Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series **X**

PAPER–II **Question Booklet No.**

(Identical with OMR Answer Sheet Number)

Subject Code : 15

MATHEMATICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet No. is identical with OMR Answer Sheet No.; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

PAPER II

(MATHEMATICAL SCIENCES)

1. If $u_1 = \sqrt{6}$ and $u_{n+1} = \sqrt{6 + u_n}$ for $n \geq 1$, then the sequence $\{u_n\}$ converges to

(A) 0

(B) 3

(C) 6

(D) 9

2. The radius of convergence of $\sum_{n=0}^{\infty} a_n z^n$, where $a_n = \cos\frac{n\pi}{4}$ is

(A) larger than 1.

(B) smaller than 1.

(C) equal to 1.

(D) equal to 0.

3. Let A be a matrix with complex entries. If A is Hermitian as well as unitary and α is an eigenvalue of A, then

(A) α can be any real number.

(B) $\alpha = 1$ or $\alpha = -1$.

(C) α can be any complex number of absolute value 1.

(D) α will have unique value 1.

4. Let $S = \{(0, 1, \alpha), (\alpha, 1, 0), (1, \alpha, 1)\}$. Then S is a basis for $\mathbb{R}^3(\mathbb{R})$ if and only if

(A) $\alpha \neq 0$

(B) $\alpha \neq 1$

(C) $\alpha \neq 0$ and $\alpha^2 \neq 2$

(D) $-1 \leq \alpha \leq 1$

5. If x is a feasible solution to the primal *max* $z = cx$, subject to $Ax \leq b$, $x \geq 0$ and v be the feasible solution to the dual problem *min* $u = b^T v$ subject to $A^T v \geq c^T$, $v \geq 0$, then

(A) $cx = b^T v$

(B) $cx \geq b^T v$

(C) $cx \leq b^T v$

(D) $c^T x = bv$

6. Consider the linear programming problem:

max $z = \alpha x_1 + x_2$

subject to $2x_1 + x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_1 + x_2 \leq 4$

$x_1, x_2 \geq 0$,

where α is a constant. If $(3, 0)$ is the only optimal solution, then

(A) $\alpha < -2$

(B) $-2 < \alpha < 0$

(C) $\alpha > 2$

(D) $\alpha < 2$

7. If in the Rolle's theorem, the third condition (*i.e.* $f(a) = f(b)$) is replaced by $\frac{f(a)}{a} = \frac{f(b)}{b}$ $(a, b \neq 0)$, then

(A) the conclusion of the Rolle's theorem remains true.

(B) $f'(\xi) = \xi$, $a < \xi < b$.

(C) $\frac{f'(\xi)}{f(\xi)} = \xi$, $a < \xi < b$.

(D) $\xi f'(\xi) = f(\xi)$.

8. The function $f(x) = \sin\left(\frac{x}{2}\right)$, $x \in [0, \pi]$ is

(A) monotonically increasing in $[0, \pi]$.

(B) monotonically decreasing in $[0, \pi]$.

(C) monotonically increasing in $\left[0, \frac{\pi}{2}\right]$ and then decreasing in $\left[\frac{\pi}{2}, \pi\right]$.

(D) neither monotonically increasing nor decreasing.

9. For $\alpha > 0$ and $\beta > 0$, the function

$$f(x) = \begin{cases} x^{\alpha} \sin \dfrac{1}{x^{\beta}}, & \text{when } x \neq 0 \\ 0 \quad , & \text{when } x = 0 \end{cases}$$

is of bounded variation in [0, 1] if

(A) $\alpha > \beta$

(B) $\alpha < \beta$

(C) $1 + \alpha < \beta$

(D) $1 - \alpha > \beta$

10. Let $f(z) = \begin{cases} \dfrac{\overline{z}^2}{z} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$,

then

(A) Cauchy–Riemann equations are satisfied at the origin and $f'(0) = 0$.

(B) Cauchy–Riemann equations are satisfied at the origin but $f'(0)$ does not exist.

(C) Cauchy–Riemann equations are not satisfied at the origin.

(D) Cauchy–Riemann equations are not satisfied at the origin and $f'(0) = 0$.

11. Inverse point of $z = 1 + i$ with respect to $|z| = 2$ is

(A) (2, 2)

(B) $(\sqrt{2}, \sqrt{2})$

(C) (1, 1)

(D) (–1, –1)

12. Which of the following functions is not harmonic?

(A) $u(x, y) = \sin hx \cos y$

(B) $u(x, y) = \dfrac{1}{2} \log (x^2 + y^2)$

(C) $u(x, y) = x^2 - y^2$

(D) $u(x, y) = x^2 + y^2$

13. The set of all matrices of the form $\begin{pmatrix} 0 & x \\ 0 & y \end{pmatrix}$, where $x, y \in \mathbb{Q}$ under the operation of matrix addition and multiplication is

(A) a ring with identity.

(B) a commutative ring.

(C) a non-commutative ring with zero divisors.

(D) a ring without zero divisors but with unity.

