A

24721 120 Minutes

1. If the lines $\frac{x+2}{4} = \frac{y-2}{3k} = \frac{z+4}{9}$ and $\frac{3x+6}{4} = \frac{5y-10}{2} = \frac{2z+8}{6}$ are parallel, then the value of k is

(A) $\frac{5}{2}$ (B) $\frac{3}{2}$ (C) $\frac{2}{5}$ (D) $\frac{2}{3}$

2. Let A and B be the conics with equations $2x^2 + 3y + 5x + 6 = 0$ and $2x^2 + 3y^2 + x + y + 1 = 0$ respectively. Then

(A) A and B are ellipses

(B) A is an ellipse and B is a circle

(C) A is a parabola and B is a hyperbola

(D) A is a parabola and B is an ellipse

3. The equation of the plane passing through the points $(1, 1, -1), (4, 1, -4), (4, 4, 2)$ is

(A) $x - 2y + z + 2 = 0$

(B) $x - 2y + 2z + 2 = 0$

(C) $2x - y + z + 2 = 0$

(D) $2x - y + 2z - 2 = 0$

4. The smallest value of the polynomial in $x^3 - 3x^2 + 3x + 6$ the interval $[0, 2]$ is

(A) 6 (B) 0 (C) 7 (D) 8

5. The function $f(x) = \dfrac{x}{1+x} - log(1 + x)$ for $x > 0$

(A) Always increases

(B) Always decreases

(C) Increases in some finite interval

(D) None of these

6. The value of $\lim_{x \to 0} (x^2)^{\frac{1}{x}}$ is

(A) 0 (B) e (C) $\frac{1}{e}$ (D) 1

7. There are 10 eggs in a refrigerator, of which 2 are rotten. If 3 eggs are taken for cooking, what is the probability that at least one of the eggs are rotten?

(A) $\frac{7}{15}$ (B) $\frac{8}{15}$ (C) $\frac{488}{1000}$ (D) $\frac{512}{1000}$

8. How many distinct 4 digit even number can be formed using the digits 0,1, 2, 3, 4, 5 without repetition of digits?

(A) 144 (B) 156 (C) 180 (D) 216

9. The series $\sum_{n=1}^{\infty}(-1)^n \sin(\frac{1}{n})$ is

(A) absolutely convergent

(B) convergent but not absolutely convergent

(C) absolutely convergent but not convergent

(D) diverges

10. Let $\{f_n(x)\}$ be a sequence of real functions defined by $f_n(x) = \dfrac{\sin nx}{\sqrt{n}}$. Then $\{f_n(x)\}$

(A) converges pointwise but not uniformly

(B) converges uniformly but $\{f_n'(x)\}$ does not converge for each x

(C) converges uniformly and $\{f_n'(x)\}$ converges

(D) not pointwise convergent

11. Consider the function $f : [0, \infty) \to \mathbb{R}$ defined by $f(x) = \begin{cases} \dfrac{\sin x}{x}, & \text{if } x > 0 \\ 1, & \text{if } x = 0 \end{cases}$

(A) f is Riemann integrable but not Lebesgue integrable.

(B) f is Lebesgue integrable but not Riemann integrable.

(C) f is both Riemann integrable and Lebesgue integrable.

(D) f is neither Riemann integrable nor Lebesgue integrable.

12. If the imaginary part of $\dfrac{2z+1}{iz+1}$ is -2, then the locus of the point representing z is

(A) Circle (B) Straight line (C) Parabola (D) Hyperbola

13. The value of the integral $\int_c \tan z \, dz$, where C is the circle $|z| = 2$ is

(A) $4\pi i$ (B) $-4\pi i$ (C) $2\pi i$ (D) $-2\pi i$

14. Find the bi-linear transformation which maps the points $z = 1, i, -1$ in to the points $w = i, 0, -i$

(A) $\dfrac{1+iz}{1-i\bar{z}}$ (B) $\dfrac{1-iz}{1+i\bar{z}}$ (C) $\dfrac{1+z}{1-z}$ (D) $\dfrac{1-z}{1+z}$

15. Which of the following is true

(A) $(\mathbb{Z}_4, +)$ is isomorphic to $(\mathbb{Z}_5^*, \times)$ where $\mathbb{Z}_5^*$ is the non zero elements of $\mathbb{Z}_5$

(B) $(\mathbb{Z}_4, +)$ has an element of order 3

(C) An infinite cyclic group may have three generators

(D) If G is an infinite cyclic group, then there exists no isomorphism from G to G other than identity isomorphism.

