A

Paper – II

Mathematical Sciences

Booklet Code

A

Subject Code : 0 4

TEST BOOKLET NO.

Roll No. :

(Figures as per admission card)

Roll No. (in words) : ____________________

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OMR Sheet No. :

Name and Signature of Invigilator/s

Signature : ____________________

Name : ____________________

Time : 2 Hours | Maximum Marks : 200

Number of Pages in this Booklet : 32 | Number of Questions in this Booklet : 175

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of one hundred seventy five (175) multiple-choice type of questions. These questions are divided as Section – A, Section – B and Section – C. Section – A is **compulsory**. Section – B and Section – C are optional. Candidate must choose either Section – B or Section – C as per his/her discipline and mark it correctly on OMR answer sheet at appropriate box. Questions attempted from both the Section – B and Section – C will not be evaluated.
3. At the commencement of examination, the test booklet will be given to you. In the first 5 minutes, you are requested **to open the booklet and compulsorily examine it as below** :
   (i) To have access to the Test Booklet, tear off the paper seal on the edge of the cover page. Do not accept a booklet without sticker seal or open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Test Booklet will be replaced nor any extra time will be given.**
   (iii) After the verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.

   Example : (A) (B) ● (D)

   where (C) is the correct response.
5. Your responses to the questions are to be indicated in the **OMR Sheet kept inside this Booklet**. If you mark at any place other than in the circles, the OMR Sheet will not be evaluated.
6. Read the instructions given in OMR carefully. Fill the Booklet Code of Paper – II in OMR Sheet **Compulsorily**.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space provided for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the OMR Answer Sheet to the invigilators at the end of the examination compulsorily and must NOT carry it with you outside the Examination Hall.
10. You can take away test booklet and carbon copy of OMR Answer Sheet after the examination.
11. **Use only Blue/Black Ball point pen.**
12. **Use of any calculator, electronic gadgets or log table, etc. is prohibited.**
13. **There is no negative mark for incorrect answer.**

A

MATHEMATICAL SCIENCES

Paper – II

Note : This paper contains 175 multiple choice questions of 2 marks each, in THREE (3) SECTIONS . Attempt all the questions either from SECTION – A and SECTION – B only OR from SECTION – A and SECTION – C only. The OMR Sheet with questions attempted from both Sections viz. Section – B and Section – C will not be evaluated. Number of questions Section wise :

Section – A : Q. No. 1 to 25 ; Section – B : Q. No. 26 to 100 ; Section – C : Q. No. 101 to 175.

SECTION – A

1. The rank of the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{pmatrix}$ is equal to

(A) 3

(B) 2

(C) 1

(D) 4

2. Let $L : R^3 \to R$ be given by $L(x, y, z) = 3x - 2y + z$, then

(A) dim (image of L) = 0

(B) dim (kernel of L) = 1

(C) dim (kernel of L) = 2

(D) dim (image of L) = dim (kernel of L)

3. Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}_{2\times 2}$,

where $a, b, c, d \in \{0, 1\}$. The number of such matrices which have inverse is

(A) 5 (B) 6

(C) 7 (D) 8

4. Let V = $\mathbb{R}^2$ be a vector space over $\mathbb{R}$. Let $\alpha = (1, 2)$, $\beta = (2, 1)$ and $\gamma = (2, 4)$ in V. Then

(A) α, β are linearly independent and α, γ are linearly independent

(B) α, β are linearly independent and α, γ are linearly dependent

(C) α, β are linearly dependent and α, γ are linearly independent

(D) α, β are linearly dependent and α, γ are linearly dependent

5. Let $u_n = \sin n\pi$ and $u_n = \left(1+\frac{1}{n}\right)\cos n\pi$, then which of the following options is correct ?

(A) $< u_n >$ is convergent sequence

(B) $\lim_{n\to\infty} \inf u_n = \lim \inf v_n$

(C) $\lim \sup u_n > \lim \sup v_n$

(D) $\lim \inf u_n > \lim \inf v_n$

6. $\lim_{x\to 4} \frac{\sqrt{1-\cos 2(x-4)}}{x-4}$

(A) is equal to $\sqrt{2}$ (B) is equal to 1

(C) does not exist (D) is equal to $-\sqrt{2}$

7. Which of the following statements is not correct ?

(A) The set of all rational numbers is countable

(B) Countable union of countable sets is countable

(C) $\{x \in R \,;\, 0 < x < 1\}$ is uncountable

(D) The set of all irrational numbers in [0, 1] is countable

8. Let $\{f_n\}$ be a sequence of functions defined by $f_n(x) = x^n$. Then this sequence

(A) converges uniformly in [0, 1]

(B) converges uniformly in [0, k] where $0 < k < 1$

(C) converges uniformly in [–1, 1]

(D) does not converge uniformly on any subset of $\mathbb{R}$

9. Let $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$ be any two sets of non-negative real numbers and let $p > 1$. Then Minkowski's inequality states that

(A) $\sum_{i=1}^{n}(a_i + b_i)^p \le \sum_{i=1}^{n}(a_i)^p + \sum_{i=1}^{n}(b_i)^p$

(B) $\left(\sum_{i=1}^{n}(a_i + b_i)^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^{n}(a_i)^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^{n}(b_i)^p\right)^{\frac{1}{p}}$

(C) $\left(\sum_{i=1}^{n}(a_i + b_i)^p\right)^{\frac{1}{p}} \le \left(\sum_{i=1}^{n}(a_i)^p\right)^{\frac{1}{p}} . \left(\sum_{i=1}^{n}(b_i)^p\right)^{\frac{1}{p}}$

(D) $\sum_{i=1}^{n} a_i\, b_i \le \left(\sum_{i=1}^{n}(a_i)^p\right)^{\frac{1}{p}} . \left(\sum_{i=1}^{n}(b_i)^p\right)^{\frac{1}{p}}$

10. If P* is a refinement of the partition P of an interval [a, b], then with the usual notations the upper Riemann sum U and the lower Riemann sum L of f satisfy

(A) $L(P, f) \le L(P^*, f)$, $U(P, f) \le U(P^*, f)$

(B) $L(P^*, f) \le L(P, f)$, $U(P, f) \le U(P^*, f)$

(C) $L(P^*, f) \le L(P, f)$, $U(P^*, f) \le U(P, f)$

(D) $L(P, f) \le L(P^*, f)$, $U(P^*, f) \le U(P, f)$

11. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$. Then which of the following statements is not true ?

(A) f is bounded above on (a, ∞)

(B) f is infinitely differentiable at every non-zero $x \in R$

(C) f' is not continuous at 0

(D) f is neither convex nor concave on $(0, \delta)$

12. Which of the following statements is not true ?

(A) $\left(\frac{1}{2}, 2\right)$ and $(2, \infty)$ are homeomorphic

(B) $(0, 2)$ and $(0, \infty)$ are homeomorphic

(C) $[1, 2]$ and $(3, 4)$ are homeomorphic

(D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is homeomorphic to $(-\infty, \infty)$

13. Quadratic form corresponding to the matrix $\begin{bmatrix} 4 & -5 & 7 \\ -5 & -6 & 8 \\ 7 & 8 & -9 \end{bmatrix}$ is (variables being x_1, x_2, x_3)

(A) $4x_1^2 - 8x_1x_2 - 4x_3^2 + 14x_1x_3 + 16x_2x_3 - 2x_2^2$

(B) $4x_1^2 - 6x_1x_2 - 4x_3^2 + 8x_1x_3 + 7x_2x_3 - 9x_2^2$

(C) $4x_1^2 - 5x_1x_2 - 6x_2^2 + 7x_1x_3 + 16x_2x_3 - 9x_3^2$

(D) $4x_1^2 - 10x_1x_2 - 6x_2^2 + 14x_1x_3 + 16x_2x_3 - 9x_3^2$

14. Which of the following statements is false ?

(A) A function of bounded variation is bounded

(B) A bounded monotonic function is a function of bounded variation

(C) Every continuous function is of bounded variation

(D) A function of bounded variation need not be continuous

15. If u = (1, 2, k, 3) and v = (3, k, 7, –5) in $\mathbb{R}^4$ are orthogonal, then the value of k is

(A) $\frac{3}{4}$ (B) $\frac{4}{3}$

(C) $\frac{6}{5}$ (D) $\frac{5}{6}$

16. The function f defined by $f(x)=\frac{1}{2^n}$ when $\frac{1}{2^{n+1}}<x\leq\frac{1}{2^n},(n=0,1,2,\ldots)$ and f(0) = 0

