1. If $2x^3 + 4x^2 + 2ax + b$ is divisible by $x^2 - 1$, then a and b are respectively
   (A) 1 and 4 (B) 1 and -4 (C) -1 and 4 (D) -1 and -4

2. The distance between the parallel planes $x - 2y + 2z = 8$ and $x - 6y + 6z = 20$ is
   (A) $\frac{2}{9}$ (B) $\frac{3}{9}$ (C) $\frac{4}{9}$ (D) $\frac{5}{9}$

3. If $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$, then
   (A) $(1-x^2)y_1 - xy = 1$
   (B) $(1-x^2)y_2 - xy_1 - y = 0$
   (C) $(1-x^2)y_1 + xy = 0$
   (D) $(1-x^2)y_2 - xy_1 - y = 0$

4. $\displaystyle\int_0^1 |x-1|\, dx$ has value
   (A) 1 (B) 2 (C) 3 (D) 5

5. A fair die is thrown and a fair coin is tossed. What is the probability of getting a six on the die and head on the coin?
   (A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{1}{12}$ (D) $\frac{2}{3}$

6. A fair die is thrown six times. If getting an even number is a success, what is the probability of getting at least five successes?
   (A) $\frac{3}{32}$ (B) $\frac{1}{64}$ (C) $\frac{7}{64}$ (D) $\frac{63}{64}$

7. The centre of the circle $z\overline{z} + \overline{a}z + a\overline{z} + c = 0$ is
   (A) a (B) $-a$ (C) $\overline{a}$ (D) c

8. If $f(z) = \cos\left(\frac{1}{z}\right)$, then the point $z = 0$ is
   (A) an essential singularity (B) a simple pole
   (C) a removable singularity (D) a point of analyticity

9. The bilinear transformation that takes the points $z = 2, i, -2$ into the points $w = 1, i, -1$ respectively is given by
   (A) $w = \dfrac{2z+3i}{z+6i}$ (B) $w = \dfrac{3z+2i}{iz+6}$ (C) $w = \dfrac{3z-2i}{iz-6}$ (D) $w = \dfrac{2z-3i}{z-6i}$

10. Let $f(x) = x|x|$, $x \in R$. Then
    (A) f is not differentiable on R (B) $f'(x) = |x|$, $x \in R$
    (C) $f'(x) = 2|x|$, $x \in R$ (D) $f'(x) = 2x$, $x \in R$

11. Which of the following series is absolutely convergent?

(A) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ (B) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1}$ (C) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (D) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$

12. For $x \in [0,1]$, define $f(x) = \begin{cases} 0, x \text{ is rational} \\ 1, x \text{ is irrational} \end{cases}$

Which of the following statements is true?

(A) The Riemann integral $\int_0^1 f(x)dx$ exists and equals 1
(B) The Lebesgue integral $\int_0^1 f(x)dx$ does not exist
(C) The Lebesgue integral $\int_0^1 f(x)dx$ exists and equals 1
(D) The Lebesgue integral $\int_0^1 f(x)dx$ exists and equals 0

13. Consider the functions
$f_n(x) = \frac{1}{nx+1}$, for all $x \in (0,1)$, $n = 1, 2, ...$
$g_n(x) = \frac{x}{nx+1}$, for all $x \in (0,1)$, $n = 1, 2, ...$
Which of the following statements is not true?

(A) $\{f_n\}$ converges pointwise but not uniformly on $(0,1)$
(B) $\{g_n\}$ converges uniformly on $(0,1)$
(C) Both $\{f_n\}$ and $\{g_n\}$ converge uniformly on $(0,1)$
(D) $\{g_n\}$ converges to 0

14. Which of the following statements is not true?

(A) There exist non-abelian group(s) of order 6
(B) There exist non-abelian group(s) of order 8
(C) A_4 has a subgroup of order 6
(D) The class equation of any non-abelian group of order 6 is $6 = 1 + 2 + 3$

15. Which of the following is an integral domain?

(A) Z_9 (B) Z_{10} (C) Z_{11} (D) Z_{12}

16. Let F be a field and char(F) denote the characteristic of F.
Which of the following statements is true?

(A) char(F) is always a prime number
(B) char(F) is always zero
(C) char(F) is either zero or a prime number
(D) char(F) may be a composite number

17. What is $[Q(\sqrt{2}, \sqrt{3}, \sqrt{6}) : Q]$?

(A) 2 (B) 4 (C) 6 (D) 8

18. The minimal polynomial of $\sqrt{2} + \sqrt{3}$ over $Q[\sqrt{3}]$ is

(A) $x^2 - 2\sqrt{3}x + 1$ (B) $x^2 + 2\sqrt{3}x + 1$ (C) $x^2 + 2\sqrt{3}x - 1$ (D) $x^2 - 2\sqrt{3}x - 1$

19. What is the rank of the martrix $\begin{pmatrix} 1 & 1 & -3 & -1 \\ 4 & -2 & 6 & 8 \\ 15 & -3 & 9 & 21 \end{pmatrix}$

(A) 1 (B) 2 (C) 3 (D) 4

20. For what values of λ and μ will the system of equations
$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$
have no solution?

