A

24121 **120 MINUTES**

1. The function $f(x) = x^2$ from the set of positive real numbers to positive real numbers is:
   A) Injective but not surjective
   B) Surjective but not injective
   C) Both injective and surjective
   D) Neither injective nor surjective

2. Maximum value of $2\ sin\theta + 3\ cos\theta$ is:
   A) $\sqrt{13}$ B) 2 C) 3 D) 5

3. If G is a group $a, b \in G$ and n is any integer, then $(a^{-1}ba)^n$ is equal to:
   A) aba^{-1} B) ab^na^{-1} C) $a^{-1}b^na$ D) $a^{-1}ba$

4. Let ϕ be a group of homomorphism from Z_{24} on to Z_8. Then the kernel of the homomorphism ϕ is the group.
   A) $\{0, 4, 8, 12, 16, 20\}$ B) $\{0, 6, 12, 18\}$
   C) $\{0,\ 8,\ 16\}$ D) $\{0\}$

5. If m is the number of abelian groups (up to isomorphism) of order 24 and n is the number of abelian groups (up to isomorphism) of order 25, then
   A) $m = 1, n = 2$ B) $m = 3, n = 2$
   C) $m = 2, n = 1$ D) $m = 2, n = 3$

6. The differential equation of all straight lines touching the circle $x^2 + y^2 = a^2$ is:
   A) $\left(x - y\frac{dy}{dx}\right)^2 = a^2\left(1 - (\frac{dy}{dx})^2\right)$
   B) $\left(x + y\frac{dy}{dx}\right)^2 = a^2\left(1 + (\frac{dy}{dx})^2\right)$
   C) $\left(y - x\frac{dy}{dx}\right)^2 = a^2\left(1 + (\frac{dy}{dx})^2\right)$
   D) $\left(y + x\frac{dy}{dx}\right)^2 = a^2\left(1 - (\frac{dy}{dx})^2\right)$

7. Particular integral of the differential equation $\frac{d^2y}{dx^2} + y = \sin x$ is:
   A) $-\frac{x}{2}\cos x$ B) $\frac{x}{2}\sin x$ C) $-\frac{x}{2}\sin x$ D) $\frac{x}{2}\cos x$

8. Primitive of the differential equation $(y^2 + yz)dx + (z^2 + xz)dy + (y^2 - xy)dz = 0$ is:
   A) $y(x - z) = c(y - z)$ B) $y(x + z) = c(y + z)$
   C) $y(x - z) = c(y + z)$ D) $None\ of\ these$

9. In a colony, there are 55 families. Each family posts a greeting card to each other family. How many greeting cards will be posted by them?

A) 2970 B) 109 C) 2790 D) 2079

10. A survey determined that in a locality 33% go to work by car, 42% by bus and 12% use both. Probability that a person selected at random uses neither car nor bus is:

A) 0.67 B) 0.37 C) 0.77 D) 0.53

11. The value of $\lim_{x\to\infty} \frac{2\,sinx - \sin 2x}{x - sinx}$ is equal to:

A) 5 B) 6 C) 3 D) 0

12. Area enclosed by the curve $y^2 = 2 - x^2$ between $x = -\sqrt{2}$ and $x = \sqrt{2}$ is equal to:

A) 2π B) $\sqrt{2}\,\pi$ C) 3π D) $\sqrt{3}\,\pi$

13. $\int x\, logx dx$ is equal to:

A) $\frac{x^2 logx}{4} - \frac{x^2}{2} + C$ B) $\frac{x^2 logx}{4} + \frac{x^2}{2} + C$

C) $\frac{x^2 logx}{2} + \frac{x^2}{4} + C$ D) $\frac{x^2 logx}{2} - \frac{x^2}{4} + C$

14. If the series $\sum a_{n,} \sum b_{n,} \sum c_n$ converges to *A, B, C* respectively where $c_n = a_0 b_n + \cdots + a_n b_{0,}$ then C is equal to:

A) $A + B$ B) $A - B$ C) AB D) $\frac{A}{B}, B \neq 0$

15. The sequence $\{S_n\}$ defined by $S_{n+1} = \sqrt{7 + S_n}$ and $S_1 = \sqrt{7}$ converges to:

A) positive root of $x^2 - x - 7 = 0$

B) negative root of $x^2 - x - 7 = 0$

C) positive root of $x^2 + x - 7 = 0$

D) negative root of $x^2 + x - 7 = 0$

16. Which among the following statements is **false**?

A) Topological product of spaces is T_2 if and only if each coordinate space is T_2.

