A

21721 120 MINUTES

1. If $f(x) = \dfrac{x-1}{x+1}$, then $f(f(f(x)))$ is equal to

   A) $f(x)$ B) $\dfrac{1}{f(x)}$ C) $-f(x)$ D) $\dfrac{-1}{f(x)}$

2. The trigonometric expression $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$ is equal to

   A) $2\sin x$ B) $2\cos x$ C) $2\operatorname{cosec} x$ D) $2\sec x$

3. Which of the following sets of four points are collinear?

   A) $\{(0,1), (-1,0), (1,2), (2,4)\}$ B) $\{(0,1), (2,4), (-2,0), (-1,0)\}$
   C) $\{(-1,0), (3,2), (2,1), (0,1)\}$ D) $\{(1,2), (-2,-1), (0,1), (-1,0)\}$

4. Equation to the parabola whose focus is $(3,0)$ and directrix $3x+4y=1$ is

   A) $16x^2-9y^2-24xy-144x+8y+224=0$ B) $16x^2+9y^2-24xy-144x+8y+224=0$
   C) $16x^2+9y^2-24xy-144x+8y-224=0$ D) $16x^2-9y^2-24xy-144x-8y+224=0$

5. Area enclosed by the curve $y^2 = 2-x^2$ between $x=-\sqrt{2}$ and $x=\sqrt{2}$ is equal to

   A) 2π B) $\sqrt{2}\pi$ C) 3π D) $\sqrt{3}\pi$

6. In a colony, there are 55 families. Each family posts a greeting card to each other family. How many greeting cards will be posted by them?

   A) 2970 B) 109 C) 2790 D) 2079

7. A survey determined that in a locality 33% go to work by car, 42% by bus and 12% use both. Probability that a person selected at random uses neither car nor bus is

   A) 0.67 B) 0.37 C) 0.77 D) 0.53

8. Let A be the set of all sequences whose elements are the digits 0 and 1. Then the set A is

   A) empty B) finite C) countable D) uncountable

9. If the series $\sum a_n$ converges, then $\lim_{n\to\infty} a_n$ is equal to

   A) 0 B) 1 C) ∞ D) indeterminate

10. In the set of real numbers $\mathbb{R}$, define the function $f(x) = \begin{cases} 0 & \text{if } x \text{ irrational} \\ 1 & \text{if } x \text{ rational,} \end{cases}$

then the lower Riemann integral of f namely, $\mathbb{R}\underline{\int}_a^b f(x)dx$ is equal to

A) 1 B) 0 C) $b-a$ D) $b+a$

11. If ω denotes one of the cube roots of unity, then $(1+\omega-\omega^2)^7$ equals

A) $128\,\omega$ B) $-128\,\omega$ C) $128\,\omega^2$ D) $-128\,\omega^2$

12. For the function of the complex variable $f(z) = |z|^2$, which of the following statements is correct?

A) $f(z)$ is analytic for all z

B) Cauchy Riemann equations are satisfied for all z, but $f(z)$ not analytic.

C) Cauchy Riemann equations are satisfied at $z = 0$ only

D) None of the above three statements is correct.

13. If G is a region and $f : G \to \mathcal{C}$ is a complex valued function which is analytic such that there is a point a in G with $|f(a)| \geq |f(z)|$ for all z in G, then

A) f is positive always B) f is negative always

C) f is the zero function D) f is a constant function

14. Value of the $\oint_C \frac{1}{z-z_0}dz$ where C is the circle centered at $z = z_0$ and of any radius, the path being traced out once in the anticlockwise direction is

A) $2\pi i$ B) πi C) $\frac{\pi}{2}i$ D) $\frac{\pi}{4}i$

15. The set of all $n \times n$ matrices of real numbers is not a group under matrix multiplication because

A) matrix multiplication is not a binary operation

B) matrix multiplication is not associative

C) unit matrix has no multiplicative inverse

D) singular matrices have no multiplicative inverse

16. Suppose a group contains elements a, b such that $O(a) = 4$, $O(b) = 2$ and $a^3b = ba$. Then $O(ab)$ is equal to

A) 2 B) 4 C) 1 D) None of these

17. Let H and K be subgroups of a group G such that $O(H) = 24$ and $O(K) = 55$, then $O(H \cap K)$ is equal to

A) 24 B) 55 C) 31 D) 1

18. Let ϕ be a group homomorphism from Z_{24} onto Z_8 Then the kernel of the homomorphism ϕ is the group

A) $\{0, 4, 8, 12, 16, 20\}$ B) $\{0, 6, 12, 18\}$ C) $\{0, 8, 16\}$ D) $\{0\}$

19. If for every element x in a ring R, $x^2 = x$, then which of the following is true?

A) R is a field B) R is an integral domain

C) R is a commutative ring D) R is a non-commutative ring

20. (a) Ring of rational numbers is an integral domain.
(b) The ring $\mathcal{Z}_6$ is an integral domain.
Which of the above statements is true?

