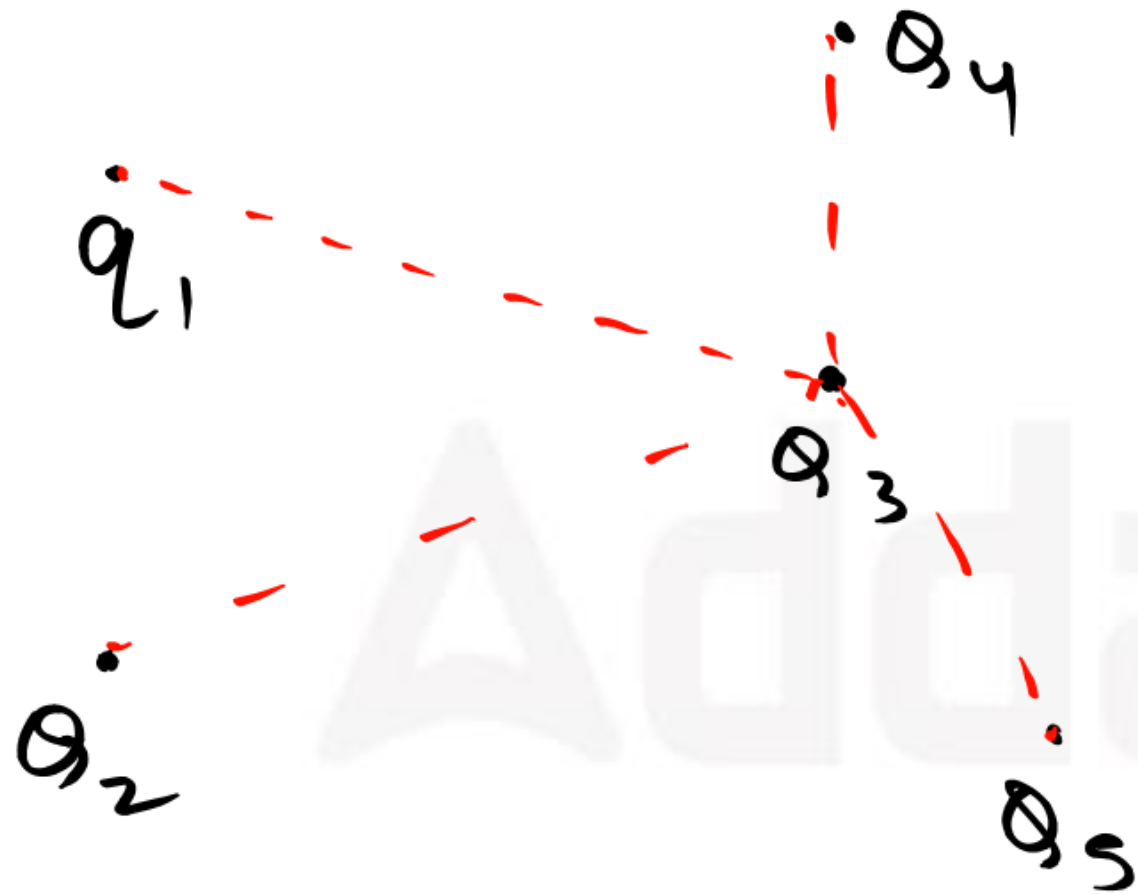
$$F_e = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \frac{\vec{R}}{R} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^3} \vec{R}$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

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Electric Force due to multiple Charges

