DO NOT OPEN THE SEAL UNTIL INSTRUCTED TO DO SO

Question Booklet No.

QUESTION BOOKLET

APPLIED SCIENCE (MATHEMATICS)

Booklet Series

A

Roll No. | | | | | |

(Enter your Roll number in the above space)

Time Allowed : 2 Hours **Maximum Marks : 100**

INSTRUCTIONS FOR CANDIDATES

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS QUESTION BOOKLET **DOES NOT** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR QUESTIONS ETC. IF SO, GET IT REPLACED BY A COMPLETE QUESTION BOOKLET.
2. Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Question Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the OMR Answer Sheet liable for rejection.
3. This Question Booklet contains **100** questions. Each question is printed in **English** only. Each question comprises four responses (answers). You will select the response which you want to mark on the OMR Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each question.
4. You have to mark all your responses **ONLY** on the separate OMR Answer Sheet provided. See Instructions at the backside of the OMR Answer Sheet.
5. **All** questions carry equal marks.
6. Before you proceed to mark in the OMR Answer Sheet the response to various questions in the Question Booklet, you have to fill in some particulars in the OMR Answer Sheet as per instructions mentioned on the OMR Answer Sheet.
7. After you have completed filling in all your responses on the OMR Answer Sheet and the examination has concluded, you should hand over to the Invigilator **only the OMR Answer Sheet**. You are permitted to take away with you the **Question Booklet**, along with candidate's copy of **OMR Answer Sheet**.
8. Sheets for rough work are appended in the Question Booklet at the end.
9. **Penalty for wrong answers :**

   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE AS UNDER.

   (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **0.25 mark** assigned to that question will be deducted as penalty.

   (ii) If a candidate gives more than one answer, it will be treated as a wrong answer even if one of the given answers happens to be correct and there will be same penalty as above to that question.

   (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be no penalty for that question.

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1. The value of double integral

$$\int_0^{a/\sqrt{2}} \int_y^{\sqrt{a^2-y^2}} \frac{y}{\sqrt{x^2+y^2}}\, dxdy \text{ is}$$

