| FS – 25 / 15-16 |
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| Statistics |
| Paper – I |

*Time : 3 hours*

*Full Marks : 200*

*The figures in the right-hand margin indicate marks.*

*Candidates should attempt Q. No. 1 from Section – A and Q. No. 5 from Section – B which are compulsory and any **three** of the remaining questions, selecting at least **one** from each Section.*

## SECTION – A

1. Answer any **two** of the following subparts :

(a) (i) If $\{A_n\}$ is a monotone sequence of events, show that $P(\lim A_n) = \lim P(A_n)$. 10

(ii) Let $\beta_n = E|X|^n < \infty$. Then for arbitrary k, $2 \le k \le n$, prove that $\beta_{k-1}^{1/(k-1)} \le \beta_k^{1/k}$. 10

(b) (i) State and prove Chebyshev's Theorem on Weak Law of Large Numbers. 10

(ii) Obtain a large sample 95% confidence interval for the parameter $\theta$ in random sampling from the exponential distribution : 10

$$dF(x) = \theta \exp(-\theta x);\ x > 0,\ \theta > 0$$

(c) (i) The probability density function of a random variable X is given by

$$f(x) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

If $0.5 \le x$ is the critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ on the basis of a single observed value x, compute type I and type II errors, and power of the test. 4+4+2 = 10

(ii) Develop the sequential probability ratio test to test the hypothesis $H_0: \lambda = \lambda_0$ against $H_1: \lambda = \lambda_1$ for the distribution with the following probability mass function :

$$f(x, \lambda) = \lambda^x (1 - \lambda)^{1-x},\ x = 0, 1\ ;\ 0 < \lambda < 1$$

10

2. (a) Let F be the distribution function of a random variable X. Show that F is monotone and non-decreasing, $\lim_{x \to -\infty} F(x) = 0$, $\lim_{x \to +\infty} F(x) = 1$ and F is everywhere continuous to the right. 14

(b) If X and Y are independent Poisson random variables with respective parameters $\alpha$ and $\beta$, find the conditional distribution of X, given that $X + Y = n$. 13

(c) Define characteristic function of a random variable. State and prove Inversion Theorem associated with the characteristic function. 13

3. (a) State and prove Borel-Cantelli Lemma. 14

(b) State and prove Lindeberg-Levy Central Limit Theorem. Show that this theorem holds good to a sequence of iid Bernoulli variables with a common parameter p. 13

(c) Define minimum variance unbiased estimator. Show that a minimum variance unbiased estimator in unique. 13

4. (a) Explain the method of maximum likelihood estimation and point out some important properties of the maximum likelihood estimators. If a random sample of size n is drawn from a normal distribution $N(\mu, \sigma^2)$, find the maximum likelihood estimators for $\mu$ and $\sigma^2$. 14

(b) State and prove Neyman-Pearson Lemma. Using this lemma, obtain the most powerful critical region for testing the hypothesis $H_0: \theta = 2$ against the alternative $H_1: \theta = 1$ on the basis of a random sample of size n drawn from the distribution with probability density function : 13

$$f(x) = \begin{cases} \theta e^{-\theta x}, x \geq 0, \theta > 0 \\ 0, x < 0 \end{cases}$$

(c) What are the advantages of non-parametric tests over parametric tests ? Describe Sign Test stating clearly the assumptions made.

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## Section – B

5. Attempt any **two** of the following subparts :

(a) (i) Explain the terms 'estimable parametric function,. 'error function' and 'best linear unbiased estimator' in connection with the problem of linear estimation. 10

(ii) Explain the concepts of multiple and partial correlations with examples. For a tri-variate distribution in $X_1$, $X_2$ and $X_3$, show that

$$1 - R^2_{1.23} = (1 - r^2_{12})(1 - r^2_{13.2}),$$

where $R_{1.23}$ is the multiple correlation coefficient of $X_1$ on $X_2$ and $X_3$, $r_{13.2}$ is the partial correlation coefficient between $X_1$ and $X_3$, and $r_{12}$ is the simple correlation coefficient between $X_1$ and $X_2$. 10

(b) (i) If a random vector **X** follows a p – variate normal distribution with mean vector $\mu$ and dispersion matrix $\Sigma$, find the distribution of the quadratic form $\mathbf{Q} = \mathbf{X}' \Sigma^{-1} \mathbf{X}$. 10

(ii) What are the various types of non-sampling errors that arise in a survey operation ? Point out their sources. 10

(c) (i) Under the mechanism of simple random sampling without replacement, show that the probability of drawing a specified unit at any given draw is equal to the probability of drawing it at the first draw. 10

(ii) Explain various basic principles of designing of an experiment pointing out their importance. 10

6. (a) State and prove the Gauss-Markov Theorem and state its importance in linear estimation. 14

(b) Discuss on a test procedure to test for linearity of regression of the dependent variable Y on the independent variable X. 13

(c) Define Hotelling $T^2$ statistic and derive its sampling distribution. 13

7. (a) What is the problem of discrimination ? Describe the procedure for construction of Fisher's linear discriminant function ? Describe the procedure for construction of Fisher's liner disciminant function. Discuss how this discriminant function is related to the Mahalanobis $D^2$ statistic. 14

   (b) Discuss on the Horvitz-Thompson method of estimation of a finite population mean. Derive Yates and Grundy expression for the variance of the Horvitz-Thompson estimator. 13

   (c) Explain factorial method of experimentation. A complete $2^3$-experiment is replicated r times. Describe the procedure for testing the presence of different main effects and interactions. 13

8. (a) Explain a two-stage sampling method. For a two-stage sampling with unequal first stage units, define an unbiased estimator of the population mean and find its variance. 14

(b) Describe the ratio method of estimation for estimating population mean. Show that ratio estimator is biased. Derive asymptotic expressions for the bias and mean square error of the said estimator. 13

(c) Define a balanced incomplete block design with parameters v, b, r, k and $\lambda$. Prove that $bk = vr$, $r(k-1) = \lambda(v-1)$ and $b \geq v$. 13