16. Number of subgroups of a cyclic group of order 2024

(A) 16 (B) 20 (C) 200 (D) 1760

17. Number of cyclic subgroups of order 20 in $\mathbb{Z}_{100} \times \mathbb{Z}_{25}$

(A) 16 (B) 40 (C) 48 (D) 100

18. Which of the following is always true ?

(A) If R' is a sub ring of a ring R with unity then R' contains unity.

(B) If R is a ring with $a.a = a$ for every $a \in R$, then $a + a = 0$

(C) If $(R, +, .)$ is a ring and $a, b \in R$, then $(a+b)^2 = a^2 + 2a.b + b^2$, where $a^2 = a.a$

(D) Every commutative ring has a multiplicative identity.

19. The degree of the splitting field of $x^3 - 3$ over $\mathbb{Q}$ is

(A) 1 (B) 3 (C) 4 (D) 6

20. Number of elements in the field $\mathbb{Z}_3[x]/\langle x^2+1\rangle$

(A) 3 (B) 8 (C) 9 (D) infinite

21. Which of the following is irreducible in $\mathbb{Z}_3[x]$

(A) $x^5 + x^2 + 1$

(B) $2x^5 + 2x^4 + 2x - 1$

(C) $x^5 + x^4 + x^3 + 1$

(D) $x^5 + x^4 + x^3 + x^2 + 1$

22. The eigen values of a 3×3 matrix A are given by 1, -2, 3, then

(A) $A^{-1} = \frac{1}{6}(5I - 2A - A^2)$

(B) $A^{-1} = \frac{1}{6}(5I + 2A - A^2)$

(C) $A^{-1} = \frac{1}{6}(5I - 2A + A^2)$

(D) $A^{-1} = \frac{1}{6}(5I + 2A + A^2)$

23. A 2×2 real matrix A is diagonalizable if and only if

(A) $(\text{trace}\, A)^2 < 4 \det A$

(B) $(\text{trace}\, A)^2 = 4 \det A$

(C) $(\text{trace}\, A)^2 > 4 \det A$

(D) $(\text{trace}\, A)^2 = \det A$

24. Consider the three equations $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$. The system has no solutions, unless $a + b + c =$

(A) 1

(B) -1

(C) 0

(D) any odd integer

25. What is the condition on a, b, c so that $v = (a, b, c)$ in $\mathbb{R}^3$ belongs to $W = \text{span}(u_1, u_2, u_3)$ where $u_1 = (1, 2, 0)$, $u_2 = (-1, 1, 2)$ and $u_3 = (3, 0, -4)$?

(A) $4a - 2b + 3c = 0$

(B) $3a - 2b + 3c = 0$

(C) $4a - 2b + c = 0$

(D) $4a - b + 3c = 0$

26. What is the rank of the matrix A whose rows are $(1, 1, -1)$, $(2, 3, -1)$, and $(3, 1, -5)$?

(A) 1
(B) 2
(C) 3
(D) rank not defined

27. Consider the vector space $V = \mathbb{P}(t)$ of polynomials over the field of real numbers. Let $H : V \to V$ be the third derivative operator, then the nullity of H is

(A) 1 (B) 2 (C) 3 (D) 4

28. Let L be the linear transformation on $\mathbb{R}^2$ that reflects each point P across the line $y = kx$, where $k > 0$. Which of the following are the eigen vectors of T,

(A) $(k, 1)$ (B) $(-k, 1)$ (C) $(2k, 5)$ (D) $(k, -1)$

29. If the determinant of a 3×3 matrix is 11, then the value of the square of determinant formed by the co-factors will be

(A) 11 (B) 121 (C) 1331 (D) 14641

30. The remainder obtained on dividing 2^{1000} by 77 is

(A) 24 (B) 75 (C) 26 (D) 23

31. The differential equation of all parabolas having their axis along $x - axis$ and focus at origin is

(A) $2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2 - y = 0$

(B) $2x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} - y = 0$

(C) $2x\frac{dy}{dx} - y\left(\frac{dy}{dx}\right)^2 - y = 0$

(D) $2x\frac{dy}{dx} + y\left(\frac{dy}{dx}\right)^2 + y = 0$

32. Which of the following is an integrating factor of the differential equation $(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$

