Roll No. | | | | | | | | |

(Write Roll Number from left side exactly as in the Admit Card)

Signature of Invigilator

Question Booklet Series X

PAPER–II **Question Booklet No.**

Subject Code : 16

PHYSICAL SCIENCES

Time : 2 Hours *Maximum Marks: 200*

Instructions for the Candidates

1. Write your Roll Number in the space provided on the top of this page as well as on the OMR Sheet provided.
2. At the commencement of the examination, the Question Booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and verify it:
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page.
   (ii) Faulty booklet, if detected, should be got replaced immediately by a correct booklet from the invigilator within the period of 5 (five) minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (iii) Verify whether the Question Booklet Number is identical with OMR Sheet Number; if not, the full set is to be replaced.
   (iv) After this verification is over, the Question Booklet Series and Question Booklet Number should be entered on the OMR Sheet.
3. This paper consists of One Hundred (100) multiple-choice type questions. All the questions are compulsory. Each question carries *two* marks.
4. Each Question has four alternative responses marked: (A) (B) (C) (D) . You have to darken the circle as indicated below on the correct response against each question.

   Example: (A) (B) ● (D) , where (C) is the correct response.
5. Your responses to the questions are to be indicated correctly in the OMR Sheet. If you mark your response at any place other than in the circle in the OMR Sheet, it will not be evaluated.
6. Rough work is to be done at the end of this booklet.
7. If you write your Name, Phone Number or put any mark on any part of the OMR Sheet, except in the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
8. Do not tamper or fold the OMR Sheet in any way. If you do so, your OMR Sheet will not be evaluated.
9. You have to return the Original OMR Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are, however, allowed to carry question booklet and duplicate copy of OMR Sheet after completion of examination.
10. **Use only Black Ball point pen.**
11. **Use of any calculator, mobile phone, electronic devices/gadgets etc. is strictly prohibited.**
12. **There is no negative marks for incorrect answer.**

The Question Booklet is encrypted with QR code for security purpose.

Physical Sciences

1. Consider N electrons at $T = 0$K in a $2d$ box of size $L \times L$. Its Fermi energy is E_F . Now, the box size is changed to $2L \times 2L$, then the new Fermi energy will be

(A) $4E_F$

(B) $\frac{E_F}{4}$

(C) $\frac{E_F}{2}$

(D) $2E_F$

2. For any vector $\vec{A}$, the value of $|\vec{A}\times\vec{i}|^2 + |\vec{A}\times\vec{j}|^2 + |\vec{A}\times\vec{k}|^2$ will be

(A) $|\vec{A}|^2$

(B) $2|\vec{A}|^2$

(C) $3|\vec{A}|^2$

(D) $4|\vec{A}|^2$

3. For the transformation $u = x + y, v = x / y$ with $x \geq 0$ and $y \geq 0$, the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ is given by

(A) 0

(B) 1

(C) $-\frac{u}{(v+1)^2}$

(D) $\frac{v}{(u+1)^2}$

4. If the eccentricity of a planet's orbit is e, then the ratio of maximum to minimum speeds of the planet in its orbit is

(A) $\frac{1+e}{1-e^2}$

(B) $\frac{1+e^2}{1+e}$

(C) $\frac{1+e}{1-e}$

(D) $\frac{1+e^2}{1-e^2}$

5. The canonical transformation from the phase space variables (q, p) to a new one (Q, P) is described by $P = \frac{p^2 + q^2}{2}$, $Q = \tan^{-1}\left(\frac{q}{p}\right)$. Now the generating function is

(A) $F = \frac{p^2}{2}.\tan Q$

(B) $F = -\frac{p^2}{2}.\tan Q$

(C) $F = \frac{q}{2}.\tan Q$

(D) $F = -\frac{q^2}{2}.\tan Q$

6. A solid homogeneous cylinder of mass m and radius r rolls without slipping inside a hollow cylinder of radius 7r. If the solid cylinder executes small oscillations about the stable point, what is the frequency?

(A) $\frac{1}{3}\sqrt{\frac{g}{r}}$

(B) $\sqrt{\frac{g}{r}}$

(C) $\frac{1}{2}\sqrt{\frac{g}{r}}$

(D) $2\sqrt{\frac{g}{r}}$

7. The corresponding magnetic field due to the magnetic vector potential $\vec{A} = (\vec{F} \times \vec{r}) + 3y\hat{i}$, is (where $\vec{F}$ is a constant vector and $\vec{r}$ is the position vector)

(A) $\vec{F} + 3\hat{k}$

(B) $2\vec{F} - 3\hat{k}$

(C) $2\vec{F} + 3\hat{k}$

(D) $\vec{F} - 3\hat{k}$

8. The value of the integral $\int_{-\infty}^{+\infty} dx\, \delta(x^2 - \pi^2)(\sin x + \cos x)$ will be

(A) π

(B) $-\frac{1}{2\pi}$

(C) $-\frac{1}{\pi}$

(D) 0

9. A lattice is defined by the unit vectors $\vec{a_1} = a\hat{i}, \vec{a_2} = -\frac{a}{2}\hat{i} + \frac{a\sqrt{3}}{2}\hat{j}$ and $\vec{a_3} = a\hat{k}$, where $a > 0$ is a constant. The spacing between (100) planes of the lattice is

(A) $\frac{\sqrt{3}}{2}a$

(B) $\frac{a}{2}$

(C) a

(D) $\sqrt{2}\, a$

10. The potential of a diatomic molecule as a function of the distance r between the atoms is given as $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$.