14. In the group $(\mathbb{Z}, +)$, the subgroup generated by 2 and 7 is

(A) $\mathbb{Z}$

(B) $5\mathbb{Z}$

(C) $9\mathbb{Z}$

(D) $14\mathbb{Z}$

15. Let $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3) \in \mathbb{R}^3$ be linearly independent, and

$\delta_1 = x_2 y_3 - y_2 x_3$, $\delta_2 = x_1 y_3 - y_1 x_3$, $\delta_3 = x_1 y_2 - y_1 x_2$.

If V is the span of x, y, then

(A) $V = \{(u, v, w) \,!\, \delta_1 u - \delta_2 v + \delta_3 w = 0\}$

(B) $V = \{(u, v, w) \,!\, -\delta_1 u + \delta_2 v + \delta_3 w = 0\}$

(C) $V = \{(u, v, w) \,!\, \delta_1 u + \delta_2 v - \delta_3 w = 0\}$

(D) $V = \{(u, v, w) \,!\, \delta_1 u + \delta_2 v + \delta_3 w = 0\}$

16. Let T be a linear transformation on $\mathbb{R}^3$ given by $T(x, y, z) = (2x, 4x - y, 3x + 3y - z)$, then $T^{-1}(x, y, z)$ is

(A) $(\frac{x}{2}, 2x - y, 7x - 3y - z)$

(B) $(x, 2x + y, 4x + 3y - z)$

(C) $(\frac{x}{2}, 4x + 3y, 7x + 3y - z)$

(D) $(x, 3x + y, 7x - 3y + z)$

17. Let $A = [a_{ij}]$ be an $n \times n$ matrix such that $a_{ij} = 3$ for all i and j. Then nullity of A is

(A) $n - 1$

(B) $n - 3$

(C) n

(D) 0

18. The initial value problem $\frac{dy}{dx} = \cos(xy), x \in \mathbb{R}$, $y(0) = y_0$, where y_0 is a real constant, has

(A) a unique solution.
(B) exactly two solutions.
(C) infinitely many solutions.
(D) no solution.

19. The eigenvalues of the boundary value problem

$$\frac{d^2y}{dx^2} + \lambda y = 0,\ x \in [0, \pi]$$

with $y(0) = 0$ and $y(\pi) - y'(\pi) = 0$ are given by

(A) $\lambda = k_n^2$, where k_n, $(n = 1, 2, 3, ...)$ are the roots of $k + \tan(k\pi) = 0$.
(B) $\lambda = n^2$, $n = 1, 2, 3, ...$
(C) $\lambda = (n\pi)^2$, $n = 1, 2, 3, ...$
(D) $\lambda = k_n^2$, where k_n, $(n = 1, 2, 3, ...)$ are the roots of $k - \tan(k\pi) = 0$.

20. The Wronskian of the functions $f_1(x) = x^2$ and $f_2(x) = x|x|$ is zero

(A) for all x.
(B) for $x > 0$.
(C) for $x < 0$.
(D) for $x = 0$.

21. Let $f \in C^1[-1, 1]$, then $\lim_{n\to\infty} \sum_{k=1}^{n} f'\left(\frac{k}{3n}\right)$ is equal to

(A) $3\left[f\left(\frac{1}{3}\right) - f(0)\right]$
(B) $3\left[f\left(\frac{1}{3}\right) + f(0)\right]$
(C) $\left[f\left(\frac{1}{3}\right) - f(0)\right]$
(D) $\left[f\left(\frac{1}{3}\right) + f(0)\right]$

22. Which of the following improper integrals is convergent?

(A) $\int_0^\infty \frac{x\tan^{-1}x}{(1+x^4)^{1/3}}dx$
(B) $\int_{e^2}^\infty \frac{dx}{x\log x}$
(C) $\int_2^3 \frac{3x^2+1}{x^2-4}dx$
(D) $\int_0^1 \frac{\sin x}{x^{3/2}}dx$

23. Which of the following functions is Riemann integrable on [0, 1]?

(A) $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$
(B) $f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$
(C) $f(x) = \lim_{n\to\infty} \cos^{2n}(24\pi x)$
(D) $f(x) = \lim_{n\to\infty} \cos^{3n}(21\pi x)$

24. The bilinear transformation which maps the points $z = 1, z = 0, z = -1$, of z-plane into $\omega = i, \omega = 0, \omega = -i$ of ω-plane respectively is

(A) $\omega = iz$
(B) $\omega = z$
(C) $\omega = i(z + 1)$
(D) $\omega = -iz$

25. For $0 < |z| < 1$, $\frac{1+2z}{z^2+z^3}$ can be expressed as

(A) $\frac{1}{z^3} + \frac{1}{z^2} + \frac{1}{z} - 1 + z - z^2 + z^3 - ...$
(B) $\frac{1}{z^2} + \frac{1}{z} - 1 + z - z^2 + z^3 - ...$
(C) $\frac{1}{z} - 1 + z - z^2 + z^3 - ...$
(D) $\frac{1}{z} - 1 - z - z^2 - z^3 - ...$