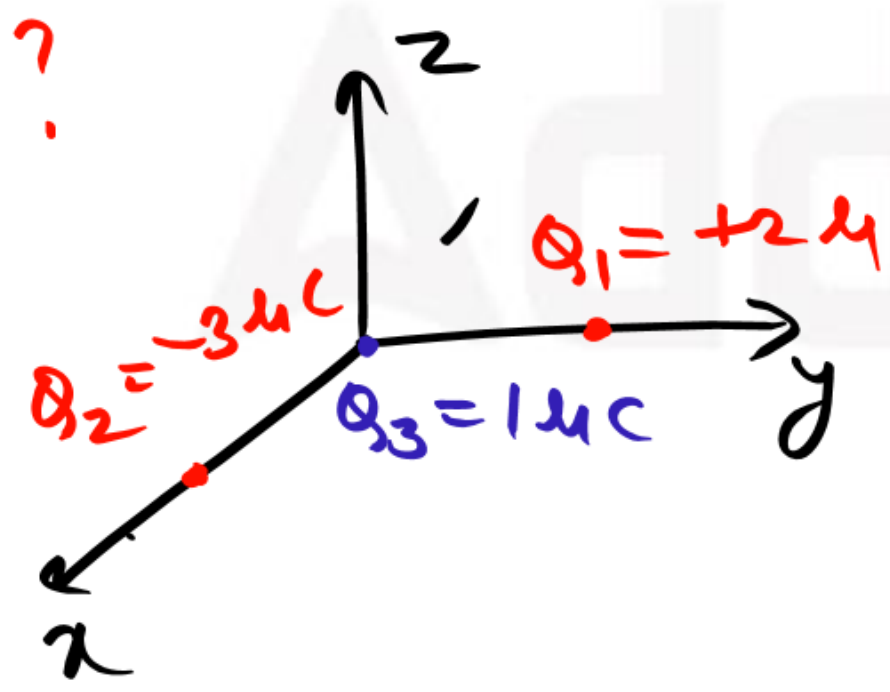


Force on q_3

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} + \vec{F}_{43} + \vec{F}_{53}$$

Q:55 If three charges of $+2\mu\text{C}$, $-3\mu\text{C}$ & $1\mu\text{C}$ are located at $(0,1,0)$, $(1,0,0)$ & $(0,0,0)$ respectively then find electric force on the charge at origin?

Sol:



$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23}$$

$$= \frac{-2 \times 10^{-12}}{4\pi \times 10^{-9} (1)^2} \hat{j} + \frac{3 \times 10^{-12}}{4\pi \times 10^{-9} (1)^2} \hat{i}$$