Now the value of the potential at equilibrium separation between the atoms is

(A) $-\frac{4a^2}{b}$

(B) $-\frac{2a^2}{b}$

(C) $-\frac{a^2}{2b}$

(D) $-\frac{a^2}{4b}$

11. If a group is defined as

$$a * b = a + b - 1$$

then inverse of the group is

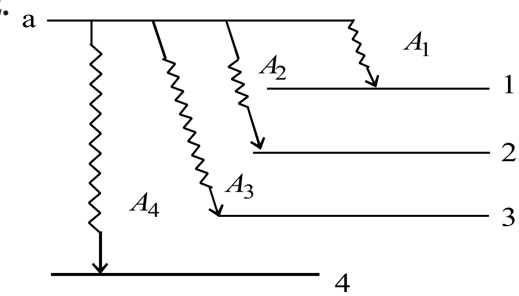
(A) $2 - a$

(B) a

(C) a^{-1}

(D) 0

12.



Let A_i be the transition rates shown in the above diagram for the atomic energy levels i from 'a' energy level. The life time of the atomic state 'a' is

(A) $\frac{1}{A_1 + A_2 + A_3 + A_4}$

(B) $\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4}$

(C) $\frac{1}{A_1 + A_2} + \frac{1}{A_3 + A_4}$

(D) $\frac{1}{A_1 + A_3} + \frac{1}{A_2 + A_4}$

13. Superconducting tin has a critical temperature 3·7 K at zero magnetic field and a critical field of 0·0306 Tesla at 0K. The critical field at 2K is

(A) 0·0216 Tesla

(B) 0·216 Tesla

(C) 0·00216 Tesla

(D) 0·025 Tesla

14. An electron in an electron microscope with an initial velocity $v_0\hat{i}$ enters a region of a stray transverse electric field $E_0\hat{j}$. The time taken for the de Broglie wavelength to change from an initial value of λ to a final value of $\lambda/3$ is proportional to

(A) E_0

(B) $\frac{1}{E_0}$

(C) $\frac{1}{\sqrt{E_0}}$

(D) $\sqrt{E_0}$

15. A Dirac spinor will change its sign under a spatial rotation by an angle

(A) $\frac{\pi}{2}$

(B) π

(C) $\frac{3\pi}{2}$

(D) 2π

16. An unstable neutral particle decays into two charged particles of kinetic energies 190 MeV and 30 MeV and corresponding momenta 300 MeV/c and 240 MeV/c, respectively. If the angle between the decay particles is 45°, then the rest mass of the neutral particle is

(A) 450 MeV/c

(B) 500 MeV/c

(C) 550 MeV/c

(D) 600 MeV/c

17. Let $q_{Cl}(t)$ be a trajectory in the interval [0, *T*] for a system defined by the Lagrangian $L=\frac{1}{2}\left(\dot{q}^2-q^2\right)$. Let $\tilde{q}(t)\equiv q_{Cl}(t)+\in\sin\frac{\pi t}{T}$ be another this is path infinitesmally close to $q_{Cl}(t)$. The second order deviation in the action,

$$\delta^2 S\equiv \delta S\left[\tilde{q}(t)\right]-\delta S\left[q_{Cl}(t)\right]$$

(A) is positive for $T<\pi$ and negative for $T>\pi$

(B) is always positive

(C) vanishes

(D) is negative for $T>\pi$ and positive for $T<\pi$

18. Given the following results:

(i) $\frac{1}{2}+\sum_{n=1}^{n}\cos nx=\frac{\sin\left(n+\frac{1}{2}\right)x}{2\sin x}$

(ii) $\frac{1}{2\pi}\int_0^{\pi}\frac{\sin\left(n+\frac{1}{2}\right)x}{\sin\left(\frac{x}{2}\right)}=\frac{1}{2}$

Choose the correct option.

(A) Only Statement (i) is correct

(B) Only Statement (ii) is correct

(C) Statements (i) and (ii) both are correct

(D) Statements (i) and (ii) both are incorrect

19. The eigenvalues of the Matrix

$$M = \begin{bmatrix} 0 & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \end{bmatrix} \text{ is}$$

(A) 0, 1, – 1

(B) 0, 0, 0

(C) 0, 1 + i, – 1 –i

(D) 0, i, – i

20. For a gas of non-interacting massive bosons, Bose Condensation can take place in

(A) one-dimension only

(B) three-dimensions only

(C) three or higher dimensions

(D) all dimensions

21. A LASER of 5 kW contains high intensity parallel beams. It is concentrated into a 10^{-4} cm^2 area. What is the magnitude of the Poynting vector?

(A) 5×10^7 W/m^2

(B) 5×10^9 W/m^2

(C) 5×10^{10} W/m^2

(D) 5×10^{11} W/m^2

22. How many Flips-Flops are required to build a binary counter circuit to count from 0 to 1023?

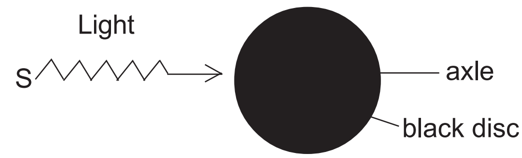
(A) 1

(B) 16

(C) 10

(D) 24

23. A special visible light (6000 Å) of 2 Watts power falls on a black disc, which can rotate an axle parallel to the light beam as shown in figure:



Consider that the black disc absorbs the light totally and the velocity of light is c = 3×10^8 m/s.

