WELCOME

TO Adda247

GATE 2024

प्रचण्ड Batch

ENGINEERING MATHEMATICS

CAYLEY HAMILTON THEOREM

TIME- 10:00AM

ANANT SIR

125

14-Apr

Friday - App

11.00 AM

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Cayley Hamilton Theorem :→

$A_{n\times n}$ → charactoric matrix $[A - \lambda I]$

$|A - \lambda I| = 0$ ← characteristic eqⁿ

↳ roots / solution = eigen values.

$\lambda_1, \lambda_2 - \ldots \lambda_n$

Any square matrix 'A' itself satisfies its own charactoristic equation.

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$$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2}$$

$$A_{2\times 2} \times A_{2\times 2} \longrightarrow A^2_{2\times 2}$$

$$[A - \lambda I] = \begin{bmatrix} -2-\lambda & 1 \\ 3 & 2-\lambda \end{bmatrix}$$

$$7 \longrightarrow 7I$$

$$|A - \lambda I| = (-2-\lambda)(2-\lambda) - 3 = 0$$

$$\lambda^2 - 4 - 3 = 0 \Rightarrow \lambda^2 - 7 = 0$$

$$BI = B$$

with applying Caley Hamilton th.

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$$A^2 - 7I = 0$$

$$A^{-1} = \frac{\text{adj} A}{|A|}$$

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Applications of C.H. theorem

① To find A^{-1} in terms of A (polynomial of A)

$$A^2 - 7I = 0$$

$$A^2A^{-1} - 7IA^{-1} = 0 \qquad IA^{-1} = A^{-1}$$

$$A - 7A^{-1} = 0$$

$$\Rightarrow \boxed{A^{-1} = \frac{A}{7}} \Rightarrow A^{-1} = \begin{bmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} \qquad \text{adj}A = \begin{bmatrix} 2 & -1 \\ -3 & -2 \end{bmatrix}$$

$$|A| = -4 - 3 = -7$$

$$A^{-1} = \frac{1}{-7}\begin{bmatrix} 2 & -1 \\ -3 & -2 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{2}{7} & \frac{1}{7} \\ \frac{3}{7} & \frac{2}{7} \end{bmatrix}$$

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② To find higher powers of 'A' in terms of 'A'

$$A^2 - 7I = 0$$

$$A^2 = 7I$$

$$\Rightarrow A^4 = A^2 A^2 = (7I)\times(7I) = 49I$$

$$\Rightarrow A^8 = A^4 A^4 = 49I \times 49I = (49)^2 I$$

$$\Rightarrow A^9 = A^8 \cdot A = (49)^2 I \cdot A = (49)^2 A$$

$$A^9 = (49)^2 \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} = 2401 \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -4802 & 2401 \\ 7203 & 4802 \end{bmatrix}$$

Q: Find A^9 ??

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Consider a matrix

$$A = \begin{bmatrix} -3 & 2 \\ -1 & 0 \end{bmatrix}$$

Q:126 A satisfies the relation

(a) $A + 3I + 2A^{-1} = 0$

(b) $A^2 + 2A + 2I = 0$

(c) $(A + I)(A + 2I) = I$

(d) $\exp(A) = 0$

$|A - \lambda I| = 0$

$\begin{vmatrix} -3-\lambda & 2 \\ -1 & -\lambda \end{vmatrix} = 0$

$\lambda^2 + 3\lambda + 2 = 0$

Ch. $A^2 + 3A + 2I = 0$

$A + 3I + 2A^{-1} = 0$

$A^2 + 2A + A + 2I = I$

$A^2 + 3A + 2I = I$

$\exp(A) = e^A$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

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Q:127 A^9 equals

(a) 511 A + 501 I

(b) 309 A + 104 I

(c) 154 A + 155 I

(d) exp (9A)

$A^2 + 3A + 2I = 0$

$A^2 = -(3A + 2I)$

$\Rightarrow A^4 = A^2 \cdot A^2 = [-(3A+2I)]^2 = (3A+2I)^2$

$A^4 = 9A^2 + 4I + 12A$

$= 9[-3A - 2I] + 4I + 12A$

$A^4 = -27A - 18I + 4I + 12A = -15A - 14I$

$A^8 = A^4 \cdot A^4 = [-(15A + 14I)]^2 = 225A^2 + 196I + 420A$

$= 225(-3A - 2I) + 196I + 420A$

$A^8 = -675A - 450I + 196I + 420A = -255A - 254I$

$A^9 = -255A^2 - 254A = -255(-3A - 2I) - 254A$

$= 511A + 510I$

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Q:128 The characteristic equation of a (3×3) matrix P is defined as

