A

24731 **120 MINUTES**

1. In the measurement of trend, the moving average method:
   A) gives the trend in a line B) measures the seasonal variations
   C) smooth out the time series D) none of these

2. Let X $\sim N_p(\mathbf{0}, \mathbf{I})$. Then $X'AX \sim \chi^2$ with k degrees of freedom if and only if:
   A) A is symmetric B) A is idempotent
   C) A has rank k D) All the above

3. If U $\sim N_p(\mu, \Sigma)$, where **a** is a p × 1 real vector and B is p × p matrix, then the distribution of V = a + BU is:
   A) $N_p(a + B\mu, B\Sigma B^T)$ B) $N_p(B\mu, B\Sigma B^T)$
   C) $N_p(a + B\mu, a + B\Sigma B^T)$ D) $N_p(\mu, \Sigma)$

4. Let $X = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} \sim N_3(0, \Sigma)$, where $\Sigma = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$. Then the conditional distribution of $\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$ given X_3 has covariance matrix
   A) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ D) $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}$

5. Let $\boldsymbol{X_1}, \boldsymbol{X_2}, \ldots, \boldsymbol{X_n}$ be a random sample from $N_p(\mu, \Sigma)$. Let $\overline{\boldsymbol{X}} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{X_i}$, **S**$= \frac{1}{N-1}\sum_{i=1}^{N}(\boldsymbol{X_i} - \overline{\boldsymbol{X}})(\boldsymbol{X_i} - \overline{\boldsymbol{X}})'$, and $T = N(\overline{\boldsymbol{X}} - \mu_0)'\boldsymbol{S^{-1}}(\overline{\boldsymbol{X}} - \mu_0)$, where μ_0 is a given value of μ. Then the distribution of T as N → ∞ is:
   A) $N_p(\mu, \Sigma)$
   B) Wishart's distribution $W(\Sigma, \mathrm{p})$
   C) Chi square with p degrees of freedom
   D) F distribution with (p, p-1) degrees of freedom

6. Let a simple random sample of size n is drawn from a population of size N with replacement and let T_i denote the number of times the i^{th} unit occurs in the sample. Then covariance between T_i and T_j is:
   A) 0 B) $\frac{n}{N}$ C) $-\frac{n}{N}$ D) $-\frac{n}{N^2}$

7. For a simple random sample of size 5 from a population of size 50 without replacement, probability that any two specified units are included in the sample is:
   A) $\frac{1}{2}$ B) $\frac{1}{49\times50}$ C) $\frac{5\times4}{49\times50}$ D) $\frac{1}{5\times4}$

8. A population of size N is divided into k strata. A sample of size n is to be chosen and N_i is the size of the i^{th} stratum . Then sample size n_i to be selected from the i^{th} stratum as per proportional allocation is given by:

A) $n_i = nN$ B) $\frac{n_i}{N_i} = \frac{n}{N}$ C) $n_i N_i = nN$ D) none of these

9. If n is the sample size, then ratio estimator has a bias of order:

A) $\frac{1}{n}$ B) $\frac{1}{\sqrt{n}}$ C) $\frac{1}{n^2}$ D) None of these

10. In sampling with probability proportional to size, the units are selected with probability proportional to:

A) size of the unit B) size of the sample
C) population size D) none of these

11. Each contrast among r treatments has:

A) 1 degrees of freedom B) $r - 2$ degrees of freedom
C) $r - 1$ degrees of freedom D) r degrees of freedom

12. In an RBD, a missing observation can be estimated by the method of:

A) minimizing the MSE B) ANACOVA
C) Both A and B D) neither A nor B

13. In a BIBD, assume that each block is replaced by another block containing those treatments which are not included in the original block. Then:

A) the resulting design will be a BIBD with same parameters
B) the resulting design will be a BIBD with a different set of parameters
C) the resulting design must be a symmetric BIBD
D) the resulting design will not be a BIBD

14. Let the composite treatments of a 2×2 factorial design with factors A and B are denoted by a_{11}, a_{12}, a_{21} and a_{22} where a_{ij} stands for the composite treatment at level i of A and level j of B. Then the interaction effect AB can be represented by the contrast

A) $(a_{11} + a_{12}) - (a_{21} + a_{22})$ B) $(a_{11} + a_{21}) - (a_{12} + a_{22})$
C) $(a_{11} - a_{22}) - (a_{21} - a_{12})$ D) $(a_{11} - a_{21}) - (a_{12} - a_{22})$

15. An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a colour television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. Which of the following design is suitable for the study?