(A) $\lambda = 3$, $\mu = 10$ (B) $\lambda = 3$, $\mu \neq 10$ (C) $\lambda \neq 3$, μ any value (D) $\lambda \neq 3$, $\mu \neq 10$

21. Consider R^2 with the standard inner product. Which of the following elements of R^2 form an orthonormal basis?

(A) $\{(1,-1),(-1,-1)\}$ (B) $\{(-1,0),(0,-1)\}$ (C) $\{(1,0),(0,1),(1,1)\}$ (D) None of these

22. Which of the following statements is true?

(A) The subspace $\{(x,x,x)\}$ of R^3 is of dimension 3
(B) Every spanning set contains a basis
(C) If U and W are subspaces of a finite dimensional vector space, then dim $(U) \leq$ dim $(W) \Longrightarrow U \subseteq W$
(D) If $\{x,y,z\}$ is a basis of R^3 and w is any non-zero element of R^3, then $\{w+x,y,z\}$ note is also a basis

23. The linear map $f : R^3 \to R^3$ is defined by $f(x,y,z) = (x+y, y+z, z+x)$
What are $\text{Im}(f)$ and $\ker(f)$ respectively?

(A) R^3 and $\{(0,0,0)\}$ (B) R^2 and $\{(0,0)\}$
(C) R^3 and R^2 (D) $\{(0,0,0)\}$ and $\{(0,0)\}$

24. The linear map $f : R^2 \to R^3$ defined by $f(x,y,z) = (y,0,x)$ is

(A) injective and surjective (B) injective but not surjective
(C) surjective but not injective (D) neither injective nor surjective

25. Which of the following statements is not true?

(A) The eigen values of an orthogonal matrix have absolute value 1
(B) The eigen vectors corresponding to distinct eigen values of a real symmetric matrix are orthogonal
(C) An eigen vector may correspond to two different eigen values
(D) The eigen vectors corresponding to distinct eigen values of a matrix are linearly independent

26. The least non-negative residue of $2^{68} \pmod{19}$ is

(A) 2 (B) 3 (C) 4 (D) 6

27. The general solution of the congruence $12x \equiv 9 \pmod{15}$ is

(A) $x \equiv 1 \pmod 5$ (B) $x \equiv 2 \pmod 5$ (C) $x \equiv 3 \pmod 5$ (D) There is no solution

28. What is the integrating factor of the differential equation
$\dfrac{dy}{dx} + y\sec x = \tan x$, $0 < x < \frac{\pi}{2}$?

(A) $\sec x$ (B) $\sec x + \tan x$ (C) $\tan x$ (D) $\sec x \tan x$

29. The differential equation $(ax+hy+g)dx+(hx+by+f)dy=0$ has solution

(A) $ax^2+2hxy+by^2=c$
(B) $ax^2+2hxy+by^2+2gx+2fy=c$
(C) $ax^2+hxy+by^2=c$
(D) $ax^2+2hxy+by^2+gx+fy=c$

30. The equation $(1+x^2)\dfrac{d^2y}{dx^2}-4(1+x)\dfrac{dy}{dx}+6y=0$ reduces to which of the following linear differential equations with the substitution $1+x=e^z$ and $D\equiv\dfrac{d}{dx}$?

(A) $(D^2-5D+6)y=e^z-1$
(B) $(D^2-5D+6)y=e^z$
(C) $(D^2+5D+6)y=e^z-1$
(D) $(D^2+5D+6)y=e^z$

31. The complementary function of the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y$$

is given by

(A) $f_1(y-x)+f_2(y+x)$
(B) $f_1(y-x)+f_2(y+2x)$
(C) $f_1(y+x)+f_2(y+2x)$
(D) $f_1(y-x)+f_2(y-2x)$

32. Which of the following metric spaces is not complete?

(A) R with the Euclidean metric
(B) R with the discrete metric
(C) $C[0,1]$ with the supremum metric
(D) The metric space $P[0,1]$ of polynomials with the supremum metric

