A

23721 **120 MINUTES**

1. The maximum value of $2 \sin^2 \theta + 3 \cos^2 \theta$ is:
   A) 1 B) 2 C) 3 D) 5

2. Consider the statements:
   1. In a group the two cancellation laws hold
   2. A closed finite subset of a group is a group
   3. The set of all automorphisms of a group is a group

   A) 1 and 2 are true but 3 is not true
   B) 2 and 3 are true but 1 is not true
   C) 3 and 1 are true but 2 is not true
   D) All the three statements are true

3. If $n \geq 2$, then collection of all even permutations of $\{1, 2, \ldots, n\}$ forms a subgroup of the symmetric group S_n of order:
   A) n B) $\frac{n!}{2}$ C) $n!$ D) $2n$

4. If p and q are prime numbers, the number of generators of the cyclic group $\mathcal{Z}_{pq}$ is equal to:
   A) pq B) $(p-1)(q-1)$
   C) $pq-1$ D) $pq(p-1)(q-1)$

5. The coordinates of the foot of the normal from $P(-4, -2)$ on the line $2x + 3y + 1 = 0$ is:
   A) $(2, 1)$ B) $(2, -1)$ C) $(-2, -1)$ D) $(-2, 1)$

6. Integrating factor of the differential equation $(1-y^2)\frac{dx}{dy} + yx = ay$ is:
   A) $\frac{1}{\sqrt{1-y^2}}$ B) $\frac{1}{\sqrt{1+y^2}}$ C) $\frac{1}{\sqrt{1-x^2}}$ D) $\frac{1}{\sqrt{1+x^2}}$

7. The general solution of the differential equation $x^3 dy + (3x^2 y - e^x)dx = 0$ is:
   A) $xy = e^x + C$ B) $x^2 y = e^x + C$
   C) $x^3 y = e^x + C$ D) $x^3 y = 2e^x + C$

8. Two cards are drawn from a well shuffled pack of 52 cards. The probability that both are spades is:
   A) $\frac{1}{2}$ B) $\frac{1}{4}$ C) $\frac{1}{17}$ D) None of these

9. If the diagonal of a square is the line joining the points (1,3,2) and (2,1,3),then the area of the square is:
A) 3 square units B) 4 square units
C) 2 square units D) 1 square unit

10. The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which the area increases when the side is 10 cm is:
A) $10 cm^2/sec$ B) $10\sqrt{3}\ cm^2/sec$
C) $\sqrt{3}\ cm^2/sec$ D) $\frac{10}{\sqrt{3}}\ cm^2/sec$

11. The Value of the definite integral $\int_0^1 x(1-x)^{99}dx$ is equal to:
A) $\frac{1}{11000}$ B) $\frac{1}{10010}$ C) $\frac{1}{10001}$ D) $\frac{1}{10100}$

12. Which among the following statements is **not** true?
A) Product topology is the weak topology determined by the projection functions.
B) Every open,surjective map is a quotient map.
C) Every quotient map is either open or closed.
D) Every closed, surjective map is a quotient map.

13. Taylor series of $\frac{1}{z}$ about $z = 1$ is:
A) $1 + (z-1) + (z-1)^2 + (z-1)^3 + \cdots$
B) $1 - (z-1) - (z-1)^2 - (z-1)^3 - \cdots$
C) $1 - (z-1) + (z-1)^2 - (z-1)^3 + \cdots$
D) $1 - (z+1) + (z+1)^2 - (z+1)^3 + \cdots$

14. If $x + iy = \frac{a+ib}{a-ib}$,then x and y satisfy the equation:
A) $x^2 + y^2 = 0$ B) $x^2 + y^2 = 1$
C) $x^2 - y^2 = 0$ D) $x^2 - y^2 = 1$

15. The inverse of the Mobius transformation $f(z) = \frac{az+b}{cz+d}$, where $ad - bc \neq 0$ is:
A) $\frac{az-b}{cz-d}$ B) $\frac{dz-b}{a-cz}$ C) $\frac{dz-b}{cz+a}$ D) $\frac{dz+b}{-cz+a}$

16. If E_2 denotes the complex plane, among the following statements which is **not** true?
A) The function $f(z) = z^n$ where n is a positive integer is analytic at all Points in E_2
B) Polynomials are analytic everywhere in E_2
C) The exponential function $f(z) = e^x$ is analytic everywhere in E_2
D) None of these

17. A homomorphism f of a group G into a group G' is one-one if and only if the kernel of the function f

A) is empty
B) contains only one element
C) contains G
D) contains only the identity element of G'