A) Both (a) and (b) B) (a) only C) (b) only D) Neither (a) nor (b)

21. (a) $\mathcal{Z}_4$ is isomorphic to $\mathcal{Z}_2 \times \mathcal{Z}_2$.
(b) $\mathcal{Z}_6$ is isomorphic to $\mathcal{Z}_2 \times \mathcal{Z}_3$.
Which of the above statements is true?

A) Both (a) and (b) B) (a) only C) (b) only D) Neither (a) nor (b)

22. If A is any square matrix whose transpose is denoted by A^T, then $A - A^T$ is a

A) diagonal matrix B) non-singular matrix

C) symmetric matrix D) skew symmetric matrix

23. If the matrix $\begin{pmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{pmatrix}$ is singular, then x is equal to

A. Zero B. $\dfrac{-25}{13}$ C. $\dfrac{-35}{12}$ D. none of these

24. If $M = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$, then inverse of M denoted by M^{-1} is equal to

A) $\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$ B) $\dfrac{1}{17}\begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$ C) $\dfrac{1}{17}\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$ D) $\dfrac{1}{17}\begin{pmatrix} 4 & 3 \\ -3 & 2 \end{pmatrix}$

25. Rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{pmatrix}$ is equal to

A) zero B) one C) two D) three

26. In a vector space $\mathcal{V}$, let x, y and z be such that $x+y+z = 0$. Also let the subspace spanned by $\{x, y\}$ be M and subspace spanned by $\{y, z\}$ be N. Then which of the following is true

A) $M = N$ B) $M \subset N$ C) $N \subset M$ D) $M \cap N = \phi$

27. If A and B are linear operators on $\mathbb{R}^2$ such that $A(x, y) = (y, x)$ and $B(x, y) = (0, x)$. Then the linear operator BA is such that

A) $BA(x, y) = (0, y)$ B) $BA(x, y) = (y, x)$ C) $BA(x, y) = (0, x)$ D) $BA(x, y) = (x, 0)$

28. Let A be a linear transformation over a finite dimensional vector space $\mathcal{V}$ over some field $\mathcal{F}$ and $R(A)$ and $N(A)$ are the range space and null space of A respectively. Then A is invertible if and only if

A) $R(A) = \mathcal{V}$ and $N(A) = \{0\}$ B) $R(A) = \mathcal{V}$ and $N(A) = \phi$

C) $R(A) = \{0\}$ and $N(A) = \mathcal{V}$ D) $R(A) = \phi$ and $N(A) = \mathcal{V}$

29. gcd of the numbers 630 and 196 is

A) 21 B) 14 C) 17 D) none of these

30. If $\mathcal{N}$ is the set of all positive integers such that $n^2 + 1$ is divisible by $n + 1$, then

A) $\mathcal{N}$ = the set of all odd positive integers

B) $\mathcal{N}$ = the set of all even positive integers

C) $\mathcal{N} = \phi$, the empty set

D) $\mathcal{N} = \{1\}$

31. The differential equation with primitive $y = Ae^{-x} + Be^{2x}$ is

A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 2y = 0$ B) $\frac{d^2y}{dx^2} - \frac{dy}{dx} + 2y = 0$

C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 0$ D) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$

32. Orthogonal trajectories of the family of parabolas $y^2 = 4ax$ will be the

A) family of lines $y = mx$, passing through the origin

B) family of circles $x^2 + y^2 = a^2$

C) family of curves $2x^2 + y^2 = c$

D) family of rectangular hyperbolas $xy = c^2$

33. Primitive of the differential equation $(y^2 + yz)dx + (z^2 + xz)dy + (y^2 - xy)dz = 0$ is

A) $y(x - z) = c(y - z)$ B) $y(x + z) = c(y + z)$

C) $y(x - z) = c(y + z)$ D) none of these

34. The complete integral of the partial differential equation $q = 3p^2$ is

A) $z = ax + 3a^2y + b$ B) $z = ax - 3a^2y + b$ C) $z = -ax + 3ay + b$ D) none of these

35. In a metric space (X, d), $B(a, r)$ denotes the open ball centered at a and radius r. Consider the statements
(I) $B(a, r)$ is contained in a closed ball centered at a
(II) $B(a, r)$ contains a closed ball centered at a
Then which is the correct choice?

