A

22731 **120 MINUTES**

1. Which of the following process is a process with independent increments
   A) Poisson process B) Brownian motion process
   C) Both A and B D) None of these

2. In an irreducible Markov chain:
   A) All states are transient
   B) All states are persistent
   C) Some states are transient and others are persistent
   D) Either all states are transient or all states are persistent

3. Consider two independent series of events A and B occurring in accordance with Poisson process with mean λt and μt respectively. Then the number N of occurrences of A between two successive occurrences of B has:
   A) Geometric distribution B) Exponential distribution
   C) Uniform distribution D) Binomial distribution

4. Arrivals at a telephone booth are considered to be Poisson with an average time 10 minutes between one arrival and the next. The length of phone calls is assumed to be distributed exponentially with a mean of 3 minutes. What is the probability of that a person arriving at the booth will have to wait?
   A) 0.7 B) 0.03 C) 0.3 D) 0

5. Yule-Furry process is an example for a:
   A) Birth and death process B) Pure death process
   C) Birth immigration process D) Pure birth process

6. In time-series analysis, which source of variation can be estimated by the ratio-to-trend method?
   A) Trend B) Cyclical variation
   C) Irregular variation D) Seasonal variation

7. In the measurement of secular trend, method of moving average:
   A) Measure the seasonal variation
   B) Smooth out the time series
   C) Give the trend in a straight line
   D) None of these

8. All the index numbers are affected by:
   A) Formula error B) Sampling error
   C) Homogeneity error D) All the above

9. Simple aggregative type of index number satisfies:
   A) Time reversal and factor reversal tests
   B) Time reversal and circular tests
   C) Factor reversal and circular tests
   D) None of the three tests

10. Which of the following statements are true?

1. A set may have no limit point, unique limit point or any finite or infinite number of limit points
2. limit point of a set may or may not be a member of the set

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

11. The limit points of the set $\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, \ldots\right\}$ is/are:

A) Does not exist B) Only 1 and -1
C) (-1, 1) D) Only 0

12. If $f(x) = \begin{cases} 1, if\ x\ is\ irrational \\ 0, if\ x\ is\ rational \end{cases}$, then the value of $(f\ o\ f)(\sqrt{3})$ is:

A) 0 B) 1 C) $\sqrt{3}$ D) 3

13. Which of the following sets has measure zero?

1. The set of all rational numbers
2. The set of all irrational numbers
3. Every countable subset of R

A) 1 and 2 only B) 1 and 3 only
C) 2 and 3 only D) 3 only

14. If $f(x) = \begin{cases} x + x^2, when\ x\ is\ rational \\ x^2 + x^3, when\ x\ is\ irrational \end{cases}$, then the value of upper Reimann integral in (0, 2) is:

A) 0 B) $\frac{53}{12}$ C) 12 D) $\frac{83}{12}$

15. If ν is a signed measure and $E_1 \subseteq E_2 \subseteq \cdots$, then $\nu(\bigcup_{i=1}^{\infty} E_i) =$

A) $\lim \nu(E_i)$ B) $\nu(\bigcap_{i=1}^{\infty} E_i)$ C) $\sum_{i=1}^{\infty} \nu(E_i)$ D) None of these

16. Generalised variance is --------- of covariance matrix

A) Trace B) Product of the diagonal elements
C) Determinant D) None of these

17. Let $X = \binom{X_1}{X_2}$ be a random vector with mean vector $\mu = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$ and variance covariance matrix $\Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Then the covariance matrix of $Z_1 = X_1 - X_2$ and $Z_2 = X_1 + X_2$ is:

A) $\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 4 & 1 \\ 1 & 4 \end{bmatrix}$

18. Let $X = \begin{bmatrix} X_1 \\ - \\ X_2 \end{bmatrix}$ be distributed as $N_p(\mu, \Sigma)$ with $\mu = \begin{bmatrix} \mu_1 \\ - \\ \mu_2 \end{bmatrix}$, $\Sigma = \begin{bmatrix} \Sigma_{11} & | & \Sigma_{12} \\ - & | & - \\ \Sigma_{21} & | & \Sigma_{22} \end{bmatrix}$ and