(A) has only finitely many points of discontinuity

(B) is integrable on [0, 1]

(C) is not integrable

(D) is not integrable on [0, 1], but has infinitely many points of discontinuity

17. Let $A=\left\{\left[1,1+\frac{1}{n}\right]:n\in N\right\}$, then

(A) $\bigcap_{S\in A} S$ has at least two points

(B) $\bigcap_{S\in A} S$ is not closed in R

(C) $\bigcap_{S\in A} S$ is a non-compact subset of R

(D) $\bigcap_{S\in A} S$ is a bounded closed subset of R

18. The parallelogram equality, which a normed linear space induced by an inner product space is given by

(A) $||x+y||^2 + ||x-y||^2 = 2(||x||^2 + ||y||^2)$

(B) $||x+y||^2 - ||x-y||^2 = 2(||x||^2 - ||y||^2)$

(C) $||x+y||^2 + ||x-y||^2 = (||x||^2 + ||y||^2)$

(D) $||x+y||^2 + ||x-y||^2 = (||x|| + ||y||)^2$

19. For a vector space V over complex field, which of the following is true ?

(A) Every inner product on V induces a norm on V

(B) Every norm on V induces an inner product on V

(C) Scalar multiplication on V is continuous with respect to every norm on V

(D) Every metric on V is induced by a norm on V

20. If x = u is a point of discontinuity of f(x) = $\lim_{n\to\infty}\cos^{2n}x$, then the value of cos u is

(A) 0

(B) ½

(C) 1

(D) $(-1)^n$

21. The value of x for which the matrix

$$A = \begin{bmatrix} \frac{2}{x} & -1 & 2 \\ 1 & x & 2x^2 \\ 1 & \frac{1}{x} & 2 \end{bmatrix}$$ is singular, is

(A) ± 1 (B) ± 2

(C) ± 3 (D) 4

22. Let $f(x) = \frac{1}{x}, x \in (0, 1]$ and $g(x) = x^2$, $x \in [-1, 1]$. Then

(A) f is not uniformly continuous on (0, 1], g is uniformly continuous on [–1, 1]

(B) f is uniformly continuous on (0, 1], g is uniformly continuous on [–1, 1]

(C) f is not uniformly continuous on (0, 1], g is not uniformly continuous on [–1, 1]

(D) f is uniformly continuous on (0, 1], g is not uniformly continuous on [–1, 1]

23. The series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2 + n}{n^2}$

(A) Converges uniformly in [–1, 1] and also converges absolutely at x = –1

(B) Converges uniformly in [–1, 1] and does not converge absolutely at x = –1

(C) Does not converge uniformly in [–1, 1] and does not converge absolutely at x = –1

(D) Does not converge uniformly in [–1, 1] , converges absolutely at x = –1

24. The radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ and the series $\sum_{n=0}^{\infty} n!x^n$ respectively are

(A) $0, \infty$

(B) $\infty, 0$

(C) 1, 1

(D) 1, 2

25. Let $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$.

Then

(A) both partial derivatives exist at (0, 0), but the function is not continuous at (0, 0)

(B) both partial derivatives exist at (0, 0) and the function is continuous at (0, 0)

(C) both partial derivatives do not exist at (0, 0), but the function is continuous at (0, 0)

(D) both partial derivatives do not exist at (0, 0), and the function is also not continuous at (0, 0)

SECTION – B

26. Which of the following is incorrect ?

(A) Unit circle and boundary of the square $[0, 1] \times [0, 1]$ are homeomorphic

(B) Closed interval [0, 1] and the cantor set are not homeomorphic

(C) The subspace Z of integers and the subspace Q of rational numbers of the usual space R of real numbers are homeomorphic

(D) A continuous self-map on the closed interval [0, 1] has a fixed point

27. Let X be an uncountable set and τ be the family consisting of empty set and all complements of countable sets. Then the topological space (X, τ) is

(A) T_1 space but not T_2 space

(B) T_2 space but not T_3 space

(C) a T_3 space but not a normal space

(D) a normal space

28. Which of the following statements is true for topological spaces ?

(A) Every separable space is second countable

(B) Every second countable space is separable

(C) Every first countable space is separable

(D) Every first countable space is second countable

29. Which of the following is true ?

(A) A discrete topological space having more than one element is connected

(B) A connected topological space has exactly two sets which are open and closed

(C) Compact and connected subset of the usual real line are closed intervals

(D) All open sets of the real line are open intervals

30. In a topological space, which of the following statements is correct ?

(A) a compact set is connected

(B) a connected set is compact

(C) a continuous image of connected set is connected

(D) inverse image of a compact set under a continuous map is compact

31. Let G be a group and let H be any subgroup of G. If N is any normal subgroup of G, then

(A) $\frac{HN}{H}$ is isomorphic to $\frac{H}{H \cap N}$

(B) $\frac{HN}{N}$ is isomorphic to $\frac{H}{H \cap N}$

(C) $\frac{HN}{N}$ is isomorphic to $\frac{H \cap N}{N}$

(D) $\frac{HN}{H}$ is isomorphic to $\frac{H \cap N}{H}$

32. Let A_n be the subgroup of even permutations of S_n, then the order $O(A_n)$ is

(A) n! (B) n!/2

(C) 2n (D) n^2

33. Which of the following statements is not correct ?

(A) For any group G, Z(G) the centre of G is a normal subgroup of G

(B) The subgroup of rotations in D_n is normal in D_n

(C) $\{A \in GL(n, R) : \det A \text{ is a power of } 2\}$ is a normal subgroup of GL(n, R)

(D) In A_4 of even permutations of {1, 2, 3, 4}, $k = \{\alpha, \alpha_5, \alpha_9\}$ is a normal A_4, where $\alpha = (1)$, $\alpha_5 = (1, 2, 3)$, $\alpha_9 = (1, 3, 2)$

34. For each integer n, define $f_n(x) = x + n$, $x \in R$ and let $G = \{f_n : n \text{ is an integer}\}$, then

(A) G does not form a group under composition

(B) G is a cyclic group under composition

(C) G is a non-cyclic group under composition

(D) G is non-abelian group under composition

35. Consider the following two statements :

i. Every group of order 159 is cyclic.

ii. Every group of order 42 has a normal subgroup of order 8. Then

(A) (i) is true (ii) is true

(B) (i) is false (ii) is false

(C) (i) is false (ii) is true

(D) (i) is true (ii) is false

36. A force $5\hat{i} - 2\hat{j} + 3\hat{k}$ acts on a particle with position vector $2\hat{i} + \hat{j} - 2\hat{k}$. The torque of the force about the origin is

(A) $\hat{i} + 16\hat{j} + 9\hat{k}$

(B) $-\hat{i} + 16\hat{j} - 9\hat{k}$

(C) $\hat{i} + 16\hat{j} - 9\hat{k}$

(D) $-\hat{i} - 16\hat{j} - 9\hat{k}$

37. If $\frac{\partial L}{\partial q_n} = 0$, where L is the Lagrangian for a conservative system without constraints and q_n is a generalized coordinate, then the generalized momentum is

(A) A cyclic coordinate

(B) A constant of motion

(C) Equal to $\frac{d}{dt}\left(\frac{dL}{dq_n}\right)$

(D) Undefined

38. A linear transformation of a generalized coordinate q and corresponding momentum p to Q and P are given by Q = q + p; P = q + αp is canonical if the value of the constant α is