33. Which of the following is a connected space that is not path-connected?

(A) The Moore plane (B) The topologist's sine curve
(C) The Cantor set (D) R with the usual topology

34. Which of the following statements is not true?

(A) A finite product of compact spaces is compact
(B) Continuus images of compact spaces is compact
(C) R^n is compact
(D) Every closed and bounded subset of R^n is compact

35. Let A and B be self-adjoint operators on a Hilbert space. Then

(A) AB is self-adjoint
(B) AB is never self-adjoint
(C) AB is self-adjoint iff $A=B$
(D) AB is self-adjoint iff $AB=BA$

36. The Hilbert space in which Legendre polynomials are orthogonal is
(A) $L^2[-1,1]$ (B) $L^2[-\pi,\pi]$ (C) $L^2[-\frac{\pi}{2},\frac{\pi}{2}]$ (D) $L^2[0,\infty)$

37. Let C be the positively oriented circle $|z-i|=2$. Then
$\int_C \frac{e^z}{z^2+4}$ equals
(A) $\frac{\pi}{2}e^{2i}$ (B) $2\pi i e^{2i}$ (C) $\pi i e^{i}$ (D) 0

38. The number of group homomorphisms from the cyclic group Z_4 to the cyclic group Z_7 is
(A) 1 (B) 2 (C) 3 (D) 4

39. The number of elements of order 3 in A_4 is
(A) 8 (B) 7 (C) 2 (D) 1

40. Which of the following statements is true?
(A) l^p is an inner product space for $1<p<\infty$
(B) l^p is a Hilbert space for $1<p<\infty$
(C) l^2 is an inner product space, but not a Hilbert space
(D) l^2 is a Hilbert space

41. The function $f(x)=x^2$ from the set of positive real numbers to positive real numbers is
(A) injective but not surjective
(B) surjective but not injective
(C) neither injective nor surjective
(D) both injective and surjective

42. The sides of an equilateral triangle are shortened by 12 units, 13 units and 14 units respectively and a right angle triangle is formed. Then length of each side of the equilateral triangle is
(A) 13 (B) 15 (C) 17 (D) 19

43. Equation to the line whose gradient is $\frac{3}{2}$ and passes through the point P which divides the line segment joining the points $A(-2,6)$ and $B(3,-4)$ in the ratio $2:3$ internally is
(A) $2x-3y+4=0$ (B) $3x-2y+4=0$ (C) $2x+3y+4=0$ (D) $3x+2y-4=0$

44. Equation of the circumcircle of the triangle whose vertices are $(1,0),(0,-6)$ and $(3,4)$ is
(A) $x^2+y^2+142x+47y-143=0$ (B) $4x^2+4y^2-142x+47y+138=0$
(C) $x^2+y^2+42x+2y-24=0$ (D) $4x^2+4y^2+47x-142y+138=0$

45. Equation of the ellipse whose centre is the origin, $x-$ axis is the major axis and which passes through the points $(-3,1)$ and $(2,-2)$ is
(A) $3x^2+5y^2=32$ (B) $3x^2+5y^2=23$ (C) $5x^2+3y^2=32$ (D) $5x^2+3y^2=23$

46. The value of $\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x-\sin x}$ is equal to
(A) 5 (B) 6 (C) 3 (D) 0

47. Maximum value of y in the equation $9x^2+4y^2-72=0$ is
(A) zero (B) infinity (C) $2\sqrt{3}$ (D) $3\sqrt{2}$

48. For the series $\Sigma_{n=1}^{\infty}\dfrac{(-1)^{n+1}}{n}$, which among the following is a true statement?

(A) The series is convergent and absolutely convergent.
(B) The series is convergent but not absolutely convergent.
(C) The series is absolutely convergent but not convergent.
(D) The series is neither convergent nor absolutely convergent.

49. Define $f : (0, 1) \to \mathcal{R}$ by the rule

$$f(x) = \begin{cases} \dfrac{1}{k} & \text{if } x = \dfrac{m}{k} \quad \text{where } m \text{ and } k \text{ are integers and have no common factor} \\ 0 & \text{if } x \text{ is irrational,} \end{cases}$$

then which among the following is true?

(A) f is continuous at all rationals and irrationals in $(0, 1)$.
(B) f is discontinuous at all rationals and irrationals in $(0, 1)$.
(C) f is continuous at all rationals and discontinuous at all irrationals in $(0, 1)$.
(D) f is discontinuous at all rationals and continuous at all irrationals in $(0, 1)$.

50. If the function $f(x)$ is defined by the rule $f(x) = \begin{cases} 0 & \text{if } x \text{ is irrational} \\ 1 & \text{if } x \text{ is rational,} \end{cases}$

then which among the following is true?

