A

23121 120 Minutes

1. The integral surface satisfying the differential equation $\frac{dy}{dx}$ - ytanx = secx and passing through the point (0, 2) is

   A) ycosx $= x^2 + 2$ B) ycosx $= e^x + 1$ C) ycosx $= x + 2$ D) ycosx $= e^{sinx} + 1$

2. Any compact subset of a Hausdorff space is

   A) compact B) open C) closed D) connected

3. If A, B, C are three sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$, then which of the following is true

   A) $A = B$ B) $B = C$ C) $C = A$ D) none of these

4. The intersection of all intervals $(-1/n, 1/n), n = 1, 2, \cdots$ is the set

   A) null set B) $\{0\}$ C) $(-\epsilon, +\epsilon)$ where ϵ is very small but positive D) $\{1/n\}$

5. Consider the statements

   (a) If f is monotonic on $[a, b]$, then the set of discontinuities of f is empty.
   (b) If f is monotonic on $[a, b]$, then f is of bounded variation on $[a, b]$.

   Here which of the following is correct?

   A) (a) is true and (b) is false B) (a) is false and (b) is true
   C) both (a) and (b) are true D) both (a) and (b) are false

6. If $\vec{a}, \vec{b}, \vec{c}$ are three orthonormal vectors, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ is equal to

   A) 0 B) 1 C) -1 D) ± 1

7. Let G be a finite group. Then G is a p-group if and only if order of G is

   A) p B) a power of p C) p^2 D) none of these

8. Two cards are drawn from a well shuffled pack of 52 cards. Probability that both are spades is

   A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{17}$ D) $\frac{1}{6}$

9. Value of the definite integral $\int_0^1 \frac{(\tan^{-1} x)^2}{1+x^2} dx$ is

A) $\frac{\pi^3}{192}$ B) $\frac{\pi}{192}$ C) $\frac{\pi^2}{192}$ D) $\frac{1}{192}$

10. Equation to the parabola whose vertex is the origin, symmetric about $y-$axis and passing through the point $(2, -3)$ is

A) $4x^2 + 3y = 0$ B) $3x^2 + 4y = 0$ C) $4x^2 - 3y = 0$ D) $3x^2 - 4y = 0$

11. If $s_n = n^2$, then the sequence $\{s_n\}$ is

A) bounded and convergent B) bounded and divergent
C) unbounded and convergent D) unbounded and divergent

12. If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence such that $x_n \geq 0$ and $\lim_{n\to\infty} \sqrt{x_n} = l$, then $\sqrt{\lim_{n\to\infty} x_n}$ is equal to

A) $\sqrt{l}$ B) l^2 C) l D) zero

13. In the co countable topology only convergent sequences are

A) Cauchy sequences B) constant sequences
C) eventually constant sequences D) none of these

14. For the complex number i, $\frac{3i^{30} - i^{19}}{2i - 1}$ is equal to

A) i B) $-i$ C) $1 + i$ D) $1 - i$

15. Polar form of the Cauchy Riemann equations is

A) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial r}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial r}{\partial \theta}$ B) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

C) $\frac{\partial u}{\partial r} = -\frac{1}{r}\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial u}{\partial \theta}$ D) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta}, \quad \frac{\partial v}{\partial r} = -r\frac{\partial u}{\partial \theta}$

16. $\int_C \frac{\cos z}{z} dz$ where C is the unit circle is

A) $2\pi i$ B) πi C) $\frac{\pi}{2} i$ D) $\frac{\pi}{4} i$

17. Residue of the function $\cot z$ at the singular points is

A) 0 B) i C) $-i$ D) 1

18. In the group $G = \{1, -1, i, -i\}$ under multiplication, inverse of the element $-i$ is

A) 1 B) -1 C) i D) $-i$

19. Let G be a finite group on 84 elements. If H is a proper subgroup of G, then maximum number of elements possible in H is

A) 84 B) 42 C) 12 D) 22

20. Identity of the group $G = \{2, 4, 6, 8\}$ under multiplication modulo 10 is

A) 2 B) 4 C) 6 D) 8

21. (a) Every finite integral domain is a field.
(b) $\mathbb{Z}_n$ is a field if and only if n is a prime number.
Which of the above two statements is true?