26. If g is analytic at $z = a$ and f has a pole of order k at $z = a$, then $res_{z=a} \dfrac{g(z) f'(z)}{f(z)}$ is

(A) $kg(a)$

(B) $-\dfrac{1}{k} g(a)$

(C) $-kg(a)$

(D) $-k!g(a)$

27. The number of elements of a principal integral domain can be

(A) 15

(B) 25

(C) 35

(D) 36

28. Let $\langle p(x) \rangle$ denote the ideal generated by the polynomial $p(x)$ in $\mathbb{Q}[x]$. If $f(x) = x^3 + x^2 + x + 1$ and $g(x) = x^3 - x^2 + x - 1$, then

(A) $\langle f(x) \rangle + \langle g(x) \rangle = \langle x^3 + x \rangle$

(B) $\langle f(x) \rangle + \langle g(x) \rangle = \langle f(x)\, g(x) \rangle$

(C) $\langle f(x) \rangle + \langle g(x) \rangle = \langle x^2 + 1 \rangle$

(D) $\langle f(x) \rangle + \langle g(x) \rangle = \langle x^4 - 1 \rangle$

29. The number of elements in the field $\dfrac{\mathbb{Z}[i]}{\langle 2+i \rangle}$ is

(A) 2

(B) 3

(C) 5

(D) ∞

30. Let $f_n : [0, 1] \to \mathbb{R}$ be defined by

$$f_n(x) = \begin{cases} 2n, & \text{if } \dfrac{1}{2n} \le x \le \dfrac{1}{n} \\ 0, & \text{otherwise} \end{cases}.$$

Then the values of $\int_0^1 \lim_{n\to\infty} f_n \, d\mu$ and $\lim_{n\to\infty} \int_0^1 f_n \, d\mu$, where μ is the Lebesgue measure on $\mathbb{R}$, are respectively

(A) 0 and 0

(B) 0 and 1

(C) 1 and 0

(D) 1 and 1

31. Let $X = \mathbb{N}$, the set of positive integers. Consider the metrics d_1 and d_2 on X given by

$d_1(m, n) = |m - n|,\ m, n \in X$

$d_2(m, n) = \left| \dfrac{1}{m} - \dfrac{1}{n} \right|,\ m, n \in X.$

Let X_1 and X_2 denote the metric spaces (X, d_1) and (X, d_2) respectively. Then

(A) X_1 is complete.

(B) X_2 is complete.

(C) X_1 is totally bounded.

(D) X_2 is not totally bounded.

32. The degree of the extension of $\mathbb{Q}\left(\sqrt{2} + \sqrt[3]{2}\right)$ over the field $\mathbb{Q}\left(\sqrt{2}\right)$ is

(A) 1

(B) 2

(C) 3

(D) 6

33. Which of the following statements is correct?

(A) Sylow–2 subgroup of group $S_3 \times \mathbb{Z}_{10}$ is normal.

(B) Sylow–3 subgroup of group $S_4 \times \mathbb{Z}_5$ is normal.

(C) Sylow–p subgroup of group $G_1 \times G_2$ is normal, where G_1 and G_2 are finite abelion groups.

(D) Sylow–3 subgroup of group $S_4 \times \mathbb{Z}_3$ is normal.

34. Let $A : H \to H$ be any bounded linear operator on a complex Hilbert space H such that $\| Ax \| = \| A^*x \|$ for all x in H, were A^* is the adjoint of A. If there is a non-zero x in H such that $A^*(x) = (2 + 3i)x$, then A is

(A) a unitary operator on H.

(B) a self-adjoint operator on H but not unitary.

(C) a self-adjoint operator on H but not normal.

(D) a normal operator.

35. Consider the Banach space $C[0, \pi]$ with the sup norm. The norm of the linear functional $l : C[0, \pi] \to \mathbb{R}$, given by $l(f) = \int_0^{\pi} f(x) \sin^2 x \, dx$ is

(A) 1

(B) $\frac{\pi}{2}$

(C) π

(D) 2π

36. Let X be the space of bounded real sequences with sup norm. Define a linear operator $T : X \to X$ by

$$T(x) = \left(\frac{x_1}{1}, \frac{x_2}{2}, \ldots\right) \text{ for } x = (x_1, x_2, \ldots) \in X.$$

Then

(A) T is bounded but not one-to-one.

(B) T is one-to-one but not bounded.

(C) T is bounded and its inverse (from range of T) exists but not bounded.

(D) T is bounded and its inverse (from range of T) exists and bounded.

37. Let $X = \{1, 2, 3, 4, 5\}$ and $\tau = \{\varphi, \{1\}, \{1, 2\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 2, 3, 4\}, X\}$ be a topology on X. Which is not a τ-neighbourhood of 3?

(A) $\{1, 3, 4\}$

(B) $\{1, 3, 4, 5\}$

(C) $\{1, 3\}$

(D) $\{1, 2, 3, 4\}$

38. Nagata–Smirnov metrization theorem of G_δ set states : A subset A of topological space X is G_δ set in X

(A) if it is equal to the union of a countable collection of open subsets of X.

(B) if it is equal to the intersection of a countable collection of open subsets of X.

(C) if it is equal to the union of a uncountable collection of open subsets of X.

(D) if it is equal to the uncountable collection of closed subsets of X.

39. Let $X = \mathbb{R}$ with cofinite topology. Then X is a

(A) first countable space

(B) T_1 space

(C) regular space

(D) normal space

40. Let us consider the following two Sturm-Liouville type ordinary differential equations:

(1) $x^2u'' + 2xu' = x^2$, $0 \le x \le 1$ with the boundary conditions $u(0)$ is finite and $u(1) + u'(1) = 0$.