A) completely randomised design
B) randomised block design
C) Latin-square design
D) Graeco-Latin square design

16. Consider a one factor experiment with model $y_{ij} = \mu + \alpha_i + e_{ij}$, $i = 1,2,...,v$, $j = 1,2,...,n$, where e_{ij} are $NID(0,\sigma^2)$ and assume that the model is a fixed effect model. Then the expected value of mean squares of treatments is given by:

A) $E(MST) = \sigma^2$

B) $E(MST) = \sigma^2 + \frac{\sum_{i=1}^{v} \alpha_i^2}{v-1}$

C) $E(MST) = \sigma^2 + \frac{n\sum_{i=1}^{v} \alpha_i^2}{v-1}$

D) $E(MST) = \sigma^2 + \frac{(n-1)\sum_{i=1}^{v} \alpha_i^2}{v}$

17. $\frac{d}{dx}\int_{1/x}^{2/x} \frac{sinxt}{t} dt =$

A) 0 B) 1 C) $\frac{(sin1-sin2)}{x}$ D) $\frac{(cos1-cos2)}{x}$

18. Consider the function $f(x) = \begin{cases} \frac{x^3+2x}{|x|}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$. Then

A) the function is continuous if $k = 2$
B) the function is continuous if $k = -2$
C) the function is continuous if $k = 0$
D) there is no value of k that makes the function continuous

19. Consider the function $f(x) = x^2\log(x) - \frac{5}{2}x^2$. Then the function is convex when

A) $x < \frac{1}{e}$ B) $x > \frac{1}{e}$ C) $x < e$ D) $x > e$

20. Let $f(x)$ be continuous and differentiable function on the interval $[5,15]$. If $f(5) = 4$ and $f'(x) \leq 10$, then the maximum value of $f(15)$is:
A) 95 B) 99 C) 104 D) 111

21. Let (X, d) be a metric space. Then which among the following statements is/are true:
1. The whole space X is open
2. The union of any collection of open subsets of X is open
3. The intersection of any collection of open subsets of X is open

A) 1 only B) 1 & 2 only C) 2 & 3 only D) 1, 2 & 3

22. The magnitude of projection of the vector $[5\ -2\ 3\ 6]$ onto the vector $[1\ 2\ 7\ 3]$ is:
A) 2.32 B) 5.03 C) 4.91 D) 6.12

23. If 0 is an Eigen value of a matrix A, then the columns of A are:
A) linearly independent
B) linearly dependent
C) either linearly independent or linearly dependent
D) Cannot be determined

24. A system of linear equations $\boldsymbol{Ax} = \boldsymbol{b}$ is consistent if and only if:
A) $\boldsymbol{b}$ is in the null space of $\boldsymbol{A}$
B) $\boldsymbol{b}$ is in the row space of $\boldsymbol{A}$
C) $\boldsymbol{b}$ is in the column space of $\boldsymbol{A}$
D) none of the above

25. Consider the following system of linear equations:
$$x + ay = 1$$
$$ax + y = 1$$
Then which of the following is true?
A) The system has no solution when $a = +1$.
B) The system has infinitely many solutions when $a = -1$.
C) The system has one solution when $a \neq -1,1$.
D) None of the given statements is true

26. Let A be a square matrix with determinant 7. If $E_{ij}(r)$ is the elementary matrix for adding r times the j^{th} row of A to the i^{th} row, then determinant of $E_{23}(-4)A$ is equal to:
A) -28 B) -7 C) 7 D) 28