A) Both (a) and (b) B) (a) only C) (b) only D) Neither (a) nor (b)

22. If p is a prime number and S is the set of all divisors of zero in $\mathbb{Z}_p$, then

A) $S = \{1\}$ B) $S = \{1, 2\}$ C) S is empty D) none of these

23. $A = \begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix}$ is a symmetric matrix if

A) $x = 2$ B) $x = -2$ C) $x = 4$ D) $x = -4$

24. If A is a square matrix, then $A - A^T$ is

A) zero matrix B) unit matrix
C) skew symmetric matrix D) symmetric matrix

25. If $A = \begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$, then $x =$

A) 1 B) 2 C) $\frac{1}{2}$ D) 0

26. If λ denotes the eigen values of the matrix $\begin{pmatrix} 5 & 2 & 2 \\ 3 & 6 & 3 \\ 6 & 6 & 9 \end{pmatrix}$, then one of the values of λ is

A) $\lambda = 14$ B) $\lambda = 2$ C) $\lambda = 1$ D) $\lambda = 7$

27. Let S be the sphere with centre origin and finite radius $r(\neq 0)$ in $\mathbb{R}^3$, the three dimensional Euclidean space. Then which of the following statement is true?

A) S is an empty set in $\mathbb{R}^3$
B) S is a subspace of dimension one in $\mathbb{R}^3$
C) S is a subspace of dimension two in $\mathbb{R}^3$
D) None of these

28. Which of the following is a basis for M_{22}, the vector space of all 2×2 real matrices?

A) $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 3 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}.$ B) $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 3 & 0\end{pmatrix}, \begin{pmatrix}0 & 2\\ 0 & 0\end{pmatrix}.$

C) $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}3 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}.$ D) $\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 3 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}.$

29. If A is an invertible linear transformation and $\alpha \neq 0$, then $(\alpha A)^{-1}$ is equal to

A) $\frac{1}{\alpha}A^{-1}$ B) $-\alpha A^{-1}$ C) $\alpha^{-1}A$ D) none of these

30. If ϕ denotes the Euler's totient function, then value of $\phi(1200)$ is

A) 320 B) 360 C) 400 D) 350

31. Let Fermat's little theorem be stated as, if p is a prime number and a is a number such that p does not divide a, then $a^{p-1} \equiv y(\mathrm{mod}\, m)$. In this case

A) $y = 1,\ m = p$ B) $y = 1,\ m = a$ C) $y = p,\ m = 1$ D) $y = a,\ m = 1$

32. Integrating factor of the differential equation
$x^2y^3 + x(1+y^2)\frac{dy}{dx} = 0$ is

A) $\frac{x}{y^3}$ B) $\frac{1}{xy^3}$ C) $\frac{1}{y^3}$ D) $\frac{x}{y}$

33. The closed unit ball in a normed linear space X is compact if and only if

A) X is equal to $\mathbb{R}$
B) X is equal to $\mathbb{C}$
C) X is any finite dimensional normed linear space
D) X is any normed linear space

34. Let X and Y be linear spaces and $A : X \to Y$ be a linear operator. Then rank(A) + nullity(A) is equal to

A) dimension of X B) dimension of Y
C) dimension of X + dimension of Y D) none of these

35. If Y is a finite dimensional subspace of a normed linear space X, then Y is

A) open B) closed C) both open and closed D) neither open nor closed

36. If X and Y are Hilbert spaces and $A \in B(X,Y)$, then null space of A, $N(A)$ is equal to

A) $R(A)$ B) $R(A^*)$ C) $R(A)^{\perp}$ D) $R(A^*)^{\perp}$

37. Let X be a normed linear space over the field K and $F(\not\equiv 0) : X \to K$ is a linear map such that F is bounded on some closed ball about 0 of positive radius. Then which among the following is **not** true?