$$a(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$$

If I denotes identity matrix, then the inverse of matrix P will be

(a) $(P^2 + P + 2I)$

(b) $(P^2 + P + I)$

(c) $-(P^2 + P + I)$

(d) $-(P^2 + P + 2I)$

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Q:129 Given that

$$A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \text{ and } l = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

the value A^3 is

(a) 15 A + 12 l

(b) 19 A + 30 l

(c) 17 A + 15 l

(d) 17 A + 21 l

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -5-\lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + 5\lambda + 6 = 0$$

$$A^2 + 5A + 6I = 0$$

$$A^2 = -(5A + 6I)$$

$$A^3 = A^2 A = -(5A^2 + 6A) = -[5(-5A - 6I) + 6A]$$

$$= 25A + 30I - 6A$$

$$= 19A + 30I$$

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Q:130 A 3 × 3 matrix P is such that, $P^3 = P$. Then the eigenvalues of P are

(a) 1, 1, -1

(b) 1, 0.5 + j 0.866, 0.5 - j 0.866

(c) 1, - 0.5 + j 0.866, - 0.5 - j 0.866

(d) 0, 1, -1

$P^3 = P$

$\Rightarrow P^3 - P = 0$

$\Rightarrow \lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = 0, \lambda^2 = 1$

$\lambda = \pm 1$

0, 1, -1

3 Hrs

11.00 A.M

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Q:131 . A sequence x(n) is specified as

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \text{ for } n \geq 2.$$

The initial conditions are x[0] = 1, x[1] = 1, and x[n] = 0 for n < 0. The value of x[12] is 233 Ans

$$\begin{bmatrix} x[n] \\ x[n-1] \end{bmatrix}_{2\times 1} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n_{2\times 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1} \quad n \geq 2$$

for $n = 12$

$$\begin{bmatrix} x[12] \\ x[11] \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{12} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

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let $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

$$\begin{vmatrix} 1-\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda - 1 = 0$$

$$A^2 - A - I = 0$$

$$A^2 = A + I$$

$$A^4 = (A+I)^2 = A^2 + I + 2A = (A+I) + I + 2A = 3A + 2I$$

$$A^8 = (3A+2I)^2 = 9A^2 + 4I + 12A = 9(A+I) + 4I + 12A$$

$$= 21A + 13I$$

:

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$$A^{12} = A^{8} \cdot A^{4}$$

$$= (21A + 13I)(3A + 2I)$$

$$= 63A^{2} + 42A + 39A + 26I$$

$$= 63(A + I) + 42A + 39A + 26I$$

$$A^{12} = 144A + 89I$$

$$\begin{bmatrix} x(12) \\ x(11) \end{bmatrix} = \left(\begin{bmatrix} 144 & 144 \\ 144 & 0 \end{bmatrix} + \begin{bmatrix} 89 & 0 \\ 0 & 89 \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 233 \\ . \end{bmatrix}$$

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ALL PSUs	ENGINEERING MATHS	10:00 AM	ANANT SIR
GATE 2024-25	NETWORK THEORY	6:00 PM	RAVI SIR
GATE 2024-25	ELECTRICAL MACHINE	7:30 PM	SANTAN SIR
GATE 2024-25	COMMUNICATION	9:00 PM	RENU SIR

YouTube Classes Schedule

CIVIL ENGINEERING

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ALL PSUs	ENGINEERING MATHS	10:00 AM	ANANT SIR
ALL PSUs	GEOTECHNICAL	1:00 PM	RUDRA SIR
GATE 2024-25	STEEL STRUCTURE	6.00 PM	REHAN SIR
GATE 2024-25	ENVIRONMENT	8:00 PM	PRATIK SIR
GATE 2024-25	SOM	9:00 PM	MUKESH SIR

YouTube Classes Schedule

MECHANICAL ENGINEERING

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ALL PSUs	ENGINEERING MATHS	10:00 AM	ANANT SIR
ALL PSUs	PRODUCTION	11:30 PM	GAURAV SIR
ALL PSUs	THERMODYNAMICS	3:00 PM	KANISTH SIR
GATE 2024-25	HMT	4:30 PM	YOGESH SIR
GATE 2024-25	SOM	9:00 PM	MUKESH SIR


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MECHANICAL ENGINEERING

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HMT	MONDAY Live @11AM	YOGESH SIR
PRODUCTION	TUESDAY Live @11AM	GAURAV SIR
SOM	WEDNESDAY Live @8PM	MUKESH SIR
THERMODYNAMICS	THURSDAY Live @11AM	KANISTH SIR
ENGINEERING MATHEMATICS	FRIDAY Live @11AM	ANANT SIR