(2) $u'' + u = f(x)$, $0 \le x \le \pi$, with $u(0) = \alpha$ and $u(\pi) = \beta$.

Consider the following two statements.

(i) Green's function for the first problem exists.

(ii) Green's function for the second problem exists for any arbitrary function $f(x)$.

Then,

(A) both the statements (i) and (ii) are correct.

(B) only (i) is correct.

(C) only (ii) is correct.

(D) both (i) and (ii) are not correct.

41. The solution of $x\, u_x + y\, u_y = 0$ is of the form

(A) $f\left(\frac{x}{y}\right)$

(B) $f(x + y)$

(C) $f(x - y)$

(D) $f(xy)$

42. Consider the congruence $x^n \equiv 2 \pmod{13}$. This congruence has a solution for x if

(A) $n = 8$

(B) $n = 6$

(C) $n = 7$

(D) $n = 9$

43. Let $N = a_m 10^m + a_{m-1} 10^{m-1} + ... + a_1 10 + a_0$ be the decimal expansion of the positive integer N, $0 \leq a_n \leq 10$, $(n = 1, 2, ..., m)$ and let $T = a_0 - a_1 + a_2 + ... + (-1)^n a_n$. Then $11/N$ if and only if

(A) $11/T+1$

(B) $11/T$

(C) $11/T-1$

(D) $T/11$

44. Let H, T and V denote the Hamiltonian, the kinetic energy and the potential energy respectively of a mechanical system at time t. If H depends on t explicitely, the $\frac{\partial H}{\partial t}$ is equal to

(A) $\frac{\partial T}{\partial t} + \frac{\partial V}{\partial t}$

(B) $\frac{\partial T}{\partial t} - \frac{\partial V}{\partial t}$

(C) $\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$

(D) $-\frac{\partial V}{\partial t} - \frac{\partial T}{\partial t}$

45. A rigid body, symmetrical about the z-axis, has one point fixed on this axis. Let $\vec{\omega} = \omega_x \hat{i} = \omega_y \hat{j} + \omega_z \hat{k}$ be the angular velocity of the body. Then

(A) ω_x and ω_y are constant.

(B) ω_z is constant.

(C) ω_z must be zero.

(D) $\omega_x^2 + \omega_y^2 = \omega_z^2$.

46. The deformation of a body is defined by the displacement components $u_1 = k(3x_1^2 + x_2)$, $u_2 = k(2x_2^2 + x_3)$, $u_3 = k(4x_3 + x_1)$, where k is a positive constant. The extension of a line element that passes through the point (1, 1, 1) in the direction $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is

(A) $\frac{16k}{3}$

(B) $\frac{17k}{3\sqrt{3}}$

(C) $\frac{17k}{3}$

(D) $\frac{22k}{3\sqrt{3}}$

47. The stress field for an isotropic solid is given by

$\tau_{11} = \tau_{22} = \tau_{33} = \tau_{12} = 0$

$\tau_{13} = -\mu\alpha x_2$, $\tau_{23} = \mu\alpha x_1$,

$\alpha \neq 0$ is a constant and μ is the Lame's constant. The strain energy function is equal to

(A) $\frac{1}{4\mu\alpha^2}\left(x_1^2 + x_2^2\right)$

(B) $\frac{\mu\alpha^2}{4}\left(x_1^2 + x_2^2\right)$

(C) $\mu\alpha^2\left(x_1^2 + x_2^2\right)$

(D) $4\mu\alpha^2\left(x_1^2 + x_2^2\right)$

48. If $\vec{v} = 2t\hat{i} + x\hat{j}$ is the velocity of two dimensional fluid flow and $\psi(x, y, t)$ is the stream function, then

(A) $\psi(x, y, t) = -2yt + \frac{1}{2}x^2 + g(t)$,

(B) $\psi(x, y, t) = -2t + \frac{1}{2}x^2 + g(t)$,

(C) $\psi(x, y, t) = -2yt + \frac{1}{2}x + g(t)$,

(D) $\psi(x, y, t) = -2y + \frac{1}{2}x^2 + g(t)$,

where $g(t)$ is an arbitrary function of t.

49. A circular cylinder of radius a is fixed across a stream of fixed velocity U and circulation k round the cylinder. Then the maximum velocity is

(A) $2U + \frac{k}{\pi a}$

(B) $U + \frac{k}{\pi a}$

(C) $3U + \frac{k}{2\pi a}$

(D) $2U + \frac{k}{2\pi a}$

50. Directed graph is a graph in which

(A) edges do not have a direction.

(B) set of vertices are connected by directed edges.

(C) the connection between two nodes is necessarily reciprocated.

(D) the edges indicate a two-way relationship.