[A] $\left(\frac{2-\sqrt{2}}{4}\right)a^2$

[B] $\left(\frac{2-\sqrt{2}}{2}\right)a^2$

[C] $\left(\frac{2+\sqrt{2}}{4}\right)a^2$

[D] $\left(\frac{2+\sqrt{2}}{2}\right)a^2$

2. Find a vector normal to the surface $2x^2+3y^2+z=6$ at the point (1,–1,1).

[A] $4\hat{i}+6\hat{j}+\hat{k}$

[B] $-4\hat{i}-6\hat{j}+\hat{k}$

[C] $4\hat{i}-6\hat{j}+\hat{k}$

[D] $4\hat{i}+6\hat{j}-\hat{k}$

3. If $A=3x\hat{i}+4y\hat{j}+z\hat{k}$, then the divergence of curl A is

[A] 8

[B] $3\hat{i}+4\hat{j}+\hat{k}$

[C] $\vec{0}$

[D] 0

4. Find the maximum directional derivative of the function $f(x,y)=x\ln y+x^2y^2$ at the point (1,1).

[A] 0

[B] 1

[C] $\sqrt{13}$

[D] $\sqrt{8}$

5. Find α so that the vector $\vec{F}=\alpha y\hat{i}+4x\hat{j}+2z\hat{k}$ is irrotational.

[A] 2

[B] 4

[C] –2

[D] –4

6. If $A=\begin{pmatrix}1 & 1\\ 2 & 0\end{pmatrix}$, then A^5 is

[A] $\begin{pmatrix}-21 & 11\\ 22 & -10\end{pmatrix}$

[B] $\begin{pmatrix}21 & 11\\ 22 & 10\end{pmatrix}$

[C] $\begin{pmatrix}21 & -11\\ -22 & 10\end{pmatrix}$

[D] $\begin{pmatrix}-21 & 11\\ -22 & 10\end{pmatrix}$

7. The characterization of an orthogonal matrix A is

[A] $A^{-1}A=I$

[B] $AA^{-1}=I$

[C] $AA^T=I$

[D] $A^{-1}A^T=I$

8. The function $f(x)=x\sin(1/x)$ at $x=0$ is

[A] not continuous

[B] continuous and bounded

[C] continuous but unbounded

[D] bounded but not continuous

9. The infimum and supremum for the set $D = \{x \in R : x^2 - 2x - 5 < 0\}$ are

[A] $-6, 0$

[B] $-6, \infty$

[C] $-1 \cdot 45, 3 \cdot 45$

[D] $-\infty, \infty$

10. Which of the following functions is ***not*** entire?

[A] $\cos z$

[B] $\sin z$

[C] $\tan z$

[D] e^z

11. The family of conics represented by the solution of differential equation $(4x + 3y + 1)\,dx + (3x + 2y + 1)\,dy = 0$ is

[A] circles

[B] parabolas

[C] hyperbolas

[D] ellipses

12. Let $f(x)$ be a function defined on $[a, b]$ satisfying the following conditions :

1. $f(x)$ is continuous on $[a, b]$.
2. $f(x)$ is differentiable on (a, b).

Then which of the following is ***true***?

[A] There exists a number $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

[B] There exists a number $c \in (a, b)$ such that $f'(c) = \dfrac{f(b) + f(a)}{b + a}$

[C] There exists a number $c \in (a, b)$ such that $f'(c) = 0$

[D] There exists a number $c \in [a, b]$ such that $f'(c) = \dfrac{f(b) - f(c)}{b - a}$

13. The function $f(z) = z \sin \dfrac{1}{z}$ has

[A] pole of order 3 at $z = 0$

[B] removable singularity at $z = 0$

[C] essential singularity at $z = 0$

[D] None of the above

14. The orthogonal trajectories of the family of curves $y^3 - 3x^2 y = k_1$ are

[A] $-3xy^2 + x^3 = k_2$

[B] $3x^2 + y^2 = k_2$

[C] $3x^2 + 2y^2 = k_2$

[D] $xy^2 + x^3 = k_2$

15. The maximum number of linearly independent solutions of the differential equation

$$\frac{d^4 y}{dx^4} = 0, \quad y(0) = 0$$

is

[A] 4

[B] 3

[C] 2

[D] 1

16. For the differential equation

$$x^2(1 - x)\frac{d^2 y}{dx^2} + x\frac{dy}{dx} + y = 0$$

[A] $x = 1$ is an ordinary point

[B] $x = 1$ is a regular singular point

[C] $x = 1$ is an irregular singular point

[D] None of the above

17. The number of surjective maps from a set of 4 elements to a set of 3 elements is

[A] 64

[B] 69

[C] 36

[D] 81

18. Let G be a non-Abelian group. Then its order can be

[A] 25

[B] 55

[C] 35

[D] 49

19. Which one of the following is a ring homomorphism $f: Z_{20} \to Z_{30}$?

[A] $x \to 6x$

[B] $x \to 10x$

[C] $x \to 20x$

[D] All of the above

20. A complete normed linear space is called

[A] Banach space

[B] Hausdorff space

[C] Hilbert space

[D] metric space

21. Which of the following is ***true***?

[A] $d(x, y) = |x - y|$ is not a metric on R

[B] $d(x, y) = |x - y| + 1$ is not a metric on R

[C] $d(x, y) = \begin{cases} 0, & x = y \\ 1, & x \neq y \end{cases}$ is not a metric on R

[D] $d\{(x_1, x_2), (y_1, y_2)\} = |x_1 - x_2| + |y_1 - y_2|$ is not a metric on R^2

22. The global truncation error in the classical Runge-Kutta fourth order method, with step-size h is

[A] $0(h^4)$

[B] $0(h^5)$

[C] $0(h^6)$

[D] $0(h)$

23. Gauss N-point integration formula yields exact value of the integral if the integrand is a polynomial of degree less than, or equal to