A) the hyperspace $Z(F)$ is open in X B) F is continuous at 0
C) F is continuous on X D) F is uniformly continuous on X

38. Let $f : X \to Y$, $g : Y \to Z$ be two functions. Then which of the following is false ?

A) If f and g are one-to-one then $g \circ f$ is one-to-one.
B) If f and g are onto then $g \circ f$ is onto.
C) If $g \circ f$ is one-to-one then g is one-to-one.
D) If $g \circ f$ is one-to-one then f is one-to-one.

39. Let $A = \{(x, \log x) : x \in \mathbb{R}^+\}$, $B = \{(x, \exp x) : x \in \mathbb{R}\}$, and $C = \{(x, -x) : x \in \mathbb{R}\}$. Then

A) $|A \cap B| = 1$. B) $|A \cap C| = 1$. C) $C \cap B = \phi$. D) $|A \cap B \cap C| = 2$.

40. The distance between $(1, -1, -2)$ and the plane $x - 2y + z = 6$ is

A) $\frac{5}{\sqrt{6}}$. B) $\frac{5\sqrt{3}}{6}$. C) $\frac{5}{6}$. D) $\frac{5}{2}$

41. The angle between the lines $2x + y - 8 = 0$ and $x + 3y + 4 = 0$ is

A) π B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) 2π

42. The minimum value of $f(x) = xlogx$ is

A) e B) $-e$ C) $\frac{1}{e}$ D) $-\frac{1}{e}$

43. A fair coin is tossed 4 times. The probability of getting at most two Heads is

A) $\frac{13}{16}$ B) $\frac{15}{16}$ C) $\frac{9}{16}$ D) $\frac{11}{16}$

44. A box contains 6 red balls and 4 blue balls. Two balls are drawn at random. The probability that both balls are of same color is

A) $\frac{2}{15}$ B) $\frac{7}{15}$ C) $\frac{11}{15}$ D) $\frac{8}{15}$

45. Let $f_n(x) = \dfrac{x}{1+nx^2}$ $0 \le x \le 1$, $n = 1, 2, 3, \ldots$

A) f_n converges uniformily to a function f and $\lim\limits_{n\to\infty} f_n'(x) = f'(x)$ for every $x \neq 0$
B) f_n converges pointwise but not uniformily
C) f_n converges uniformily to a function f and $\lim\limits_{n\to\infty} f_n'(x) = f'(x)$ for every x
D) f_n converges uniformily to a function f and $\lim\limits_{n\to\infty} f_n'(x) = f'(x)$ for $x = 0$

46. Let (x_n) and (y_n) be two real sequences with $x_n = \dfrac{3^n}{n!}$, $\quad y_n = n^{\frac{1}{n}}$, $\quad n \ge 1$.

A) (x_n) converges to 0 and (y_n) converges to 1
B) (x_n) converges to 1 and (y_n) converges to 1
C) (x_n) converges to 1 and (y_n) converges to 0
D) Both (x_n) and (y_n) converges to 0

47. Let $f(x) = \begin{cases} 0, x \in [0, \frac{1}{2}] \\ 1, x \in (\frac{1}{2}, 1] \end{cases}$

$\alpha(x) = \begin{cases} 0, x \in [0, \frac{1}{2}) \\ 1, x \in [\frac{1}{2}, 1] \end{cases}$

Then $\int_0^1 f d\alpha$ is

A) 0 B) 2 C) 3 D) 4

48. Which of the following is true?

A) There exists a measurable function$f$ such that $|f|$ is not measurable.
B) If f and g are measurable then fg need not be measurable
C) Every continuous real valued function is measurable
D) There exists a non measurable set with measure zero.

49. Let $z \in \mathbb{C}$ be such that $|z| < 1$. If $w = \dfrac{5+3z}{5(1-z)}$ then

A) $4\ \mathrm{Im}(w) > 5$ B) $5\ \mathrm{Re}(w) > 4$ C) $5Re(w) > 1$ D) $5\ \mathrm{Im}(w) < 1$

50. If $z = a$ is an isolated singularity of $f(z)$, then a is a pole of f, if

A) $\lim\limits_{z\to a} |f(z)| = 0$ B) $\lim\limits_{z\to a} |f(z)| = a$

C) $\lim\limits_{z\to a} |f(z)| = \infty$ D) $\lim\limits_{z\to a} |f(z)|$ is not defined

51. $f(z) = \dfrac{\sin z}{(z-\pi)^2}$ has the pole of order

A) 1 B) 2 C) 3 D) 0

52. Let ϕ be a homomorphism from a group G to G', H a subgroup of G and $g \in G$. Then which of the following is **not** true?