The torque exerted on the disc by the light is

(A) $\frac{10^{-15}}{2\pi}$ N.m.

(B) $\frac{2\times10^{-15}}{\pi}$ N.m.

(C) $\frac{6\times10^{-7}}{\pi}$ N.m.

(D) 4×10^{-15} N.m.

24. In a thermocouple pressure gauge, the temperature of heater element is a function of pressure for pressure range

(A) above atmospheric pressure

(B) below 10^{-5} mm of Hg

(C) above 1 mm of Hg

(D) below 10^{-3} mm of Hg

25. An electric field vector of an electromagnetic wave travelling in vacuum is represented as $\vec{E} = \vec{E_0}\left(\cos\left(5x + 4y - wt\right)\right)$ where w is a constant. The valid choice for the electric field vector $\vec{E_0}$ is

(A) $\hat{i} + \frac{4}{5}\hat{j}$

(B) $\hat{i} - \frac{4}{5}\hat{j}$

(C) $\hat{i} + \frac{5}{4}\hat{j}$

(D) $\hat{i} - \frac{5}{4}\hat{j}$

26. If $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, then the value of $A^{25} + A^{102}$ will be

(A) $\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

27. For Dy^{3+} ion with electron configuration $[Xe]4f^9$, the values of S, L and J are respectively

(A) $\frac{5}{2}, 3, \frac{11}{2}$

(B) $\frac{7}{2}, 3, \frac{13}{2}$

(C) $\frac{5}{2}, 5, \frac{15}{2}$

(D) $\frac{5}{2}, 7, \frac{19}{2}$

28. 1 mole of an ideal gas with initial volume V_1 is first expanded to volume V_2 and then to volume V_3. The temperature T is kept constant throughout the process. The net work done is

(A) $RT \ln \frac{V_3}{V_1}$

(B) $RT \ln \left(\frac{V_2}{V_1} + \frac{V_3}{V_2}\right)$

(C) $RT \ln \frac{V_1}{V_3}$

(D) $RT \ln \left(\frac{V_1}{V_2} + \frac{V_2}{V_3}\right)$

29. Let U be an $n \times n$ unitary martix. Which among the following cannot be a possible value of det U?

(A) $\frac{1 \pm i}{\sqrt{2}}$

(B) ± 1

(C) $e^{i\theta}$ (θ is a real no.)

(D) $e^{\pm 1}$

30. Consider a one-dimensional system with Hamiltonian $H = \frac{p^2}{2m} + \lambda x^6 (\lambda > 0)$ in equilibrium with a heat bath kept at temperature T. The average thermal energy of the system is (K_B = Boltzmann constant)

(A) $\frac{3}{4} K_B T$

(B) $\frac{1}{6} K_B T$

(C) $\frac{5}{6} K_B T$

(D) $\frac{2}{3} K_B T$

31. After every second, a particle jumps one lattice point to the immediate right (probability 5/8) or left (probability 3/8) along a very long chain. If it starts at the origin at $t = 0$, what is the mean displacement after one hour?

(A) 900 units to the right

(B) 600 units to the right

(C) 200 units to the right

(D) 0 unit

32. The magnitude of the directional derivative of $\phi(x, y) = \frac{x}{x^2 + y^2}$ at (0,3) along a line making an angle 45° with the positive x-axis is

(A) $\frac{\sqrt{2}}{9}$

(B) $\frac{1}{9\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{1}{81\sqrt{2}}$

33. Given $f(x) = \begin{cases} 1 & |x| < 2 \\ -1 & 2 \le |x| \le 5 \\ 0 & |x| > 5 \end{cases}$

The Fourier transform of $f(x)$ is denoted by $F(k)$. For small k, $F(k)$ is given by

(A) $-2 + \frac{109}{3} k^2$

(B) $-2 + \frac{80}{3} k^2$

(C) $k - \frac{40}{3} k^3$

(D) $k + \frac{20}{3} k^3$

34. Which of the following is an analytic function of the complex variable $Z = x + iy$ in the domain $|Z| < 3$?

(A) $(6 + x - iy)^9$

(B) $(x + iy - 1)^{\frac{1}{2}}$

(C) $(4 - 2x - iy)^4 (3 - x - iy)^3$

(D) $(1 + x + iy)^3 (7 - x - iy)$

35. A class is composed of 60% boys and 40% girls. It is known that, in a given examination, 12% of the girls got an 'A' grade while only 6% of the boys did so. If a randomly selected student got an 'A' grade, the probability that it is a girl is closest to

(A) 0·57

(B) 0·60

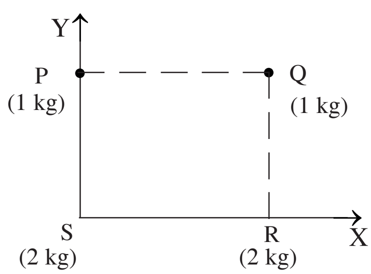
(C) 0·42

(D) 0·67

36. An electromagnetic wave incident from a medium of $\varepsilon = 3\varepsilon_0$ and $\mu = \mu_0$ to its boundary with air, at $x = 0$. If the magnetic field of the medium is $\overline{H} = \hat{k} H_0 \cos\left(\omega t - kx - \frac{k}{\sqrt{3}} y\right)$ where, ω and k are positive constants, the angle of refraction is

(A) 90º
(B) 60º
(C) 45º
(D) 30º

37. Particle of masses 1kg, 1kg, 2kg and 2kg are placed respectively at the corners P, Q, R, S of a square of side L as shown below in the figure.