[A] $N - 1$

[B] $2N - 1$

[C] N

[D] $2N$

24. The recursion relation to solve $x = e^{-x}$ using Newton-Raphson method is

[A] $x_{n+1} = e^{-x_n}$

[B] $x_{n+1} = x_n - e^{-x_n}$

[C] $x_{n+1} = (1 + x_n)\dfrac{e^{-x_n}}{1 + e^{-x_n}}$

[D] $x_{n+1} = \dfrac{x_n^2 - e^{-x_n}(1 + x_n) - 1}{x_n - e^{-x_n}}$

25. Let $u(x, t) = e^{iwx}v(t)$ with $v(0) = 1$ be a solution to $\dfrac{\partial u}{\partial t} = \dfrac{\partial^3 u}{\partial x^3}$. Then $u(x, t)$ is

[A] $e^{iw(x - w^2 t)}$

[B] $e^{iwx - w^2 t}$

[C] $e^{iw(x + w^2 t)}$

[D] $e^{iw^3(x - t)}$

26. The second-order partial differential equation

$$\left(1 - \sqrt{xy}\right)\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} = -\left(1 + \sqrt{xy}\right)\frac{\partial^2 u}{\partial y^2}$$

is

[A] hyperbolic in the second and the fourth quadrants

[B] elliptic for all x, y

[C] hyperbolic in the second and elliptic in the fourth quadrants

[D] hyperbolic in the first and the third quadrants

27. The equation $\frac{\partial z}{\partial x}e^y = e^x \frac{\partial z}{\partial y}$ gives the general solution

[A] $z = ae^x - be^x$

[B] $z = ae^x + be^y$

[C] $z = a(e^x + e^y) + b$

[D] $z = a(e^x - e^y)$

28. The following partial differential equation

$$xy\frac{\partial z}{\partial x} = 5\frac{\partial^2 z}{\partial y^2}$$

is classified as

[A] elliptic

[B] parabolic

[C] hyperbolic

[D] None of the above

29. Solve the partial differential equation

$x^3\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$ using method of separation of variables if $u(0, y) = 10e^{5/y}$.

[A] $10e^{5/2x^2}e^{5/y}$

[B] $10e^{-5/2x}e^{5/y}$

[C] $10e^{-5/2y^2}e^{-5/x}$

[D] $10e^{-5/2x^2}e^{5/y}$

30. Which of the following properties of a topological space remains preserved under taking arbitrary subsets?

[A] Separability

[B] Connectedness

[C] Normality

[D] Regularity

31. How many distinct topologies are there on the collection of real numbers $\Re$?

[A] Exactly two

[B] Finitely—many may be more than two

[C] Infinitely—many but there are only countably many

[D] Uncountably many

32. Which of the following statements is ***true*** with respect to the optimal solution of an LP problem?

[A] Every LP problem has an optimal solution

[B] Optimal solution of an LP problem always occurs at an extreme point

[C] At optimal solution, all resources are completely used

[D] If an optimal solution exists, there will always be at least one at a corner

33. Consider the following pay-off matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -2 & 2 \end{bmatrix}$$

If solved as game of pure strategies, then the game value is

[A] -2

[B] 1

[C] 2

[D] -1

34. The partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$, is known as

[A] two-dimensional Laplace equation

[B] two-dimensional wave equation

[C] one-dimensional heat flow equation

[D] two-dimensional heat flow equation

35. Maximum value of $Z = 3x + 4y$ subject to the constraints, $4x + 2y \le 80;\ 2x + 5y \le 180;\ x, y \ge 0$, is

[A] 147·5 at $x = 2{\cdot}5,\ y = 35$

[B] 150 at $x = 2{\cdot}5,\ y = 36$

[C] 200 at $x = 20,\ y = 35$

[D] 147·5 at $x = 1{\cdot}5,\ y = 35$

36. The radius of convergence of power series $\sum_{n=1}^{\infty} z^{n!}$ is equal to

[A] 0

[B] 1

[C] ∞

[D] Any real value greater than 1

37. In the Laurent series expansion of $f(z) = \frac{1}{z(z-1)}$ valid for $|z-1| > 1$, the coefficient of $\frac{1}{(z-1)^3}$ is equal to