(A) y^2 (B) $\frac{1}{y^2}$ (C) $\frac{1}{y}$ (D) y

33. The particular integral of the differential equation $\frac{d^2y}{dx^2} + 25y = \cos 5x$ is

(A) $\frac{x \cos 5x}{10}$ (B) $\frac{-x \sin 5x}{10}$ (C) $\frac{x \cos 5x}{10}$ (D) $\frac{\cos 5x + \sin 5x}{10}$

34. The Partial Differential equation of the family of curves given by $z = axy + b$, where a and b are arbitrary constants is

(A) $px + qy = 0$ (B) $px^2 + qy^2 = 0$ (C) $px - qy = 0$ (D) $py + qx = 0$

35. The solution of the linear Partial Differential equation
$\frac{\partial^2 z}{\partial x^2} - 6\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} + 8\frac{\partial^2 z}{\partial y^2} = e^{2x+y}$ is

(A) $z = f_1(y + 3x) + f_2(y + 2x) + \frac{1}{25}e^{x+2y}$

(B) $z = f_1(y - 4x) + f_2(y - 2x) + \frac{1}{21}e^{x+2y}$

(C) $z = f_1(y + 4x) + f_2(y + 2x) + \frac{1}{21}e^{x+2y}$

(D) $z = f_1(y - 4x) + f_2(y + 2x) + \frac{1}{25}e^{x+2y}$

36. Let $S = \{n + \frac{1}{n} : n \in \mathbb{N}\}$ be a subset of $\mathbb{R}$. Then

(A) S has a limit point and is not compact

(B) S has no limit point and is compact

(C) S has no limit point and is not compact

(D) S has a limit point and is compact

37. Which of the following sequence of sets in $\mathbb{R}$ satisfy the hypothesis of Cantor's intersection theorem?

(A) $A_n := [0, n]; n \in \mathbb{N}$

(B) $A_n := [0, \frac{1}{n}); n \in \mathbb{N}$

(C) $A_n := (0, \frac{1}{n}); n \in \mathbb{N}$

(D) $A_n := [0, 1 - \frac{1}{n}]; n \in \mathbb{N}$

38. Let $X = \left\{\frac{1}{n} : n \in \mathbb{N}\right\} \cup \{0\}$ be endowed with discrete topology. Which of the following is false?

(A) X is closed

(B) X is not connected

(C) X is compact

(D) $X = \{\frac{1}{n} : n \in \mathbb{N}\}$ is not dense in X

39. Let $S_1 = (\mathbb{R}, \tau_1)$ and $S_2 = (\mathbb{R}, \tau_2)$ be such that τ_1 is the usual topology and τ_2 is the discrete topology on $\mathbb{R}$. If $f : S_1 \to S_2$ and $g : S_2 \to S_1$ are two functions, then necessarily

(A) f is continuous

(B) g is continuous

(C) f is not continuous

(D) g is not continuous

40. Which among the following pairs are homeomorphic?

(A) $[0,1]$ and $\{z \in \mathbb{C} : |z| = 1\}$

(B) $[0,1)$ and $(1,2) \cup \{3\}$

(C) $(0,1)$ and $(\sqrt{2}, \infty)$

(D) $\mathbb{R}$ and $\mathbb{R} \setminus \mathbb{Q}$

41. Which among the following is a norm that does not arise from an innerproduct?

(A) $\mathbb{R}^2$ with $\|(x_1, x_2)\| = \sqrt{x_1^2 + x_2^2}$

(B) l^2 with $\|(x_1, x_2, \ldots)\| = \left(\sum\limits_{n=0}^{\infty} |x_n|^2\right)^{\frac{1}{2}}$

(C) $C[0,2]$ with $\|f\| = \left(\int\limits_0^2 |f(x)|^2 dx\right)^{\frac{1}{2}}$

(D) $C[0,1]$ with $\|f\| = \sup\limits_{x \in [0,1]} |f(x)|$

42. Which of the following is not an orthonormal basis for $\mathbb{R}^3$?

(A) $\{(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (0,0,1)\}$

(B) $\{(1,0,0), (0,-1,0), (0,0,1)\}$

(C) $\{(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}})\}$

(D) $\{(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}), (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), (0,1,0)\}$

43. What will be the range of the function $f(x) = 2 - |x - 5|$ is

A) [-∞, 1] B) (-∞, 2] C) (-∞, 2) D) (-∞, 1)