18. If U and V are vector spaces of dimensions m and n respectively, then the vector space Hom(U, V) is of dimension:

A) $m + n$
B) $m - n$
C) $m \times n$
D) m^n

19. Any basis $\mathcal{B}$ for the three dimensional Euclidean space $\mathcal{R}^3$ over $\mathcal{R}$, will contain:

A) Exactly three elements
B) Less than three elements
C) At least three elements
D) Infinite number of elements

20. If A and B are symmetric matrices of the same order, then which of the following is true?

A) AB is always symmetric
B) AB is never symmetric
C) AB is skew symmetric
D) AB is symmetric if and only if $AB = BA$

21. If $\mathcal{U}$ and $\mathcal{V}$ are two subspaces of a vector space $\mathcal{W}$, then which one of the following results is **not** equivalent to the other three?

A) $\mathcal{W} = \mathcal{U} \oplus \mathcal{V}$, $\oplus$ denotes the direct sum
B) $\mathcal{U} \cap \mathcal{V} = \phi$ and $\mathcal{U} + \mathcal{V} = \mathcal{W}$
C) $\mathcal{U} \cap \mathcal{V} = \{0\}$ and $\mathcal{U} + \mathcal{V} = \mathcal{W}$
D) Every vector $z \epsilon \mathcal{W}$ may be written in the form $z = x + y$, $x \epsilon \mathcal{U}$, $y \epsilon \mathcal{V}$, in one and only one way.

22. The matrix $A = \begin{pmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{pmatrix}$ is singular if:

A) $x = 0$
B) $x = 2$
C) $x = 4$
D) $x = 8$

23. If the characteristic equation of a matrix M is $\lambda^2 - \lambda - 1 = 0$, then

A) $M^{-1} = M$
B) $M^{-1} = M + I$
C) $M^{-1} = M - I$
D) None of these

24. If θ is the angle between the two planes $x + 2y + z = 7$ and $2x - y + z = 13$, then θ is equal to:

A) $cos^{-1}(\frac{1}{2})$
B) $cos^{-1}(\frac{1}{3})$
C) $cos^{-1}(\frac{1}{4})$
D) $cos^{-1}(\frac{1}{6})$

25. The product $gcd(197, 48) \times lcm\ (197,48)$ is equal to:

A) 197
B) 48
C) 9654
D) 9456

26. The value of x satisfying the equation $150x \equiv 35(mod 31)$ is:

A) 14 B) 22 C) 24 D) 12

27. If E_0 and E_1 are disjoint closed subsets of a metric space X, then Urysohn's lemma states that there exists a continuous function $F : X \to [0,1]$ such that:

A) $F|_{E_0} = 0$ and $F|_{E_1} = 0$ B) $F(x) = 0$ for all $x \in X$

C) $F(x) = 1$ for all $x \in X$ D) none of these

28. For $1 \leq p < \infty$ consider the sequence space l^p with p norm. If $x, y \in l^p$ such that $x = (1, 0, 0, \ldots)$ and $y = (0, 1, 0, \ldots)$, then $\|x - y\|_p$ is equal to:

A) 1 B) 2 C) 2^p D) $2^{\frac{1}{p}}$

29. Consider the statements:

1. A discrete metric space is complete
2. The set of real numbers with usual metric is complete

A) 1 is true but 2 is false B) 2 is true but 1is false

C) Both 1 and 2 are true D) Both 1 and 2 are false

30. Let x and y be measurable functions with measure m on a set E, $1 < p < \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$.Then Holder's inequality is:

A) $\int_E |xy| dm \leq (\int_E |x| dm)(\int_E |y| dm)$

B) $\int_E |xy| dm \leq (\int_E |x|^p dm)^{\frac{1}{p}} (\int_E |y|^q dm)^{\frac{1}{q}}$

C) $(\int_E |x + y|^p dm)^{\frac{1}{p}} \leq (\int_E |x|^p dm)^{\frac{1}{p}} + (\int_E |y|^p dm)^{\frac{1}{p}}$

D) $(\int_E |x + y|^q dm)^{\frac{1}{q}} \leq (\int_E |y|^q dm)^{\frac{1}{q}} + (\int_E |y|^q dm)^{\frac{1}{q}}$

31. Let X be a normed space over K, Y be a subspace of X and $g \in Y'$, the dual space of Y. Then Hahn Banach Extension theorem states that there is some $f \in X'$ *such that:*