(A) –1 (B) 0

(C) 1 (D) 2

39. The Lagrangian for the Kopler problem is given by

$$L = \frac{1}{2}(\dot{r}^2 + r^2\dot{\theta}^2) + \frac{M}{r} (M > 0)$$

Where (r, θ) denote the polar coordinates and the mass of the particle is unity. Then

(A) $P_\theta = 2r^2\dot{\theta}$

(B) $P_r = 2\dot{r}$

(C) The angular momentum of the particle about the centre of attraction is a constant

(D) The total energy of the particle is time dependent

40. The Lagrangian for a three particle system is given by

$$L = \frac{1}{2}(\dot{n}_1^2 + \dot{n}_2^2 + \dot{n}_3^2) - \alpha^2(n_1^2 + n_2^2 + n_3^2 - n_1 n_3)$$

where α is real. Then one of the normal coordinates has a frequency w given by

(A) $w^2 = \alpha^2$

(B) $w^2 = \frac{\alpha^2}{2}$

(C) $w^2 = 2\alpha^2$

(D) $w = \sqrt{2}\alpha^2$

41. In the ring $\mathbb{Z}$, which of the following is not true ?

(A) J = $\{7r : r \in \mathbb{Z}\}$ is a prime ideal

(B) J = $\{7r : r \in \mathbb{Z}\}$ is maximal ideal

(C) K = $\{14r : r \in \mathbb{Z}\}$ is a prime ideal

(D) K = $\{14r : r \in \mathbb{Z}\}$ is not maximal ideal

42. For k $\in$ {9, 10, 11, 12}, if the ring $\mathbb{Z}_k$ is an integral domain, then k =

(A) 9 (B) 10

(C) 11 (D) 12

43. Which of the following statements is not correct ?

(A) The ring Z[x] of polynomials with integers coefficients is an integral domain

(B) The ring Z_p of integers modulo a prime p is an integral domain

(C) The ring of integers is an integral domain

(D) $M_2(Z)$ of 2×2 matrices over integers is an integral domain

44. Which of the following statements is not correct ?

(A) {0, 2, 4} is a subring of the ring Z_6, the integer modulo 6

(B) {a + bi | a, b are integers} is a subring of the complex numbers

(C) The set of diagonal matrices over the set of integers Z is a subring of $M_2(Z)$

(D) If $R = Z \oplus Z \oplus Z$ and $S = \{(a, b, c) \in R;\ a + b = c\}$, then S is a subring of R

45. Which of the following statements is not correct ?

(A) nZ is a prime ideal in Z (set of all integers) if and only if n is prime

(B) There are exactly three maximal ideals in Z_{36}

(C) The ideal $<x^2 + 1>$ is not prime in $Z_2[x]$

(D) The ideal $<x^2 + 1>$ is maximal in R[x] [R[x] is the ring of polynomials over R]

46. The variational problem of extremizing the functional

$$I(y(x)) = \int_0^{2\pi} \left[\left(\frac{d}{dx} y \right)^2 - y^2 \right] dx;$$

$y(0) = 1$, $y(2\pi) = 1$ has

(A) a unique solution

(B) exactly two solutions

(C) exactly three solutions

(D) infinite number of solutions

47. The extremal of the functional $I = \int_0^{x_1} y^2 (y')^2 \, dx$ that passes through (0, 0) and (x_1, y_1) is

(A) part of a parabola

(B) a linear function of x

(C) part of an ellipse

(D) a constant function

48. The infimum of $\int_0^1 (u'(t))^2 \, dt$ on the class of functions $\{u \in C^1[0, 1] : u(0) = 0$ and $\max_{[0, 1]} |u| = 1\}$ is equal to

(A) 0

(B) $1/2$

(C) 2

(D) 1

49. Consider the functional

$$I(y(x)) = \int_{x_0}^{x_1} f(x, y) \sqrt{1 + y'^2} \, e^{\tan^{-1} y'} \, dx$$

where $f(x, y) = 0$. Let the left end of the extremal be fixed at the point $A(x_0, y_0)$ and the right end $B(x_1, y_1)$ be movable along the curve $y = \varphi(x)$. Then the extremal $y = y(x)$ intersects the curve $y = \varphi(x)$ along which the boundary point $B(x_1, y_1)$ slides at an angle

(A) $\pi/4$

(B) $\pi/2$

(C) $\pi/3$

(D) $\pi/6$

50. The resolvent kernel for the integral equation $u(x) = f(x) + \int_{ln2}^{x} e^{t-x} u(t)\,dt$ is

(A) $\cos(x - t)$

(B) 1

(C) $e^{(t-x)}$

(D) $e^{2(t-x)}$

51. If γ is the circle $|z| = \frac{1}{2}$, then $\int_{\gamma} \frac{dz}{(z-1)(z+1)} =$

(A) $2\pi i$

(B) $-2\pi i$

(C) 0

(D) πi

52. The number of zeros of the polynomial $z^{10} - 6z^7 + 3z^3 + 1$ in $|z| < 1$ is

(A) 10

(B) 7

(C) 3

(D) 0

53. The value of the integral $\int_C \bar{z}\,dz$ from $z = 0$ to $z = 4 + 2\pi i$ along the curve C given by $z = t^2 + it$, is

(A) $10 - \frac{8i}{3}$

(B) $8 - \frac{10i}{3}$

(C) $8 + \frac{10i}{3}$

(D) $10 + \frac{8i}{3}$

54. The function $f(z) = e^{1/(z-2)}$ has

(A) removable singularity at $z = 2$

(B) pole at $z = 2$

(C) essential singularity at $z = 2$

(D) non-isolated singularity at $z = 2$

55. If $f(z)$ is analytic within and on the boundary C of a simply connected region D and $a \in D$, then $\int_C \frac{f(z)}{(z-a)^{n+1}} dz =$

(A) $\frac{n!}{2\pi i} f^n(a)$

(B) $\frac{2\pi i}{n!} f^n(a)$

(C) $\frac{2\pi i}{n} f^n(a)$

(D) $f^n(a)$

56. The general solution of the partial differential equation $xzp + yzq = xy$ is $\left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right)$

(A) $\varphi\left(xy - z^2, \frac{x}{y}\right) = 0$

(B) $\varphi\left(xz - y^2, \frac{x}{y}\right) = 0$

(C) $\varphi\left(xz - x^2 - y, \frac{x}{y}\right) = 0$

(D) $\varphi\left(xy - z^2, \frac{y}{z}\right) = 0$

57. The particular integral of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - z\frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$ is a

(A) Bounded function

(B) Unbounded function

(C) Discontinuous function

(D) Continuous everywhere except some finite number of points

58. Using Newton iteration, the second iteration of square root of $2(x_0 = 1)$ is (upto three decimal place)

(A) 1.501

(B) 1.421

(C) 1.427

(D) 1.417

59. The bound on error of approximation in Simpson's rule for integration $\int_a^b f(x)\,dx$ is

(A) $\frac{(b-a)^5}{2880}\max\left\{\left|f^{(iv)}(x)\right| : x \in [a, b]\right\}$

(B) $\frac{(b-a)^5}{2280}\max\left\{\left|f^{(iii)}(x)\right| : x \in [a, b]\right\}$

(C) $\frac{(b-a)^5}{2880}\max\left\{\left|f'(x)\right| : x \in [a, b]\right\}$

(D) $\frac{(b-a)^5}{2880}\max\left\{\left|f''(x)\right| : x \in [a, b]\right\}$

60. Which of the following function is the solution of the Fredholm integral equation ?

$$u(x) + \frac{1}{2}\int_0^1 e^{x-t}\,u(t)\,dt = 2xe^x$$

(A) $u(x) = e^x(2x - 2/3)$

(B) $u(x) = e^x(x^2 - x)$

(C) $u(x) = e^x(x - 3)$

(D) $u(x) = e^x(3x^2 - 3x - 1)$

61. Consider the integral equation

$y(x) = x^3 + \int_0^x \sin(x-t)y(t)dt \; x \in [0, \pi]$.