The centre of mass is at

(A) $\frac{L}{2}\hat{i} + \frac{L}{2}\hat{j}$
(B) $\frac{L}{2}\hat{i} + \frac{L}{3}\hat{j}$
(C) $\frac{L}{3}\hat{i} + \frac{L}{2}\hat{j}$
(D) $\frac{L}{3}\hat{i} + \frac{L}{3}\hat{j}$

38. The Lagrangian of a particle of mass m moving in one dimension is given by, $L = \frac{1}{2} m\dot{x}^2 - bx$, where b is a positive constant. The co-ordinate of the particle $x(t)$ at time t is given as

(A) $c_1 t + c_2$
(B) $-\frac{b}{2m} t^2 + c_1 t + c_2$
(C) $c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$
(D) $c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$

39. Consider two rockets 1 and 2 moving with a velocity $w = (w_x, 0, 0)$ and $-w = (-w_x, 0, 0)$, respectively, as measured from the frame S. Now the velocity of rocket 2 as measured from the frame of rocket 1 is

(A) $\left(\frac{-2w_x}{1+\frac{w_x}{c}}, 0, 0\right)$
(B) $\left(\frac{-2w_x}{1+\frac{w_x^2}{c^2}}, 0, 0\right)$
(C) $\left(\frac{-2w_x}{1-\frac{w_x^2}{c^2}}, 0, 0\right)$
(D) $\left(\frac{2w_x}{1+\frac{w_x^2}{c}}, 0, 0\right)$

40. Consider a sample of radiation contained in a volume V undergoing a quasistatic adiabatic change. If S denotes the entropy, then $T(S, V)$ will vary with volume V as

(A) $\frac{C(S)}{V^{4/3}}$

(B) $\frac{C(S)}{V^{1/3}}$

(C) $\frac{C(S)}{V}$

(D) $\frac{C(S)}{V^{2/3}}$

C(S) is a constant depending on entropy.

41. For two ultrarelativistic particles constrained to move in a one-dimensional segment of length R, the Hamiltonian is given by $H(p_1, p_2) = c\left[\left|\overrightarrow{p_1}\right| + \left|\overrightarrow{p_2}\right|\right]$. The volume of the phase space enclosed by a surface of energy E is given by

(A) $2R^2E^2/c^2$

(B) $4R^2E^2/c^2$

(C) R^2E^2/c^2

(D) $R^2E^2/2c^2$

42. Two bodies of mass $2m$ and $4m$ are connected by a spring of spring constant K. The frequency of normal mode is

(A) $\sqrt{\frac{K}{6m}}$

(B) $\sqrt{\frac{6K}{m}}$

(C) $\sqrt{\frac{3K}{4m}}$

(D) $\sqrt{\frac{4K}{3m}}$

43. The spin-parity of the ground state of a nucleus is $\frac{1}{2}^-$ and that of its first excited state is $\frac{5}{2}^-$. The polarity of the gamma radiation emitted by the excited nucleus is

(A) E2 or E3

(B) M2 or M3

(C) E2 or M3

(D) E3 or M2

44. In a medium, an electromagnetic wave has a field given by

$\vec{E} = 20e^{-0{\cdot}02x}\sin\left[2\times10^8 t - 4x\right]\hat{z}\frac{V}{m}$ where x, t are measured in SI units. The speed of light v in the medium and the skin depth δ are respectively given by

(A) $v = \frac{c}{6}, \quad \delta = 50\, m$

(B) $v = c, \quad \delta = 0{\cdot}02\, m$

(C) $v = \frac{2}{3}c, \quad \delta = 0{\cdot}02\, m$

(D) $v = c, \quad \delta = 50\, m$

45. An electromagnetic wave is represented as

$\vec{E}(x,t) = 2E_0 \cos(kx + wt)\hat{j} + E_0 \sin(kx + wt)\hat{k}$

where w, k are positive constants. This electromagnetic wave represents

(A) a circularly polarised wave travelling in the negative x direction

(B) an elliptically polarised wave travelling in the negative x direction

(C) a linearly polarised wave travelling in the negative x direction

(D) a circularly polarised wave travelling in the positive x direction

46. Let $|p\rangle$ and $|n\rangle$ denote the isospin (I, I_Z) states of proton and neutron respectively. Which of the following two-nucleon states corresponds to $I = 0$ and $I_Z = 0$?

(A) $\frac{1}{\sqrt{2}}\left[|n, n\rangle + |p, p\rangle\right]$

(B) $\frac{1}{\sqrt{2}}\left[|n, p\rangle + |p, n\rangle\right]$

(C) $\frac{1}{\sqrt{2}}\left[|n, n\rangle - |p, p\rangle\right]$

(D) $\frac{1}{\sqrt{2}}\left[|n, p\rangle - |p, n\rangle\right]$

47. The ground state spin-parity of ^{209}Bi -nucleus is $\frac{9}{2}^-$, and its radius is 7.12 fm. The shell model prediction of the quadrupole moment of ^{209}Bi -nucleus is

(A) – 0·11 barn

(B) – 0·22 barn

(C) – 0·33 barn

(D) – 0·44 barn

48. Assume that free charges are present in a material of dielectric constant 11 and resistivity $10^{11}\Omega$ m. The time required for the charge density inside the material to decay by $\frac{1}{e}$ is closest to

(A) $10^{-10} s$

(B) $11\times10^{-11} s$

(C) $11\times10^{11} s$

(D) 10 s

49. If the frequencies of vibrations of a solid are all equal and $\frac{h\nu}{K_B} = 100K$, then the molar specific heat of the solid at 500 K will be

(A) infinite

(B) 2R

(C) 3R

(D) 1·5 R

(R is the Universal gas constant.)