51. A poset in which every pair of elements has both *lub* and *glb* is termed as

(A) sublattice

(B) trail

(C) work

(D) lattice

52. The asymptotic lines on the surface $x = 3u(1 + v^2) - u^3, y = 3v(1 + u^2) - v^3, z = 3u^2 - 3v^2$ are

(A) $u + v$ = constant

(B) $u - v$ = constant

(C) $\pm\frac{u}{2v}$ = constant

(D) Both (A) and (B)

53. For the curve $x = a \tan\theta, y = a \cot\theta, z = a\sqrt{2} \log(\tan\theta)$, the radius of curvature is

(A) $\frac{2\sqrt{2}\, a}{\sin^2\theta}$

(B) $\frac{2\sqrt{2}\, a}{\sin^2 2\theta}$

(C) $\frac{2\sqrt{2}\, a}{\sin\theta}$

(D) $\frac{2\sqrt{2}\, a}{\sin 2\theta}$

54. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $n = 0, 1, 2, ...$ with $x_0 = 0.5$ obtained from the Newton–Raphson method. The series converges to

(A) 1.5

(B) $\sqrt{2}$

(C) 1.6

(D) 1.4

55. Trapezoidal rule for evaluation of $\int_a^b f(x)\,dx$ requires the interval (a, b) to be divided into

(A) $2n$ subintervals of equal width.

(B) $2n + 1$ subintervals of equal width.

(C) any number of subintervals of equal width.

(D) $3n$ subintervals of equal width.

56. The extremal of the functional

$$I[y(x)] = \int_a^b \frac{\sqrt{1 + y'^2}}{y}\,dx$$

are

(A) circles

(B) parabolas

(C) ellipses

(D) straight lines

57. The variational problems of extremizing the functional $I[y(x)] = \int_1^3 y(3x - y)\,dx$, $y(3) = 4\frac{1}{2}$, $y(1) = 1$ has

(A) a unique solution.

(B) exactly two solutions.

(C) infinitely many solutions.

(D) no solution.

58. The homogeneous integral equation $y(x) - \lambda \int_0^1 (3x - 2)t\; y(t)\; dt = 0$ has

(A) one characteristic number.

(B) two characteristic numbers.

(C) three characteristic numbers.

(D) no characteristic number.

59. Let $y(s)$ be the solution of the integral equation $y(s) = s + \int_0^1 su^2 y(u)\; du$.

Then $y(3)$ is given by

(A) 1

(B) 2

(C) 3

(D) 4

60. Let $I = [2, 3)$. J be the set of all rational numbers in the interval $[4, 6]$. K be the cantor (ternary) set, and let $L = \{7 + x \; ! \; x \in K\}$. Then what is the Lebesgue measure of the set $IUJUL$?

(A) 0

(B) 1

(C) 2

(D) 3

61. Let f and g be measurable real-valued function on $\mathbb{R}$ such that

$$\int_{-\infty}^{\infty} (f(x)^2 + g(x)^2)\,dx = 2\int_{-\infty}^{\infty} f(x)g(x)\,dx.$$

If $E = \{x \in \mathbb{R} \; ! \; f(x) \neq g(x)\}$, then which of the following statements is not true?

(A) E is a non-empty set.

(B) E is measurable.

(C) E has Lebesgue measure zero.

(D) For almost all $x \in \mathbb{R}$, we have $f(x) = 0$ and $g(x) = 0$.

62. Fourier cosine transform of e^{-x^2} is

(A) $\frac{1}{2} e^{-\lambda^2/4}$

(B) $\frac{1}{\sqrt{2}} e^{-\lambda^2/4}$

(C) $\frac{1}{2} e^{-\lambda^2/2}$

(D) $\frac{1}{\sqrt{2}} e^{-\lambda^2/2}$

where λ is the transformed variable.

63. If the partial differential equation $(x - 1)^2 u_{xx} - (y - 2)^2 u_{yy} + 2xu_x + 2yu_y + 2xyu = 0$ is parabolic in $S \subseteq \mathbb{R}^2$, then S is

(A) $\{(x, y) \in \mathbb{R}^2 \; ! \; x = 1 \text{ or } y = 2\}$

(B) $\{(x, y) \in \mathbb{R}^2 \; ! \; x = 1 \text{ and } y = 2\}$

(C) $\{(x, y) \in \mathbb{R}^2 \; ! \; x = 1\}$

(D) $\{(x, y) \in \mathbb{R}^2 \; ! \; y = 2\}$

64. Which of the following is the Fourier transform of

$$F(x) = \begin{cases} 1 - x^2, & \text{if } |x| \le 1 \\ 0, & \text{if } |x| > 1 \end{cases}$$

(λ is the transformed variable)?

(A) $\frac{4}{\lambda^2}(\sin\lambda - \lambda\cos\lambda)$

(B) $\frac{4}{\lambda^3}(\sin\lambda - \lambda\cos\lambda)$

(C) $\frac{4}{\lambda^2}(\cos\lambda - \lambda\cos\lambda)$

(D) $\frac{4}{\lambda^3}(\cos\lambda - \lambda\sin\lambda)$

65. If f is a differentiable function on the real line such that $\lim_{x\to\infty} f(x)=1$ and $\lim_{x\to\infty} f'(x)=a$, then

(A) $a = 0$
(B) $a \neq 0$ but $|a|<1$
(C) $a > 1$
(D) $a < -1$

66. Suppose $(X_1, X_2, ..., X_n)$ is a random sample from a Cauchy distribution with pdf

$$f(x)=\frac{1}{\pi}.\frac{1}{1+x^2}, -\infty < x < \infty.$$

Consider the following statements:

1. X_1 and $-X_1$ have the same distribution.
2. X_1 and $\overline{X}$ have the same distribution.
3. $\overline{X}$ and $\frac{1}{\overline{X}}$ have the same distribution.