A) If H is abelian, then $\phi(H)$ is cyclic.
B) $\phi(g^n) = \phi(g)^n$
C) If H is cyclic, then $\phi(H)$ is cyclic.
D) If H is normal in G, then $\phi(H)$ is normal in $\phi(G)$

53. Number of group homomorphisms from S_3 to $\mathbb{Z}_3$ is

A) 1 B) 2 C) 3 D) 0

54. Let R be the ring of all real valued continuous functions on the closed interval $[0,1]$ and $I = \{f \in R : f(\frac{1}{2}) = 0\}$. Then

A) I is not an ideal
B) I is a maximal ideal
C) I is an ideal but not maximal
D) $R = I$

55. Which of the following ring is isomorphic to $\{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

A) $\left\{\begin{bmatrix} x & y \\ y & x \end{bmatrix} : x, y \in \mathbb{Z}\right\}$ B) $\left\{\begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix} : x, y \in \mathbb{Z}\right\}$

C) $\left\{\begin{bmatrix} x & 0 \\ 0 & 0 \end{bmatrix} : x, y \in \mathbb{Z}\right\}$ D) $\left\{\begin{bmatrix} x & 2y \\ y & x \end{bmatrix} : x, y \in \mathbb{Z}\right\}$

56. Number of irreducible polynomials of degree atmost 4 in $\mathbb{Z}_2[x]$ is

A) 2 B) 4 C) 6 D) 8

57. Let F be a field with 32 elements. Number of subfields of F is

A) 1 B) 2 C) 3 D) 4

58. If F, E and K are fields such that $K \subseteq E \subseteq F$, then which of the following is **not** true?

A) $[F : E] = 1$ if and only if $E = F$
B) If $[F : E] = p$, a prime, then there is no intermediate fields between F and K
C) If $\alpha \in F$ has degree m over K, then m divides $[F : E]$
D) If F is a finite extension of E then F need not be algebraic over E.

59. Under which one of the following conditions does the system of equations

$$\begin{pmatrix} 1 & 2 & 4 \\ 2 & 1 & 2 \\ 1 & 2 & a-4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ a \end{pmatrix}$$

have a unique solution?

A) for all $a \in \mathbb{R}$ B) $a = 8$ C) for all $a \in \mathbb{Z}$ D) $a \neq 8$

60. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $a + d = 1 = ad - bc$, then A^3 equals

A) 0 B) $-I$ C) $2I$ D) I

61. The dimension of the vector space of all real numbers '$\mathbb{R}$' over the field of rational numbers is

A) 1 B) 2 C) 3 D) infinite

62. The dimension of the subspace $\{(x_1, x_2, x_3, x_4, x_5) : 3x_1 - x_2 + x_3 = 0\}$ of $\mathbb{R}^5$

A) 1 B) 2 C) 3 D) 4

63. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a map defined by

$$T(x, y) = (x + y, x - y)$$

which of the following statement is correct?

A) Nullity of T is zero
B) T is linear and its kernel has infinite number of elements of $\mathbb{R}^2$
C) T is not linear
D) The kernel of T consists of only two elements of $\mathbb{R}^2$

64. Let $T : \mathbb{R}^7 \to \mathbb{R}^7$ be a linear transformation such that $T^2 = 0$, then $rank(T)$ is

A) $= 5$ B) > 3 C) ≤ 3 D) $= 6$

65. Let $V = \{(x_1, x_2, \cdots, x_{100}) \in \mathbb{R}^{100} : x_1 = 2x_2 = 3x_3 \text{ and } x_{51} - x_{52} - \cdots - x_{100} = 0\}$, then $dim\, V$ is