50. Consider the band energies $E(k) = \pm V_0(1 + 2\cos ka), V_0$ is a constant. The two bands will cross each other at the value ka equal to

(A) $\frac{\pi}{4}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

51. A simple pendulum is being operated in vacuum. Let θ_{max} be the maximum amplitude such that its motion is described by simple harmonic oscillations to an accuracy of 5%. Then θ_{max} is best approximated by

(A) 30º

(B) 10º

(C) 5º

(D) 45º

52. At normal incidence, a beam of light propagating in vacuum reflects off an interface with a medium of refractive index n = 4. The fraction of energy reflected R is given by $R = \left(\frac{n-1}{n+1}\right)^2$. If the fractional error in the value of n is 2%, then the fractional error in the estimation of R is

(A) 0·53%

(B) 1·4%

(C) 1·54%

(D) 2%

53. Given $Y = ABC + A\overline{B}C + \overline{A}B\overline{C} + \overline{A}\overline{B}\overline{C}$. The simplified form of the given logic expression will be

(A) $Y = A$ XOR B

(B) $Y = A$ XNOR C

(C) $Y = A$ XOR C

(D) $Y = A$ XNOR B

54. χ is a spin -1 neutral particle. Consider the decays

(i) $\chi \rightarrow \gamma\,\gamma$

(ii) $\chi \rightarrow e^{+}e^{-}\,\gamma$

(iii) $\chi \rightarrow \gamma\,\gamma\,\gamma$

(iv) $\chi \rightarrow \nu\,\bar{\nu}\,\gamma$

Which decays (is) are not allowed at all?

(A) Only (i)

(B) Only (i) and (iii)

(C) Only (iii) and (iv)

(D) All of (i), (iii) and (iv)

55. A disk of radius 4m is kept at a plane perpendicular to the z-axis. An electromagnetic wave travelling in free space crosses the disk. The electric field of the wave is represented as $\vec{E}(z,t) = E_0 \cos(wt - kz)\,\hat{i}$, where $E_0 = 45$ V / m. Find the value of the average power crossing the disk along z-direction.

(A) 60 W

(B) 90 W

(C) 135 W

(D) 180 W

56. For a gas obeying $P(V-b) = RTe^{-a/RTV}$, the value of $\frac{\partial}{\partial V}(C_V)_T$ is

(A) $\frac{Pa}{RVT^2}$

(B) $\frac{2Pa^2}{RV^2T^2}$

(C) $\frac{Pa^2}{R^2V^2T^3}$

(D) $\frac{Pa^2}{R^2V^2T^2}$

57. Imagine a world with 4 spatial dimensions. For a monoatomic gas, the ratio of specific heats C_P/C_V is

(A) $\frac{3}{2}$

(B) $\frac{5}{3}$

(C) 2

(D) $\frac{4}{3}$

58. In a Geiger-Muller detector the central anode wire is surrounded by a metallic cylindrical cathode tube. If the polarities of the wire and the tube are interchanged, then the system will

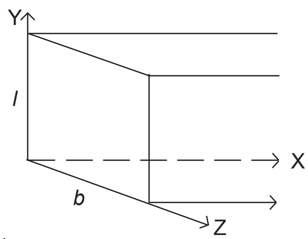
(A) continue to detect in the same way as the original

(B) continue to detect with an increasing efficiency compared to the original

(C) continue to detect with a decreasing efficiency compared to the original

(D) cease to detect

59. Consider a rectangular waveguide of dimension ($l \times b$) as shown in the figure. The X-component of the magnetic field corresponding to the TE_{11} mode is given by $H_x = H_0 \cos(2\pi y)\cos(6\pi z)$. y & z are in cm. The area of cross-section of the waveguide is



(A) $\frac{1}{12}$ cm^2

(B) $\frac{1}{6}$ cm^2

(C) 6 cm^2

(D) 12 cm^2

60. A particle governed by the one-dimensional Hamiltonian $\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$ is in a state $|\psi\rangle$ such that $\langle\psi|\hat{\Pi}|\psi\rangle = 0$ where $\hat{\Pi}$ is the parity operater. What is the lowest possible value of $\langle\psi|\hat{H}|\psi\rangle = 0$?

(A) $\hbar\omega$

(B) $\frac{3}{2}\hbar\omega$

(C) $2\hbar\omega$

(D) $\frac{5}{2}\hbar\omega$

61. The Hamiltonian for a particle in the $l = 3$ state is given by $H = \frac{1}{\hbar^2}\left[2\hat{L}_x^2 + 2\hat{L}_z^2 + 2\hat{L}_y^2\right] + \frac{1}{20\hbar}\hat{L}_x$. To 1% accuracy, the energy of the ground state is

(A) 15·0

(B) 14·5

(C) 15·6

(D) 16·5

62. A particle of mass m is confined in the ground state of a one-dimensional box extending from $x = -2L$ to $x = 2L$. The wavefunction of the particle in this state is $\Psi(x) = \Psi_0 \cos\frac{\pi x}{4L}$, where ψ_0 is constant. The energy eigenvalue corresponding to this state is