B) Topological product of spaces is regular if and only if each coordinate space is regular.

C) Topological product of spaces is completely regular if and only if each coordinate space is completely regular.

D) None of these

17. Which among the following statement is true for a Cantor set?

A) The set is finite B) The set is countable

C) The set is uncountable D) None of these

18. If E_1 and E_2 are any two measurable sets, then $m\,(E_1 \cup E_2) + m(E_1 \cap E_2)$ is equal to:

A) 0 B) $m\,(E_1) + m(E_2)$

C) $m\,(E_1) - m(E_2)$ D) $m\,(E_1) \times m(E_2)$

19. Singularities of the function $f(z) = \frac{1}{\cos\left(\frac{1}{z}\right)}$ are at the points:

A) $z = \frac{1}{(2n+1)\pi}$ for $n = 0, \pm 1, \pm 2, \ldots$

B) $z = \frac{2}{(2n+1)\pi}$ for $n = 0, \pm 1, \pm 2, \ldots$

C) $z = \frac{2}{(2n-1)\pi}$ for $n = 0, \pm 1, \pm 2, \ldots$

D) None of these

20. If $x + iy = \frac{a+ib}{a-ib}$, then x and y satisfies the equation:

A) $x^2 + y^2 = 0$ B) $x^2 + y^2 = 1$

C) $x^2 - y^2 = 0$ D) $x^2 - y^2 = 1$

21. Value of z in the equation $\sin z - \cos z = 0$ is:

A) $z = \frac{\pi}{2} + k\pi$ where k is an integer

B) $z = \frac{\pi}{2} - k\pi$ where k is an integer

C) $z = \frac{\pi}{4} + k\pi$ where k is an integer

D) None of these

22. If ω denotes one of the cube roots of unity, then $(1 + \omega - \omega^2)^7$ equals:

A) $128\,\omega$ B) $-128\,\omega$ C) $128\,\omega^2$ D) $-128\,\omega^2$

23. A non-empty subset H of a group G is a subgroup of G if and only if :

A) $HH = G$ B) $HH = H$

C) $H^{-1} = H$ D) $HH = H \text{ and } H^{-1} = H$

24. Number of generators of a cyclic group of order 60 is :

A) 16 B) 32 C) 60 D) 1

25. If V is a vector space of finite dimension and W is a subspace of V, then $\dim V/W$ is equal to:

A) $\dim V + \dim W$ B) $\dim V - \dim W$

C) $\dim V \times \dim W$ D) None of these

26. In the vector space of polynomials $\mathcal{P}$ in the variable t, the polynomials x, y, z are defined by $x(t) = 1 - t, y(t) = t(1 - t)$ and $z(t) = 1 - t^2$. Then for the set $\mathcal{B} = \{x, y, z\}$, which of the following statements is true?

A) $\mathcal{B}$ is linearly dependent B) $\mathcal{B} = \mathcal{P}$
C) $\mathcal{B}$ is a basis for $\mathcal{P}$ D) None of these

27. If A is an invertible linear transformation and $\alpha \neq 0$, then $(\alpha A)^{-1}$ is equal to:

A) $-\alpha A^{-1}$ B) $\alpha^{-1} A$ C) $\frac{1}{\alpha} A^{-1}$ D) None of these

28. If A is a square matrix of order 3 and value of the determinant of A is -4, then value of the determinant of the adjoint of A is equal to:

A) 8 B) −8 C) 16 D) −16

29. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is equal to:

A) $a + b + c$ B) abc C) zero D) None of these

30. Eigen values of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{pmatrix}$ are:

A) $1, 4, 7$ B) $-1, 4, 7$ C) $1, -4, 7$ D) $1, 4, -7$

31. Equation to the parabola whose focus is (3,0) and directrix $3x + 4y = 1$ is:

A) $16x^2 - 9y^2 - 24xy - 144x + 8y + 224 = 0$
B) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
C) $16x^2 + 9y^2 - 24xy - 144x + 8y - 224 = 0$
D) $16x^2 - 9y^2 - 24xy - 144x - 8y + 224 = 0$