Then the value of y(1) is

(A) $\frac{19}{20}$

(B) $\frac{21}{20}$

(C) $\frac{31}{20}$

(D) $\frac{17}{20}$

62. If ϕ is the solution of $\int_0^x (1 - x^2 + t^2)\phi(t)dt = \frac{x^2}{2}$, then $\phi(\sqrt{2})$ is equal to

(A) $\sqrt{2e^2}$

(B) $\sqrt{2}\, e^{\sqrt{2}}$

(C) $\sqrt{2e^{2\sqrt{2}}}$

(D) $2e^4$

63. For the homogeneous Fredholm integral equation $\phi(x) = \lambda\int_0^1 e^{x+t}\phi(t)dt$, a non-trivial solution exists, when λ has the value

(A) $\frac{2}{e-1}$

(B) $\frac{2}{e^2-1}$

(C) $\frac{1}{e^2+1}$

(D) $\frac{1}{e+1}$

64. Solution of the Fredholm integral equation $y(s) = s + 2\int_0^1 (st^2 + s^2t)y(t)dt$ is

(A) $y(s) = -(50s + 40s^2)$

(B) $y(s) = -(30s + 40s^2)$

(C) $y(s) = (30s + 40s^2)$

(D) $y(s) = (60s + 50s^2)$

65. A bounded harmonic function in the unit disc centered at origin and taking the value sin 2θ on the boundary is

(A) $\frac{1}{r^2}\sin 2\theta$

(B) $r \sin 2\theta$

(C) $\frac{1}{r}\sin 2\theta$

(D) $r^2 \sin 2\theta$

66. The heat equation and wave equation respectively are given by

(A) $\frac{\partial u}{\partial x} = \frac{1}{k}\frac{\partial u}{\partial t}$; $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$

(B) $\frac{\partial^2 u}{\partial t^2} = \frac{1}{k}\frac{\partial u}{\partial x}$; $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$

(C) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial^2 u}{\partial t^2}$; $\frac{\partial u}{\partial x} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$

(D) $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$; $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k^2}\frac{\partial^2 u}{\partial t^2}$

67. For each eigenvalue of a Strum-Liouville problem, the number of independent eigenfunctions that exists is

(A) equal to 2

(B) equal to 1

(C) greater than 2 but finite

(D) infinite

68. A general solution of the differential equation $xy'' - (2x + 1)y' + (x + 1)y = 0$ in any interval not containing the origin is (where c_1 and c_2 are constants)

(A) $y(x) = c_1e^x + c_2x^2$

(B) $y(x) = c_1e^x + c_2\frac{x^2e^x}{2}$

(C) $y(x) = c_1e^x + c_2e^y$

(D) $y(x) = c_1e^x + c_2$

69. A particle of mass m moves in a force field of potential V. Then the Hamiltonian in rectangular co-ordinates (x, y, z) is

(A) $H = \left(p_x^2 + p_y^2 + p_z^2\right) + V(x, y, z)$

(B) $H = \left(p_x^2 + p_y^2 + p_z^2\right) + \frac{V(x, y, z)}{2m}$

(C) $H = \frac{\left(p_x^2 + p_y^2 + p_z^2\right)}{2m} + \frac{V(x, y, z)}{m}$

(D) $H = \frac{\left(p_x^2 + p_y^2 + p_z^2\right)}{2m} + V(x, y, z)$

70. The initial value problem $\frac{dy}{dx} = y^{\frac{1}{3}}$, $y(0) = 0$ has

(A) Exactly one solution

(B) Exactly two solutions

(C) Exactly three solutions

(D) Infinitely many solutions

71. Let $f : C \to C$ be defined by $f(z) = |z|$, then f is

(A) differentiable only for all $z \neq 0$ but continuous in C

(B) continuous in C but nowhere differentiable

(C) continuous in C and differentiable only for $z = 0$

(D) discontinuous in C

72. Let $f(z) = z^3 + \sin z + 4z^{-2}$, then

(A) f is analytic in C

(B) f is analytic in $\{z : |z| < 1\}$

(C) f is not analytic

(D) $f(z) + \cos z$ is analytic in C

73. If $f(z) = u(x, y) + iv(x, y)$ is analytic with $\frac{\partial u}{\partial y} = -6xy + 6y$, then for some constant c

(A) $v(x, y) = -6xy + c$

(B) $v(x, y) = -6xy + 6y + c$

(C) $v(x, y) = 3x^2y - 6xy + c$

(D) v(x, y) cannot be found out with only this much information

74. Radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n^p} z^n$, where $p > 1$ is

(A) 0

(B) 1

(C) p

(D) ∞

75. Let γ be a simple closed contour. Then $\int_{\gamma} \left\{ e^z - (\sin z + \cos z) \right\} dz =$

(A) $2\pi i$

(B) $\frac{1}{2\pi i}$

(C) 2

(D) 0

76. Let G be a group of order 23. Then the number of proper non-trivial subgroups of G are

(A) 0

(B) 1

(C) 2

(D) 23

77. Under the usual addition and multiplication, the set E of all even integers is

(A) a commutative ring with unity

(B) a commutative ring without unity

(C) not a ring

(D) a field

78. If (Q, +, .) is the ring of rational numbers, then the set I of integers with the above operations forms

(A) subring but not an ideal of Q

(B) an ideal but not a subring of Q

(C) a subring as well as an ideal of Q

(D) neither a subring nor an ideal of Q

79. The set T = {a, b, c, d} with addition and multiplication defined by

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

.	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	c	a	c
d	a	d	a	d

is

(A) not a ring

(B) commutative ring

(C) non-commutative ring

(D) ring with multiplicative identity element

80. Let $G = \{a + ib \mid a, b \in \mathbb{Z}\}$ be the Gaussian ring of integers, under usual addition and multiplication. Then the residue classes modulo 3 of G is

(A) K = {[0], [1], [i], [2], [1 + i], [2 + i], [1 + 2i], [2 + 2i]}

(B) K = {[0], [1], [2], [1 + i], [2 + i], [1 + 2i]}

(C) K = {[0], [1], [2], [3], [1 + i], [2 + i], [3 + i]}

(D) K = {[1], [2], [3], [1 + i], [2 + i], [3 + i]}

81. The solutions to the differential equation $2xy\frac{dy}{dx} = y^2 - x^2$ represents

(A) family of circles having centres at origin

(B) family of circles with centres on y-axis and all passing through origin

(C) family of circles with centres on x-axis and all passing through origin

(D) family of parabolas with focus on x-axis

82. The solution of the differential equation $\frac{dy}{dx} - Sy = -Ty^2$ (S, T > 0)

(A) $y = \dfrac{1}{\left(\frac{T}{S}\right) + Pe^{-Sx}}$

(B) $y = \dfrac{P}{T}e^{-Sx}$

(C) $y = \dfrac{P}{T}\,\dfrac{1}{1 + e^{-Sx}}$

(D) $y = \dfrac{P}{\left(\frac{S}{T}\right) + e^{-Tx}}$

83. Solution of $\frac{d^2y}{dx^2} + y = \sec x$ using method of variation of parameters satisfying y(0) = 0, y(π/2) = π is

(A) $y = \log|\cos x| \cos x + 2x \sin x$

(B) $y = \log|\sec x| \sec x + \left(\frac{\pi}{2} + x\right)\sin x$

(C) $y = \log|\cos x| \cos x + \left(\frac{\pi}{2} + x\right)\sin x$

(D) $y = \log|\sec x| \cos x + 2x \sin x$

84. Consider the Strum – Liouville problem

$$\frac{d^2y}{dx^2} + \lambda y = 0\,;\ y(0) = 0,\ y(\pi) = 0$$

and consider the following statements :

i. $\lambda = n^2$, n = 1, 2, ... are characteristic values of the problem under consideration.

ii. Characteristic functions are $C_n \sin nx$ ($C_n \neq 0$) corresponding to $\lambda_n = n^2$.

iii. $\lambda \in N \sim \{n^2, n \in N\}$ are characteristic values.

iv. Characteristic functions are $C_n \cos nx$ ($C_n \neq 0$) corresponding to $\lambda_n \in N \sim \{n^2, n \in N\}$.

