Eigen Values and Eigen vectors Introduction

Rank of Matrix

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GATE 2024

प्रचण्ड Batch

Engineering Mathematics

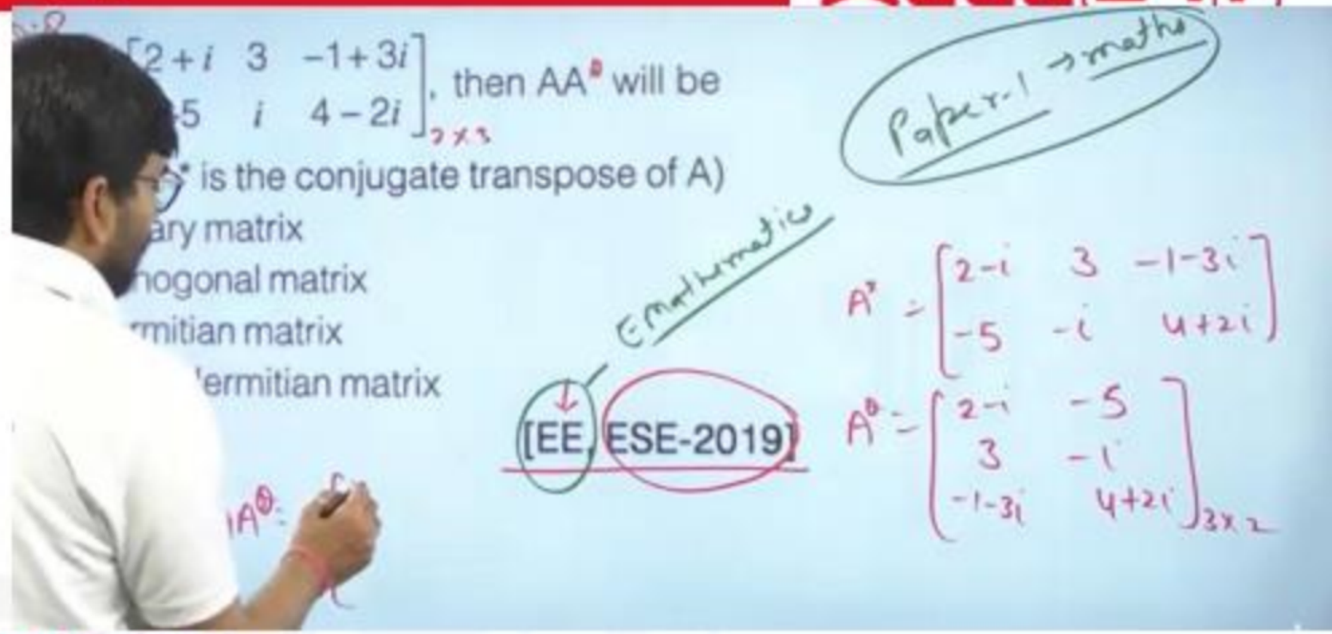
LINEAR ALGEBRA

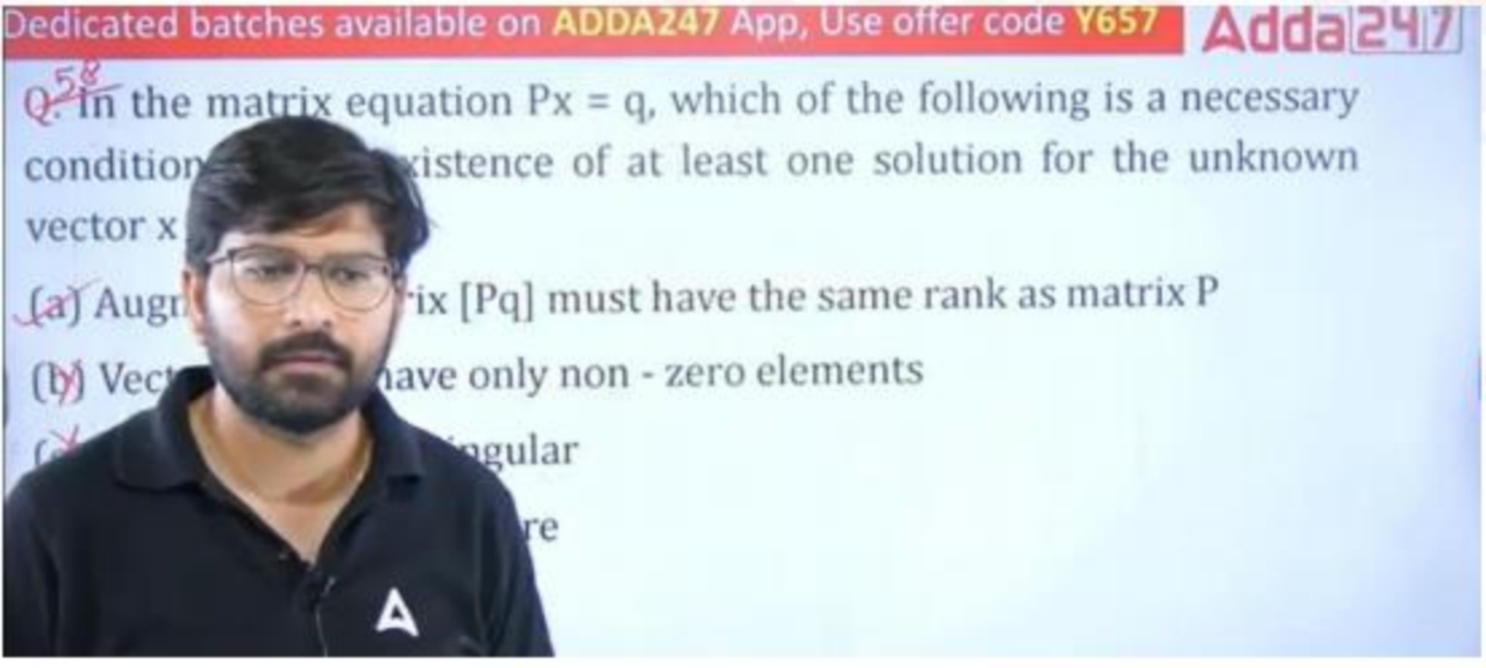
QUESTION PRACTICE ON EIGEN VALUES AND EIGEN VECTORS PART-1

LEC-09

Recap

ESE Question Practice →





← GATE Question Practice

Number of questions covered-79

1. Introduction to Linear Algebra
2. Classification of Matrices
3. Transpose, Determinant, Inverse of a matrix
4. Question practice on Basics of Matrices
5. Rank and dimension of null space of Matrix
6. System of linear simultaneous equations

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Questions on Eigen Values and Eigen vectors

Properties of Eigen Values :→

① Addition of eigen values is always equal to trace of matrix.

$A_{n\times n} \longrightarrow \lambda_1, \lambda_2, \lambda_3 \cdots \lambda_n$

$\lambda_1 + \lambda_2 + \lambda_3 + \cdots \lambda_n =$ trace(A)

② Multiplication of eigen values is equal to determinant of matrix.

$\lambda_1 \times \lambda_2 \times \lambda_3 \times \cdots \lambda_n = |A|$

$5 \times 2 = 10$

$5 + 2 = 7$

$5, 2$

Properties of Eigen Values

③ For a singular at least one eigen value is '0'

④ For a triangular matrix or diagonal matrix eigen values are same as diagonal elements of the matrix.

e.g. $A = \begin{bmatrix} -3 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$

$\lambda_1 = -3,\ \lambda_2 = 2,\ \lambda_3 = 0$

Q:80 The eigen values of the matrix

$$\begin{bmatrix} 4 & -2 \\ -2 & 1 \end{bmatrix}_{2\times 2}$$

$|A| = 4-4 = 0$ $[A-\lambda I] = \begin{bmatrix} 4-\lambda & -2 \\ -2 & 1-\lambda \end{bmatrix}$