32. If the lines $\frac{x-2}{2k} = \frac{y-3}{3} = \frac{z+2}{-1}$ and $\frac{x-2}{8} = \frac{y-3}{6} = \frac{z+2}{-2}$ are parallel, then the value of k is equal to:

A) −2 B) 2 C) 4 D) −4

33. If $gcd(a, b) = d$, then $gcd\left(\frac{a}{d}, \frac{b}{d}\right)$ is equal to:

A) $\frac{ab}{d}$ B) d C) d^2 D) 1

34. The last digit of 6^{500} is :

A) 0 B) 5 C) 6 D) None of these

35. If ϕ is the Euler's totient function, then ϕ (ϕ (1001)) is equal to:

A) 144 B) 192 C) 298 D) 96

36. In the metric space $(\mathcal{R}^2, d_1)$, where $d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|$ for $x = (x_1, x_2)$ and $y = (y_1, y_2)$, the sequence $\{(\frac{1}{n}, \frac{2n+1}{n+1})\}$ converges to:

A) (1, 0) B) (0, 1) C) (0, 2) D) (2, 0)

37. Let X and Y be Banach spaces and $B(X, Y)$ denotes the set of bounded linear maps from X to Y. Then which of the following statements is **not** true?

A) Every closed linear map $A: X \to Y$ is continuous
B) If $A \in B(X, Y)$is surjective, then A is an open map
C) If $A \in B(X, Y)$ is bijective, then $A^{-1} \in B(Y, X)$
D) None of these

38. Let $\{u_1, u_2, \dots, u_m\}$ be an orthonormal set in an inner product space X. Then for $x \in X, \sum_{n=1}^{m} |\langle x, u_n \rangle|^2 = \|x\|^2$ if and only if :

A) $x \in \{u_1, u_2, \dots, u_m\}$ B) $x \notin \{u_1, u_2, \dots, u_m\}$

C) $x \in span\{u_1, u_2, \dots, u_m\}$ D) $x \notin span\{u_1, u_2, \dots, u_m\}$

39. If θ is the angle between the vectors (2,3,5) and (1,−4, 3) in the real inner product space $\mathcal{R}^3$ over $\mathcal{R}$, then which of the following is correct?

A) $sin^2\theta = 0$ B) $sin^2\theta = \frac{25}{988}$

C) $sin^2\theta = \frac{963}{988}$ D) $sin^2\theta = 1$

40. Let X be a normed linear space over the field K and $F(\not\equiv 0) : X \to K$ is a linear map such that F is bounded on some closed ball about 0 of positive radius. Then which among the following is **not** true?

A) The hyperspace $Z(F)$ is open in X
B) F is continuous at 0
C) F is continuous on X
D) F is uniformly continuous on X

41. If V is the vector space of all 2×2 matrices over the field of complex numbers, then the dimension of V is:

A) 1 B) 2 C) 3 D) 4

42. If α and $\beta$are the roots of the equation $2x^2 - ax + b = 0$ then the equation whose roots are 2α and 2β is:

A) $x^2 - ax + 2b = 0$ B) $x^2 - 2ax + 4b = 0$
C) $2x^2 - ax + 2b = 0$ D) $x^2 - ax + b = 0$

43. $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n$ is:

A) 1 B) 0 C) e D) ∞

44. A box contains 2 white balls,3 black balls and 3 red balls. The number of ways in which we can select three balls from the box if at least one red ball included is :
A) 10 B) 46 C) 56 D) 66

45. The number of common tangents to the circles $x^2 + y^2 = 16$ and $x^2 - 4x + y^2 = 0$ is:
A) 0 B) 1 C) 2 D) 4

46. If the plane $2x + 3y - 5z = 0$ contains the straight line $\frac{x}{l} = \frac{y}{4} = \frac{z}{2}$ then the value of l is:
A) 1 B) -1 C) 2 D) -2

47. Total number of subsets of a finite set A has 56 more elements of total number of subsets of B then the number of elements in A is:
A) 3 B) 6 C) 8 D) 9

48. $f(x) = \begin{cases} x\sin\frac{\pi}{x} & when\ 0 < x \le 1 \\ 0 & when\ x = 0 \end{cases}$ then in [0,1]:

A) $f(x)$is continuous on$[0,1]$ and f is not bounded variation on $[0,1]$
B) $f(x)$is continuous on$[0,1]$ and f is bounded variation on $[0,1]$
C) $f(x)$is not continuous on$[0,1]$ and f is not bounded variation on $[0,1]$
D) $f(x)$is not continuous on$[0,1]$ and f is bounded variation on $[0,1]$