$|\Sigma_{22}| > 0$.Then the conditional distribution of X_1, given that $X_2 = x_2$ is:

A) Normal having mean μ and covariance Σ

B) Normal having mean $\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2)$ and covariance Σ_{11}- $\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

C) Normal having mean $\mu_1 - \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2)$ and covariance Σ_{11}- $\Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

D) Normal having mean $\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - \mu_2)$ and covariance Σ_{22}- $\Sigma_{12}\Sigma_{11}^{-1}\Sigma_{21}$

19. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N_p(0, \Sigma)$. Then the distribution of $\sum_{j=1}^{n} X_j X_j'$ is:

A) Chi square distribution with n degrees of freedom

B) Wishart distribution with n-1 degrees of freedom

C) Chi square distribution with n-1 degrees of freedom

D) Wishart distribution with n degrees of freedom

20. If $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$ respectively denote the product moment correlation coefficients between X_1 and X_2, between X_2 and X_3, and between X_1 and X_3, then the multiple correlation of X_1 on X_2 and X_3 is

A) $\frac{2}{3}$ B) $\sqrt{\frac{2}{3}}$ C) 0 D) 1

21. If A and B are any two subspaces of a vector space V over a field F, then which of the following statements are true?

1. A + B is a subspace
2. A∩B is a subspace
3. A∪B is a subspace

A) 1 and 2 only B) 1 and 3 only

C) 2 and 3 only D) 3 only

22. The eigen values of the matrix D= $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ are:

A) 0 B) -1, 2 C) 0,1 D) Does not exist

23. If $\rho(A) = rank\ of\ a\ matrix\ A$, then which of the following is/are true?

1. $\rho(A) = \rho(A^T)$ 2. $\rho(A) = \rho(AA^T)$ 3. $\rho(A) = \rho(A^TA)$

A) 1 only B) 1 and 2 only

C) 1 and 3 only D) 1, 2 and 3

24. $\begin{vmatrix} 1+x_1 & x_2 & \cdots & x_n \\ x_1 & 1+x_2 & \cdots & x_n \\ \vdots & \vdots & \ddots & \vdots \\ x_1 & x_2 & \cdots & 1+x_n \end{vmatrix} =$

A) 0 B) 1
C) $1+x_1+x_2+\cdots+x_n$ D) None of the above

25. The quadratic form $2x^2+5y^2+2z^2+4xy+2xz+2yz$ is:
A) Positive definite B) Positive semidefinite
C) Negative definite D) indefinite

26. If G is the generalized inverse of a matrix A, then which of the following statements is/are true?
1. G^T is a generalized inverse of A^T
2. AG is idempotent

A) 1 only B) 2 only
C) Both 1 and 2 D) Neither 1 nor 2

27. Let a ball be drawn from an urn containing four balls, numbered 0, 1, 2, 3.
Let A ={0, 1}, B= {0, 2}, C={0, 3}. If four outcomes are assumed equally likely, then the events A,B, C are:
A) Only pairwise independent B) Independent
C) Disjoint D) None of these

28. Let A and B be mutually exclusive events in the sample space of a random experiment. Suppose the experiment is repeated until either event A or B occurs. Then the probability that the event A occurs before the event B is:

A) $\frac{P(A)}{P(B)}$ B) $\frac{P(A)}{P(A)+P(B)}$ C) $\frac{P(B)}{P(A)+P(B)}$ D) $P(A)P(B)$

29. Two dice are rolled. If the two faces are different, what is the probability that at least one is a six?
A) 11/36 B) 11/30 C) 1/3 D) 1/4

30. A laboratory test result is 90% effective in detecting a certain disease, when it is, infact, present. However, the test also yields a false positive result for 1 percent of the healthy persons tested. If 10% of the population actually has disease, what is the probability that a person has disease given that his test is positive?