(A) $f(x)$ is of bounded variation on the set of rational points in $[0, 1]$
(B) $f(x)$ is of bounded variation on the set of irrational points in $[0, 1]$
(C) $f(x)$ is of bounded variation on any interval
(D) $f(x)$ is not of bounded variation on any interval

51. If $f \in \mathcal{R}(\alpha)$ on $[a, b]$ and if $\displaystyle\int_a^b f d\alpha = 0$ for every f which is monotonic on $[a, b]$, then

(A) α can be an arbitrary function on $[a, b]$ (B) α must be a constant function on $[a, b]$
(C) α must be a positive function on $[a, b]$ (D) α must be a zero function on $[a, b]$

52. For the complex number i, the value of i^{-999} is equal to

(A) i (B) $-i$ (C) 1 (D) -1

53. If ω is the cube root of unity $(\omega \neq 1)$, then the least value of n where n is positive integer such that $(1 + \omega^2)^n = (1 + \omega^4)^n$ is

(A) one (B) two (C) three (D) four

54. If R is the radius of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$, then

(A) $R = 1$ (B) $R = 1/2$ (C) $R = 1/4$ (D) none of these

55. If G is a group $a, b \in G$ and n is any integer, then $(a^{-1}ba)^n$ is equal to

(A) aba^{-1} (B) $ab^n a^{-1}$ (C) $a^{-1}b^n a$ (D) $a^{-1}ba$

56. Number of generators of a cyclic group of order 60 is

(A) 16 (B) 32 (C) 60 (D) 1

57. If p is a prime number and G is a group of order p^2, then which of the following is true?

(A) G is a trivial group
(B) G is an abelian group
(C) G is a non abelian group
(D) None of these

58. A sylow 3-subgroup of a group of order 12 has order

(A) 2 (B) 3 (C) 6 (D) 8

59. Let D be an integral domain. a and b be two elements of D such that $a^n = b^n$ and $a^m = b^m$ for two relatively prime positive integers m and n then

(A) $a = b$ (B) $na = mb$ (C) $ma = nb$ (D) None of these

60. Solutions of the equation $x^2 - 5x + 6 = 0$ in $\mathcal{Z}_{12}$ are

(A) 0 and 1 (B) 2 and 7 (C) $2, 3, 6$ and 9 (D) $2, 3, 6$ and 11

61. Let $f(x) \in F[x]$, and $f(x)$ be of degree 2 or 3. Consider the statements
(1) If $f(x)$ is reducible over F, then it has a zero in F
(2) If $f(x)$ has a zero in F, then $f(x)$ is reducible over F
Then which is the correct choice?

(A) both (1) and (2) are true
(B) (1) is true and (2) is false
(C) (1) is false and (2) is true
(D) both (1) and (2) are false

62. If r is the remainder when 8^{103} is divided by 13, then r is equal to

(A) 1 (B) 5 (C) 7 (D) 11

63. Value of x for which the $M = \begin{pmatrix} 6-x & 4 \\ 3-x & 1 \end{pmatrix}$ is a singular matrix is

(A) -1 (B) 1 (C) -2 (D) 2

64. If $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$, then $\dfrac{A^2 - 3I}{2}$ is equal to

(A) zero (B) A (C) A^{-1} (D) none of these

65. Minor of the element 6 and co-factor of the element 4 in the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ are respectively

(A) 6, 6 (B) 6, -6 (C) -6, 6 (D) -6, -6

66. The three dimensional Euclidean space $\mathcal{R}^3$ over $\mathcal{R}$ has

(A) one basis (B) two bases (C) three bases (D) infinite number of bases

67. In the vector space of polynomials $\mathcal{P}$ in the variable t, the polynomials x, y, z are defined by $x(t) = 1 - t, y(t) = t(1-t)$ and $z(t) = 1 - t^2$. Then for the set $\mathcal{B} = \{x, y, z\}$, which of the following statements is true?