Which of the following options is true ?

(A) Only (i) and (ii) are true

(B) Only (ii) and (iv) are true

(C) Only (i), (ii) and (iii) are true

(D) Only (iii) and (iv) are true

85. Solution of $(x-4)\, y^4 dx - x^3(y^2-3)\, dy = 0$, $y(1) = 1$ is

(A) $-\frac{1}{x} + \frac{2}{x^2} - \frac{1}{y} + \frac{1}{y^3} = 1$

(B) $-\frac{1}{x} + \frac{2}{x^2} + \frac{1}{y} - \frac{1}{y^3} = 1$

(C) $-\frac{2}{x} + \frac{3}{x^2} + \frac{1}{y} - \frac{1}{y^3} = 1$

(D) $\frac{2}{x} - \frac{3}{x^2} + \frac{(y^2-1)}{y^3} + 1 = 0$

86. If D_n is the dihedral group and $Z(D_n)$ is its centre, then

(A) $Z(D_5)$ has only two elements

(B) $Z(D_7)$ has only two elements

(C) $Z(D_8)$ has only one element

(D) $Z(D_{10})$ has exactly two elements

87. Let (1, 4, 5, 6) and (2, 1, 5) be cycles in the group S_6 of all permutations of {1, 2, 3, 4, 5, 6}. Then (1, 4, 5, 6) (2, 1, 5) =

(A) (2, 1, 5) (1, 4, 5, 6)

(B) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 5 & 6 & 2 & 5 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 4 & 5 & 6 & 2 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 6 & 5 \end{pmatrix}$

88. The permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 2 & 4 & 3 & 1 \end{pmatrix}$ can be written as product of disjoint cycles as

(A) (1, 6) (2, 5, 3)

(B) (1, 6) (2, 5) (3, 2) (4, 4) (5, 3) (6, 1)

(C) (6, 5, 2) (4, 3, 1)

(D) (4, 3, 1) (6, 5, 2)

89. If a and b are elements of a non-abelian group G, then for any integer n, $(b\,a\,b^{-1})^n =$

(A) $b^n a^n b^{-n}$

(B) $b\,a^n b^{-1}$

(C) $(b^{-1})^n a\,b^n$

(D) a^n

90. A group of order 17 is

(A) cyclic but not abelian

(B) cyclic and also abelian

(C) abelian but not cyclic

(D) neither cyclic nor abelian

91. The initial value problem

$$\frac{d^2y}{dx^2} + xy = 1,\ y(0) = y'(0) = 0$$

is equivalent to the Volterra's integral equation

(A) $y(x) = 1 - \int_0^x x(t+x)\,y(t)\,dt$

(B) $y(x) = 1 - \int_0^x x(t-x)\,y(t)\,dt$

(C) $y(x) = 1 - \int_0^x x(x-t)\,y(t)\,dt$

(D) $y(x) = 1 + \int_0^x xty(t)\,dt$

92. The Lagrange's equation of motion of the bob of a simple pendulum is

(l being the length of the pendulum)

(A) $\ddot{\theta} + \left(\frac{1}{l}\right)\sin\theta = 1$

(B) $\ddot{\theta} + \left(\frac{2g}{l}\right)\sin\theta = 0$

(C) $\ddot{\theta} + \left(\frac{2g}{l}\right)\cos\theta = 0$

(D) $\ddot{\theta} + \left(\frac{g}{l}\right)\sin\theta = 0$

93. A simple pendulum has a bob of mass m with a mass m_1, at the moving support (pendulum with moving support) which moves on a horizontal line in the vertical plane in which the pendulum oscillates. The Lagrangian equation of motion (x and θ are the generalized co-ordinates and l being the length of pendulum)

(A) $(m + m_1)\,\ddot{x} + ml\,(\sin\theta)\,\ddot{\theta} - ml\,(\cos\theta)\,\dot{\theta}^2 = 0$

(B) $(m + m_1)\,\ddot{x} + ml\,(\cos\theta)\,\ddot{\theta} - ml\,(\sin\theta)\,\dot{\theta} = 0$

(C) $(m + m_1)\,\ddot{x} - ml\,(\cos\theta)\,\ddot{\theta} + ml\,(\sin\theta)\,\dot{\theta}^2 = 0$

(D) $(m + m_1)\,\ddot{x} - ml\,(\sin\theta)\,\ddot{\theta} + ml\,(\cos\theta)\,\dot{\theta}^2 = 0$

94. The Lagrangian of a simple pendulum of length L of mass m is given

by $L = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl\,(1 - \cos\theta)$.

Hamiltonian equation are

(A) $\dot{P}_\theta = -mgl\sin\theta,\ \dot{\theta} = \frac{P_\theta}{ml^2}$

(B) $\dot{P}_\theta = mgl\sin\theta,\ \dot{\theta} = \frac{P_\theta}{ml^2}$

(C) $\dot{P}_\theta = -m\dot{\theta},\ \dot{\theta} = \frac{P_\theta}{ml}$

(D) $\dot{P}_\theta = -\left(\frac{g}{l}\right)\theta,\ \dot{\theta} = \frac{P_\theta}{ml}$

95. A solution u(x), v(x) to the following system of equations

$u' + u - v = 0$

$v' - u + v = 2$

$u(0) = 1, v(0) = 2$ is

(A) $u(x) = 1 + x, v(x) = 2 + x$

(B) $u(x) = 1 - x, v(x) = 2 - x$

(C) $u(x) = 1 + x, v(x) = 2 - x$

(D) $u(x) = 1 - x, v(x) = 2 + x$

96. The bilinear transformation which maps $z = 0, -i, -1$ into $w = i, 1, 0$ is

(A) $i\frac{z+1}{z-1}$

(B) $-i\frac{z+1}{z-1}$

(C) $i\frac{z-1}{z+1}$

(D) $\frac{z+1}{z-1}$

97. Taylor's series representation of the function $f(z) = \frac{z-1}{z+1}$ about $z = 0$ is

(A) $1-2\sum_{n=0}^{\infty}(-1)^n z^n$

(B) $1+2\sum_{n=0}^{\infty}(-1)^n z^n$

(C) $1-2\sum_{n=0}^{\infty} z^n$

(D) $1-\sum_{n=0}^{\infty}(-1)^n z^n$

98. The Laurent series expansion of $f(z) = \frac{z-1}{z+1}$ in $1 < |z| < \infty$ is

(A) $1-\frac{2}{z}\sum_{n=0}^{\infty}\frac{1}{z^n}$

(B) $1-\frac{2}{z}\sum_{n=0}^{\infty}(-1)^n\frac{1}{z^n}$

(C) $\frac{2}{z}\sum_{n=0}^{\infty}(-1)^n\frac{1}{z^n}$

(D) $1-\sum_{n=0}^{\infty}(-1)^n\frac{1}{z^n}$

99. The function $f(z) = \frac{(z-2)}{z^2}\sin\left(\frac{1}{z-1}\right)$ has

(A) $z = 0$ as pole of order 2 and simple pole at $z = 1$

(B) $z = 0$ as pole of order 2 and isolated essential singularity at $z = 1$

(C) $z = 0$ as pole of order 2 and non-isolated singularity at $z = 1$

(D) no singularities

100. The residues of $\frac{z+3}{z^2-2z}$ at $z = 0$ and $z = 2$ respectively are

(A) $-\frac{3}{2}, \frac{5}{2}$

(B) $\frac{3}{2}, \frac{5}{2}$

(C) $\frac{3}{2}, -\frac{5}{2}$

(D) $\frac{2}{5}, -\frac{2}{3}$

SECTION – C

101. What is the probability of extinction of a Galton Watson branching process with offspring distribution $P_0 = \frac{1}{4}, P_1 = \frac{1}{4}, P_2 = \frac{1}{2}$?