Which of the above statements is/are true?

(A) 1 only
(B) 2 only
(C) 1 and 2 only
(D) All of 1, 2 and 3

67. Let the joint m.g.f. of (X, Y) be given by $M_{X,Y}(s,t)=a(e^{s+t}+1)+b(e^s+e^t), a>0, b>0$ and $a+b=\frac{1}{2}$. Then the correlation between X and Y is

(A) $4a$
(B) $4a - 1$
(C) $4b$
(D) $4b - 1$

68. Let X_1, X_2 and X_3 be i.i.d. random variables with common pdf $f_X(x) = exp(-x), x > 0$.

Then the pdf of $Y=\frac{X_1+X_2}{X_1+X_2+X_3}$ is

(A) $f_Y(y) = 1, 0 < y < 1$
(B) $f_Y(y) = \frac{3}{2}, 0 < y < \frac{2}{3}$
(C) $f_Y(y) = 2y, 0 < y < 1$
(D) $f_Y(y) = 3y^2, 0 < y < 1$

69. If X is a non-negative integer-valued random variable and $P(s) = E(s^X)$, then $\int_0^1 P(s^2)\,ds$ equals

(A) $E(X^2)$
(B) $E(\sqrt{X})$
(C) $E\left(\frac{1}{X+1}\right)$
(D) $E\left(\frac{1}{2X+1}\right)$

70. In terms of the incomplete beta function defined as $I_u(x,y)=\frac{\int_0^u t^{x-1}(1-t)^{y-1}\,dt}{\int_0^1 t^{x-1}(1-t)^{y-1}\,dt}$, the distribution function of the r-th order statistic when the random sample of size n is drawn from the distribution function $F(x)$ is

(A) $I_{F(x)}(r-1,\ n-r)$
(B) $I_{F(x)}(r-1,\ n-r+1)$
(C) $I_{F(x)}(r,\ n-r+1)$
(D) $I_{F(x)}(r,\ n-r)$

71. A coin is tossed repeatedly until a head turns up. Let X be the number of tosses it takes until this happens. The probability of getting a head is p. If it is known that tail has fallen 3 times, the probability that head will fall within next two tosses is

(A) $(1-p)^2p^3$
(B) $(1-p)^3p^2$
(C) $(2-p)p$
(D) $(2-p)p^2$

72. In a series of houses actually invaded by small pox, 70% of the inhabitants are attacked and 85% have been vaccinated. The percentage of the vaccinated that have been attacked cannot be smaller than

(A) 64.7%
(B) 70.2%
(C) 80%
(D) 58.3%

73. Let X be a random variable with $E(X) = 0$ and $\text{Var}(X) = \sigma^2$. Then

(A) $\Pr(X > x) \le \frac{\sigma^2}{\sigma^2 + x^2}$, $x > 0$

(B) $\Pr(X > x) \ge \frac{\sigma^2}{\sigma^2 + x^2}$, $x > 0$

(C) $\Pr(X > x) \le \frac{\sigma^2}{\sigma^2 + x^2}$, $x < 0$

(D) $\Pr(X > x) \le \frac{x^2}{\sigma^2 + x^2}$, $x < 0$

74. Suppose independent samples are available from k univariate normal populations $N(\mu_1, \sigma_1^2)$, $N(\mu_2, \sigma_2^2)$,..., $N_k(\mu_k, \sigma_k^2)$, where all parameters are unknown. Let λ_1 be LR statistic for testing equality of the k means under the assumption of homoscedasticity and λ_2 be the LR statistic for testing homoscedasticity. If λ be the LR statistic for testing equality of all the k populations, then

(A) λ cannot be computed from λ_1 and λ_2

(B) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(C) $\lambda = \min(\lambda_1, \lambda_2)$

(D) $\lambda = \lambda_1 \lambda_2$

75. In estimating the location parameter of a distribution under the transformation of location, if $\bar{x}$ and $\tilde{x}$ denote, respectively, the sample mean and sample median, then

(A) both $\bar{x}$ and $\tilde{x}$ are location equivariant.

(B) $\bar{x}$ is location equivariant, but $\tilde{x}$ is not.

(C) $\tilde{x}$ is location equivariant, but $\bar{x}$ is not.

(D) neither $\bar{x}$ nor $\tilde{x}$ is location equivariant.