(A) $\frac{\hbar^2\pi^2}{2mL^2}$

(B) $\frac{\hbar^2\pi^2}{4mL^2}$

(C) $\frac{\hbar^2\pi^2}{16\,mL^2}$

(D) $\frac{\hbar^2\pi^2}{32\,mL^2}$

63. The number of ways by which N identical bosons can be distributed in 4 different energy levels is

(A) $\frac{N!}{4!(N-4)!}$

(B) $\frac{N(N-4)}{4!}$

(C) $\frac{(N+3)!}{N!\ 3!}$

(D) $\frac{N!}{4!}$

64. A spin $\frac{1}{2}$ particle is in the state $\frac{1}{\sqrt{6}}\begin{pmatrix}1+i\\2\end{pmatrix}$. What is the probability of getting $\frac{\hbar}{2}$ when we measure S_x?

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{5}{6}$

(D) $\frac{3}{8}$

65. The ratio of the decay lifetimes $T\left(\mu^- \rightarrow e^- \overline{\nu}_e \nu_\mu\right) : T\left(\tau^- \rightarrow e^- \overline{\nu}_e \nu_\tau\right)$ is approximately

(A) 7×10^{-7}

(B) $1{\cdot}4 \times 10^{6}$

(C) 8×10^{4}

(D) 1×10^{-5}

66. If the mean square fluctuation in the energy of a system at an equilibrium temperature T is proportional to T^n, then the energy of the system is proportional to

(A) T^{n-2}

(B) T^{n-1}

(C) $T^{n/2}$

(D) T^{n}

67. For some event, observer X measures $\vec{E} = (\alpha, 0, 0)$ and $\vec{B} = (\alpha, 0, 2\alpha)$ in units of c = 1 and observer Y measures $\vec{E}' = (E_x', \alpha, 0)$ and $\vec{B}' = (\alpha, B_y', \alpha)$. Then, the value of E_x' and B_y' will be respectively

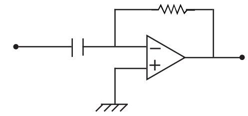
(A) $2\alpha, -\alpha$

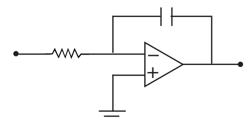
(B) $-\alpha, 2\alpha$

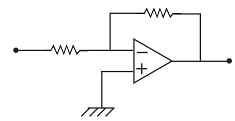
(C) $\alpha, -2\alpha$

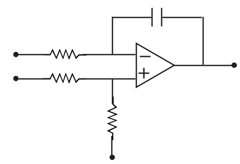
(D) $-\alpha, -2\alpha$

68. Which of the following represents a differentiator?

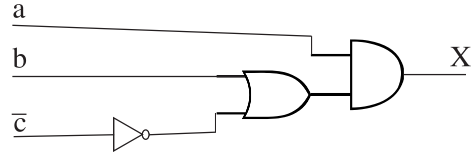
(A) 

(B) 

(C) 

(D) 

69. Which Boolean expression describes the output 'X' in the arrangement given below?



(A) $X = a + (b.c)$

(B) $X = a.(b + c)$

(C) $X = (a.b) + c$

(D) $X = a + b + c$

70. At time t = 0, the normalized wavefunction for a particle confined to $0 \le x \le L$ is given by $\Psi(x) = N \sum_{n=1}^{\infty} 2^{-n/2} \Psi_n(x)$ where $\Psi_n(x)$ are the normalized energy eigenstates. Then $N\left(\theta \in [0, 2\pi]\right)$ is given by

(A) $e^{i\theta}$

(B) $2\,e^{i\theta}$

(C) $\sqrt{\frac{2}{L}}\, e^{i\theta}$

(D) $2\sqrt{\frac{2}{L}}\, e^{i\theta}$

71. Two particles of mass m are confined in a one-dimensional length L. Their self-interaction is described by a hard-core potential of the form $V_0\delta(x_1 - x_2)$. The first order shift in the ground state energy is

(A) $\frac{3V_0}{2L}$

(B) $\frac{V_0}{L}$

(C) $\frac{2V_0}{L}$

(D) $\frac{V_0}{2L}$

72. An LED operates at 2 V and 16 mA in forward bias. Assuming a 90% external efficiency of the LED, the number of photons emitted per second is

(A) 9×10^{14}

(B) 2×10^{30}

(C) 9×10^{16}

(D) 2×10^{21}

73. The wave function of a particle moving under the influence of a spherically symmetric potential $V(r)$ is given by, $\Psi(r, \theta, \varphi) = z\, f(r)$. The eigenvalue of the L^2 operator for this state is

(A) 0

(B) $\hbar^2/4$

(C) $2\hbar^2$

(D) $\hbar^2/2$

74. Which of the following gates can be used as a parity checker?

(A) An OR gate

(B) A NOR gate

(C) An XOR gate

(D) An AND gate

75. Let Ψ and $\bar{\Psi}$ represent respectively the 4-component Dirac spinor and its conjugate for a free electron. Which among the following quantities is a Lorentz scalar?

(A) $\Psi\bar{\Psi}$

(B) $\bar{\Psi}\Psi$

(C) $\Psi^{\dagger}\Psi$

(D) $\bar{\Psi}\gamma^{\mu}\Psi$

76. A particle of rest mass $m_0 = 9 \times 10^{-3}$ kg is moving with a velocity $\frac{4}{5}$c. The de-Broglie wavelength of the particle is

(A) $\frac{4}{5}\frac{h}{m_0 c}$

(B) $\frac{3}{4}\frac{h}{m_0 c}$

(C) $\frac{4}{3}\frac{h}{m_0 c}$

(D) $\frac{5}{4}\frac{h}{m_0 c}$

[Consider h is Planck's constant and c is the velocity of light in vaccum.]