(A) 1 (B) $\frac{3}{4}$

(C) $\frac{1}{2}$ (D) $\frac{1}{4}$

102. What is the stationary distribution of a Markov chain with transition probability matrix $\begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix}$?

(A) $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$

(B) $\left(0, \frac{1}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{2}{3}, 0, \frac{1}{3}\right)$

(D) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

103. If $\{X(t), t > 0\}$ is a Birth-Death process with birth rate $\lambda_n = n\lambda$ and death rate $\mu_n = n\mu$, then what is the distribution of the Sojourn time of the process at state 1 ?

(A) Exponential with mean μ^{-1}

(B) Exponential with mean λ^{-1}

(C) Exponential with mean $(\mu + \lambda)^{-1}$

(D) Exponential with mean $\lambda(\mu + \lambda)^{-1}$

104. Consider an aperiodic Markov chain with state space S and with Transition Probability Matrix (TPM), $P = ((P_{ij}))$, $i, j \in S$. Let the n-step TPM be denoted by $P^n = ((P^n_{ij}))$, $i, j \in S$. Then which of the following statement is true ?

(A) $\lim_{n\to\infty} P^n_{ii} = 0$ if the state i is transient

(B) $\lim_{n\to\infty} P^n_{ii} > 0$ if and only if the state i is recurrent

(C) $\lim_{n\to\infty} P^n_{ij} = \lim_{n\to\infty} P^n_{ji}$ if i and j are in the same communicating class

(D) $\lim_{n\to\infty} P^n_{ii} = \lim_{n\to\infty} P^n_{jj}$ if i and j are in the same communicating class

105. Which of the following statements is not true ?

(A) Homogeneous Poisson process is Markovian

(B) Inter-arrival time between Poisson events has an exponential distribution

(C) Difference of two independent Poisson processes is again a Poisson process

(D) Poisson process is an example of a renewal process

106. Which one of the following is viewed as a multivariate version of chi-square distribution ?

(A) Wishart

(B) Multivariate normal

(C) Hotelling's T^2-distribution

(D) Mardia's distribution

107. Which one of the following is a method of classification for an observation into one or more known probability distribution(s) ?
(A) Method of principal components
(B) Discriminant analysis
(C) Regression analysis
(D) Correlation analysis

108. Which one of the following is a method of dimension reduction of the data ?
(A) Simple random sampling
(B) Stratified sampling
(C) Principal component analysis
(D) *k*-means method of cluster analysis

109. If X is a random variable with distribution function F(.), then what is the distribution of F(X) ?
(A) Normal (B) Uniform
(C) Exponential (D) Beta

110. Which one of the following distributions possesses a bath tub shaped hazard function ?
(A) Exponential (B) Pareto
(C) Weibull (D) Uniform

111. What type of testing problems are described by Neyman-Pearson fundamental lemma ?
(A) Testing simple null hypothesis against composite alternative hypothesis
(B) Testing composite null hypothesis against composite alternative hypothesis
(C) Testing simple null hypothesis against simple alternative hypothesis
(D) Testing of any arbitrary hypotheses

112. What is the advantage of forming the homogeneous blocks in the design of experiments ?
(A) For increasing the mean effect
(B) For increase of the treatment effect
(C) For controlling the error
(D) For making the execution of design easy

113. Let X be a random variable following a standard Cauchy distribution. Which of the following is the characteristic function of X ?
(A) $f(t) = \frac{1}{1+t^2}$ (B) $f(t) = e^{-|t|}$
(C) $f(t) = e^{-t^2}$ (D) $f(t) = e^{-t^4}$

114. If $\{A_n\}$ is a sequence of independent events and $\sum_{n=1}^{\infty} P(A_n) = \infty$, then which is the correct statement ?
(A) $P(\lim\sup A_n) = 1$
(B) $P(\lim\sup A_n) = 0$
(C) $P(\lim\inf A_n) = 1$
(D) $P(\lim\inf A_n) = 0$

115. In a simple linear regression model y = a + bx + e, where e is a random error, what is the correct hypothesis to test for "significance of regression" ?
(A) $H_0 : a = 0$ versus $H_1 : a \neq 0$
(B) $H_0 : a > 0$ versus $H_1 : a < 0$
(C) $H_0 : b > 0$ versus $H_1 : b < 0$
(D) $H_0 : b = 0$ versus $H_1 : b \neq 0$

116. Read the following two statements and then identify the correct answer of the question.

i. Every field is a sigma field.

ii. Power set of a sample space is a sigma field.

Which one of the following is correct ?

(A) (i) is true and (ii) is wrong

(B) (ii) is true and (i) is wrong

(C) Both (i) and (ii) are wrong

(D) Both (i) and (ii) are true

117. Answer the question based on the following statements.

i. The distribution function is a probability measure induced by a random variable.

ii. The random variable is a real-valued measurable function.

Which one of the following is the correct ?

(A) (i) is true and (ii) is wrong

(B) (ii) is true and (i) is wrong

(C) Both (i) and (ii) are wrong

(D) Both (i) and (ii) are true

118. In a 2^n factorial design, which of the following is the correct option ?

(A) It is a factorial experiment with 2 factors with n levels

(B) It is a factorial experiment with n factors with 2 levels

(C) It is a factorial experiment with 2^n factors

(D) It is a factorial experiment with 2^n levels

119. When do you say that an estimator T_n is unbiased for a parameter θ ?

(A) If $E(T_n) = \theta$ for a specific θ

(B) If $E(T_n) = \theta$ for some θ in the parameter space

(C) If $E(T_n) = \theta$ for every θ in the parameter space

(D) If $E(T_n) \to \theta$ for a specific θ

120. When do you say that an estimator T_n is consistent estimator of θ ?

(A) If T_n converges in probability to θ

(B) If $E(T_n) = \theta$ for every n

(C) If $E(T_n) \to \theta$ as n tends to infinity

(D) If T_n converges in distribution to normal

121. Which one of the following statements is not true for a uniform distribution ?

(A) It is a finite range distribution.

(B) It is not a member of exponential family.

(C) It is infinitely divisible.

(D) It is a special case of beta distribution.

122. A sufficient condition for the existence of a stationary distribution for a Markov chain is that

(A) All its states are periodic

(B) All its states are null

(C) None of the states is recurrent

(D) All its states are ergodic

123. For a negatively skewed distribution of a set of observations, the correct relation between mean, median and mode is

(A) mean = median = mode

(B) mean < median < mode

(C) median < mean < mode

(D) mode < mean < median

124. Let the random variable X follows uniform distribution $U\left(\theta - \frac{1}{2}, \theta + \frac{1}{2}\right)$.

Let $y_{(1)} = \min(X_1, X_2, ..., X_n)$ and $y_{(n)} = \max(X_1, X_2, ..., X_n)$. The maximum likelihood estimator, denoted as $\hat{\theta}$ is

(A) Any statistic satisfying $y_{(n)} \leq \hat{\theta} y_{(1)}$

(B) Any statistic satisfying $\hat{\theta} \leq y_{(1)} + \frac{1}{2}$

(C) Any statistic satisfying $y_{(n)} - \frac{1}{2} \leq \hat{\theta}$

(D) Any statistic satisfying $y_{(n)} - \frac{1}{2} \leq \hat{\theta} \leq y_{(1)} + \frac{1}{2}$

125. If an estimator T is consistent for θ and $f(\theta)$ is a real-valued function, then

(A) f(T) is consistent for $f(\theta)$ for any arbitrary function f

(B) f(T) is consistent for $f(\theta)$ for any continuous function f

(C) f(T) is consistent for $f(\theta)$ only for a differentiable function f

(D) f(T) can't be consistent for $f(\theta)$ for any function f

126. For a random variable X with mean μ and finite variance, identify the correct form of the Chebyshev's inequality where a > 0.