A) (I) is true and (II) is false B) (I) is false and (II) is true

C) both (I) and (II) are true D) both (I) and (II) are false

36. In the set of real numbers $\mathbb{R}$, consider the metric defined by the rule, for $x, y \in \mathbb{R}$,
$$d(x,y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}.$$
Then the open ball $B(0, 1)$, with centre origin and radius one unit is equal to

A) $\{0\}$ B) $(0, 1)$ C) $[0, 1]$ D) $\{1\}$

37. Let X be an infinite set and T_f be the collection of all subsets U of X such that either $X - U$ is finite or all of X. Similarly T_c is the collection of all subsets V of X such that either $X - V$ is countable or all of X. Which of the following statements is true?

A) T_c and T_f are topologies on X and T_c is finer than T_f

B) T_c and T_f are topologies on X and T_f is finer than T_c

C) T_c and T_f are topologies on X and $T_c = T_f$

D) T_c and T_f are not topologies on X.

38. Which among the following is a false statement?

A) Metrizability is a hereditary property

B) Second countability is a hereditary property

C) Compactness is a hereditary property

D) Connectedness is not a hereditary property

39. Consider the two statements
(I) Every separable topological space is second countable
(II) Every second countable topological space is separable
Which of the following is the correct option?

A) both (I) and (II) are true

B) (I) is true but (II) is false

C) (I) is false but (II) is true

D) both (I) and (II) are false

40. Let X be a normed linear space over the field K. Then which of the following is not true?

A) If $E_1 \subset X$ is open and $E_2 \subset X$, then $E_1 + E_2$ is open

B) If $E \subset X$ is convex, then so are E° and $\overline{E}$

C) If Y is a subspace of X, then $Y \neq X$ if and only if $Y^\circ = \phi$

D) If $f : X \to K$ is any linear functional, then f maps open sets in X onto open sets in K.

41. If E is an orthonormal subset of an inner product space X, then for $u, v \in E$ with $u \neq v$, the value of $\|u - v\|$ is

A) 2

B) $\sqrt{2}$

C) 2^p

D) $2^{\frac{1}{p}}$

42. The domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right)$ is

A) $(-\infty, 6)$

B) $[-6, 6]$

C) $[6, \infty)$

D) $(-6, 6)$

43. The shortest distance from the plane $x + y + z = 4$ to the sphere $x^2 + y^2 + z^2 - 2x = 0$ is:

A) $\sqrt{3} - 1$

B) $\sqrt{3} + 1$

C) $\sqrt{3} - 2$

D) $\sqrt{3} + 2$

44. The angle between the normals at the two extremes of the latus rectum of the parabola $y^2 = 8x$ is:

A) π

B) $\frac{\pi}{2}$

C) $\frac{\pi}{3}$

D) $\frac{\pi}{4}$

45. Arc length of the paramentric curve $x = \cos t + t \sin t,\ \ y = \sin t - t\cos t$, $0 \le t \le \pi$ is

A) $\dfrac{\pi^2}{4}$ B) $\dfrac{\pi}{2}$ C) $\dfrac{\pi}{4}$ D) $\dfrac{\pi^2}{2}$

46. The value of the integral $\displaystyle\int_0^{\pi} \frac{dx}{1+3^{\cos x}}$ is

A) π B) $\frac{\pi}{4}$ C) 0 D) $\frac{\pi}{2}$

47. If $P(A) = \frac{1}{4}$, $P(B^c) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{9}$, then $P(A/B)$ is

A) $\frac{7}{72}$ B) $\frac{7}{9}$ C) $\frac{7}{36}$ D) $\frac{7}{18}$

48. $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x+y) = f(x) + f(y)$ and $f(1) = 5$. Then,

A) $f(x) = 5x$ for all $x \in \mathbb{R}$

B) f is not uniquely defined even if f is continuous.

C) $f(x) = 5$ for all $x \in \mathbb{R}$

D) $f(x) = 5x$ for all $x \in \mathbb{R}$ if f is continuous on $\mathbb{R}$

49. The series $\sum\limits_{n=1}^{\infty} \dfrac{x^2}{(1+x^2)^n}$

A) uniformly convergent in $[0, 1]$

B) uniformly convergent in $[0, \frac{1}{2}]$

C) not pointwise convergent

D) converges pointwise but not converges uniformly in $[0, 1]$

50. The value of the integral $\int_0^2 (x - [x])\, dx^2$, where $[x]$ denote the greatest integer less than or equal to x.