A) 1/ 2 B) 1/3 C) 9/11 D) 10/11

31. If the joint pdf of (X_1, X_2) is $f(x_1,x_2)=\frac{1}{2}x_2e^{-x_1x_2}, 0<x_1<\infty, 0<x_2<2$, then $E\{e^{X_1/2}|X_2=1\}$ is:

A) 1 B) 0 C) 2 D) Does not exists

32. Consider a system of n identical components operating independently. Suppose that the length of life of components has common density

$$f(x) = \begin{cases} \frac{1}{\lambda}\ e^{-x/\lambda}, & x > 0,\ \lambda > 0 \\ 0 & , \quad otherwise \end{cases}$$

If the components operate in series, then the mean life of the system is:

A) λ B) $n\lambda$ C) $\frac{\lambda}{n}$ D) $\frac{\lambda^2}{n}$

33. Let $P(s)$ be the probability generating function of a nonnegative integer valued random variable X. Then which among the following statements are true:

1 $\left.\frac{d^k P(s)}{ds^k}\right|_{s=0} = k!\,P(X = k)$

2. $\left.\frac{d^k P(s)}{ds^k}\right|_{s=1} = E[X^k]$, when $E[X^k]$ exists.

3. $P(s)$ does not determine the distribution of X uniquely.

4. If $X_1, X_2, \ldots, X_n$ be independent random variables, then the probability generating function of sum of X_i's is the product of probability generating functions of X_i's.

A) 1 and 2 only B) 2 and 3 only C) 3 and 4 only D) 1 and 4 only

34. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from $N(\mu, \sigma^2)$. If $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$, then the distribution of $X_1 - \bar{X}$ is:

A) $N\left(0\,, \frac{n+1}{n}\sigma^2\right)$ B) $N\left(0\,, \frac{n-1}{n}\sigma^2\right)$

C) $N\left(0\,, \frac{\sigma^2}{n}\right)$ D) $N\left(0\,, \frac{2\sigma^2}{n}\right)$

35. If X follows t- distribution with 1 degree of freedom then:

A) $E(X) = 0$ and $V(X) = \frac{1}{2}$

B) $E(X) = 0$ and $V(X) = 1$

C) $E(X) = 0$ and $V(X)$ does not exist

D) $E(X)$ and $V(X)$ do not exists.

36. Let (X, Y) be a bivariate normal (BN) random variable with parameters $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ and ρ, and let $U = aX + b$, $a \neq 0$, $V = cY + d$, $c \neq 0$. Then the distribution of (U, V) is:

A) $BN(a\mu_1,\ c\mu_2\ a^2\sigma_1^2,\ c^2\sigma_2^2,\ \rho)$

B) $BN\left(a\mu_1 + b,\ c\mu_2 + d,\ a^2\sigma_1^2,\ c^2\sigma_2^2,\ \frac{\rho}{|ac|}\right)$

C) $BN(a\mu_1 + b,\ c\mu_2 + d,\ a\sigma_1^2,\ c\sigma_2^2,\ |ac|\rho)$

D) $BN(a\mu_1 + b,\ c\mu_2 + d,\ a^2\sigma_1^2,\ c^2\sigma_2^2,\ \rho)$

37. In a multiple choice oral examination, the grade is based on the number of questions asked until he gets one correct answer. Suppose that a student guesses at each answer and there are 4 choices for each answer. If the trials on assumed to be independent, then the average number of questions required for the first correct answer is:

A) 4 B) 5 C) 8 D) 12

38. If (X, Y) has trinomial distribution with parameters (n, p_1, p_2), then the conditional distribution of $Y|X = x$ is:

A) Binomial B) Hypergeometric

C) Geometric D) Poisson

39. An urn contains N marbles numbered 1 through N. Suppose n marble are drawn with replacement. Let M_n be the largest number drawn. Then:

A) $P(M_n = k) = \left(\frac{k}{N}\right)^n$

B) $P(M_n = k) = n\left(\frac{k}{N}\right)^{n-1}$

C) $P(M_n = k) = \left(\frac{k}{N}\right)^n - \left(\frac{k-1}{N}\right)^n$

D) $P(M_n = k) = \frac{\binom{k-1}{n-1}}{\binom{N}{n}}, \quad k = n, \; n+1, \ldots, N$

40. Suppose the survival times of patients who have advanced cancer of the bladder can be modelled by exponential distribution with mean λ. Then the time by which 25% of the patients will die is:

A) $\lambda \log\left(\frac{1}{2}\right)$ B) $\lambda \log\left(\frac{4}{3}\right)$ C) $\lambda \log\left(\frac{1}{4}\right)$ D) $\lambda \log\left(\frac{3}{4}\right)$