27. Consider the vectors $\boldsymbol{x} = (\alpha, 1,0)$, $\boldsymbol{y} = (1, \alpha, 1)$ and $\boldsymbol{z} = (0,1, \alpha)$. The values of α which make $\{\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}\}$ a linearly dependent subset of R^3 are:

A) $-1,0,1$ B) $-\sqrt{2}, 0, \sqrt{2}$ C) $-\sqrt{3}, 0, \sqrt{3}$ D) $-2,0,2$

28. Which among the following statements are true?
1. The outer measure of an interval is its length
2. Outer measure is translation invariant
3. Outer measure is finitely additive

A) 1& 2 only B) 2 & 3only C) 1 & 3 only D) 1, 2 & 3

29. The population of a country was increased by 9% in 2015 and 16% in 2016. Then the average increase over these years is:
A) 11% B) 12% C) 12.5% D) 14%

30. If the correlation coefficient is a positive value, then the slope of the regression line:
A) can be zero B) must be negative
C) must be positive D) can be either positive or negative

31. If $P(s)$ is the probability generating function of a non-negative integer valued random variable and Y is the random variable such that $P(Y = n) = P(X \leq n)$, then the probability generating function of Y is:

A) $\frac{P(s)}{1-s}$ B) $\frac{sP(s)}{1-s}$ C) $\frac{1-P(s)}{1-s}$ D) $\frac{1-sP(s)}{1-s}$

32. In the sex distribution of families each having a fixed number n of children, it is known that the probability of a child being a boy is a constant p. Assuming that the sex of a child is independent of the sex distribution of the other children in the family, then the probability that a randomly selected family with n children has the first $k(\leq n)$ children are of the same sex and rest of the children are of the other sex is:

A) $p^k + (1-p)^{n-k}$
B) $p^k(1-p)^{n-k}$
C) $p^k(1-p)^{n-k} + p^{n-k}(1-p)^k$
D) $nC_k p^k(1-p)^{n-k}$

33. Consider a set of 10 objects. A sample of size 4 is drawn at random with replacement. Then the probability that no element appears more than once is:

A) 0.021 B) 0.211 C) 0.504 D) 0.612

34. Five percent of patients suffering from a certain disease are selected to undergo a new treatment that is believed to increase the recovery rate from 30 percent to 50 percent. A person is randomly selected from these patients after the completion of the treatment and is found to have recovered. Then the probability that the patient received the new treatment is:

A) 0.04 B) 0.08 C) 0.12 D) 0.23

35. Let A, B and C be three events such that $P(A|C) \geq P(B|C)$ and $P(A|\overline{C}) \geq P(B|\overline{C})$, $P(C) > 0$, then which of the following is always true:

A) $P(A) = P(B)$ B) $P(A) \leq P(B)$
C) $P(A) \geq P(B)$ D) None of the above

36. Let f_i, $i = 1,2,\ldots$, be Borel measurable functions on Ω with respect to a σ-field $\mathcal{F}$. Consider the following functions:

$g_1 = \sup_n f_n$ $g_2 = \inf_n f_n$ $g_3 = \limsup_n f_n$ $g_4 = \liminf_n f_n$

Then which of the following is true?

A) only g_1 and g_3 are Borel measurable
B) only g_2 and g_4 are Borel measurable
C) only g_3 and g_4 are Borel measurable
D) All the functions are Borel measurable

37. Let (X, Y) be a bivariate random vector with the probability density function

$$f(x,y) = \begin{cases} \pi^{-1}, & x^2+y^2 \le 1 \\ 0\ , & x^2+y^2 > 1 \end{cases}$$

Then which of the following is FALSE:

A) $E(X) = 0$
B) $E(Y) = 0$
C) X and Y are uncorrelated
D) X and Y are independent

38. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} \frac{1}{2}, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Let $Y = g(X)$, where $g(x) = 1$ if $x > 0$ and $g(x) = -1$ if $x \le 0$. If $H(y)$ is the distribution function of Y then the value of $H(0.5)$is:

A) 0
B) 0.5
C) 0.75
D) 1

39. Consider the function $F(x) = \frac{1}{\pi}\tan^{-1}x, -\infty < x < \infty$. Then:

A) F is the distribution function of a continuous random variable which is symmetric about zero
B) F is the distribution function of a non symmetric continuous random variable
C) F is the distribution function of a discrete random variable
D) F is not a distribution function

40. Consider a discrete bivariate random vector (X, Y) with joint probability mass function

$$p(x,y) = \begin{cases} e^{-\lambda}\dfrac{\lambda^y}{(y+1)!}, & x = 0,1,\dots,y;\, y = 0,1,2,\dots \\ 0, & \text{otherwise} \end{cases}$$

where $\lambda > 0$ is the parameter of the distribution. Then $E(X|Y)$ is given by:

A) $E(X|Y=y) = \frac{y+1}{2}, y = 0,1,2,\dots$
B) $E(X|Y=y) = \frac{y}{2}, y = 0,1,2,\dots$
C) $E(X|Y=y) = \frac{1}{y+1}, y = 0,1,2,\dots$
D) $E(X|Y=y) = \frac{1}{y}, y = 0,1,2,\dots$

41. Let $X_1, X_2, \dots, X_n$ be i.i.d. random variables with common variance σ^2. Then the correlation between X_1 and $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ is:

A) $\frac{\sigma}{n}$
B) $\frac{\sigma^2}{n}$
C) $\frac{1}{n}$
D) $\frac{1}{\sqrt{n}}$

42. Let (X, Y) be a continuous random vector with joint probability density function

$$f(x,y) = \begin{cases} 2, & 0 \le x \le y \le 1 \\ 0, & \textit{otherwise} \end{cases}.$$

Then $E(X+Y)$is:

A) 0 B) 0.5 C) 1 D) 2

43. The length of service of a component can be modelled by density function

$$f(x) = \begin{cases} (x/\beta)^2 \exp(-x/\beta), & x > 0, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Then the uniformly most powerful critical region for testing $H_0: \beta \le \beta_0$ against $H_1: \beta > \beta_0$ based on a random sample $(X_1, X_2, \ldots, X_n)$ has the form

A) $\sum_{i=1}^{n} X_i \ge k$ B) $\sum_{i=1}^{n} X_i \le k$ C) $\prod_{i=1}^{n} X_i \ge k$ D) $\prod_{i=1}^{n} X_i \le k$

44. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from the probability mass function $P(X = j) = 1/N, j = 1,2,\ldots,N, N \ge 1$ is an integer. The critical region of the likelihood ratio test for testing $H_0: N \le N_0$ against $H_1: N > N_0$ has the form:

A) $\max_i X_i \ge N_0$ B) $\max_i X_i \le N_0$

C) $\max_i X_i \le k, k > N_0$ D) $\max_i X_i \ge N_0$ or $\max_i X_i < k, k > N_0$

45. The level of measurement that allows for the rank ordering of data items is:

A) nominal measurement B) ratio measurement

C) interval measurement D) ordinal measurement

46. Which of the following statement is true about the Kolmogorov-Smirnov goodness of fit test?

A) The hypothesized distribution function should be continuous to perform the test

B) The hypothesized distribution can be discrete or continuous but the null distribution of the test statistics in the discrete case is no longer exact

C) The hypothesized distribution can be discrete or continuous and the null distribution of the test statistics are exact in both cases

D) None of the above statements is true

47. Suppose that each of the 15 randomly chosen female registered voters was asked to indicate if she was going to vote for candidate A or candidate B in an upcoming election. The results show that 11 of the respondents preferred A. Is this information is sufficient to use any of the following test to conclude that candidate A is preferred to B by female voters?

A) sign test B) Wilcoxon signed rank test

C) both A and B D) neither A nor B

48. For testing the null hypothesis of identical populations with two independent samples, the asymptotic efficiency of median test relative to t-test for normal populations is:

A) less than 1 B) greater than 1
C) equal to 1 D) None of the above

49. The stationary distribution of a Markov chain with transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is:

A) $\left[\frac{1}{2}, \frac{1}{2}\right]$ B) $\left[\frac{1}{3}, \frac{2}{3}\right]$ C) $[0, 1]$ D) Does not exist

50. Defects occur along an undersea cable according to a Poisson process of rate 0.05 per kilometer. What is the probability that no defects appear in the first 3 km of the cable?