76. Let $X_1, X_2, ..., X_n$ be a random sample from Uniform $\left(\mu - 3\sqrt{\sigma}, \mu + 3\sqrt{\sigma}\right)$. Then MLEs of μ and σ are respectively [$X_{(r)}$ stands for r-th order statistic]

(A) $\bar{X}$ and $\frac{1}{n}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2$

(B) $X_{(1)}$ and $X_{(n)}$

(C) $\frac{X_{(1)} + X_{(n)}}{2}$ and $\frac{X_{(n)} - X_{(1)}}{2\sqrt{3}}$

(D) $\frac{X_{(1)} + X_{(n)}}{2\sqrt{3}}$ and $\frac{X_{(n)} + X_{(1)}}{2}$

77. Let X have a binomial $(5, p)$ distribution and p has prior distribution Beta (2, 3). An observation $X = 4$ is obtained from this set up. Then the posterior distribution of p is

(A) Beta (6, 4)

(B) Uniform (0, 1)

(C) Beta (3, 2)

(D) Uniform (2, 3)

78. Let $X_1, X_2, ..., X_n$ be a random sample from $f(x, \theta) = \theta \, exp(-\theta x)$, $x > 0$. Then the UMVUE's of θ and $\frac{1}{\theta}$ are respectively

(A) $\bar{X}, \frac{1}{\bar{X}}$

(B) $\frac{1}{\bar{X}}, \bar{X}$

(C) $\bar{X}, \frac{n-1}{n\bar{X}}$

(D) $\frac{n-1}{n\bar{X}}, \bar{X}$

79. For a two-sample location problem based on independent samples of size n_1 and n_2 respectively, the Wilcoxon–Mann–Whitney test statistic for testing equality of two distribution functions F_1 and F_2 has (mean, variance) under the null hypothesis as

(A) $\left(\frac{n_1(n_1 + n_2 + 1)}{2}, \frac{n_1 n_2 (n_1 + n_2)}{4}\right)$

(B) $\left(\frac{n_2(n_1 + n_2 + 1)}{2}, \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}\right)$

(C) $\left(\frac{n_1 n_2(n_1 + n_2 + 1)}{2}, \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}\right)$

(D) $\left(\frac{n_1(n_1 + n_2 + 1)}{2}, \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}\right)$

80. Thirty students wrote a mathematics examination having a maximum of 50 marks. The marks distribution is given in the following stem and leaf plot:

0	9
1	2 2 5
2	0 1 3 3 3 5 8 8 9
3	0 0 1 3 6 6 7 9
4	0 2 2 4 4 4 7 8
5	0

The median mark is

(A) 30.5

(B) 30.0

(C) 25.0

(D) 28.5

81. The frequency distribution of the radius of a large number of spherical balls is bell-shaped and symmetric about its median. Then the frequency distribution of the volume of those balls will be

(A) symmetric bell-shaped

(B) negatively skewed

(C) positively skewed

(D) J-shaped

82. The joint pdf of X, Y is

$$f(x,y) = \frac{3x+y}{4} \cdot \exp(-x-y), x > 0,\ y > 0.$$

Then the covariance between X and Y is

(A) $\frac{3}{16}$

(B) $-\frac{3}{16}$

(C) $\frac{5}{16}$

(D) $-\frac{5}{16}$

83. Consider a linear model with observations Y_1, Y_2, Y_3, Y_4 such that

$$E(Y_1) = \theta_1 + \theta_2 + \theta_3$$
$$E(Y_2) = \theta_1$$
$$E(Y_3) = \theta_2$$
$$E(Y_4) = \theta_1 - \theta_2 - \theta_3.$$

Which of the following statements is/are true?

1. $\theta_2 + \theta_3$ is not estimable.
2. θ_1, θ_2 and θ_3 are all estimable.
3. $\theta_1 + \theta_2 + \theta_3$ is estimable.
4. Y_2 is the BLUE of θ_1.

(A) 1 only

(B) 1 and 3 only

(C) 2 and 3 only

(D) 2 and 4 only

84. Let D be a BIBD with parameters b, v, r, k and λ. Which of the following statements is/are true?

1. D is connected if $k \geq 2$.
2. The covariance between the BLUE's of a pair of orthogonal treatment contrasts is zero.
3. The efficiency factor of D relative to a RBD with replication r is strictly smaller than unity.

(A) 1 only

(B) 1 and 2 only

(C) 3 only

(D) 2 and 3 only

85. Suppose that events occur following Poisson process with average time between occurrence of events calculated at 60 minutes. Given that one event just occurred at 12 noon, what is the probability that the next occurrence of the event will take place after 2 p.m.?

(A) e^{-120}

(B) e^{-12}

(C) e^{-2}

(D) e^{-4}

86. For the Markov chain with state space $\{1, 2, 3, 4\}$ and the transition probability matrix

$$\begin{pmatrix} \frac{1}{4} & \frac{1}{2} & 0 & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{pmatrix}$$

consider the following statements:

1. The chain is irreducible.
2. All the states are recurrent.
3. State 4 is transient.

Which of the above statement(s) is/are true?