(A) $P(|X - \mu| < a) < \frac{E(X-\mu)^2}{a^2}$

(B) $P(|X - \mu| < a) > \frac{E(X-\mu)^2}{a^2}$

(C) $P(|X - \mu| > a) > \frac{E(X-\mu)^2}{a^2}$

(D) $P(|X - \mu| > a) < \frac{E(X-\mu)^2}{a^2}$

127. Which one of the following statements is not true ?

(A) Convergence almost surely implies convergence in probability

(B) Convergence in distribution always implies convergence in probability

(C) Convergence in mean implies convergence in probability

(D) Convergence in probability implies convergence in distribution

128. Let $\{X_n\}$ be a sequence of random variables with probability mass functions

$P(X_n = 0) = 1 - \frac{1}{n}$ and $P(X_n = n) = \frac{1}{n}$,

for n = 1, 2, . . .

Then identify the correct statement

(A) $\{X_n\}$ does not converge in probability but in converges in means square

(B) $\{X_n\}$ converges in probability and also converges in means square

(C) $\{X_n\}$ converges in probability and does not converge in means square

(D) $\{X_n\}$ neither converges in probability nor converge in means square

129. Which one of the following statements is not true for Lindberg-Feller Central Limit Theorem ?

(A) It requires the existence of third moments

(B) It requires finiteness of variance

(C) It is proved for independent random variables

(D) It gives a necessary and sufficient condition

130. What is the maximum likelihood estimator of m based on a random sample of size n from a pdf

$$f(x, m) = \begin{cases} e^{-(x-m)} & \text{if } x \geq m \\ 0, & \text{otherwise} \end{cases} ?$$

(A) Maximum of the sample observations

(B) Minimum of the sample observations

(C) Sample mean

(D) Sample median

131. Suppose the eigenvalues of a sample covariance matrix are 10, 15 and 25. Then the variance carried by the first principle component is

(A) 10%

(B) 15%

(C) 25%

(D) 50%

132. Let X and Y be two independent identically and independently distributed continuous random variables. Then X + Y and X – Y are independent if and only if X and Y have

(A) Normal distribution

(B) Exponential distribution

(C) Cauchy distribution

(D) Laplace distribution

133. In an M/M/1 queue, what is the distribution of the interval between the arrival of two successive customers ?

(A) Gamma distribution

(B) Erlang distribution

(C) Exponential distribution

(D) Poisson distribution

134. What is the arrival process in an M/G/1 queue discipline ?

(A) General (B) Poisson

(C) Erlang (D) Degenerate

135. In the context of a queueing model, let λ = Arrival rate, L = Expected number of units in the system and W = Expected waiting time in the system in steady state. Then the Little's formula is

(A) $\lambda = L\,W$ (B) $L = \lambda\,W$

(C) $W = L\,\lambda$ (D) $L = \lambda^2\,W$

136. If X follows normal distribution and Y follows log-normal distribution, then what is the correct relationship between X and Y ?

(A) X = log (Y) (B) X = exp (Y)

(C) X = 1/Y (D) $X = Y^2$

137. Suppose that in an independent Bernoulli trial the *P*(success) = *P*(failure) = $\frac{1}{2}$. If the experiment is conducted with *n* trials, then the probability that there is *k* successes is

(A) $\left(\frac{1}{2}\right)^n$ (B) $\binom{n}{k}\left(\frac{1}{2}\right)^n$

(C) $\binom{n}{k}\left(\frac{1}{2}\right)^{n-k}$ (D) $\left(\frac{1}{2}\right)^{n-k}$

138. Identify the correct statement.

(A) Maximum likelihood estimators are always unbiased estimators of the parameter.

(B) Maximum likelihood estimators are always inconsistent estimators of the parameter.

(C) Moment estimators are consistent estimators of the parameter, if exist.

(D) Moment estimators of the parameter always uniformly minimum variance unbiased estimators.

139. In a Sequential Probability Ratio Test (SPRT) with the stopping bound (a, b) and strength (α, β), which is the correct condition ?

(A) $a \le \frac{1-\alpha}{\alpha}$ and $b \ge \frac{\beta}{1-\beta}$

(B) $a \le \frac{1-\beta}{\alpha}$ and $b \ge \frac{\beta}{1-\beta}$

(C) $a \le \frac{\beta}{1-\alpha}$ and $b \ge \frac{1-\alpha}{\beta}$

(D) $a \le \frac{\alpha}{1-\beta}$ and $b \ge \frac{\beta}{1-\beta}$

140. Type I error is the probability of

(A) Not rejecting H_0 when it is false

(B) Rejecting H_0 when it is false

(C) Not rejecting H_0 when it is true

(D) Rejecting H_0 when it is true

141. Identify the wrong statement.

(A) Transportation problem is a Linear Programming Problem (LPP).

(B) Vogel method gives the optimum solution for a transportation problem.

(C) Solution set for an LPP is a convex set.

(D) Dual of dual is primal.

142. Consider a BIBD with parameters (v, b, r, k, λ), where v denotes the number of treatments, b denotes the number of blocks, r denotes the number of replications, k denotes the number of plots in a block, and λ denotes the number of times a pair of treatment occurs in the blocks. Then, under usual notations, which of the following are the correct statements ?

i. bk = vr

ii. $\lambda(v-1) = k(r-1)$

iii. $b \ge v$

(A) Only (i) and (iii)

(B) Only (ii) and (iii)

(C) Only (i) and (ii)

(D) (i), (ii) and (iii)

143. Queuing process is an example of which one of the following ?

(A) Binomial process

(B) Poisson process

(C) Birth-death process

(D) Branching process

144. Which of the following is not true for Latin square designs with p rows ?

(A) The number of columns is p

(B) The replication of a treatment is p

(C) The degrees of freedom corresponding to error sum of squares is p – 1

(D) It eliminates two sources of variability

145. If X_1, X_2, X_3 are three variables, the partial correlation between X_2 and X_3 eliminating the effect of X_1 in terms of simple correlation coefficient is given by the which of the following statements ?

i. $r_{23.1} = \dfrac{r_{23} - r_{12}r_{13}}{\sqrt{\left(1-r_{12}^2\right)\left(1-r_{13}^2\right)}}$

ii. $r_{23.1} = \dfrac{r_{32} - r_{21}r_{31}}{\sqrt{\left(1-r_{21}^2\right)\left(1-r_{31}^2\right)}}$

iii. $r_{23.1} = \dfrac{r_{32} - r_{12}r_{13}}{\sqrt{\left(1-r_{21}^2\right)\left(1-r_{31}^2\right)}}$

(A) Only (i)

(B) Only (ii) and (iii)

(C) Only (i) and (ii)

(D) (i), (ii) and (iii)

146. Which of the following is true for type 2 censoring ?

(A) Number of failed units is fixed

(B) Number of failed units is random

(C) The study period is fixed

(D) Number of censored units is random

147. Three customers simultaneously approach a gas station with two filling pumps. The first and second customers occupy booths 1 and 2 respectively and the third customer must wait. Suppose that the lengths of time T_1, T_2, T_3 taken by the customers are independent and exponentially distributed with parameter μ, then the probability that the third customer gets in to booth 1 is

(A) 1/3 (B) 2/3

(C) 1/2 (D) 0

148. Which of the following is not true for randomized block designs ?

(A) The number of replications in each block is the same

(B) The number of replications for different treatments is the same

(C) It eliminates three sources of variations

(D) It has two-way ANOVA model

149. Confounding is useful in design of experiment when

(A) Block size is less than number of treatment combinations in one replication

(B) Block size is greater than number of treatment combinations in one replication

(C) Block size is less than number of replications in each treatment combination

(D) Block size is greater than number of replications in each treatment combination

150. A population is divided into two strata. From first stratum a 50% sample is drawn while the second stratum is completely enumerated. The allocations is optimum when population variances S_1^2 and S_2^2 of two strata are related as

(A) $S_2^2 \leq S_1^2$

(B) $4S_1^2 = S_2^2$

(C) $2S_2^2 \leq S_1^2$

(D) $S_2^2 > 2S_1^2$

151. Suppose that in a service station, customers arrive at a rate of 8 per hour and each customer takes an average 6 minutes for service. Then what is the traffic intensity ?