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NETWORK THEORY	SATURDAY Live @11AM	RAVI SIR
COMMUNICATION	WEDNESDAY Live @8PM	RENU SIR
ANALOG ELECTRONICS	THURSDAY Live @8PM	LAWRENCE SIR
ENGINEERING MATHEMATICS	FRIDAY Live @11AM	ANANT SIR
ELECTRICAL MACHINE	MONDAY Live @8PM	SANTAN SIR

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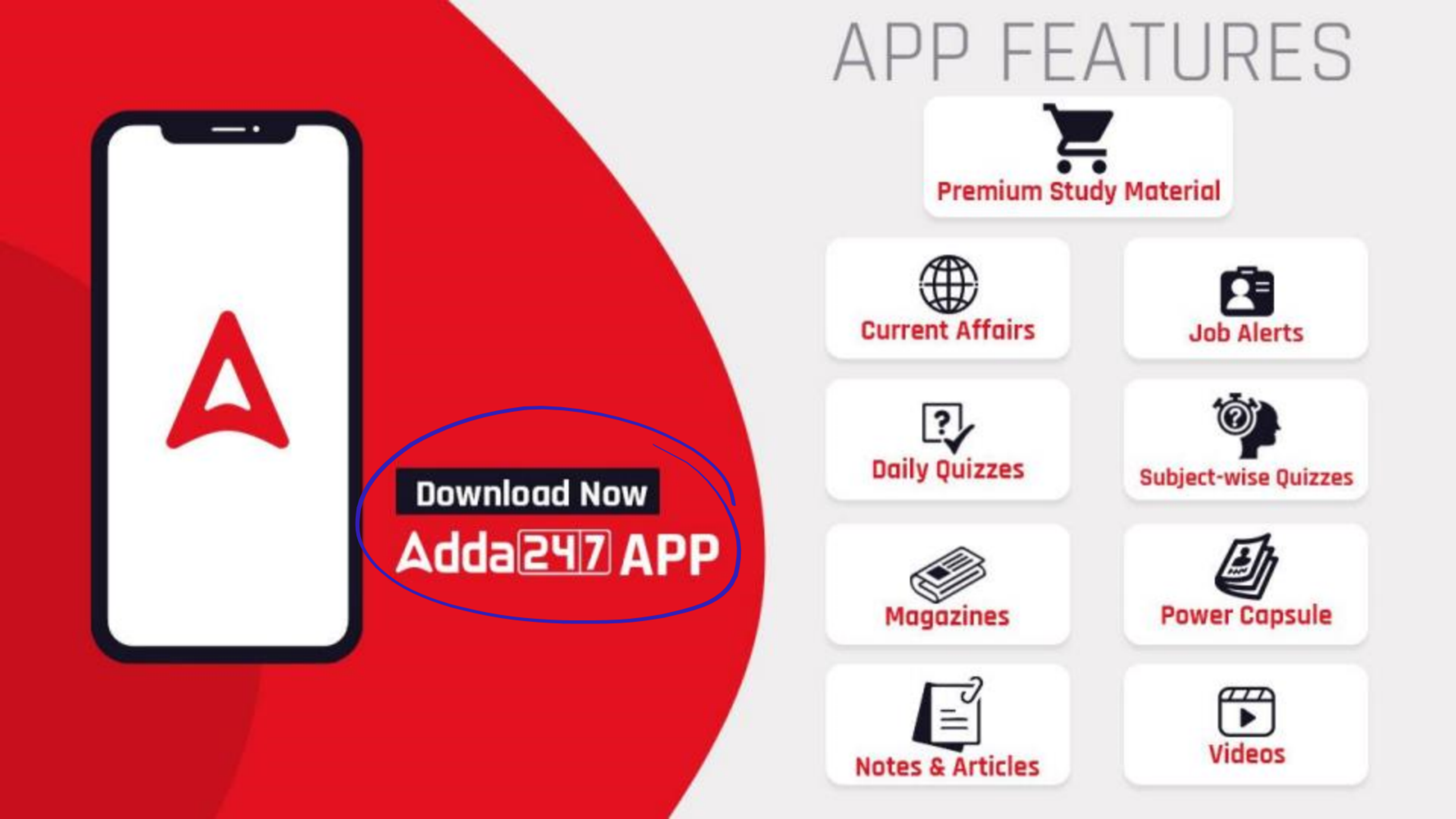
GEOTECHNICAL	SATURDAY Live @11AM	RUDRA SIR
ENGINEERING MATHEMATICS	FRIDAY Live @11AM	ANANT SIR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