41. Consider the linear model

$$\underline{y}_{n\times1} = A_{n\times p}\beta_{p\times1} + e_{n\times1},$$

where $e_{n\times1} \sim N(0, \sigma^2 I)$. Assume that the rank of A is p. If $l_1'\hat{\beta}$ and $l_2'\hat{\beta}$ are the best linear estimates of the estimable functions $l_1'\beta$ and $l_2'\beta$, then the covariance between $l_1'\hat{\beta}$ and $l_2'\hat{\beta}$ is given by:

A) $\operatorname{cov}\left(l_1'\hat{\beta},\; l_2'\hat{\beta}\right) = \sigma^2(l_2'(A'A)^{-1}l_1)$

B) $\operatorname{cov}\left(l_1'\hat{\beta},\; l_2'\hat{\beta}\right) = \sigma^2(l_1'(A'A)^{-1}l_2)$

C) $\operatorname{cov}\left(l_1'\hat{\beta},\; l_2'\hat{\beta}\right) = \sigma^2(l_1'(A'A)^{-1}l_1)$

D) $\operatorname{cov}\left(l_1'\hat{\beta},\; l_2'\hat{\beta}\right) = \sigma^2(l_2'(A'A)^{-1}l_2)$

42. In a 2^2 design having factors A and B, replicated 4 times, the total of the observations from all replications corresponding to the treatment combinations (1), a, b and ab are respectively -10, -4, -10, and 24. Then the value of the sum of squares due to the interaction effect AB is:

A) 28.5 B) 49 C) 100 D) 229.5

43. For a symmetric BIBD with parameters (v, b, r, k, λ), the number of treatments common between any two blocks is:
A) λ B) $r-\lambda$ C) $\lambda(r-1)$ D) b/r

44. In $p \times p$ Graeco-Latin square design, the degrees of freedom of the error sum of square is equal to:
A) $(p-3)(p-2)$ B) $(p-1)^2$
C) $(p-2)(p-1)$ D) $(p-3)(p-1)$

45. What would happen if multiple t-test is performed instead of an ANOVA to compare 10 groups?
A) No change in results, except that making multiple comparisons with a t-test requires more computation than doing a single ANOVA
B) Making multiple comparisons with a t-test increases the probability making a type I error
C) There is no difference between using ANOVA and using t-test
D) None of the above

46. What effect does increasing the sample size have upon the sampling error?
A) It reduces the sampling error
B) It increases the sampling error
C) It has no effect on the sampling error
D) None of the above

47. The number of possible samples of size n out of N population size in SRSWR is equal to:
A) N^n B) NC_n C) $\frac{N-n}{N}$ D) $\frac{N}{n}$

48. Suppose that a simple random sample of n units is taken from a population of size N. If V_{srswor} and V_{srswr} respectively denote the variance of the sample mean in SRSWOR and SRSWR, then:
A) $V_{srswor} = \frac{(N-n)}{N} V_{srswr}$ B) $V_{srswor} = \frac{(N-n)}{(N-1)} V_{srswr}$
C) $V_{srswor} = \frac{(N-1)}{(N-n)} V_{srswr}$ D) None of the above

49. If the sampling frame of the elements are not available, then the sampling technique usually used is:
A) Systematic sampling B) Stratified sampling
C) Simple random sampling D) Cluster sampling

50. In cluster sampling, clusters are formed in such a way that
A) Variation within clusters is minimum while variation between clusters is maximum
B) Variation within clusters is maximum while variation between clusters is minimum
C) Variation within clusters is minimum while variation between clusters is minimum
D) Variation within clusters is maximum while variation between clusters is maximum

51. Which of the following is a procedure for selecting ppswor sample?

A) Cumulative total method B) Lahiri's method
C) Midzuno-Sen method D) All the above

52. Which of the following statement(s) is/are true?

1. Ratio estimator make use of auxiliary information
2. Ratio estimator provides a precise estimate of the population mean if the regression is linear and passes through origin
3. Ratio estimator is biased