44. If points (a,0), (0,b) and (1,1) are collinear then what is the value of a+b

A) -ab B) ab C) 1/ab D) -1/ab

45. Find the minimum value of the function $f(x) = x^2 - x + 2$ is

A) 1/2 B) 3/4 C) 7/4 D) 1/4

46. The value of the integral $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x} dx\, dy$ is equal to

A) 1 B) 2 C) 3 D) -1

47. The number of surjective maps from a set of 4 elements to a set of 3 elements is

A) 36 B) 64 C) 69 D) 81

48. A man is known to speak truth 2 out of 3 times. He throws a die and reports that it is a 6. Then the probability that it is actually a 6 is

A) 2/3 B) 3/4 C) 3/8 D) 2/7

49. $\lim_{n\to\infty} \{\sqrt{n+1} - \sqrt{n}\}$

A) 0 B) 1/2 C) 1 D) ∞

50. $\int_{-1}^{1} |x|\, dx$ is

A) -1 B) 1 C) 0 D) 2

51. From the following function, pick the function which is uniformly continuous on (0,1)

A) $f(x) = \frac{1}{x}$ B) $f(x) = \frac{1}{x^2}$ C) $f(x) = \frac{\sin x}{x^2}$ D) $f(x) = \frac{1-\cos x}{x^2}$

52. The harmonic conjugate of $x^2 - y^2$ is

A) $x^2 + y^2$ B) $4xy$ C) $2xy$ D) $y^2 - x^2$

53. Let $f(z)$ and $\overline{f(z)}$ be analytic in a domain D. Then

A) $f(z)$is zero for all z

B) $f(z)$is a constant function

C) $f(z)$is a real valued function but not constant

D)$f(z)$is imaginary valued function but not constant

54. $\int_C \frac{3z^2+z}{z^2-1} dz$ where C is the circle $|z| = 2$

A) $4\pi i$ B) πi C) 0 D) $2\pi i$

55. The function $f(z) = \frac{1}{e^{1/z}+1}$ has at z=0 is

A) a removable singularity
B) an isolated essential singularity
C) a pole
D) a non-isolated essential singularity

56. Let $Z_2(\alpha)$ be an extension of the field Z_2, where α is a zero of $x^2 + x + 1 \in Z_2(x)$. Then $\alpha^2 + \alpha + 1 =$

A) α^2
B) α^3
C) α^4
D) α^5

57. Which of the following is **not** a class of cyclic group

A) all groups of order 4
B) all groups of order
C) all groups of order 33
D) all groups of order 15

58. Which of the following is a generator of the group $Z_5 \, X \, Z_{12}$

A) (2,6)
B) (2,3)
C) (3,4)
D) (3,5)

59. Let $f(x) = x^3 + 2x^2 + 1$ and $g(x) = 2x^2 + x + 2$. Then over Z_3

A) $f(x)$ and $g(x)$ are irreducible
B) $f(x)$ is irreducible and $g(x)$ is not
C) $g(x)$ is irreducible and $f(x)$ is not
D) Neither $f(x)$ nor $g(x)$ is irreducible

60. Which of the following is a subspace of $M_n(R)$, the vector space of n x n matrices

A) The set of all non-invertible real matrices
B) The set of all matrices A with det A=0
C) The set of all matrices A with trace A=0
D) None of the above

61. Let $T: R^2 \to R^2$ by T(x,y)=(2x+y,3x+2y),then inverse of T is

A) $T^{-1}(x, y) = (2x - y, 3x - 2y)$
B) $T^{-1}(x, y) = (-2x + y, 3x - 2y)$
C) $T^{-1}(x, y) = (2x - y, -3x + 2y)$
D) $T^{-1}(x, y) = (-2x + y, -3x + 2y)$

62. The minimal polynomial of $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$ is

A) $(x + 1)(x - 2)$
B) $(x + 1)(x - 2)^2$
C) $(x + 1)^2(x - 2)^2$
D) $(x + 1)^2(x - 2)$

63. For a positive integer n, let p_n denote the vector space of polynomials in one variable x with real coefficients and with degree $\leq n$. Consider the map T:$P_2 \to P_4$ defined by $T(p(x)) = p(x^2)$. Then

A) T is a linear transformation and rank (T)=5

B) T is a linear transformation and rank (T)=3

C) T is a linear transformation and rank (T)=2

D) T is not a linear transformation

64. The determinant $\begin{bmatrix} 1 & 1+x & 1+x+x^2 \\ 1 & 1+y & 1+y+y^2 \\ 1 & 1+z & 1+z+z^2 \end{bmatrix}$ is equal to