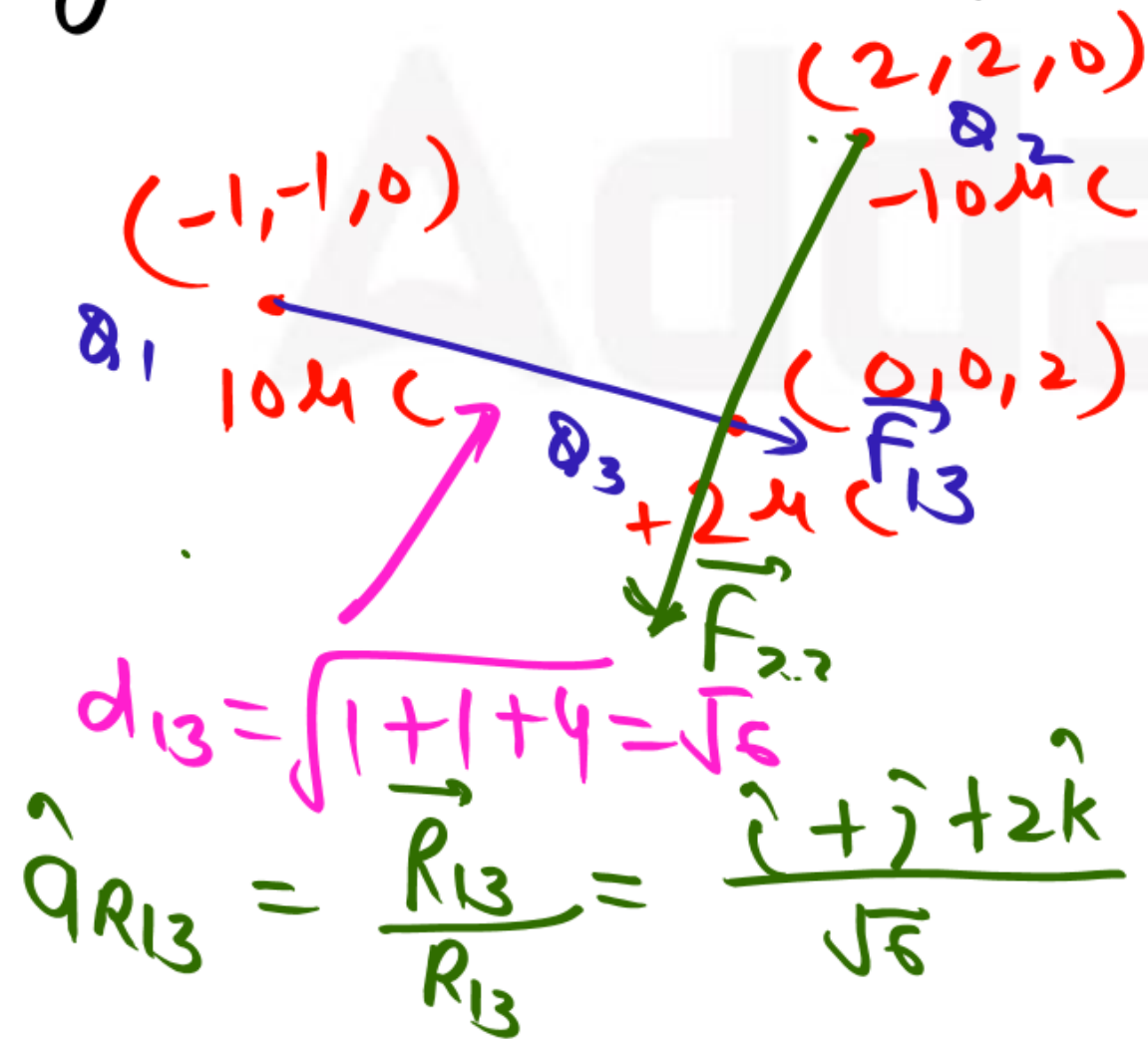
$\frac{36\pi}{36\pi}$

∴ $(-18\hat{j} + 27\hat{i}) \times 10^{-3}$ newton

∴ $(27\hat{i} - 18\hat{j})$ mili Newton

Q.56 If two charge of $10\mu\text{C}$ & $-10\mu\text{C}$ are located at $(-1, -1, 0)$ & $(2, 2, 0)$ then find force on a charge of $+2\mu\text{C}$ placed at $(0, 0, 2)$.

Sol:

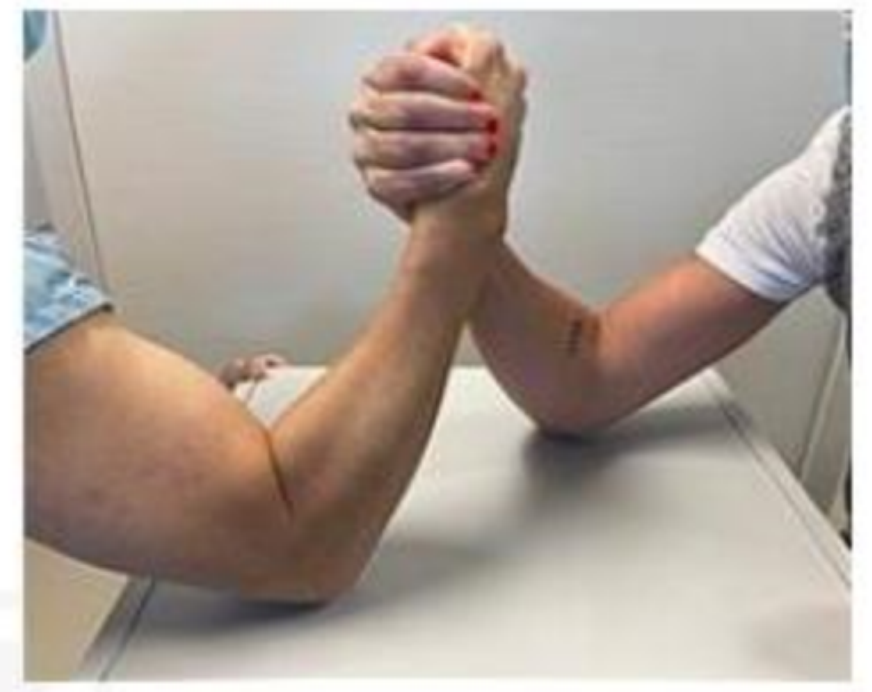


$$\begin{aligned}
 \vec{F}_3 &= \vec{F}_{13} + \vec{F}_{23} \\
 &= \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 6} \frac{(\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{6}} \\
 &\quad - \frac{20 \times 10^{-12}}{4\pi \times 10^{-9} \times 12} \frac{(-2\hat{i} - 2\hat{j} + 2\hat{k})}{\sqrt{12}}
 \end{aligned}$$

$$30 \times 10^{-3} \frac{(\hat{i} + \hat{j} + 2\hat{k})}{\sqrt{6}} + 15 \times 10^{-3} \frac{(2\hat{i} + 2\hat{j} - 2\hat{k})}{\sqrt{12}}$$
$$= \left(30 \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}} \right) \hat{i} + 30 \left(\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{12}} \right) \hat{j} + 30 \left(\frac{2}{\sqrt{6}} - \frac{1}{\sqrt{12}} \right) \hat{k} \right) \text{ mH}$$

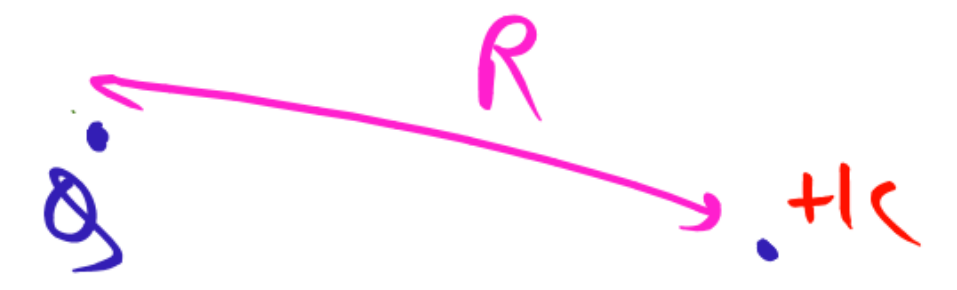
Electric Field Intensity

$$\vec{F}_e = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \hat{q}_R$$



Electric field intensity due to a charge Q at a point in space is electric force on a test charge of +1 coulomb's at that point.