A) 1 and 3 only B) 2 only C) 3 only D) 1, 2 and 3

53. If $E(Y|X) = 1$, then:

A) $V(XY) \geq V(X)$ B) $V(XY) \leq V(X)$
C) $V(XY) = V(X)$ D) None of these

54. Which of the following is not a probability density function?

A) $f(x) = 1, 1 < x < 2$ B) $f(x) = x(2 - x), 0 < x < 2$
C) $f(x) = 2, -\frac{1}{4} < x < \frac{1}{4}$ D) $f(x) = 1, -\frac{1}{2} < x < \frac{1}{2}$

55. The function defined by $F(x_1, x_2) = \begin{cases} 1, x_1 + 2x_2 \geq 1 \\ 0, x_1 + 2x_2 < 1 \end{cases}$ represents:

A) The distribution function of discrete bivariate random variables X_1 and X_2
B) The distribution function of continuous bivariate random variables X_1 and X_2
C) The distribution function of bivariate random variables X_1 and X_2 of mixed type
D) Not a distribution function

56. Let X be an integer valued random variable with probability generating function (pgf) P(s). Then the pgf of 2X +1 is:

A) P(s) B) 2P(s) + 1 C) $sP(s^2)$ D) Does not exist

57. A random variable X has pdf $f(x) = 1, 0 < x < 1$. Then $P\left[\left|X - \frac{1}{2}\right| > \frac{1}{\sqrt{3}}\right] \leq$ ----.

A) 0.25 B) 0.05 C) 0.2 D) 0.01

58. If X is a random variable with $\beta_n = E|X|^n < \infty$, then for $2 \leq k \leq n$, which of the following is true?

A) $(\beta_{k-1})^{\frac{1}{k-1}} \leq (\beta_k)^{\frac{1}{k}}$ B) $(\beta_{k-1})^{\frac{1}{k-1}} \geq (\beta_k)^{\frac{1}{k}}$
C) $(\beta_{k-1})^{\frac{1}{k-1}} = (\beta_k)^{\frac{1}{k}}$ D) $(\beta_{k-1})^{\frac{1}{k}} \leq (\beta_{k-1})^{\frac{1}{k-1}}$

59. Let X_n be a random variable defined by $P(X_n = n^2) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Then which of the following is true?

A) $X_n \xrightarrow{P} 0$ and $E(X_n) \to 0$ B) $X_n \xrightarrow{P} 0$ and $E(X_n) \to \infty$

C) $X_n \xrightarrow{P} 1$ and $E(X_n) \to 0$ D) None of the above

60. Let $\{X_n\}$ be a sequence of iid random variables with mean μ and finite variance σ^2. If $S_n = X_1 + X_2 + \cdots + X_n$, then $P\left[\lim_{n\to\infty} \frac{S_n}{n} = \mu\right]=$

A) 0   B) $\frac{1}{2}$ C) 1   D) $\frac{\sigma^2}{\mu^2}$

61. Which of the following is not a characteristic function?

A) e^{-t^4} B) $e^{-|t|^4}$ C) $(1 + t^4)^{-1}$ D) All of these

62. Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$, σ^2 is known. It is also known that $\mu \in (\theta_1, \theta_2)$, $\theta_1 < \theta_2$. If $\bar{X}$ is the sample mean and

$$T = \begin{cases} \theta_1, \; if\ \bar{X} < \theta_1 \\ \bar{X}, \; if\ \theta_1 \le \bar{X} \le \theta_2 \\ \theta_2, \; if\ \bar{X} > \theta_2 \end{cases}$$

Then which of the following is true?

A) T is unbiased estimator for μ with $MSE(\bar{X}) = MSE(T)$
B) T is biased estimator for μ with $MSE(\bar{X}) < MSE(T)$
C) T is unbiased estimator for μ with $MSE(\bar{X}) > MSE(T)$
D) T is biased estimator for μ with $MSE(\bar{X}) > MSE(T)$

63. If $X_{(1)}$ and $X_{(n)}$ are the 1st and n^{th} order statistics of a random sample of size n from the rectangular distribution $U(a, \theta)$, where a is known. Then,

A) $X_{(n)}$ is sufficient for θ
B) $X_{(1)}$ and $X_{(n)}$ are jointly sufficient for θ
C) $\min(-X_{(1)}, X_{(n)})$ is sufficient for θ
D) $X_{(n)} - X_{(1)}$ is sufficient for θ