A) $f = g$ B) $f|X = g$ and $\|f\| = \|g\|$

C) $f|Y = g$ and $\|f\| = \|g\|$ D) none of these

32. Let X be a normed linear space. Consider the statements:

1. If $E_1 \subset X$ is an open set and $E_2 \subset X$, *then* $E_1 + E_2$ *is open.*
2. If $E \subset X$ is convex, then so is E^o and $\bar{E}$
3. If Y is a subspace of X, then $Y \neq X$ if and only if $Y^o = \Phi$

A) 1 is a false statement B) 2 is a false statement

C) 3 is a false statement D) None of 1, 2 or 3 is a false statement

33. A linear operator on $\mathcal{R}^2$ with standard inner product is defined by $T(x, y) = (x + 2y, x - y)$. Then the adjoint T^* is given by $T^*(x, y) =$

A) $(x + y, 2x - y)$ B) $(x - y, 2x + y)$

C) $(x - y, 2x - y)$ D) $(x + y, 2x + y)$

34. If the sets A and B are defined by $A = \{(x, y): y = e^x, x \in \mathbb{R}\}$ and $B = \{(x, y): y = x, x \in \mathbb{R}\}$. Then

A) $A \subseteq \boldsymbol{B}$ B) $B \subseteq \boldsymbol{A}$ C) $A \cap B = \varphi$ D) $A \cup B = A$

35. If $f(x) = \frac{a^x + a^{-x}}{2}$ then $f(x + y) + f(x - y)$ is:

A) $f(x)f(y)$ B) $2f(x)f(y)$ C) $\frac{f(x)}{f(y)}$ D) None of these

36. $\int \frac{\sin x}{\sin x - \cos x} dx$ is:

A) $\frac{x}{2} - \frac{1}{2}\ln |\sin x - \cos x| + c$

B) $\frac{x}{2} + \frac{1}{2}\ln |\sin x - \cos x| + c$

C) $\frac{x}{2} + \frac{1}{2}\ln |\sin x + \cos x| + c$

D) $\frac{x}{2} - \frac{1}{2}\ln |\sin x + \cos x| + c$

37. The solution of $\frac{dy}{dx} = 1 + \tan(y - x)$ is:

A) $\sin(y + x) = e^{-x} + c$ B) $\sin(y - x) = e^x + c$

C) $\cos(y - x) = e^{-x} + c$ D) $\cos(y - x) = e^x + c$

38. The equation of the plane which passes through the points (1,1,1),(3,-1,2) and (-3,5,-4) is:

A) $x + 2y - 3z = 7$ B) $x + 5y - 3z = 1$

C) $x - 4y + z = 3$ D) $x + y - 2 = 0$

39. The slope of the curve $y^3 - xy^2 = 4$ at the point $y = 2$ is:

A) 2 B) $-\frac{1}{2}$ C) $\frac{1}{2}$ D) -2

40. A linear transformation maps a point (x, y) in the plane to another point $(\hat{x}, \hat{y})$ according to the rule $\hat{x} = 3y$ and $\hat{y} = 2x$. Then the disc $x^2 + y^2 \leq 1$ gets transformed into the region with an area equal to:

A) 12π B) 6π C) 60π D) 8π

41. What is the value of the definite integral $\int_{-2}^{3} |x + 1| \, dx$?

A) $\frac{5}{2}$ B) $\frac{11}{2}$ C) $\frac{7}{2}$ D) $\frac{17}{2}$

42. Two persons sit in at a round table along with 10 others. What is the probability that the two persons always sit together?

A) $\frac{2}{11}$ B) $\frac{1}{11}$ C) $\frac{2}{9}$ D) $\frac{5}{9}$

43. Which of the following is **not** true?

A) Subset of a countable set is countable
B) Superset of an uncountable set is uncountable
C) Countable union of countable set is countable
D) Countable product of countable set is countable

44. If $\{y_n\}$ is strictly increasing and diverges to $+\infty$ and $\frac{x_n - x_{n-1}}{y_n - y_{n-1}} \to l$ then $\frac{x_n}{y_n}$ converges to:

A) 0 B) $+\infty$ C) 1 D) l

45. The supremum and infimum of the set $\left\{\frac{1}{m} - \frac{1}{n} \middle| m, n \in \mathbb{N}\right\}$ is respectively:

A) $2, 0$ B) $2,1$ C) $1, -1$ D) None of these

46. Which of the following is **not** true?

A) Strictly monotonic functions are always one-one
B) Strictly monotonic functions on an interval are always continuous
C) There always exists continuous onto function $f: (0,1) \to [0,1]$
D) There exists no continuous onto function $f: [0,1] \to (0,1)$