A) $e^{-0.05}$ B) $e^{-0.15}$ C) $e^{-0.5}$ D) $\frac{e^{-0.5}}{3}$

51. Which of the following statement(s) is/ are true?
1. Gaussian process is always covariance stationary.
2. Poisson process is always covariance stationary.

A) 1 only B) 2 only C) Both 1&2 D) Neither 1 nor 2

52. Customers arrive at a bank counter manned by a single cashier according to Poisson distribution with mean arrival rate of 6 per hour. The cashier attends the customers on first come, first served basis at an average rate of 10 customers per hour with the service time exponential distribution. The time a customer should expect to spend in the queue is:

A) 9 minutes B) 9 hours C) 0 D) 15 minutes

53. In a finite irreducible Markov chain, all states are:

A) null persistent B) non null persistent
C) transient D) none of these

54. If Laspeyre's index is 289 and Fisher's index is 204, then Paasche's index is:

A) 169 B) 180 C) 200 D) 144

55. Purchasing power of money can be assessed through:

A) Fisher's index number B) Value index number
C) Simple index number D) Consumer price index number

56. Prosperity, recession and depression in business is an example for:

A) secular trend B) seasonal variation
C) cyclical variation D) irregular variation

57. The distribution of salaries on a particular year for professional sports players in a country had mean 1.6 million dollars and standard deviation 0.7 million dollars. Suppose a sample of 100 major players was taken. The approximate probability that the average salary of the 100 players in that year exceeded 1.1 million dollars is:

A) 0.6923 B) 0.8122
C) approximately 0 D) approximately 1

58. Suppose a die is rolled n times. Let X_i be the outcome of the i^{th} roll. Then if $S_n = X_1 + X_2 + \ldots + X_n$ is the sum for first n rolls, then which of the following is true?

A) $P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| < \epsilon\right) \to 1$ B) $P\left(\left|\frac{S_n}{n} - \frac{5}{2}\right| \geq \epsilon\right) \to 0$

C) $P\left(\left|\frac{S_n}{n} - \frac{7}{2}\right| \geq \epsilon\right) \to 0$ D) $P\left(\left|\frac{S_n}{n} - \frac{5}{2}\right| < \epsilon\right) \to 1$

59. Consider the following statements:
1. Two random variables X and Y have the same moment generating function, then X and Y must have the same distribution
2. If moments of all order exist, the moment generating function exist in some open neighbourhood of zero

Then which of the following is true?

A) 1 only B) 2 only C) Both 1 &2 D) neither 1 nor 2

60. Let X and Y be random variables which represents the number of failures that precedes the first success in two independent series of dichotomous trials with constant probability of successes $\frac{1}{3}$ and $\frac{1}{2}$, respectively. If $Z = \min(X, Y)$, then $E(Z)$is:

A) 2 B) $\frac{1}{2}$ C) 3 D) $\frac{1}{3}$

61. Consider the trinomial distribution with parameters (n, p_1, p_2, p_3) with $p_1, p_2, p_3 > 0$ and $p_1 + p_2 + p_3 = 1$. Then the conditional expectation of X given $Y = y$ is:

A) $E(X|y) = (n-y)\frac{p_1}{1-p_2}$ B) $E(X|y) = (n-y)\frac{1-p_1}{p_2}$
C) $E(X|y) = y\frac{p_1}{1-p_2}$ D) $E(X|y) = y\frac{p_1}{p_2}$

62. Suppose that the amount of time a customer spends at a service centre has an exponential distribution with a mean of 12 minutes. Then the probability that a randomly selected customer will spend more than 18 minutes given that the customer has been there for more than six minutes is:

A) e^{-2} B) $e^{-1.5}$ C) e^{-1} D) $e^{-0.5}$

63. Which among the following statements are true for log-normal distribution?
1. The distribution has the support $(-\infty, +\infty)$.
2. The distribution is always positively skewed.
3. The distribution is unimodal.

