FS – 19 / 15-16
Mathematics
Paper – I

Time : 3 hours

Full Marks : 200

The figures in the right-hand margin indicate marks.

Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any ***three*** *of the remaining questions, selecting at least* ***one*** *from each Section.*

SECTION – A

1. Answer any **five** of the following : 8×5 = 40

   (a) Find an orthogonal matrix P for which $P^{-1}AP$ is a diagonal matrix where $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ is a matrix.

(b) Determine whether in the following two cases W is subspace of the vector space V or not ?

(i) $W = \{f : f(5) = 2 - f(7)\}$, if V is vector space of all functions from real field R to R

(ii) $W = \{(a, b, c) ; a + b + c = 0\}$, where $V = R^3$ is a vector space

(c) Using $\varepsilon - \delta$ definition, find the limit of $f(x, y) = x^2 - 2y$ as $x \to 2$ and $y \to 1$.

(d) Define and trace Gamma function of n that is $\Gamma(n)$ for positive and negative values in the domain $-3 \le n \le +3$. Also find the value of $\Gamma\left(-\frac{3}{2}\right)$.

(e) Find the equation of a conic in polar coordinates and discuss that in case the eccentricity is less than unity it will be an ellipse.

(f) Trace the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2}$ accurately, and name it.

2. (a) If A and B are finite dimensional subspaces of a vector space V, then show that A + B is finite dimensional and dim(A + B) = dim(A) + dim(B) – dim(A $\cap$ B). 20

(b) Prove that intersection of two subspaces of a vector space is also a vector subspace. 20

3. (a) Let $f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & \text{Otherwise} \end{cases}$.

Show whether the function is continuous at (0, 0) or not ? Also show whether the two first order partial derivatives of the function exist or not at (0, 0) ? 13

(b) Let $U = x^2y$, $x^5 + y = t$, $x^2 + y^3 = t^2$, then find $\dfrac{dU}{dt}$. 13

(c) Evaluate the following : 7+7 = 14

(i) $\int_0^1 \int^{\sqrt{2x - x^2}} \left(x^2 + y^2\right) dxdy$ by changing to polar coordinates

(ii) $\int_0^1 \int_{\sqrt{x}}^1 \exp\frac{x}{y}\,dxdy$ by changing the order of integration

4. (a) Trace the conic $3(3x - 2y + 4)^2 + 2(2x + 3y - 5)^2 = 39$. 14

(b) Find the equation of sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z = 8$, $x + y + z = 3$ as its great circle. 13

(c) Find the equations of lines of intersection of the plane $3x + 4y + z = 0$ with the cone $15x^2 - 32y^2 - 7z^2 = 0$. 13

SECTION – B

5. Answer any five of the following : $8 \times 5 = 40$

(a) Write a homogeneous linear third order ordinary differential equation with variable coefficients whose ordinary point is $x = 0$ and (i) a boundary value problem and (ii) an initial value problem associated with it.

(b) Find the particular integral of the ordinary differential equation $y'' + 4y = \sec 2x$.

(c) In a central orbit at its one point (r, θ) law of central force is inversely proportional to (r^3). Find the pedal equation of the central orbit.

(d) Write down the equation of Poinsot's central axis for the system of two forces (1, –1, 2) and (2, 3, 4) acting at points (2, –1, 1) and (4, 3, 2) respectively.

(e) Derive gradient of a scalar function in cylindrical coordinate system.

(f) Determine the unit vector perpendicular to the plane containing the two vectors $2\vec{i} - 6\vec{j} - 3\vec{k}$ and $4\vec{i} + 3\vec{j} - \vec{k}$.

6. Solve the following **four** ordinary differential equations : 10×4 = 40

(a) $\dfrac{dy}{dx} = \dfrac{y^2 + 2x^2y}{xy - 2x^3}$

(b) Find the general solution and singular solution of $p^3 + 8y^2 - 4xyp = 0$, where $p = (dy / dx)$.

(c) $(xD)^3y + 3(xD)^2y + (xD)y + y = x \log x$, where $D = (d / dx)$.

(d) $y'' + 9y = \tan 3x$ using the method of variation of parameters.

7. (a) Discuss the stability of a rigid body placed at the top of a fixed rigid body and displaced slightly from rest position, the portions of both the rigid bodies are circular in the vicinity of the point of contact. 14

(b) Find the time period and amplitude of a rectilinear simple harmonic motion in an unresistive medium in terms of velocities and accelerations at any two positions of the path. 13

(c) A thin hollow circular cone with its base below the vertex floats completely immersed in water. Find the vertical angle of the cone. 13

8. (a) State and prove Gauss divergence theorem of vector calculus. 14

(b) State and prove Serret-Frenet's formule. 13

(c) State and prove the identity for curl of cross product of two vectors. 13