47. If f and g are uniformly continuous on a set A, then which of the following is **not** true?

A) $f + g$ is uniformly continuous on A
B) $f - g$ is uniformly continuous on A
C) fg is uniformly continuous on A
D) $\frac{1}{f}$ is uniformly continuous on A if f is bounded away from 0

48. Which of the following is **not** true?

A) G_δ sets are Borel set
B) F_σ sets are Borel set
C) F_σ sets need not be a Borel set
D) A countable intersection of open sets is a Borel set

49. The radius of convergence of the series $f(z) = \sum 4^n z^{2n}$

A) 1 B) $\frac{1}{2}$ C) 2 D) ∞

50. Which one of the following is true?
A) The Cantor set C is open, uncountable set of measure zero
B) The Cantor set C is closed, uncountable set of measure zero
C) The Cantor set C is open, countable set of measure zero
D) The Cantor set C is closed, countable set of measure zero

51. Which of the following is true?
A) If f is differentiable at z_0 then $\overline{f(\bar{z})}$ is differentiable at z_0
B) If f is continuous at z_0 then $\overline{f(\bar{z})}$ is continuous at z_0
C) If f is analytic in an unit disc, then $\overline{f(\bar{z})}$ is analytic in that unit disc
D) None of these

52. Which of the following functions are continuous at origin?
A) $f(x,y) = \frac{xy}{x^2+y^2}$ B) $f(x,y) = \frac{x^2y^2}{(x+y^2)^3}$
C) $f(x,y) = \frac{x^3-2y^3}{x^2+y^2}$ D) $f(x,y) = \frac{x^2y^2}{x^4+y^4}$

53. The residue of $cot\ z$ at $z = 0$ is:
A) 0 B) 1 C) i D) $\sqrt{2}$

54. Number of homomorphisms from $\mathbb{Z}_4 \to \mathbb{Z}_2 \times \mathbb{Z}_2$:
A) 1 B) 2 C) 3 D) 4

55. The number of Sylow-3 subgroups of D_6:
A) 1 B) 2 C) 3 D) 4

56. Which of the following is **not** true?
A) Every group of order 99 is abelian.
B) Every group of order 255 is cyclic.
C) Every group of order 25 is simple.
D) The centre of a group is always normal and abelian.

57. Number of zero divisors of $\mathbb{Z}_{18}$:
A) 2 B) 4 C) 9 D) 11

58. Which of the following is **not** true?
A) If R is an integral domain and I is an ideal, then R/I is also an integral domain
B) If R is a finite commutative ring with unity then every maximal ideal is a prime ideal
C) $< 2 + 2i >$ is not a prime ideal in $\mathbb{Z}[i]$
D) $\mathbb{Z}_5[i]$ is not an integral domain

59. Which of the following is **not** true?
A) Every finite extension is an algebraic extension
B) Every simple is a finite extension
C) $\mathbb{Q}(\pi)$ is not a finite extension of $\mathbb{Q}$
D) $\mathbb{Q}(\sqrt{2})$ is a finite extension of $\mathbb{Q}$

60. Which of the following is **not** true?
A) $\mathbb{Z}[i]$ is a subring of $\mathbb{C}$
B) $\{0,2,4\}$ is a subring of $\mathbb{Z}_6$
C) $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \middle| a, b \in \mathbb{Z}\right\}$ is a subring of ring of all 2×2 matrices over $\mathbb{Z}$
D) $\{n\sqrt{2} | n \in \mathbb{Z}\}$ is a subring of $\mathbb{R}$

61. If a and b are idempotent elements in a commutative ring, then which of the following is **not** idempotent?
A) ab B) $a + b$ C) $a - a$b D) $a + b - ab$

62. Which of the following is **not** true?
A) $2x^2 + 4$ is irreducible over $\mathbb{R}$
B) $x^2 + 1$ is irreducible over $\mathbb{Z}_3$
C) $x^2 + 1$ is irreducible over $\mathbb{Z}_5$
D) $x^3 + x + 1$ is irreducible over $\mathbb{Z}_2$

63. Which of the following is **not** true?
A) Any set containing the zero vector is linearly independent
B) The empty set is linearly independent
C) The singleton set $\{x\}, x \neq 0$ is linearly independent
D) A subset of any linearly independent is linearly independent