(A) 6/8 (B) 8/10

(C) 8/6 (D) 1

152. Identify a statement which is not true for a Linear Programming Problem (LPP) ?

(A) Set of all feasible solutions is a convex set.

(B) Transportation problem is a special type of LPP.

(C) LPP is not an optimization problem.

(D) It can be solved using simplex method.

153. For the following linear programming problem
Maximize : $Z = x_1 + 2x_2$
Subject to the constraints :
$x_1 + x_2 \geq 1$, $x_1 + 2x_2 \leq 10$,
$x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$, which is the correct statement ?

(A) Optimum solution is unique.

(B) Optimum solution exists but not unique.

(C) Optimum solution is unbounded.

(D) Optimum solution does not exist.

154. When does multicollinearity arise in linear regression models ?

(A) When the regressors are linearly independent

(B) When the regressors are independent random variables

(C) When the regressors are linearly dependent

(D) When the output variables are discrete

155. What is the role of Durbin-Watson test in regression analysis ?

(A) Testing the presence of multicollinearity

(B) Testing the independence of regressors

(C) Testing the linearity of model

(D) Testing the presence of first order autocorrelation

156. Read the following statements.

i. The sum of squares of deviations is minimum when it is measured from the median.

ii. The sum of squares of deviations is minimum when it is measured from the mean.

iii. The mean absolute deviation is minimum when it is measured from the median.

iv. The mean absolute deviation is minimum when it is measured from the mean.

Choose the correct answer :

(A) Only the statements (i) and (iv) are correct

(B) Only the statement (i) is correct

(C) Only the statements (ii) and (iii) are correct

(D) Only the statement (iv) is correct

157. Let A and B be non-null independent events. Then which of the following statement is not correct ?

(A) A^c and B^c are independent

(B) A^c and B^c are dependent

(C) A and B^c are independent

(D) A^c and B are independent

158. For a set of positive real numbers, identify the correct inequality among the Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic Mean (HM)

(A) AM < HM < GM

(B) HM < GM < AM

(C) GM < HM < AM

(D) AM < GM < HM

159. Which of the following is correct for a symmetric unimodal probability distribution whose mean exists ?

(A) Mean < Median < Mode

(B) Median < Mean < Mode

(C) Mode < Median < Mean

(D) Mean = Median = Mode

160. In the two lists given below, List – I provides the list of continuous probability distributions, while List – II indicates some of the properties possessed by them. Match the two lists and choose the correct answer from the alternatives given below :

List – I (Continuous Probability Distributions)	List – II (Properties)
a. Laplace distribution	i. X^2 follows Chi-square distribution
b. Standard normal distribution	ii. Mean does not exist
c. Cauchy distribution	iii. Lack of memory property
d. Exponential distribution	iv. MLE is the median of the location parameter

	a	b	c	d
(A)	i	ii	iii	iv
(B)	ii	iii	iv	i
(C)	iii	iv	i	ii
(D)	iv	i	ii	iii

161. When does a Markov chain become irreducible ?

(A) If all states communicate each other

(B) If all states are absorbing

(C) If its transition probability matrix is non-singular

(D) If its states do not communicate each other

162. Let $(X_1, X_2, ..., X_n)$ be a multivariate normal random vector. Then which of the following statement is not true ?

(A) All marginal variables follow univariate normal distribution

(B) All linear combinations of the marginal variables follow univariate normal distribution

(C) If any two marginal variables are uncorrelated then they are not necessarily independent

(D) The sub-vector (X_1, X_5, X_8) follows a tri-variate normal distribution

163. If X and Y are two negatively correlated random variables, then which one of the following statements is correct ?

(A) Var (X + Y) = Var (X) + Var (Y)

(B) Var (X + Y) < Var (X) + Var (Y)

(C) Var (X + Y) > Var (X) + Var (Y)

(D) Var (X + Y) = Var (X) = Var (Y)

164. Which one of the following statements is correct for the power of an unbiased test ?

(A) Power is least the size

(B) Power is at most the size

(C) Power is always 1

(D) Power is either 0 or 1

165. Which of the following tests is used for testing the hypothesis about symmetry of a distribution ?

(A) Wilcoxon signed rank test

(B) Kolmogorov-Smirnov test

(C) Chi-square test

(D) Likelihood ratio test

166. Let $X_1, X_2, ..., X_n$ be i.i.d. random variables from a Poisson distribution with mean λ and $T_n = X_1 + X_2 + ... + X_n$. Then which one of the following properties is not true ?

(A) $\frac{T_n}{n}$ is the UMVUE of λ

(B) T_n is unbiased but not UMVUE for λ

(C) T_n is consistent for λ

(D) T_n is CAN (Consistent Asymptotic Normal) for λ

167. Consider the setup of a randomized block design with b blocks, v treatments and the observation are independently drawn from normal distribution with variance σ^2. Let S_e^2 be the sum of square due to errors. Then under the null hypothesis of the equality of treatment effects

(A) $E(S_e^2) = \sigma^2$

(B) $\frac{E(S_e^2)}{(b-1)(v-1)} = \sigma^2$

(C) $E(S_e^2) = \frac{\sigma^2}{(b-1)(v-1)}$

(D) $\frac{E(S_e^2)}{(b-1)(v-1)} = \sigma^2 + \text{Sum of squares}$ due to treatments

168. Which one of the following is suitable for testing the independence of two random variables ?

(A) Kolmogorov-Smirnov test

(B) Chi-square test

(C) Shapiro test

(D) Median test

169. If prior information about the parameters in a model are available, then which method of estimation is preferred ?

(A) Bootstrapping

(B) Non-parametric

(C) Bayesian

(D) Non-statistical

170. If the response variables in a regression analysis are binary, then which model is suitable ?

(A) Simple linear regression

(B) Multiple linear regression

(C) Polynomial regression

(D) Logistic regression

171. Suppose that a system consists of two parallel components having survival times X and Y respectively. Then what is the survival time of the system ?

(A) Maximum of X and Y

(B) Minimum of X and Y

(C) Sum of X and Y

(D) The difference between X and Y

172. Given the following two statements.

I. Gamma distribution is not infinitely divisible.

II. Normal distribution is a member of an exponential family.

Identify the correct answer :

(A) Statement I is wrong and statement II is correct

(B) Both statements I and II are wrong

(C) Both statements I and II are correct

(D) Statement II is wrong and statement I is correct

173. After fitting a linear regression model to a set of data if you get the value of $R^2 = 0.95$, then what does this indicate ?

(A) 95% of the observation is enough to make an inference

(B) 95% of the variation in the data is explained by the fitted model

(C) 95% of the time the model will be rejected

(D) 95% more observations are needed to make a conclusion

174. When do you say that a parameter μ is estimable ?

(A) If there exists a consistent estimator for μ

(B) If there exists an unbiased estimator for μ

(C) If the estimator of μ has finite variance

(D) If the estimator of μ is complete

175. In the two lists given below, List – I provides the list of discrete probability distributions, while List – II indicates some of the properties possessed by them. Match the two lists and choose the correct answer from the codes given below :

List – I (Discrete Probability Distributions)	**List – II (Properties)**
a. Poisson	i. Lack of memory property
b. Binomial	ii. Mean = variance
c. Negative binomial	iii. Variance less than the mean
d. Geometric	iv. Mean less than variance

Codes :

	a	**b**	**c**	**d**
(A)	i	ii	iii	iv
(B)	ii	iii	iv	i
(C)	iii	iv	i	ii
(D)	iv	i	ii	iii

Space for Rough Work

Space for Rough Work

Space for Rough Work