[A] 1

[B] −1

[C] 0

[D] $\frac{1}{3!}$

38. The bilinear transformation w which maps the points 2, i, −2 in the z-plane onto the points 1, i, −1 in the w-plane is

[A] $\frac{z+2i}{z+6}$

[B] $\frac{z-2i}{z+6}$

[C] $\frac{3z-2i}{iz+6}$

[D] $\frac{3z+2i}{iz+6}$

39. The residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = \infty$ is

[A] −1

[B] 1

[C] 0

[D] $\frac{1}{2}$

40. If the difference of interval is unity, then the value of a for which $\Delta^3(1-2x)(1-3x)(1-ax) = -36$, is

[A] −1

[B] 2

[C] 1

[D] −2

41. If the first forward difference of a function $f(x)$ with spacing h is e^x, then the function $f(x)$ is

[A] e^{x+h}

[B] e^{h-x}

[C] $\frac{e^x}{e^h+1}$

[D] $\frac{e^x}{e^h-1}$

42. The relationship between forward and backward difference operators Δ and ∇ is

[A] $\Delta = 1 + \nabla$

[B] $\Delta = 1 - \nabla$

[C] $\Delta\nabla = \Delta + \nabla$

[D] $\nabla\Delta = \Delta - \nabla$

43. Consider the linear system :

$$ax - y - z = 3$$
$$-2x + by + z = 9$$
$$-x + y + cz = -6$$

For what values of a, b, c Gauss-Seidel method surely converges?

[A] $a = 4, b = 6, c = 7$

[B] $a = 2, b = 6, c = 7$

[C] $a = 4, b = 3, c = 7$

[D] $a = 4, b = 6, c = 2$

44. An infinite space with co-finite topology is

[A] first countable space

[B] Hausdorff space

[C] regular space

[D] separable space

45. Under the usual topology on $\mathbb{R}^2$, the complement of $\mathbb{N} \times \mathbb{N}$ is

[A] open but not connected

[B] connected but not open

[C] both open and connected

[D] neither open nor connected

46. Let X and Y be topological spaces and $f : X \to Y$ be continuous and bijective map. Then f is a homeomorphism if

[A] X and Y are compact

[B] X is compact and Y is Hausdorff

[C] X and Y are Hausdorff

[D] Y is compact and X is Hausdorff

47. Which of the following pair of functions is ***not*** a linearly independent pair of solutions of $y'' + 9y = 0$?

[A] $\sin 3x, \sin 3x - \cos 3x$

[B] $\sin 3x + \cos 3x, 3\sin x - 4\sin^3 x$

[C] $\sin 3x, \sin 3x \cos 3x$

[D] $\sin 3x - \cos 3x, 3\sin x - 4\sin^3 x$

48. Which of the following is ***not*** an integrating factor of $xdy - ydx = 0$?

[A] $\frac{1}{x^2}$

[B] $\frac{1}{xy}$

[C] $\frac{x}{y}$

[D] $\frac{1}{x^2 + y^2}$

49. Linear combinations of solutions of an ordinary differential equation are also solutions if the differential equation is

[A] linear and non-homogeneous

[B] non-linear and homogeneous

[C] linear and homogeneous

[D] non-linear and non-homogeneous

50. If $y_1(x)$ and $y_2(x)$ are solutions of $y'' + x^2 y' + (1 - x)y = 0$ such that $y_1(0) = 0$, $y_1'(0) = 1$ and $y_2(0) = 1$, $y_2'(0) = 1$, then the Wronskian $W(y_1, y_2)$ will be