(A) $\mathcal{B}$ is a basis for $\mathcal{P}$ (B) $\mathcal{B}$ is linearly independent, but not a basis for $\mathcal{P}$
(C) $\mathcal{B}$ is linearly dependent (D) $\mathcal{B} = \mathcal{P}$

68. Let $T : \mathcal{R}^3 \to \mathcal{R}^3$ be a linear operator defined by $T(x_1, x_2, x_3) = (3x_1 + x_2, x_1 + x_3, x_1 - x_3)$. The matrix of T in the standard basis of $\mathcal{R}^3$ is

(A) $\begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ (C) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (D) none of these

69. Value of the sum $2 + 6 + 10 + \cdots + (4n - 2)$ is equal to

(A) n^2 (B) $(n+1)^2$ (C) $(n-1)^2$ (D) $2n^2$

70. Remainder when 97! divided by 101 is

(A) 15 (B) 16 (C) 17 (D) none of these

71. Orthogonal trajectories of the family of parabolas $y = ax^2$ is

(A) $x^2 + y^2 = a^2$ (B) $x^2 - 2y^2 = c^2$ (C) $x^2 + 2y^2 = c$ (D) none of these

72. Particular integral of the differential equation $\dfrac{d^2y}{dx^2} + y = \sin x$ is

(A) $-\dfrac{x}{2}\cos x$ (B) $\dfrac{x}{2}\sin x$ (C) $-\dfrac{x}{2}\sin x$ (D) $\dfrac{x}{2}\cos x$

73. A partial differential equation satisfying $z = ae^{by}\sin bx$ for arbitrary constants a and b is

(A) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ (B) $\dfrac{\partial^2 z}{\partial x^2} - \dfrac{\partial^2 z}{\partial y^2} = 0$ (C) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial z}{\partial y} = 0$ (D) $\dfrac{\partial z}{\partial x} + \dfrac{\partial^2 z}{\partial y^2} = 0$

74. The one dimensional wave equation $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}$ is

(A) hyperbolic (B) parabolic (C) elliptic (D) none of these

75. Let $\mathcal{Q}$ denotes the set of all rational numbers with usual metric $d(x, y) = |x - y|$, for $x, y \in \mathcal{Q}$. Consider the sequence $1.4, 1.41, 1.414, \cdots$ the sequence of the decimal approximations of $\sqrt{2}$ to one, two, three, ... decimal places. Then which of the following statements is true?

(A) sequence is Cauchy and convergent in $\mathcal{Q}$
(B) sequence is Cauchy but not convergent in $\mathcal{Q}$
(C) sequence is neither Cauchy nor convergent in $\mathcal{Q}$
(D) sequence is divergent in $\mathcal{R}$, the set of real numbers.

76. If T and T' are respectively the usual topology on the Euclidean plane $\mathcal{R}^2$ and the topology induced by the lexicographic ordering on $\mathcal{R}^2$, then which of the following is a correct statement?

(A) $T = T'$
(B) T is strictly weaker than T'
(C) T is strictly stronger than T'
(D) T and T' are non comparable

77. Let $\mathcal{B}$ and $\mathcal{B}'$ be bases for the topologies $\mathcal{T}$ and $\mathcal{T}'$ respectively on X. Consider the statements
(a) $\mathcal{T}'$ is finer than T
(b) For each $x \in X$ and each basis element $B \in \mathcal{B}$ containing x, there is a basis element $B' \in \mathcal{B}'$ such that $x \in B' \subseteq B$. Then

(A) $(a) \Rightarrow (b)$ but $(b) \nRightarrow (a)$
(B) $(a) \nRightarrow (b)$ but $(b) \Rightarrow (a)$
(C) $(a) \Rightarrow (b)$ and $(b) \Rightarrow (a)$
(D) $(a) \nRightarrow (b)$ and $(b) \nRightarrow (a)$

78. Let X be a topological space and for $x \in X$, let $\mathcal{N}_x$ be the neighbourhood system at x. Then which among the following statements is not correct?

(A) If $U \in \mathcal{N}_x$, then $x \in U$
(B) For any $U, V \in \mathcal{N}_x,\ U \cap V \in \mathcal{N}_x$
(C) If $V \in \mathcal{N}_x$ and $U \subset V$, then $U \in \mathcal{N}_x$
(D) A set G is open in X if and only if $G \in \mathcal{N}_x$ for all $x \in G$

79. In a topological space, consider the statements

1. Every open surjective map is a quotient map
2. Every closed surjective map is a quotient map.

Which of the following choices is correct?

(A) Both statements (1) and (2) are correct
(B) (1) is correct but (2) is not correct
(C) (1) not correct but (2) is correct
(D) Neither (1) nor (2) is correct

80. If θ is the angle between the vectors $(2, 3, 5)$ and $(1, -4, 3)$ in the real innerproduct space $\mathcal{R}^3$ over $\mathcal{R}$, then which of the following is correct?

(A) $\sin^2\theta = \dfrac{963}{988}$ (B) $\sin^2\theta = 0$ (C) $\sin^2\theta = \dfrac{25}{988}$ (D) $\sin^2\theta = 1$