(a) are 1 and 4

(b) are - 1 and 2

(c) are 0 and 5

(d) cannot be determined

$|A-\lambda I| = 0$

$(4-\lambda)(1-\lambda) - 4 = 0$

$\lambda^2 - 5\lambda = 0$

$\lambda(\lambda - 5) = 0 \Rightarrow \lambda = 0, \lambda = 5$

Q:81 The Eigen values of the matrix [P]

$= \begin{bmatrix} 4 & 5 \\ 2 & -5 \end{bmatrix}$ are

trace = −1

|A| = −20 − 10 = −30

(a) - 7 and 8

(b) - 6 and 5

(c) 3 and 4

(d) 1 and 2

Q:82 For the matrix $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$ the given value are

$|A| = 16 - 1 = 15$

(a) 3 and - 3

(b) - 3 and - 5

(c) 3 and 5

(d) 5 and

Q: 83 The lowest eigenvalue of the 2×2 matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ is _____ .

$\lambda_1 + \lambda_2 = 7$

$\lambda_1 \times \lambda_2 = 10$

$\lambda_1 = 2$

$\lambda_2 = 5$

lowest = 2

Q:84 What are the eigenvalues of the following 2 × 2 matrix ?

$$\begin{bmatrix} 2 & -1 \\ -4 & 5 \end{bmatrix}$$

trace = 7

|A| = 6

(a) - 1 and 1

(b) 1 and 6

(c) 2 and 5

(d) 4 and - 1

Q:85 The larger of the two eigenvalues of the matrix $\begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$ is _____ .

$\alpha_1 + \alpha_2 = 5$

$\alpha_1 \times \alpha_2 = -6$

6, -1

largest = 6

~~3, 2~~

Q: 86 Two eigenvalue of a 3×3 real matrix P are $(2 + \sqrt{-1})$ and 3. The determinant of P is _____.

Sol: $2+i$, 3 , $2-i$

$$|P| = (2+i)(2-i) \times 3$$

$$= (4+1)3 = 15$$

Q:87 Consider a 2 × 2 square matrix

$$A = \begin{bmatrix} \sigma & x \\ \omega & \sigma \end{bmatrix}$$

Where x is unknown. If the eigenvalues of the matrix A are $(\sigma + j\omega)$ and $(\sigma - j\omega)$, then x is equal to

(A) -
(B) $-\omega$
(C) -
(D) -

$\lambda_1 = \sigma + j\omega$

$\lambda_2 = \sigma - j\omega$

$\lambda_1 \times \lambda_2 = |A|$

$(\sigma + j\omega)(\sigma - j\omega) = \sigma^2 - x\omega$

$\sigma^2 + \omega^2 = \sigma^2 - x\omega$

$-x\omega = \omega^2$

$x = -\omega$

Q:88 The smallest and largest Eigen values of the following matrix are

$$\begin{bmatrix} 3 & -2 & 2 \\ 4 & -4 & 6 \\ 2 & -3 & 5 \end{bmatrix}_{3\times3}$$

(a) 1.5 and 2.5 , 0

(b) 0.5 and 2.5 , 1

(c) 1.0 and 3.0 , 0

(d) 1.0 and 2.0 , 1

$\lambda_1+\lambda_2+\lambda_3=4$

$|A| = 3(-2) + 2(8) + 2(-4) = -6+16-8 = 2$

0, 1.5, 2.5

Sum = 4

$\mu = 2$.

Q:89 How many of the following matrices have an eigenvalue 1?