[A] -1

[B] 2

[C] 0

[D] 1

51. The solution of partial differential equation $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ is of the form

[A] $f(y/x)$

[B] $f(x-y)$

[C] $f(x+y)$

[D] $f(xy)$

52. If $f(x)$ $g(y)$ are arbitrary functions, then the general solution of partial differential equation $u\frac{\partial^2 u}{\partial x \partial y}=\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}$ is

[A] $u(x,y)=f(x)g(y)$

[B] $u(x,y)=f(x+y)+g(x-y)$

[C] $u(x,y)=f(x)+g(y)$

[D] $u(x,y)=yf(x)+xg(y)$

53. If the nullity of the matrix $\begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then the value of k is

[A] -1

[B] 0

[C] 1

[D] 2

54. If

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$$

and $V=\{(x,y,z)\in R^3 : \det(A)=0\}$, then the dimension of V is

[A] 0

[B] 1

[C] 2

[D] 3

55. Let T be the linear transformation on V such that $T^3-T^2-T+I=0$. Then T^{-1} is

[A] $I-T-T^2$

[B] $I+T-T^2$

[C] $I+T+T^2$

[D] $I-T+T^2$

56. Which of the following matrices is ***not*** diagonalizable?

[A] $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

[B] $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

[C] $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$

[D] $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

57. Let $\{S_n\}$ be the sequence defined by the recursion relation $S_{n+1}=\sqrt{3S_n}$, $S_1=1$. Then, which of the following is ***incorrect***?

[A] S_n converges to 3

[B] S_n is monotonic increasing sequence

[C] S_n is monotonic decreasing sequence

[D] S_n is bounded sequence

58. The power series $\sum_0^\infty 2^{-n} z^{2n}$ converges if

[A] $|z|<2$

[B] $|z|\le 2$

[C] $|z|<\sqrt{2}$

[D] $|z|<1$

59. Let a, b and c be three distinct real numbers. Then, the number of distinct real roots of the equation $(x-a)^3+(x-b)^3+(x-c)^3=0$ is

[A] 1

[B] 2

[C] 3

[D] It depends on values of a, b and c

60. Let $f_n(x)=\dfrac{x^n}{1+x}$ and $g_n(x)=\dfrac{x^n}{1+nx}$ for $x\in[0,1]$ and $n\in\mathbb{N}$. Then on the interval [0, 1]

[A] both $\{f_n\}$ and $\{g_n\}$ converge uniformly

[B] neither $\{f_n\}$ nor $\{g_n\}$ converges uniformly

[C] $\{f_n\}$ converges uniformly but $\{g_n\}$ does not converge uniformly

[D] $\{g_n\}$ converges uniformly but $\{f_n\}$ does not converge uniformly

61. The maximum value of the function $f(x, y, z) = xyz$ subject to the constraint $xy + yz + zx = a$, $a > 0$ is

[A] $a^{3/2}$

[B] $(a/3)^{3/2}$

[C] $(2a/3)^{3/2}$

[D] $(a/2)^{3/2}$

62. The volume common to the cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$ is

[A] a^3

[B] $a^3/3$

[C] $16a^3/3$

[D] $4a^3/3$

63. Up to isomorphism, the number of Abelian groups of order 10^5 is

[A] 2

[B] 5

[C] 7

[D] 49

64. In the group $(Z, +)$, the subgroup generated by 2 and 7 is

[A] Z

[B] $5Z$

[C] $7Z$

[D] $14Z$

65. Let G be a group of order 49. Then

[A] G is Abelian

[B] G is non-Abelian

[C] G is cyclic

[D] centre of G has order 7

66. The number of 5-Sylow subgroup(s) in a group of order 45 is

[A] 1

[B] 2

[C] 3

[D] 4

67. Let $T:(C[0,1],\|\ \|_\infty)\to\mathbb{R}$ be defined by $T(f)=\int_0^1 2xf(x)\,dx$ for all $f\in C[0,1]$. Then $\|T\|$ is equal to