77. In which counter for the detection of thermal neutrons, boron is introduced in the form of BF_3?

(A) Ionization chamber

(B) Proportional counter

(C) GM counter

(D) Scintillation counter

78. A one-dimensional system is governed by Hamiltonian $H = \frac{p^2}{2m} + \lambda|x|^3$ where $\lambda > 0$ is a constant. If $E_0(\lambda)$ is the energy of the ground state for a given λ, then the ratio $E_0(32)/E_0(1)$ is

(A) $\sqrt{2}$

(B) 4

(C) $\sqrt{32}$

(D) 32

79. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{K_B T}{\hbar\omega}$, the free-energy of N such independent oscillator is

(A) $\frac{3}{2} N K_B T$

(B) $K_B T \ln \frac{\hbar\omega}{K_B T}$

(C) $N K_B T \ln \frac{\hbar\omega}{K_B T}$

(D) $N K_B T \ln \frac{\hbar\omega}{2 K_B T}$

80. X-ray of wavelength $\lambda = a$ is reflected from the [111] plane of a simple cubic lattice. If the lattice constant is a, the corresponding Bragg angle (in radian) is

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{8}$

(D) $\frac{\pi}{3}$

81. The maximum attainable temperature of an ideal gas in a process given by pressure $p = p_0 - \alpha V^2 (p_0, \alpha$ are constant) is

(A) $\frac{2}{3} p_0 \sqrt{\frac{p_0}{3\alpha}}$

(B) $\sqrt{\frac{p_0}{3\alpha}}$

(C) $\frac{1}{3}\sqrt{\frac{2p_0}{3\alpha}}$

(D) ∞

82. The change in collector current for a BJT connected in the CE configuration, when the base current changes by 0·6 mA (Given the current amplification factor $\alpha = 0{\cdot}8$) is

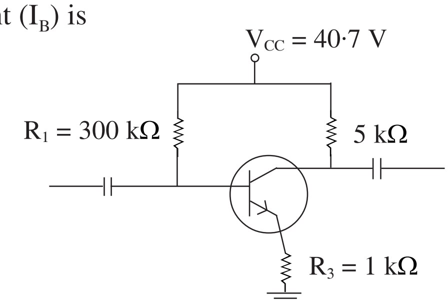
(A) 3·6 mA

(B) 1·2 mA

(C) 2·8 mA

(D) 2·4 mA

83. Consider the given circuit, where the common emitter forward amplification factor β of the transistor is 100 and $V_{BE} = 0{\cdot}7$ V, the base current (I_B) is



(A) 0·1 mA

(B) 1 μA

(C) 0·01 A

(D) 1 mA

84. A semiconductor specimen of Si has electron and hole mobilities of 0·13 and 0·05 m^2/V.s, respectively at 300 K. It is doped with P and Al with doping densities of $2{\cdot}0 \times 10^{21}/m^3$ and $2{\cdot}5 \times 10^{21}/m^3$, respectively. The conductivity of the doped Si sample at 300 K is

(A) 7·5 Ωm^{-1}

(B) 7·0 Ωm^{-1}

(C) 7·25 Ωm^{-1}

(D) 8·0 Ωm^{-1}

85. The octal equivalent of the binary number 11111001 is

(A) 761

(B) 762

(C) 371

(D) 372

86. Audio signals are not transmitted by EM waves because

(I) Antennas used will be very long.

(II) Audio signals do not radiate.

(III) Interference is due to simultaneous transmissions.

(IV) Usable frequency range is too low and narrow.

Which of the following is correct?

(A) (I) and (II)

(B) (II) and (III)

(C) (I) and (III)

(D) (I), (III) and (IV)

87. An unperturbed two-level system has energy eigenvalues E_1 and E_2 and eigenfunctions $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$ respectively. When perturbed its Hamiltonian is represented by $\begin{bmatrix}E_1 & A\\ A^* & E_2\end{bmatrix}$.

The first order correction to E_1, is

(A) 0

(B) $2A$

(C) A

(D) $\dfrac{A}{2}$

88. The mobility of the holes in a p-type semiconductor kept in a magnetic field of strength $B = 0{\cdot}25\ T$, having width and thickness equal to 5 mm each (Given, the measured values of the current and Hall voltage are $15\,\mu A$ and 75 mV respectively, and the resistivity of the material is $2{\cdot}25 \times 10^3$ ohm-m) is

(A) $0{\cdot}0044\ m^2/V\text{-}s$

(B) $0{\cdot}044\ m^2/V\text{-}s$

(C) $44\ m^2/V\text{-}s$

(D) $0{\cdot}44\ m^2/V\text{-}s$

89. The doublet-splitting of first excited state, $2\,^2P_{3/2} - 2\,^2P_{1/2}$ of He^+ is $5{\cdot}84\ cm^{-1}$. The corresponding separation for H is

(A) $1{\cdot}460\ cm^{-1}$

(B) $0{\cdot}730\ cm^{-1}$

(C) $0{\cdot}365\ cm^{-1}$

(D) $0{\cdot}182\ cm^{-1}$

90. For atomic hydrogen, the effective range of atom-atom force is about 0·4 nm. For such a gas in equilibrium, the maximum temperature below which the scattering is s-wave dominated is approximately