64. Let $X_1, X_2, \ldots, X_n$ be a random sample from pdf

$$f(x) = \frac{1}{\beta} e^{-\left(\frac{x-\alpha}{\beta}\right)}, \quad \alpha < x < \infty, \quad -\infty < \alpha < \infty, \quad \beta > 0$$

If $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$ and $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then the MLE of β is:

A) $X_{(1)}$ B) $\frac{1}{\bar{X} - X_{(1)}}$ C) $\bar{X} - X_{(1)}$ D) $\bar{X}$

65. Suppose X has density function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad x \in R$ under H_0 and $g(x) = \frac{1}{2}\exp\{-|x|\}$, $x \in R$ under H_1. Then most powerful critical region for testing H_0 against H_1 based on a single observation is of the form:

A) $|x| > k$ for a constant k

B) $|x| \le k$ for a constant k

C) $k_1 \le |x| \le k_2$ for some constant k_1 and k_2

D) $|x| \ge k_1$ or $|x| \le k_2$ for some constant k_1 and k_2

66. Which of the following family of pdf $\{f_\theta\}$ has monotone likelihood ratio property ?

1. $f_\theta(x) = \frac{1}{2}\exp(-|x-\theta|), \quad -\infty < x < \infty, \quad \theta \in R$

2. $f_\theta(x) = \frac{e^{-x-\theta}}{[1+e^{-x-\theta}]^2}, \quad -\infty < x < \infty, \quad \theta \in R$

3. $f_\theta(x) = \frac{1}{\theta}, \ 0 < x < \theta, \ \theta > 0$

A) 1 and 2 only
B) 1 and 3 only
C) 2 and 3 only
D) 1 , 2 and 3

67. Let $X_1, X_2, \ldots, X_n$ be independent observations taken from the binomial distribution $b(1, p)$. To test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$ sequential probability ratio test is applied. Let A and B, $0 < B < 1 < A$ be the boundary constants of the SPRT, then at the m^{th} stage, H_0 will be accepted if:

A) $\sum_{i=1}^{m} X_i \le \log(2^m B)$
B) $\sum_{i=1}^{m} X_i \ge \log(2^m A)$

C) $\sum_{i=1}^{m} X_i \le \log\frac{B}{2^m}$
D) $\sum_{i=1}^{m} X_i \ge \log\frac{A}{2^m}$

68. Let $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ be independent random samples from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, μ_1, μ_2 are unknown. If $\lambda\left(\underline{x}, \underline{y}\right)$ be the likelihood ratio used for testing $H_0: \sigma_1^2 = \sigma_2^2$ against $H_1: \sigma_1^2 \ne \sigma_2^2$. Then the asymptotic distribution of $-2\log\lambda\left(\underline{x}, \underline{y}\right)$ is:

A) Chi-square with 1 degree of freedom
B) Chi-square with 2 degrees of freedom
C) Chi-square with $m + n - 1$ degrees of freedom
D) Chi-square with $m + n - 2$ degrees of freedom

69. The Fisher information in the sample of size 100 taken from the Poisson distribution with mean 4 is:

A) $\frac{1}{4}$
B) $\frac{1}{25}$
C) 5
D) 25

70. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from $U(0, \theta)$. The uniformly most powerful test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$ with level of significance α is given by:

$$\varphi(\underline{x}) = \begin{cases} 1, & x_{(n)} > \theta_0 \text{ or } x_{(n)} < \theta_0 \alpha^{1/n} \\ 0, & \text{otherwise} \end{cases}$$

where $x_{(n)} = \max(X_1, X_2, \ldots, X_n)$. Then the uniformly most accurate confidence interval for θ is:

A) $\left[x_{(n)} - \alpha^{1/n}, \; x_{(n)} + \alpha^{1/n}\right]$

B) $\left[x_{(n)}\alpha^{1/n}, \; x_{(n)}\right]$

C) $\left[x_{(n)}, \; x_{(n)}\alpha^{-1/n}\right]$

D) Cannot be determined with the given information

71. Three brands of tea are rated for the taste on a scale of 1 to 10. Six persons are asked to rate each brand so that there is a total of 18 observations. The appropriate test to determine if three brand's taste equally good is:
A) One way analysis of variance
B) Friedman test
C) Kruskal-Wallis test
D) Wilcoxon rank-sum test

72. Let $X_1, X_2, \ldots, X_n$ be a random sample from a gamma distribution with PDF
$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\beta x} x^{\alpha-1}, \; x > 0, \; \alpha, \beta > 0$
If β is known, then which of the following is true?