A) 1&2 only B) 1 &3 only C) 2&3 only D) 1, 2 & 3

64. Let X follows normal distribution with mean 10 and standard deviation 2. Then the distribution of $Y = \frac{X^2}{4}$ is:
A) Chi-square with 1 degrees of freedom.
B) Non-central chi-square distribution with 1 degrees of freedom and value of non-centrality parameter is equal to 5
C) Non-central chi-square distribution with 1 degrees of freedom and value of non-centrality parameter is equal to 2.5
D) Non-central chi-square distribution with 1 degrees of freedom and value of non-centrality parameter is equal to 25.

65. If X follows standard Cauchy distribution, then the distribution of $\frac{1}{X^2}$ is:
A) Cauchy distribution B) Normal distribution
C) F distribution D) t distribution

66. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $N(\mu, \sigma^2)$ variables. Define $Y = \sum_{i=1}^{n-1} \frac{(X_{i+1} - X_i)^2}{2(n-1)}$, then:
A) $E(Y) = \frac{2\sigma^4}{n-1}$ B) $E(Y) = \frac{2\sigma^4}{n}$
C) $E(Y) = \sigma^4$ D) $E(Y) = \sigma^2$

67. Let $X_1, X_2, \ldots, X_n$ be independent random variables having uniform distribution over $(0,1)$. The distribution of $-2\log(\prod_{i=1}^{n} X_i)$ is:
A) chi-square with n degrees of freedom
B) chi-square with $2n$ degrees of freedom
C) chi-square with $n - 1$ degrees of freedom
D) chi-square with $2(n - 1)$ degrees of freedom

68. If X is an exponential random variable with parameter λ and λ is also a random variable following Gamma distribution, then the unconditional distribution of $X$is:
A) Gamma B) Pareto C) Weibull D) Beta

69. Let X_1, X_2 and X_3 be random variables with mean 0 and equal variances. The variable X_3 is uncorrelated with X_1 and X_2 and the correlation between X_1 and X_2 is denoted by r_{12}. Then the correlation between $(X_1 + X_2)$ and $(X_2 + X_3)$ is:

A) $\frac{3+r_{12}}{4}$ B) $\frac{2+r_{12}}{3}$ C) $\frac{1+r_{12}}{3}$ D) $\frac{1+r_{12}}{2}$

70. Let $X_1, X_2, \ldots, X_n$ be a random sample taken from uniform distribution over $(0,1)$. Assume that n is odd. Then the variance of the distribution of the sample median is:

A) $\frac{1}{2n}$ B) $\frac{1}{2(n+1)}$ C) $\frac{1}{4(n+2)}$ D) $\frac{1}{2(2n+1)}$

71. Suppose Cauchy distribution truncated at both sides with a resulting support $(-\alpha, \alpha)$, where α is a finite positive real number. Then the second raw moment of the distribution is:

A) $tan^{-1}\alpha$ B) $\frac{\alpha}{tan^{-1}\alpha - 1}$

C) $\frac{\alpha}{tan^{-1}\alpha} - 1$ D) does not exist

72. Let $\overline{X}_1$ and $\overline{X}_2$ be the means of two independent samples of sizes 16 and 25, respectively, taken from a normal population with mean μ and standard deviation σ. Consider (i) $P(\overline{X}_1 < \mu)$ and (ii) $P(\overline{X}_2 < \mu)$. Then which of the following is true?

A) Value of expression (i) is greater than value of expression (ii).
B) Value of expression (ii) is greater than value of expression (i).
C) Value of expression (i) is equal to value of expression (ii).
D) Insufficient data to determine the relationship between expressions (i) and (ii).