(A) 2 K

(B) 10 K

(C) 20 K

(D) 40 K

91. The spin-orbit interaction term for an electron moving in a central field is given by, $V(r) = f(r)\,\overline{L}\,.\,\overrightarrow{S}$, where r is the radial distance of the electron from the origin. If the electron is moving inside a uniformly charged sphere, then

(A) $f(r) = \frac{1}{r^3}$

(B) $f(r) = \frac{1}{r^2}$

(C) $f(r) = \frac{1}{r}$

(D) $f(r)$ = constant

92. The interplanar angle θ between the plane $(h_1\ k_1\ l_1)$ with spacing d_1 and the plane $(h_2\ k_2\ l_2)$ with spacing d_2 for a cubic lattice is

(A) $\cos\theta = \dfrac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$

(B) $\cos\theta = \dfrac{h_1^2h_2^2 + k_1^2k_2^2 + l_1^2l_2^2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$

(C) $\cos\theta = \dfrac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2) + (h_2^2 + k_2^2 + l_2^2)}}$

(D) $\cos\theta = \dfrac{h_1^2h_2^2 + k_1^2k_2^2 + l_1^2l_2^2}{\sqrt{(h_1^2 + k_1^2 + l_1^2) + (h_2^2 + k_2^2 + l_2^2)}}$

93. If the energy gap of a superconductor is $3{\cdot}4 \times 10^{-4}$ eV and the associated Fermi velocity of electrons is $2{\cdot}02 \times 10^6$ m/s, then the intrinsic coherence length of the superconductor is

(A) 3·9 μm

(B) 2·4 mm

(C) 1·8 μm

(D) 1·8 nm

94. The input impedance of a lossless transmission line is 100 Ω when terminated in a short circuit and 64 Ω when terminated in an open circuit. The characteristic impedance of the line is

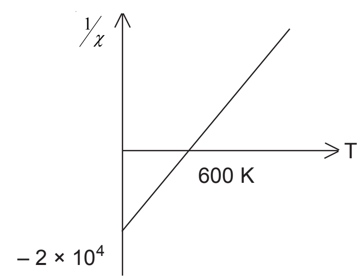
(A) 80 Ω

(B) 164 Ω

(C) 36 Ω

(D) 82 Ω

95. Inverse susceptibility $\left(1/\chi\right)$ as a function of temperature T for a material undergoing paramagnetic to ferromagnetic transition is shown in the figure:



The values of the Curie constant C and the Weiss molecular field constant λ in CGS unit are

(A) $C = 5 \times 10^{-5}$, $\lambda = 3 \times 10^{-2}$

(B) $C = 3 \times 10^{-2}$, $\lambda = 5 \times 10^{-5}$

(C) $C = 3 \times 10^{-2}$, $\lambda = 2 \times 10^{4}$

(D) $C = 2 \times 10^{4}$, $\lambda = 3 \times 10^{-2}$

96. The static dielectric constant of Ge is 15·8 and the effective mass $m_e = 0{\cdot}1$ m [$m \rightarrow$ isolated electron mass]. Then the donor ionization energy is

(A) 5 meV

(B) 9 meV

(C) 12 meV

(D) 15 meV

97. Classically estimated electronic heat capacity of a substance is $\frac{3}{2}NK_B$. If the Fermi temperature $T_F = 5 \times 10^4$ K, then at room temperature, the ratio of the electronic heat capacity to the classical value is

(A) 1·050

(B) 0·004

(C) 0·002

(D) 0·003

98. Let $\vec{L} = (L_x, L_y, L_z)$ denote the orbital angular momentum operators of a particle and let, $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$. The particle is in an eigen state of L^2 and L_z with eigenvalues $\hbar^2\, l\,(l+1)$ and $\hbar l$, respectively. The expectation value of L_+L_- in this state is

(A) $l\hbar^2$

(B) $2l\hbar^2$

(C) 0

(D) $l\hbar$

99. The Hamiltonian of two spin-$\frac{1}{2}$ particles coupled by an interaction is described by $\hat{H} = A\hat{S}_1 . \hat{S}_2$. Combination of these two spin-$\frac{1}{2}$ particles results in a joint entity with spin quantum number S_{tot} = 0 or 1. Then $\hat{S}_1 . \hat{S}_2$ is equal to $(\hbar = 1)$

(A) $\frac{1}{4}$ for $S_{tot} = 1$ and $-\frac{3}{4}$ for $S_{tot} = 0$

(B) $\frac{3}{4}$ for $S_{tot} = 1$ and $-\frac{1}{4}$ for $S_{tot} = 0$

(C) $-\frac{3}{4}$ for $S_{tot} = 1$ and $\frac{1}{4}$ for $S_{tot} = 0$

(D) $-\frac{1}{4}$ for $S_{tot} = 0$ and $\frac{3}{4}$ for $S_{tot} = 0$

100. Eigenfunctions of the Hamiltonian $\hat{H} = \hat{T} + \hat{V}$ of a harmonic oscillator are

(A) eigenfunctions of $\hat{T}$ as well as $\hat{V}$

(B) eigenfunction of $\hat{T}$ but not of $\hat{V}$

(C) eigenfunction of $\hat{V}$ but not of $\hat{T}$

(D) eigenfunction of neither $\hat{T}$ nor $\hat{V}$

Space for Rough Work

Space for Rough Work

Space for Rough Work

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