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$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$

(a) one

(b) Two

(c) Three

(d) Four

$\lambda_1 = 1, \lambda_2 = 0$

$\lambda_1 = 0, \lambda_2 = 0$

$\lambda_1 = -1, \lambda_2 = -1$

$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$\lambda_1 + \lambda_2 = 2$

$\lambda_1 \times \lambda_2 = 2$

$\lambda_1 = 1$

$\lambda_2 = 2$

if $\lambda_1 = 1$

$\lambda_2 = 1$

$\lambda_1 \times \lambda_2 \neq 2$

$\lambda_1, \lambda_2 = 1 \pm i$

$(1+i) + (1-i) = 2$

$1 + 1 = 2$

Q.90 The condition for which the eigenvalue of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & k \end{bmatrix}$ are all positive.

(a) $k > \frac{1}{2}$

(b) $k > -2$

(c) $k > 0$

(d) $k < -\frac{1}{2}$

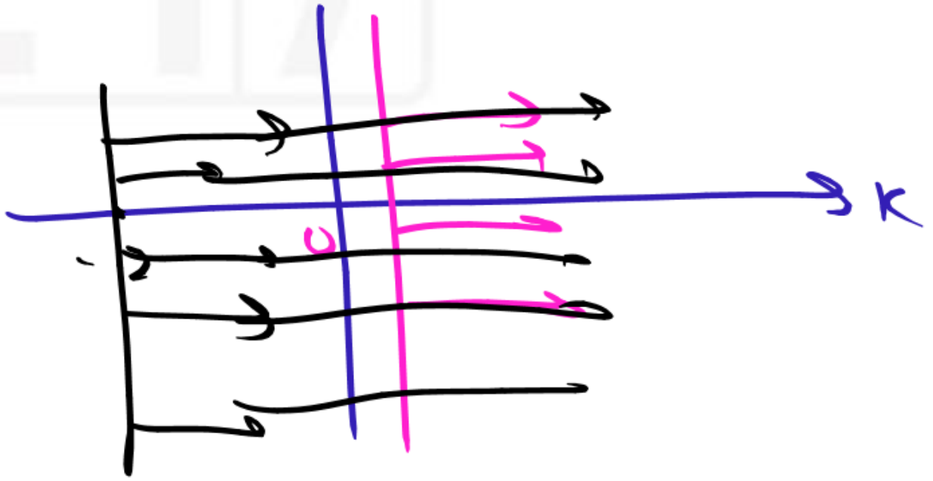
$\lambda_1 + \lambda_2 > 0$

$2 + k > 0 \Rightarrow k > -2$ — (2)

$\lambda_1 \times \lambda_2 > 0$

$2k - 1 > 0$

$k > \frac{1}{2}$ — (1)



Q:91 For a given matrix

$$A = \begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix},$$

One of the eigenvalues is 3. The other two eigenvalues are

(a) 2, - 5

(b) 3, - 5

(c) 2, 5

(d) 3, 5

$\lambda_1 + \lambda_2 + 3 = 1$

$\lambda_1 + \lambda_2 = -2$

Q:92 The sum of Eigen values of matrix, [M] is -

$$\text{Where } [M] = \begin{bmatrix} 215 & 650 & 795 \\ 655 & 150 & 835 \\ 485 & 355 & 550 \end{bmatrix}$$

(a) 915

(b) 1355

(c) 1640

(d) 2180

$N_1 + N_2 + N_3 = 215 + 150 + 550$

Q: 93 The minimum and the maximum eigen values of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}_{3\times3}$$ are – 2 and 6, respectively. What is the other eigen value ?

(a) 5

(b) 3

(c) 1

(d) - 1

$-2+6+\lambda_3 = 7$

$\lambda_3 = 3$

Q:94 The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigenvalue equal to 3. The sum of the other two eigenvalues is -

(a) p
(b) p - 1
(c) p - 2
(d) p - 3

$\lambda_1+\lambda_2+\lambda_3 = p+1$

$\lambda_1+\lambda_2+3 = p+1$

$\lambda_1+\lambda_2 = p-2$

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