64. If B is a non-singular matrix and $A = B\begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}B^{-1}$, then which of the following is true?
A) $A^2 = I$ B) $A^3 = I$ C) $A^2 = 0$ D) $A^3 = 0$

65. Let $M_n(\mathbb{R})$ be the vector space of all $n \times n$ matrices over $\mathbb{R}$. Then which of the following is a subspace of $M_n(\mathbb{R})$?
A) $\{A | Trace(A) = 0\}$ B) $\{A | det(A) = 0\}$
C) $\{A | det(A) \neq 0\}$ D) $\{A | det(A) = 1\}$

66. The dimension of the vector space $M_2(\mathbb{C})$ of all 2×2 matrices with complex entries over $\mathbb{R}$ is:
A) l B) 2 C) 4 D) 8

67. Which of the following is **not** a linear transformation?
A) $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (y, x)$
B) $T: \mathbb{R}^2 \to \mathbb{R}$ given by $T(x, y) = x$
C) $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (0, y)$
D) $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (1, x)$

68. If λ is an eigen value of a matrix A, then which of the following is **not** true?
A) $\lambda - k$ is an eigen value of $A - kI$
B) $\lambda^2 + \lambda + 1$ is an eigen value of $A^2 + A + I$
C) $\frac{1}{\lambda}$ is an eigen value of $adj(A)$
D) λ^2 is an eigen value of A^2

69. The unit digit of 7^{2023} is:
A) 3 B) 5 C) 7 D) 9

70. If $\phi$is the Euler Totient function, then $\phi(\phi(1001))$ is:
A) 192 B) 144 C) 202 D) 248

71. Integrating factor of the differential equation $ydx + (x^2y - x)dy = 0$ is
A) $\frac{1}{x}$ B) $\frac{1}{x^2}$ C) $-\frac{1}{x^2}$ D) $-\frac{1}{x}$

72. The partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0, c \neq 0$ is called:
A) Laplace Equation B) Heat Equation
C) Wave Equation D) None of these

73. The general solution of the partial differential equation
$y^2 \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} = x(z - 2y)$ is:
A) $\varphi(x^2 + y^2, y^2 - yz) = 0$
B) $\varphi(x^3 - x^2y, x + y + z) = 0$
C) $\varphi(x^2 + xy, y + z) = 0$
D) $\varphi(x^2 - y^3, y - z) = 0$

74. Let (X, d) be discrete metric space, Consider the following statements and state which is true?
1. Every open cover of X has a finite subcover
2. Every infinite subset of X is not compact.

A) Both 1 and 2 are true B) Both 1 and 2 are false
C) 1 is true and 2 is false D) 1 is false and 2 is true

75. Let $f: X \to Y$ be a closed bijective map between metric spaces X and Y such that Y is compact. Then
A) X need not be compact but f is continuous
B) X is compact but f need not be continuous
C) X need not be compact and f need not be continuous
D) X is compact and f is continuous

76. Which of the following is **not** true?
A) Let (X, d) be a metric space and $\subseteq X$. Then $a \in A$ is a boundary point of A if and only if a is a limit point of X / A
B) A connected subset A of $\mathbb{R}$ is connected in $\mathbb{R}^2$
C) A totally bounded and complete metric space is compact
D) If a metric space X is connected then there exists a non-constant continuous function$f: X \to \{0,1\}$

77. Which of the following subset of $\mathbb{R}$ with usual metric is neither open set nor closed?
A) (3,5) B) {1,2,3,4} C) $[1,2] \cup (3,4)$ D) $[1,4] \cup (2,3)$

78. Let X={a,b,c,d,e} and τ ={x,ϕ,{a,b,c},{c},{c,d,e}}.Then which of the following is true?
A) {a,d,e} is in closed set of X
B) X,ϕ are the only closed subsets of X
C) X is connected
D) X is not compact

79. Which of the following is a Banach space?
A) $\mathbb{R}^n$ B) ℓ^∞ C) $\mathbb{C}^n$ D) All of these

80. In $\mathbb{R}^2$, Which of the following norms are equivalent for $x = (x_1, x_2)$

$$\|x\|_1 = \sqrt{{x_1}^2 + {x_2}^2}$$
$$\|x\|_2 = |x_1| + |x_2|$$
$$\|x\|_3 = \max(|x_1|, |x_2|)$$

A) $\|x\|_1$and $\|x\|_2$ only B) $\|x\|_1$and $\|x\|_3$ only

C) $\|x\|_2$and $\|x\|_3$ only D) $\|x\|_1$, $\|x\|_2$ and $\|x\|_3$