A) 98 B) 49 C) 99 D) 97

66. The remainder obtained when $1! + 2! + 3! + \cdots\cdots + 1000!$ is divided by 5 is

A) 0 B) 1 C) 2 D) 3

67. The number of incongruent solutions of the linear congruence $8x \equiv 12(mod\, 20)$ is

A) 0 B) Infinite C) 4 D) 8

68. The general solution of the partial differential equation $y^2zp + x^2zq = xy^2$ is

A) $\phi(x^3 + y^3, x^2 - z^2) = 0$ B) $\phi(x^3 - y^3, x^2 - z^2) = 0$

C) $\phi(x^3 - y^3, x^2 - y^2) = 0$ D) $\phi(y^3 - z^3, x^2 - z^2) = 0$

69. The integral surface satisfying $\dfrac{\partial^2 z}{\partial x^2} + z = 0$ subject to the conditions $z = e^y$ and $\dfrac{\partial z}{\partial x} = 1$ when $x = 0$ is given by

A) $z = e^y cosx + sinx$ B) $z = e^x siny + cosy$

C) $z = e^y sinx - cosx$ D) $z = e^x cosy - siny$

70. The charactaristic of the partial differential equation $u_{xx} - x^y u_{yy} = 0$ $(y > 0)$ is

A) $\xi = x^2 + \sqrt{y}$, $\eta = x^2 - \sqrt{y}$ B) $\xi = x + \sqrt{y}$, $\eta = x - \sqrt{y}$

C) $\xi = x + 4\sqrt{y}$, $\eta = x - 4\sqrt{y}$ D) $\xi = x^2 + 4\sqrt{y}$, $\eta = x^2 - 4\sqrt{y}$

71. The integrating factor of the differential equation $(x^2 + y^2 + 2x)dx + ydy = 0$ is

A) e^x B) x^2 C) e^{2x} D) x^3

72. The orthogonal trajectory of family of curves $y = ce^x$ is

A) $y^2 = k - x$ B) $y^2 = k - 2x$ C) $y^2 = x - k$ D) $y^2 = k + 2x$

73. The particular solution of $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = x^2$ is

A) $x^2 + 4x - 6$ B) $x^2 + 4x + 8$ C) $x^2 + 4x + 6$ D) $x^2 + 4x - 10$

74. The equation of curve satisfying the differential equation $x^2\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - 15y = 0$ such that $y(1) = 0$ and $y'(1) = 8$ is

A) $x^4 - x^{-4}$ B) $x^3 - x^{-5}$ C) $x^5 - x^{-3}$ D) $x^6 - x^{-2}$

75. Let (X, d) be a metric space and $A \subseteq X$. Then which of the following is true?

A) If $x \in \bar{A}$, then there exists a sequence in A which converges to x.
B) There exists a sequence in X that converge to two distinct points in X.
C) Every Cauchy sequence in X is convergent.
D) If X is compact and A is closed, then A may not be compact.

76. A discrete space (X, τ) is second countable only if

A) X is first countable B) X is finite C) X is infinite D) X is countable

77. Which of the following is **not** true?

A) Second countability is hereditary B) First countability is hereditary
C) Normality is not hereditary D) Regularity is not hereditary

78. $\mathbb{R}^2 \setminus \{\mathbb{Q} \times \mathbb{Z}\}$ with usual topology is

A) Closed B) Bounded C) Connected D) Open

79. $S_1 : \{[a, b] : a < b; a, b \in \mathbb{Q}\}$ is a base for some topology on $\mathbb{R}$
$S_2 : \{(-\infty, a) : a \in \mathbb{R}\}$ is a base for some topology on $\mathbb{R}$. Then

A) the statements S_1 and S_2 are true
B) the statement S_1 is true but S_2 is false
C) the statement S_1 is false but S_2 is true
D) the statements S_1 and S_2 are false.

80. Let E be an orthonormal subset of an innerproduct space. Let $x, y \in E$. Then $||x - y||$ is

A) $\sqrt{2}$ B) 1 C) 2 D) 0