[A] 3/2

[B] 1

[C] 1/2

[D] 2

68. Which of the following is a unit tangent vector to the surface $x = t$, $y = t^2$, $z = t^3$ at $t = 1$?

[A] $\left(\frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

[B] $\left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$

[C] $\left(\frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$

[D] $\left(\frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}, \frac{2}{\sqrt{14}}\right)$

69. The area enclosed by the lemniscate $r^2 = 4\cos 2\theta$ is equal to

[A] 1

[B] 2

[C] 3

[D] 4

70. Let $L: R^2 \to R^2$ be the linear transformation such that $L(1,1) = (1,-2)$, $L(-1,1) = (2,3)$. Then $L(-1,5)$ is equal to

[A] (2,3)

[B] (2,–4)

[C] (6,9)

[D] (8,5)

71. Let $P_2 \to P_2$ be the linear transformation defined by $L(at^2 + bt + c) = (a + 2b)t + (b + c)$. Then which of the following elements of P_2 belongs to $\ker L$?

[A] $4t^2 - 5t + 3$

[B] $4t^2 - 2t + 2$

[C] $4t^2 - 8t + 3$

[D] $6t^2 - 3t + 5$

72. An eigenvalue and corresponding eigenvector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, respectively are

[A] 6, (1, 4)

[B] 1, (4, 4)

[C] 6, (–1, 1)

[D] 1, (1, –1)

73. Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$ is equal to

[A] 0

[B] 1

[C] 2

[D] 3

74. The series $\sum_{n=1}^{n=\infty} \frac{(-1)^n}{n}$ is

[A] absolutely convergent

[B] conditionally convergent

[C] divergent

[D] oscillating

75. Let R be the set of real numbers with usual metric. Then Q the subset of rational numbers has

[A] no limit point

[B] only irrational points as limit points

[C] every real number as a limit point

[D] only integer as limit points

76. The relation $|3 - z| + |3 + z| = 5$ represents

[A] a circle

[B] an ellipse

[C] a parabola

[D] a hyperbola

77. The value of $\int_C \frac{e^{2z}dz}{(z-1)(z-2)}$, where C is the circle $C: |z| = 3$, is

[A] $2\pi i(e^4 + e^2)$

[B] $2\pi i(e^4 + e^3)$

[C] $2\pi i(e^4 - e^2)$

[D] $2\pi i(e^4 - e^3)$

78. The function $w = \log z$ is

[A] analytic everywhere in the complex plane

[B] analytic everywhere in the complex plane except at origin

[C] analytic everywhere in the complex plane except at $z = 1$

[D] analytic nowhere in the complex plane

79. Under the transformation, $w = \cosh z$, the lines parallel to x-axis in the z-plane are mapped into

[A] ellipses in the w-plane

[B] hyperbolas in the w-plane

[C] circles in the w-plane

[D] parabolas in the w-plane

80. The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4$ is equal to

[A] $\frac{x^2}{3} + 4x$

[B] $\frac{x^2}{4} + 4x$

[C] $\frac{x^3}{3} + 4x$

[D] $\frac{x^3}{4} + x$

81. The solution of the differential equation $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ is

[A] $y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{e^x}{x^2}$

[B] $y = \frac{c_1}{x} + \frac{c_2}{x^3} + \frac{e^x}{x^2}$

[C] $y = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{e^x}{x^3}$

[D] $y = \frac{c_1}{x} + \frac{c_2}{x^3} + \frac{e^x}{x^3}$

82. The order and degree of the differential equation

$$3\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}},$$ respectively are

[A] 2, 3

[B] 3, 3

[C] 3, 2

[D] 2, 2

83. Let G be a group of order 64. Let H and K be subgroups of G of orders 16 and 32 respectively. Then $H \cap K$ is a subgroup of G with order