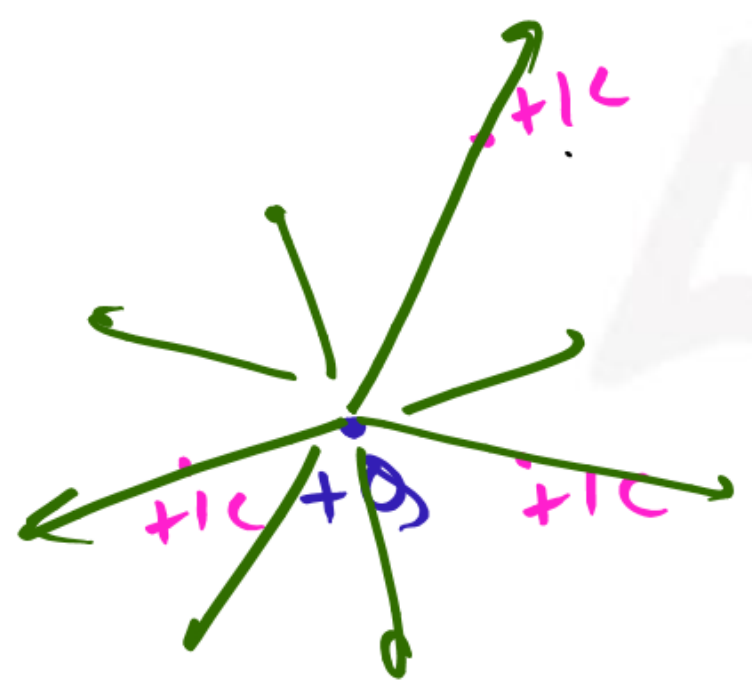
Q_1 Q_2
 3.2×10^{-4} 2.0×10^{-4}
 flux
 $\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{q}_R = \frac{1}{\epsilon_0} \dots$



Electric field intensity is force per unit charge.

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\Rightarrow \boxed{\vec{F} = q \vec{E}}$$



$$\vec{F}_e \rightarrow \text{Newton}$$
$$\vec{E} = \frac{\vec{F}_e}{q} \rightarrow \text{Newton/C}$$

Electric Flux and Electric flux density

$$\text{Electric flux} = \int \vec{E} \cdot d\vec{s}$$

$$= \int \frac{Q}{4\pi\epsilon_0 R^2} \hat{r} \cdot d\vec{s}$$

X



Note: \Rightarrow Electric flux is never given by $\psi = \int \vec{E} \cdot d\vec{s}$

Electric Flux and Electric flux density

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 R^2} \hat{r}_R$$

electric field intensity is material dependent quantity.

$$\vec{D} = \epsilon_0 \vec{E} = \frac{Q}{4\pi R^2} \hat{r}_R \Rightarrow \vec{D} \text{ is material independent.}$$

so electric flux is given by $\psi = \int \vec{D} \cdot d\vec{y}$
 $\vec{D} \cdot d\vec{y} \rightarrow$ amount of electric flux from small surface
 $\vec{D} \rightarrow$ electric flux per unit area (electric flux density)

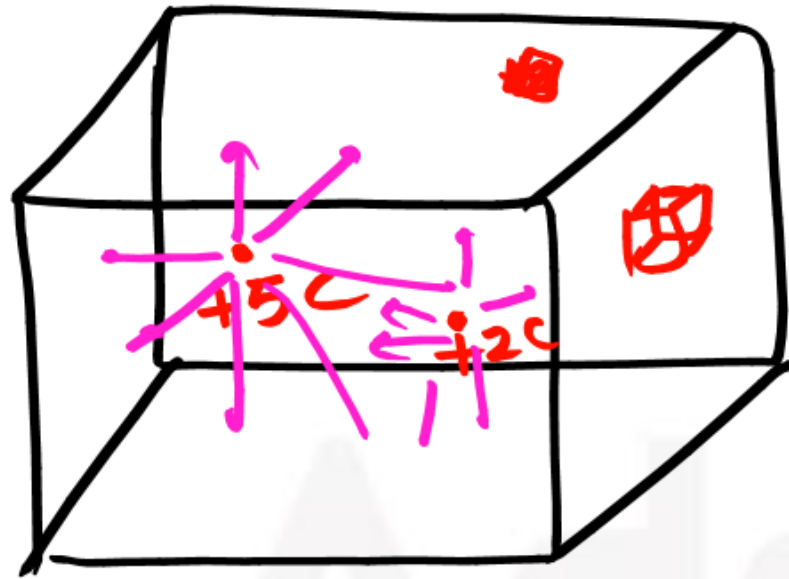
$$\vec{D} = \frac{Q}{4\pi R^2} \hat{a}_R$$