A) $(z-x)(z-y)(y-x)$

C) $(x-y)(x-z)(y-z)$

B) $(x-y)^2(y-z)^2(z-x)^2$

D) $(x^2-y^2)(y^2-z^2)(z^2-x^2)$

65. If A is a 5 X 5 real matrix with trace 15 and if 2 and 3 are eigenvalues of A, each with algebraic multiplicity 2, then the determinant of A is equal to

A) 0 B) 24 C) 120 D) 180

66. The rank of the linear transformation $T: R^4 \to R^4$ given by $T(x, y, z, w) = (x-y, x-2y, x-3y, x-4y)$is

A) 2 B) 1 C) 4 D) 3

67. Which of the following is not a linear transformation $T: R^3 \to R^3$

A)$T(x, y, z) = (y, x, 0)$

C) $T(x, y, z) = (0,0,0)$

B)$T(x, y, z) = (xy, yz, xz)$

D) $T(x, y, z) = (x+y, y+z, x+z)$

68. The number of positive divisors of 50000 is

A) 20 B) 30 C) 40 D) 50

69. The period of the function $y = |\sin x| + |\cos x|$ is

A) $\frac{\pi}{2}$ B) π C) $\frac{3\pi}{2}$ D) 2π

70. The degree of the equation $y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + x$ is

A) 0 B) 1 C) 2 D) 3

71. Solution of $(1+y^2)\,dx = (\tan^{-1} y - x)\,dy$ is

A) $x = \tan^{-1} y - 1 + Ce^{-\tan^{-1} y}$

C) $y = \tan^{-1} x - 1 + Ce^{-\tan^{-1} x}$

B) $x = \tan^{-1} y + Ce^{-\tan^{-1} x}$

D) $y = \tan^{-1} x + Ce^{-\tan^{-1} x}$

72. A particular solution of the equation $y'' + y = \sec x$ is

A) $y = x \sin x + \cos \log \sin x$

C) $y = x \cos x + \sin x \log \cos x$

B) $y = x \sin x + \cos x \log \cos x$

D) $y = x \cos x + \sin x \log \sin x$

73. The general solution of $x(y-z)\frac{\partial z}{\partial x}+y(z-x)\frac{\partial z}{\partial y}=z(x-y)$ is

A) $f(x+y+z, xyz)=0$ C) $f(xy+yz, xyz)=0$

B) $f\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}, xyz\right)=0$ D) $f(x^2-y^2, z)=0$

74 The partial differential equation representing the family of curves $Z=(x-a)^2+(y-b)^2$is

A) $Z=p^2+q^2$ C) $4Z=p^2-q^2$

B) $2Z=p^2+q^2$ D) $4Z=p^2+q^2$

75 Which of the following is false

A) Product of T_1 space is a T_1 space

B) Product of completely regular space is completely regular

C) Product of first countable space is first countable

D) Product of two second countable space is second countable

76 Let R be the set of real numbers and τ be the semi-open interval topology on R. Then which of the following is true for (R,τ)

A) (R,τ) is a second countable space C) (R,τ) is a metrizable space

B) (R,τ) is a separable space D) (R,τ) is a compact space

77. Let τ be the topology on R consisting of R,Φ,and all open intervals of the form (a,∞) where a∈R. Then the closure of the interval A=[0,1] is

A) [0,1] B) (-∞, 1) C) (0, ∞) D) R

78. Consider the norms $\|\ \|_1, \|\ \|_2, \|\ \|_\infty$ on R^n. Then for all $x\in R^2$, which one of the following is not true.

A) $\|x\|_1 \le \sqrt{2}\| x\|_2$ C) $\|x\|_2 \le \sqrt{2}\| x\|_\infty$

B) $\|x\|_\infty \le \|x\|_2 \le \|x\|_1$ D) $\|x\|_1 \le \|x\|_2 \le \|x\|_\infty$

79. Let H be a Hilbert space. If $x, y\in H$ are such that $\|x\|=6, \|x+y\|=16$ and $\|x-y\|=4$. Then $\|y\|$ is

A) 2 B) 8 C) 10 D) 12

80. Let R^2 be the usual inner product space and $u=(1,1)$. Define $f_u: R^2\to R$ by $f_u:(x)=\langle x,u\rangle$. Then $\|f_u\|$

A) 1 B) 2 C) $\sqrt{2}$ D) $\frac{1}{\sqrt{2}}$