(A) 1 only
(B) 2 only
(C) 1 and 3 only
(D) 1 and 2 only

87. For a life table, which one is correct?

(A) $L_x \cong l_x - \frac{1}{2}d_x$, $e_x^0 = \frac{T_x}{l_x}$
(B) $L_x \cong l_x + \frac{1}{2}d_x$, $e_x^0 = \frac{T_x}{l_x}$
(C) $L_x \cong l_x + \frac{1}{2}d_x$, $e_x^0 = \frac{T_x}{L_x}$
(D) $L_x \cong l_x - \frac{1}{2}d_x$, $e_x^0 = \frac{T_x}{L_x}$

88. In the usual 3σ control chart for fraction defectives, under normality, the sample size n required to detect with probability 0·5 a shift in the process fraction defective from p_0 to p_1 ($> p_0$) is

(A) $\frac{3p_0(1-p_0)}{(p_1-p_0)^2}$
(B) $\frac{9p_0(1-p_0)}{(p_1-p_0)}$
(C) $\frac{9p_1(1-p_1)}{p_1-p_0}$
(D) $\frac{9p_0(1-p_0)}{(p_1-p_0)^2}$

89. Let Λ be the likelihood ratio statistic for testing $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu}_0$ against $H : \underset{\sim}{\mu} \neq \underset{\sim}{\mu}_0$ for a p-variate normal distribution $N_p(\underset{\sim}{\mu}, \Sigma)$ based on a random sample of size n, and T^2 be the corresponding Hotellings T^2 statistic. Then

(A) $\Lambda^{n/2}$ is a linear function of T^2.
(B) $\Lambda^{2/n}$ is a linear function of T^2.
(C) $\Lambda^{-2/n}$ is a linear function of T^2.
(D) $\Lambda^{-n/2}$ is a linear function of T^2.

90. Suppose $\{X_n\}$ is a sequence of pairwise independent random variables with $P(X_n = -n^{\alpha}) = P(X_n = n^{\alpha}) = \frac{1}{2}$. Then WLLN holds for $\{X_n\}$ if

(A) $0 < \alpha < \frac{1}{2}$
(B) $\frac{1}{2} < \alpha < 1$
(C) $\alpha > 1$
(D) $0 < \alpha < \infty$

91. A time series whose properties do not depend on the time at which the series is observed

(A) is stationary.
(B) is non-stationary.
(C) has significant trend.
(D) has seasonality.

92. In a time series forecasting problem, if the seasonal indices for quarters 1, 2 and 3 are 0·80, 0·90 and 0·95 respectively, what can you say about the seasonal index of quarter 4?

(A) It will be less than 1.
(B) It will be greater than 1.
(C) It will be equal to 1.
(D) Data are insufficient to conclude.

93. A simple random sample of size 3 units is drawn with replacement from a population of N units. The probability that the sample consists of all three distinct units is

(A) $\frac{1}{N^2}$

(B) $\frac{1}{N^3}$

(C) $\frac{(N-1)^2}{N^3}$

(D) $\frac{(N-1)(N-2)}{N^2}$

94. The Hazard rate of the 2 parameter Weibull distribution (with shape parameter β and scale parameter λ) is

(A) $\frac{\beta}{\lambda}\left(\frac{t}{\lambda}\right)^{\beta-1}$

(B) $\left(\frac{t}{\lambda}\right)^{\beta-1}$

(C) $e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$

(D) $1-e^{-\left(\frac{t}{\lambda}\right)^{\beta}}$

95. In a M | M | 1 queuing model, an analyst informs that the service rate is 8/hour and $P(N \geq 2)=\frac{1}{4}$ (where N is the number of customers in the system). Then the arrival rate is

(A) 2/hour

(B) 8/hour

(C) 10/hour

(D) 4/hour

96. Let $N(t)$ denote the number of customers in a M | M | 1 | 7 queuing system with arrival rate = service rate = 3/hour.

Which of the following is true?

(A) The steady state probability distribution of number of customers in the system cannot be determined since arrival rate is equal to service rate.

(B) $\lim_{t\to\infty} P\left(N(t)=3\right)=\lim_{t\to\infty} P\left(N(t)=4\right)$

(C) $\lim_{t\to\infty} P\left(N(t)=0\right)=0$

(D) All of the above

97. The steady state system state distribution of the G | M | 1 queue is

(A) Poisson distribution

(B) Uniform distribution

(C) Binomial distribution

(D) Geometric distribution

98. In the E O Q model, assuming that order quantity (Q) arrives all at once just when desired, planned shortages are not allowed and a constant demand rate of '*a*' unit per unit time; the optimum cycle time is

(A) $\sqrt{\frac{2ah}{k}}$

(B) $\sqrt{\frac{2k}{ah}}$

(C) $\sqrt{\frac{ah}{2k}}$

(D) $\sqrt{\frac{k}{2ah}}$

where h is the unit holding cost and k is the set up cost.

99. In the context of two-person zero sum game, which of the following is correct?

(A) All mixed strategies can be obtained as a particular case of some fixed pure strategy.

(B) All pure strategies can be obtained as a particular case of mixed strategies.

(C) Pure strategies are not related to mixed strategies and vice versa.

(D) All the above are incorrect.

100. Consider a multiple regression model with r regressors and the response variable Y. Suppose $\hat{Y}$ is the fitted value of Y, R^2 is the coefficient of determination and R_a^2 is the adjusted coefficient of determination. Then which of the following statements is/are true?

1. R^2 always increases if an additional regressor is included in the model.

2. R_a^2 always increases if an additional regressor is included in the model.

3. $R^2 > R_a^2$ for all r.

4. The regression line of $\hat{Y}$ on Y has a negative slope.

(A) 2 only

(B) 1, 3 and 4 only

(C) 1 and 3 only

(D) 3 and 4 only

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work

Space for Rough Work