[A] 0

[B] > 1

[C] ≤ 1

[D] 1

84. Let F be a field and $F[x]$ be the ring of polynomials over F. Then which of the following is ***not*** true?

[A] $F[x]$ is an integral domain

[B] $F[x]$ is a Euclidean ring

[C] $F[x]$ is a principal ideal ring

[D] $F[x]$ is a field

85. If U is an ideal of a ring R and $1 \in U$, then

[A] $U = \{0\}$

[B] $U \neq R$

[C] $U = R$

[D] $U = \{1\}$

86. Let X and Y be Banach spaces and $f: X \to Y$ be a bijective, bounded linear transformation. Then f^{-1} is

[A] unbounded

[B] closed

[C] bounded

[D] open

87. The matrix L in the LU decomposition of the matrix

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

is

[A] $\begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1 & 6/5 & 1 \end{bmatrix}$

[B] $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 6/5 & 2/5 & 1 \end{bmatrix}$

[C] $\begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ -1 & 6/5 & 1 \end{bmatrix}$

[D] $\begin{bmatrix} 1 & 0 & 0 \\ -2/3 & 1 & 0 \\ 1 & -6/5 & 1 \end{bmatrix}$

88. The missing value in the following table is

x	0	1	2	3	4
y	1	3	9	--	81

[A] 31

[B] 27

[C] 21

[D] 11

89. Simpson's 1/3rd rule of numerical integration is exact for polynomials of degree

[A] greater than 3

[B] greater than 5

[C] less than or equal to 3

[D] less than or equal to 4

90. The solution of the partial differential equation $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$, given that $u = 0$, when $t = 0$ and $\frac{\partial u}{\partial t} = 0$ when $x = 0$ is

[A] $u = \sin x\,(1 - e^{-t})$

[B] $u = \sin x\,(1 + e^{-t})$

[C] $u = \sin x\,(1 - e^{t})$

[D] $u = \cos x\,(1 + e^{-t})$

91. The general solution of the partial differential equation
$z(x - y) = x(y - z)\,p + y(z - x)\,q$
is

[A] $f(xyz) = 0$

[B] $f(xyz, x + y) = 0$

[C] $f(xyz, x + y + z) = 0$

[D] $f(x + y + z, x^2 + y^2 + z^2) = 0$

92. A solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ can be found by solving

[A] $\frac{d^2X}{dx^2}+kX=0$ and $\frac{d^2Y}{dy^2}+kY=0$

[B] $\frac{d^2X}{dx^2}+kX=0$ and $\frac{d^2Y}{dy^2}-kY=0$

[C] $\frac{dX}{dx}+kX=0$ and $\frac{d^2Y}{dy^2}+kY=0$

[D] $\frac{d^2X}{dx^2}+kX=0$ and $\frac{dY}{dy}+kY=0$

93. Which of the following sets is countable?

[A] The set [0, 1]

[B] The set R of all real numbers

[C] The set Q of all rational numbers

[D] The set C of all complex numbers

94. Let D be the discrete topology and U be the usual topology for R. Then the identity map $I:\{(R, D)\to(R, U)\}$ is

[A] a continuous map

[B] a closed map

[C] a homeomorphism

[D] neither continuous map nor closed map

95. The number of basic variables in a feasible solution of a balanced transportation problem with 3 rows and 4 columns is

[A] 1

[B] 6

[C] 7

[D] 12

96. A solution to an LPP which satisfies the non-negativity restrictions of the problem is called a/an

[A] basic solution

[B] feasible solution

[C] optimal solution

[D] non-negative solution

97. A transportation problem with m sources and n destinations is degenerate if it has no initial basic solution containing

[A] $m+n$ positive values

[B] $m+n+1$ positive values

[C] $m+n-1$ positive values

[D] $mn+1$ positive values

98. The work done in moving a particle in the force field $F=3x^2I+(2xz-y)J+zK$ along the straight line from (0, 0, 0) to (2, 1, 3), is equal to

[A] 12

[B] 14

[C] 16

[D] 18

99. If $u=x\,f\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$, then the value of $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is

[A] 0

[B] 1

[C] 2

[D] 4

100. An assignment problem can be viewed as a special case of

[A] transportation problem

[B] geometric programming

[C] queuing problem

[D] simulation

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