SYLLABUS MATHEMATICS

UNIT – 1

01. Real Analysis

Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, improper integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Integral functions, line and surface integrals, Green's theorem, Stoke's theorem Metric spaces, compactness, connectedness. Normed linear spaces. Spaces of continuous functions as examples.

UNIT – 2

02. Algebra

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, symmetric groups, alternating groups, simple groups, Cayley's theorem, class equations, Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain, unique factorisation domains. Polynomial rings and irreducibility criteria. Fields, quotient fileds, finite fields, field extensions, Galois Theory.

03. Linear Algebra

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations, change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic form.

UNIT - 3

04. Functional Analysis

Banach Spaces Hahn-Banach theorem, open mapping and closed graph theorems. principal of uniform boundedness, boundedness and continuity of linear transformations, dual space, embedding in the second dual, Hilbert Spaces, projections. Orthonormal basis, Riesz-representation theorem, Bessel's Inequality, parsaval's identity, self adjoined operators, normal operators.

05. Topology

Elements of topological spaces, continuity, convergence, homeomorphism, compactness, connectedness, separation axioms, first and second countability, separability, subspaces, product spaces, quotient spaces. Tychonoft's theorem, Urysohn's metrization theorem, homotopy and fundamental group.

UNIT – 4

06. Ordinary Differential Equations

First order ordinary differential equation (ODE), singular solutions initial value problems of first order ODE, General theory of homogeneous and non-homogeneous linear ODE, Variation of Paraneters. Existence and uniqueness of solution dy/dx = f (x,y), Green's function, Sturm-Liouville boundary value problems, Cauchy problems and characteristics. Power series solutions of second order linear differential equations.

07. Partial Differential Equations

Lagrange and Charpit methods for solving first order partial differential equations (PDEs), Cauchy problem for first order PDEs. Classification of second order PDEs, general solution of higher order PDEs with constant coefficients, Monge's method, method of separation of variables for Laplace, heat and wave equations.

UNIT – 5

08. Integral Transforms and Integral Equations

Laplace transform: transform of elementary functions, transform of derivatives, inverse transform, convolution Theorem, applications, ordinary and partial differential equations.

Fourier transform: sine and cosine transform, inverse Fourier transform, application to ordinary and partial differential equations.

Linear Integral Equations: Equations of the first and second kind of Fredholm and Volterra type, solution by successive substitutions and successive approximations, solution of equations with separable kernels, the Fredholm alternative; Holbert-Schmidt theory for symmetric kernels.

UNIT – 6

09. Numerical Analysis

Finite differences, numerical solution of algebric and transcendental equations.

Iteration: Newton-Raphson method, solution on linear system of equations-direct method, Gauss elminaiton method, matrix-inversion. Eigenvalue problems, numerical differentiation and integration.

Interpolation: Newton, Lagrange and Hermite interpolations.

Numerical solution of ordinary differential equation: iteration method, Picard’s method, Euler’s method and modified Euler’s method.

UNIT – 7

10. Classical Mechanics

Generalized coordinates, Lagrange’s equations, Hamilton’s canonical equations, Hamilton’s principle and principle of least action, Two-dimensional motion of rigid bodies, Euler’s dynamical equations for the motion of a rigid body about an axis, motion of rigid body about an axis, theory of small oscillations.

11. Fluid Mechanics

Equation of continuity in fluid motion, Euler’s equations of motion for perfect fluids, Two dimensional motion-complex potential, motion of sphere in perfect fluid and motion of fluid past a sphere, vorticity, Navier-Stokes’s equations for viscous flows-some exact solutions.

UNIT – 8

12. Complex Analysis

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.

Analytic functions, Cauchy-Riemann equations.

Contour integral, Cauchy's theorem, Cauchy's integral formula, Morera's theorem, Liouville's theorem, zero sets of analytic functions, classification of singularities, Maximum modulus principle, Schwarz lemma, open mapping theorem.

Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

UNIT - 9

13. Differential Geometry

Space curves-their curvature and torsion; Serret-Frenet formula, fundamental theorem of space curves, Curves on sirfaces, First and second fundamental form, Gaussian curvatures, Principal directions and principal curvatures, goedesics, fundamental equations of surface theory.

14. Calculus of Variations

Linear functionals, minimal functional theorem, general variation of a functional, Euler-Lagrange equation; Variational methods of boundary value problems in ordinary and partial differential equations.

UNIT - 10

15. Discrete Mathematics

Elements of graph theory, Eulerian and Hamiltonian graphs, planar graphs, directed graphs, trees, permutations and combinations, Pigeonhole principle, principle of inclusion and exclusion, derangements.

Number theory: Divisibility, linear Diophantine equations, congruences. quadratic residues, sums of two squares.

16. Linear Programming

Convex sets, linear programming problem (LPP), examples of LPP, hyperplane, open and closed half-spaces, feasible, basic feasible and optimal solutions. Extreme point and graphical method, simplex method, revised simplex method, dual simplex method. Transportation and assignment problems.