$$\psi = \int \vec{D} \cdot d\vec{s}$$

flux

flux per unit area.

Gauss Law $\therefore \rightarrow$



$$\oint \vec{D} \cdot d\vec{s}$$

It states that net outward flux from a volume is equal to total enclosed charge within the volume.

$$\Psi_{\text{net}} = Q_{\text{enc}}$$

$$\oint \vec{D} \cdot d\vec{s} = \int \rho_v dv$$

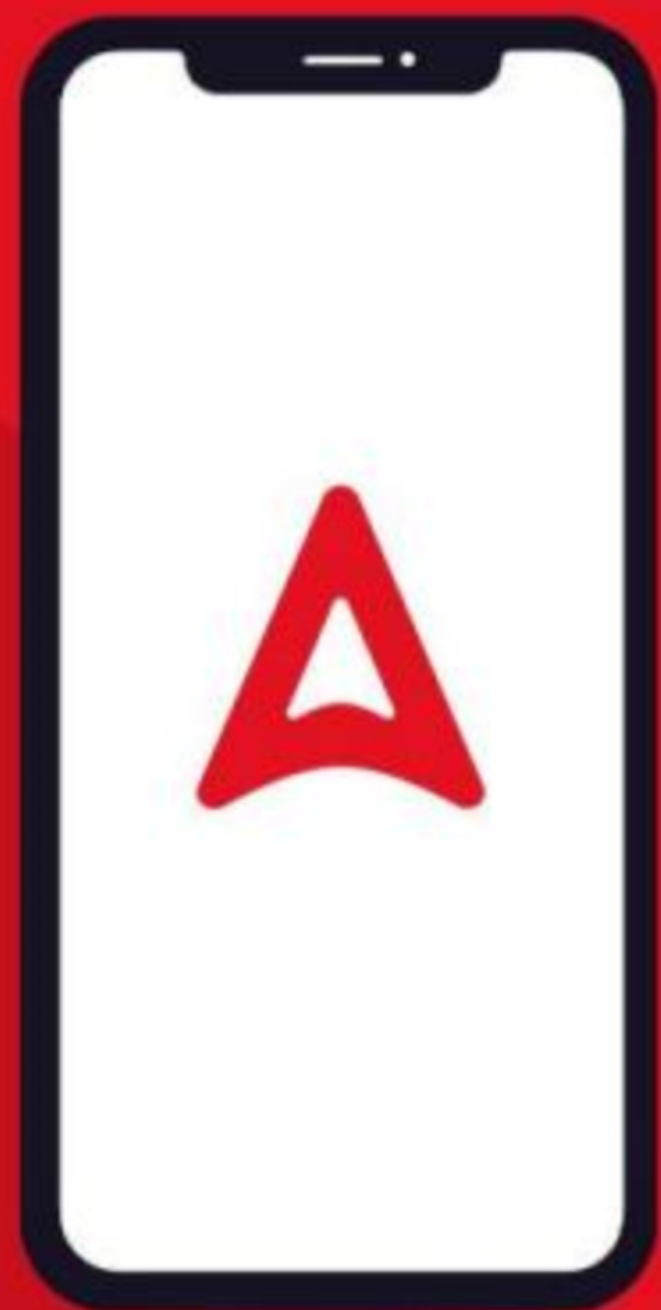
$\oint \vec{D} \cdot d\vec{s} = Q_{\text{enc}}$
 $\Psi_{\text{net}} = \text{flux} = \text{Coulomb}$
 $D \rightarrow \text{flux per unit area}$
 $D \rightarrow \text{C/m}^2$

Applying divergence theorem

$$\int (\nabla \cdot \vec{D}) dv = \int \rho_v dv \leftarrow \text{integral form}$$

$\Rightarrow \nabla \cdot \vec{D} = \rho_v \leftarrow \text{maxwell's first equation in point form or differential form}$

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