A) $\frac{5}{3}$ B) $\frac{2}{3}$ C) $\frac{7}{3}$ D) $\frac{4}{3}$

51. Which of the following statements is false:

A) If $m^*(E) = 0$, then E is measurable.

B) There exists uncountable sets of measure 0.

C) Every bounded Reimann integrable function on $[a, b]$ is Lebesgue integrable on $[a, b]$.

D) Every characteristic function is measurable.

52. The maximum number of fixed points of a non-identity Mobius transformation is

A) 1 B) 2 C) 0 D) 3

53. Let $f(z) = \dfrac{\cos z - 1}{z^2}$. Which of the following is true?

A) $z = 0$ is an essential singularity B) $f(z)$ is analytic at $z = 0$

C) $z = 0$ is a pole of order 2 D) $z = 0$ is a removable singularity

54. Let γ be any closed curve and let a and b be any two points lying incide γ. Then the value of $\int_\gamma \dfrac{1}{(z-a)(z-b)} dz$ is

A) $2\pi i$ B) $2\pi(a-b)i$ C) 0 D) $-2\pi i$

55. Which of the following is always true for a group of order 12

A) it has a subgroup of order 6 B) it is abelian

C) it has a subgroup of order 2 D) it is non abelian

56. Number of elements of order 6 in S_4, the symmetric group on 4 symbols, is

A) 0 B) 1 C) 2 D) 4

57. Which of the following is not true?

A) The alternating group A_n is simple for $n \geq 5$

B) A group of order 42 is simple

C) S_3 is simple

D) If G is a group of order 9 and H is a subgroup of order 3, then H is normal in G

58. Let G be a group of order 10. Then number of elements in the set $\{x \in G : x^2 = x\}$ is

A) 1 B) 2 C) 10 D) infinite

59. Consider the rings $\mathbb{Z}(\sqrt{2})$, $\mathbb{Z}(\sqrt{3})$ and $f : \mathbb{Z}(\sqrt{2}) \to \mathbb{Z}(\sqrt{3})$ defined by $f(a + b\sqrt{2}) = a + b\sqrt{3}$. Then

A) f is group homomorphism but not a ring homomorphism.

B) f is a ring homomorphism

C) f is neither a ring homomorphism nor a group homomorphism

D) f is the only group homomorphism from $\mathbb{Z}(\sqrt{2})$ to $\mathbb{Z}(\sqrt{3})$

60. Consider the ring $R = \mathbb{Z}[i]$, the ring of Gaussian integers and $I = \{a + ib, a, b \in 2\mathbb{Z}\}$, $J = \{a + ib, a, b \in 3\mathbb{Z}\}$ then

A) I and J are maximal ideals of R

B) I is a maximal ideal but J is not a maximal ideal of R

C) J is maximal ideal but I is not a maximal ideal of R

D) I and J are not ideals of R

61. Which of the following is not a splitting field over $\mathbb{Q}$.

A) $\mathbb{Q}(\omega)$, ω is a non-real cube root of unity.

B) $\mathbb{Q}(\alpha)$, α is the real cube root of unity

C) $\mathbb{Q}(i)$

D) $\mathbb{Q}(\sqrt{2})$

62. For which value of a, the system $x + 2y = 1, 2x + ay = 1$ is inconsistent.

A) 0 B) 5 C) -1 D) 4

63. If A is orthogonal, then A^{-1} is

A) -A B) A^T C) A^2 D) A

64. Let the minimal polynomial of a matrix A be $f(x) = x^{13} - x + 1$. Then A^{14} is

A) $A - I$ B) $I - A$ C) $A^2 - A$ D) $A - A^2$

65. Let A be a 2×2 matrix with Trace(A) $= 5$ and $|A| = 6$. Which of the following can be eigen values of A?

A) 1, 4 B) 2, 3 C) 1, 6 D) 0, 5

66. The dimension of the space of all $n \times n$ real symmetric matrices with all diagonal elements as zero is

A) $\dfrac{n(n+1)}{2}$ B) n^2 C) $\dfrac{n(n-1)}{2}$ D) $n^2 - n$

67. Let V be the set of all polynomials of degree ≤ 2. Define $T : V \to V$ by $T(p(x)) = p(1) + p(2)x + p(0)x^2$, $p(x) \in V$. Then the trace of the matrix representing T is

A) 0 B) -1 C) 1 D) 3

68. Which of the following statements is true, if a, b, c are integers.

A) If $a|bc$, then $a|c$

B) If $a|c$ and $b|c$, then $ab|c$

C) If n is any even integer, then n can be written in the form $a - b$ with $(a, n) = 1$ and $(b, n) = 1$.