49. If the derivative f exists and is bounded on$[a, b]$ then:
A) The function f is of bounded variation on$[a, b]$
B) The function f is not of bounded variation on$[a, b]$
C) The function f is not continuous
D) None of these

50. If $\left|\frac{z-2}{z-1}\right| = 2$ then $Re(z)$ is:

A) $\frac{3}{4}|z|^2$ B) $\frac{4}{3}|z|^2$ C) $\frac{3}{4}|z|$ D) $\frac{3}{4}|z-1|^2$

51. An example of a function with a non isolated essential singularity at $z = 2$ is:

A) $tan\frac{1}{z-2}$ B) $sin\frac{1}{z-2}$ C) $e^{-(z-2)}$ D) $tan\frac{z-2}{z}$

52. Let C denote the unit circle centred at the origin in C. Then $\frac{1}{2\pi i}\int_C |1 + z + z^2|^2 dz$ where the integral is taken anticlockwise along C equals
A) 2 B) 3 C) 0 D) 1

53. Consider the Power series $\sum_{n=0}^{\infty} a_n z^n$ where $a_n = \begin{cases} \frac{1}{3^n}, if\ n\ is\ even \\ \frac{1}{5^n}, if\ n\ is\ odd \end{cases}$.

The radius of convergence of the series is equal to :

A) 1 B) 2 C) 3 D) 5

54. The harmonic conjugate of the function $e^x cosy + e^y cosx + xy$ is:

A) $e^x siny + e^y sinx - \frac{1}{2}(x^2 + y^2)$

B) $e^x siny - e^y sinx + \frac{1}{2}(x^2 + y^2)$

C) $e^x siny + e^y sinx + \frac{1}{2}(x^2 + y^2)$

D) None of these

55. Which statement is **not** true?

A) Two cyclic groups of same order are isomorphic
B) Two groups of same dimensions are isomorphic
C) Isomorphic copy of a cyclic group is cyclic
D) None of these

56. Let H and K are the subgroups of order 9 and 4 respectively. If $O(G) = 36$ then possible order of $H \cap K$ is:

A) 1 B) 2 C) 3 D) 36

57. The order of the element(2,2) in the group $Z_4 \oplus Z_6$ is:

A) 2 B) 4 C) 6 D) 8

58. The number of mutually non-isomorphic abelian groups of order 72 is:

A) 1 B) 2 C) 4 D) 6

59. If ℝ is a commutative ring with unit element then M is maximal ideal of ℝ if and only if:

A) $^{M}/_{\mathbb{R}}$ is a field B) $^{\mathbb{R}}/_{M}$ is a field

C) ℝ M is a field D) None of these

60. Let G be a cyclic group of order 16.The number of subgroups of G will be:

A) 2 B) 3 C) 4 D) 5

61. The number of disjoint cycles of length$\geq$ 2 in the permutation

$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 7 & 2 & 6 & 5 & 8 & 1 \end{pmatrix}$ will be:

A) 4 B) 5 C) 2 D) 3

62. If H and K are normal subgroups of a group G then which one of the following is true?
A) HK and $H \cup K$ are normal in G
B) $H \cap K$is normal but HK is not normal in G
C) $H \cup K$ is normal but HK is not normal in G
D) $H \cap K$ and HK are normal in G

63. Let A be a non zero upper triangular matrix all of whose eigen values are0.Then $I + A$ is :
A) Singular B) Invertible C) Nilpotent D) Idempotent

64. If T be the linear operator of R^2defined by $T(x, y) = (4x - 2y, 2x + y)$ then the matrix of T in the basis $\{f_1 = (1,1), f_2 = (-1,0)\}$ is:

A) $[T]_f = \begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$ B) $[T]_f = \begin{bmatrix} 3 & -1 \\ -2 & 2 \end{bmatrix}$

C) $[T]_f = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ D) $[T]_f = \begin{bmatrix} 3 & -2 \\ 1 & -2 \end{bmatrix}$

65. Let T be a linear operator on an n-dimensional vector space having n distinct eigen values then :
A) T is diagonalizable
B) T is not diagonalizable
C) T is invertible but not diagonalizable
D) None of these