A) $\sum X_i$ is sufficient for α B) $\sum X_i^2$ is sufficient for α
C) $\prod X_i$ is sufficient for α D) $(\sum X_i, \sum X_i^2)$ is sufficient for α

73. Under the assumptions required for the Wilcoxon's signed rank test, let T^+ be the sum of the ranks of positive X_i's of a random sample $X_1, X_2, \ldots, X_n$. Then the asymptotic distribution of T^+ under the null hypothesis is:

A) $N\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{12}\right)$ B) $N\left(\frac{n(n+1)}{2}, \frac{n(n+1)(2n+1)}{24}\right)$

C) $N\left(\frac{n(n+1)}{4}, \frac{n(n+1)(2n+1)}{24}\right)$ D) $N\left(\frac{n(n+1)}{6}, \frac{n(n+1)(2n+1)}{12}\right)$

74. Let $X_1, X_2, \ldots, X_n$, $n \geq 2$ be iid observations from $N(0, \sigma^2)$, where $\sigma^2 (> 0)$ is unknown. Then the UMVUE of σ^2 is:

A) $\frac{1}{n}\sum X_i^2$ B) $\frac{1}{n-1}\sum X_i^2$

C) $\frac{1}{n}\sum(X_i - \bar{X})^2$ D) $\frac{1}{n-1}\sum(X_i - \bar{X})^2$

75. A UMP test is:

A) Always exists B) Unbiased test

C) Biased test D) None of these

76. Let $X \sim b(n,p)$ and p has the pdf $\pi(p) = 1,\ 0 < p < 1$. Then the Baye's estimator of p^2 under the quadratic error loss function is:

A) $\frac{X}{n}$ B) $\frac{X(X+1)}{n(n+1)}$ C) $\frac{(X+1)(X+2)}{(n+2)(n+3)}$ D) $\frac{X^2}{n^2}$

77. Let $X_1, X_2, \ldots, X_n$ be a sequence of iid Bernoulli random variable with probability of success p. Also, let n be a positive integer valued random variable having Poisson distribution with mean θ. Then the mean and variance of $\sum_{i=1}^{n} X_i$ are:

A) Mean= θp, variance= θp

B) Mean= p, variance= $\theta^2 p^2$

C) Mean= $n\theta p$, variance= $n\theta p^2$

D) Mean= np, variance= $n\theta p(1-p)$

78. Let $X_1, X_2, \ldots, X_n$ be iid random variables, then which of the following statement is true:

A) $min(X_1,\ X_2,\ \ldots,\ X_n)$ has Weibull distribution if and only if the common distribution of X_i's is Weibull.

B) If X_i 's are Weibull random variables, then $min(X_1,\ X_2,\ \ldots,\ X_n)$ is Weibull and the converse need not be true.

C) $min(X_1,\ X_2,\ \ldots,\ X_n)$ has Weibull distribution if and only if the common distribution of X_i 's is exponential.

D) All the above statements are wrong.

79. Let X has power series distribution with probability mass function

$$P(X = x) = \frac{a_x \theta^x}{A(\theta)},\ x = 0,\ 1,\ 2,\ \ldots;\ \theta > 0;\ a_j > 0;\ A(\theta) = \sum_{x=0}^{\infty} a_x \theta^x.$$

Then which of the following is true:

A) variance(X)= mean(X) B) variance(X)= $\theta \frac{d}{d\theta}$ (mean)

C) variance(X)= $\frac{d^2}{d\theta^2}$ (mean) D) variance(X)= $2\theta \frac{d^2}{d\theta^2}$ (mean)

80. For a set of n observations $Y_1, Y_2, \ldots Y_n$, the maximum number of mutually orthogonal contrasts among them is:

A) n B) $n-1$ C) $n-2$ D) $n-3$