D) If $a|c$ and $b|c$, then $(a + b)|c$

69. The remainder obtained when 5^{78} is divided by 12.

A) 2 B) 1 C) 3 D) 4

70. The integral curve of the Differential equation $\dfrac{dy}{dx} - \dfrac{y}{x} = 2x^2$ passing through the point $(-1, 2)$ is

A) $y^2 = x(x^2 - 5)$ B) $4y = x(x^2 - 9)$ C) $2y = x^2(x + 5)$ D) $y = x(x^2 - 3)$

71. At each point (x, y) of a curve, the intercept of the tangent on the $x - axis$ is $2x^2y$. The curve passing through $(1, -1)$ is

A) $y = xy^2 - 2x$ B) $2y = x^2y + 2x$ C) $y = x - 2y^2$ D) $2y = xy^2 - 3x$

72. The Partial Differential equation representing the family of curves $z = (x-a)^2 + (y-b)^2$ is

A) $z = p^2 + q^2$ B) $4z = p^2 - q^2$ C) $2z = p^2 + q^2$ D) $4z = p^2 + q^2$

73. The complete solution of $pq + qx = y$ is

A) $2az = 2a^2x - ax^2 - y^2 + b$ B) $2az = 2a^2x - ax^2 + y^2 + b$

C) $2az = 2a^2x + ax^2 + y^2 + b$ D) $2az = 2a^2x + ax^2 - y^2 + b$

74. The characterestics of the Partial Differential equation $u_{xx} - x^2yu_{yy} = 0 (y > 0)$ are

A) $\xi = x^2 + 4\sqrt{y}$, $\eta = x^2 - 4\sqrt{y}$ B) $\xi = y^2 + 4\sqrt{x}$, $\eta = y^2 - 4\sqrt{x}$

C) $\xi = x^2 + \sqrt{y}$, $\eta = x^2 - \sqrt{y}$ D) $\xi = y^2 + 2\sqrt{x}$, $\eta = y^2 - \sqrt{x}$

75. Let d be the metric on $\mathbb{Z}$ defined by $d(x, y) = |x - y|$, $x, y \in \mathbb{Z}$. Which of the following is true

A) The topology induced by d is the discrete topology

B) For each $x \in \mathbb{Z}$, $\{x\}$ is not open

C) For each $x \in \mathbb{Z}$, $\{x\}$ is not closed

D) The topology induced by d is the trivial topology $\{\phi, \mathbb{Z}\}$

76. Consider the following statements:
I: The cofinite topological space (X, τ) is separable
II: Every discrete space is seperable. Then

A) Both I and II are true B) Only I is true

C) Only II is true D) Both I and II are false

77. Let $\mathcal{T}$ be the cofinite topology on $\mathbb{R}$ and $\mathcal{U}$ be the usual topology on $\mathbb{R}$. Consider the identity maps $f : (\mathbb{R}, \mathcal{T}) \to (\mathbb{R}, \mathcal{U})$ and $g : (\mathbb{R}, \mathcal{U}) \to (\mathbb{R}, \mathcal{T})$. Then

    A) Both f and g are continuous

    B) Only f is continuous

    C) Only g is continuous

    D) Neither f nor g is continuous

78. Let $F : l^2 \to l^2$ be defined as $F(e_n) = (1/n)e_n$ where $\{e_n : n = 1, 2, \cdots\}$ is the orthonormal basis of l^2. Which of the following is true:

    A) F is a bijection

    B) F is injective but not surjective

    C) F is not injective but surjective

    D) F is neither injective nor surjective.

79. Let $\{e_n : n = 1, 2, \cdots\}$ be an orthonormal basis of l^2. Which of the following series is convergent in l^2?

    A) $\displaystyle\sum_{n=1}^{\infty} e_n$

    B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} e_n$

    C) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} e_n$

    D) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n} e_n$

80. Let H_1 and H_2 be Hilbert spaces over $\mathbb{K}$ and $T : H_1 \to H_2$ be an isometry. Then, for any $x, y \in H_1$, $< Tx, Ty >$ is

    A) $< x, y >$

    B) $< y, x >$

    C) $< x, -y >$

    D) $< -x, y >$