73. Let $X_1, X_2, \ldots, X_n$ be a random sample from the uniform distribution on the interval $(\theta, \theta + |\theta|)$. If $\theta \in (0, \infty)$, the MLE of θ is:

A) $\max_i X_i$ B) $\frac{\max_i X_i}{2}$

C) $\frac{\max_i X_i + \min_i X_i}{2}$ D) $\frac{\max_i X_i + \min_i X_i}{3}$

74. Let $X_1, X_2, \ldots, X_n$ be random sample from $N(\mu, \sigma^2)$, where $\mu \in R$ is unknown and $\sigma^2 > 0$ is known. Then the UMVUE of μ^3 is:

A) $\overline{X}^3$ B) $\overline{X}^3 - \frac{2\sigma^2}{n}\overline{X}^2$ C) $\overline{X}^3 - \frac{3\sigma^2}{n}\overline{X}$ D) $\overline{X}^3 - \frac{3\sigma^2}{4n}\overline{X}^2$

75. Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables taken from a population with probability density function

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}, & x > \theta \\ 0, & \text{otherwise} \end{cases}$$

where $-\infty < \theta < \infty$. If we denote $X_{(1)} = \min(X_1, X_2, \ldots, X_n)$, then an unbiased estimator of θ is:

A) $X_{(1)} - n$ B) $X_{(1)} - \frac{1}{n}$ C) $X_{(1)} + n$ D) $X_{(1)} + \frac{1}{n}$

76. Let $X_1, X_2, \ldots, X_n$ be i.i.d. as $N(0, \sigma^2)$. Consider the statistics (i) $T_1 = (X_1^2, X_2^2, \ldots, X_n^2)$, (ii) $T_2 = (X_1^2 + X_2^2 + \ldots + X_n^2)$, (iii) $T_3 = (X_1^2 + X_2^2, \ldots + X_m^2,\ X_{m+1}^2 + X_{m+2}^2 + \ldots + X_n^2)$, where m is less than n ,then which of the following is true?

A) only T_1 is sufficient for σ^2
B) only T_2 is sufficient for σ^2
C) only T_1 and T_2 are sufficient for σ^2
D) T_1, T_2 and T_3 are sufficient for σ^2

77. Consider the sampling from a negative binomial distribution with parameters (r, p), where r is known and the probability of success p $(0 < p < 1)$ is unknown. If the prior distribution of p is assumed to be beta distribution, then the posterior distribution of p is

A) uniform distribution on (0,1)
B) Beta distribution
C) truncated Gamma distribution with support on $(0,1)$
D) truncated normal distribution with support on (0,1)

78. After conducting a survey, a researcher wishes to reduce the standard error to $\frac{1}{3}$ of its original value. What will be the change in the sample size?

A) It will decrease by a factor of 3
B) It will increase by a factor of 3
C) It will increase by a factor of 9
D) Not enough information is given

79. A bottling company produces bottles that hold 14litres of liquid. Periodically, the company gets complaints that their bottles are not holding enough liquid. To test this claim, the bottling company randomly samples 64 bottles and finds the average amount of liquid held by the bottles is 13.9266 litres with a standard deviation of 0.40 litres. Suppose the p-value of this test is 0.0455. State the proper conclusion.

A) Reject the hypothesis at level of significance 0.025
B) Accept the null hypothesis at a level of significance 0.05
C) Reject the null hypothesis at a level of significance 0.05
D) Accept the null hypothesis at a level of significance 0.10

80. Let $X_1, X_2, \ldots, X_n$ be a random sample from the exponential distribution on (θ, ∞) with scale parameter 1, where $\theta \in R$ is unknown. Then the equal-tails confidence interval with confidence coefficient $(1 - \alpha)$ based on the pivotal quantity $T = \min_i X_i - \theta$ is given by

A) $\left(\min_i X_i - \frac{1}{n} log\alpha,\ \min_i X_i + \frac{1}{n} log(1 - \alpha)\right)$
B) $\left(\min_i X_i - \frac{1}{n} log(\alpha/2),\ \min_i X_i + \frac{1}{n} log(1 - \alpha/2)\right)$
C) $\left(\min_i X_i - \frac{1}{n} log(1 - \alpha/2),\ \min_i X_i + \frac{1}{n} log(\alpha/2)\right)$
D) $\left(\min_i X_i - \frac{1}{n} log\alpha,\ \min_i X_i + \frac{1}{n} log(\alpha/2)\right)$