66. The dimension of the vector space $M = \left\{ [a_{ij}]_{m \times n} \middle| a_{ij} \in \mathbb{C} \right\}$over the field $\mathbb{R}$ is:
A) $m + n$ B) mn C) $2mn$ D) $2(m + n)$

67. Let M be a real symmetric positive definite $n \times n$ matrix. Then which of the following statements is **false** ?
A) M is diagonalizable
B) M is unitary
C) The determinant of M is positive
D) There is an invertible matrix P such that $M = P^T P$

68. If A is non-scalar , non-identity idempotent matrix of order $n \geq 2$ then the minimal polynomial $m_A(x)$ is:

A) $x(x-1)$ B) $x(x+1)$ C) $x(1-x)$ D) $x^2(1+x)$

69. If four eigen values of a 4×4 matrix $B = \begin{bmatrix} A & I \\ I & A \end{bmatrix}$ are $-1, 1, 0, 4$ where are I and A are 2×2 matrices then trace of the matrix A will be:

A) 1 B) 2 C) 3 D) 4

70. The solution of the differential equation $y = e^{x-y} + x^2 e^{-y}$ is:

A) $e^{-y} = e^x + \left(\frac{x^2}{2}\right) + C$ B) $e^y = e^x + \left(\frac{x^3}{3}\right) + C$

C) $y = e^x + \left(\frac{x^3}{3}\right) + C$ D) $y = e^{-x} + \left(\frac{x^2}{2}\right) + C$

71. The orthogonal trajectories of the family of curves $2xy + y^2 - x^2 = a$,a is a parameter are given by:

A) $x^2 - y^2 + 2xy = C$ B) $x^2 - y^2 - 2xy = C$

C) $x^2 - y^2 + xy = C$ D) $x^2 + y^2 + 2xy = C$

72. The solution of the differential equation $\frac{d^2y}{dx^2} + 4y = sin^2x$ is:

A) $y(x) = C_1\, cos2x + C_2\, sin2x + \frac{1}{8} - \frac{x\, sin2x}{8}$

B) $y(x) = C_1\, e^{2x} + C_2\, e^{-2x} + \frac{1 - x\, sin2x}{8}$

C) $y(x) = C_1\, cos2x + C_2\, sin2x + 1 - x\, sin2x$

D) $y(x) = C_1\, e^{2x} + C_2\, e^{-2x} + 1 - x\, sin2x$

73. The Partial Differential Equation $y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y$

A) Parabolic B) Elliptic C) Hyperbolic D) None of these

74. The particular integral of $\left(D^2 - 2DD + D^2\right)z = \cos(x - 3y)$:

A) $\frac{1}{16}\cos(x - 3y)$ B) $\frac{-1}{16}\cos(x + 3y)$

C) $\frac{-1}{16}\cos(x - 3y)$ D) $-\cos(x - 3y)$

75. The Partial Differential Equation of $f(z - xy, x^2 + y^2) = 0$ formed by the elimination of arbitrary function is:

A) $py - qx = x^2 - y^2$ B) $px + qy = x^2 - y^2$

C) $py - qx = y^2 - x^2$ D) $px - qy = y^2 - x^2$

76. Let A and B be two non empty disjoint subsets of $\mathbb{R}$ and $d(A,B) = \inf\{|a - b|, a \in A, b \in B\}$ Then $d(A,B) > 0$ if :

A) A and B are open B) A and B are closed

C) A is closed and B is open D) None of these

77. Let $X = \mathbb{R}$ with discrete topology and $Y = \mathbb{R}$ with standard topology. Define a function from X to Y by $f(x) = x$ then :

A) f is not continuous B) f is an open map

C) f is a closed map D) None of these

78. Let X be the real line and $K = \{\frac{1}{n} : n = 1,2,3, \dots.\}$ .Then K is:

A) Open in X

B) Closed in X

C) Both open and closed in X

D) Neither open nor closed in X

79. If $X$and Y are normed spaces and if $A : X \rightarrow Y$ is a bijective bounded linear map then:

A) A is always an open map

B) A is an open map if X is a Banach space

C) A is an open map if Y is a Banach space

D) A is an open map if both X and Y are Banach spaces

80. If P is a projection on Hilbert space H Then:

A) $P^2 = P$ B) P is not a positive operator on H

C) $P^* = P$ D) $P^2 = P$ and $P^* = P$
