JEE Mains (12th)

Sample Paper - II

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(4)
2.	(1)
3.	(2)
4.	(1)
5.	(4)
6.	(2)
7.	(4)
8.	(4)
9.	(1)
10.	(3)
11.	(1)
12.	(3)
13.	(3)
14.	(2)
15.	(3)
16.	(4)
17.	(3)
18.	(2)
19.	(2)
20.	(4)
21.	(52.8)
22.	(36)
23.	(20)
24.	(15)
25.	(246)
26.	(33.33)
27.	(1)
28.	(25)
29.	(3)
30.	(5)

CHEMISTRY

31.	(1)
32.	(3)
33.	(3)
34.	(3)
35.	(1)
36.	(3)
37.	(3)
38.	(2)
39.	(4)
40.	(2)
41.	(2)
42.	(2)
43.	(4)
44.	(2)
45.	(3)
46.	(2)
47.	(3)
48.	(3)
49.	(2)
50.	(2)
51.	(13)
52.	(13)
53.	(6)
54.	(8)
55.	(3)
56.	(7)
57.	(1)
58.	(3)
59.	(18)
60.	(1)

MATHEMATICS

61.	(1)
62.	(4)
63.	(4)
64.	(2)
65.	(2)
66.	(3)
67.	(4)
68.	(2)
69.	(4)
70.	(2)
71.	(1)
72.	(3)
73.	(3)
74.	(3)
75.	(1)
76.	(4)
77.	(4)
78.	(1)
79.	(3)
80.	(3)
81.	(13)
82.	(8)
83.	(30)
84.	(11)
85.	(240)
86.	(3)
87.	(3)
88.	(3)
89.	(1)
90.	(5)

PHYSICS

1. **(4)**

$T^2 \propto r^3$ law ; $T_0^2 \propto R^3$ & $T_S^2 \propto (<7R)^3$;

$\left(\frac{T_S}{T_0}\right)^2 = 7^3$; $T_S = 7\sqrt{7}\,T_0$

2. **(1)**

Pitch = 1 mm

Number of divisions on circular scale = 200

$$L.C = \frac{\text{Pitch}}{\text{Number of divisions on circular scale}}$$

$$= \frac{1\,\text{mm}}{200} = 0.005\,\text{mm} = 0.0005\,\text{cm}$$

Diameter of the wire =

(Main scale reading + Circular scale reading × L.C.) – zero error

= 6 mm + 45 × 0.005 – (– 0.05)

= 6 mm + 0.225 mm + 0.05 mm = 6.275 mm

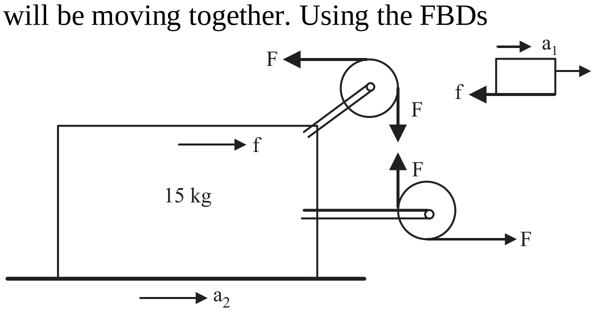
3. **(2)**

Let initial amount be 100gm.

	disintegrated	Left
$100gm \xrightarrow{5days} \frac{100\times10}{100}$	10	90
$90 \xrightarrow{\text{Next 5 days}} \frac{90\times10}{100}$	9	81
$81 \xrightarrow{\text{Next 5 days}} \frac{81\times10}{100}$	8.1	≈ 73
$73 \xrightarrow{\text{Next 5 days}} \frac{73\times10}{100}$	7.3	≈ 65

4. **(1)**

First, let us check upto what value of F, both blocks move together. Till friction becomes limiting, they will be moving together. Using the FBDs



10 kg block will not slip over the 15 kg block till acceleration of 15 kg block becomes maximum as it is created only by friction force exerted by 10 kg block on it.

$a_1 > a_{2(max)}$

$\frac{F-f}{10} = \frac{f}{15}$ = for limiting condition as f maximum is 60N.

F = 100 N

Therefore, for F = 80N, both will move together.

Their combined acceleration, by applying NLM using both as system, F = 25a

$$a = \frac{80}{25} = 3.2\ m/s^2$$

5. **(4)**

When two rods are connected in series:

$$Q = \frac{A(T_1 - T_2)t}{\frac{d_1}{K_1} + \frac{d_2}{K_1}} = \frac{A(T_1 - T_2)t}{(d_1 + d_2)/K}$$

$$\therefore \frac{d_1 + d_2}{K} = \frac{d_1}{K_1} + \frac{d_2}{K_2}$$

$$\therefore K = \frac{(d_1 + d_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$

6. **(2)**

Total energy, ; $E = \frac{1}{2}m\omega^2 a^2$;

$$K.E. = \frac{3E}{4} = \frac{1}{2}m\omega^2(a^2 - y^2)$$

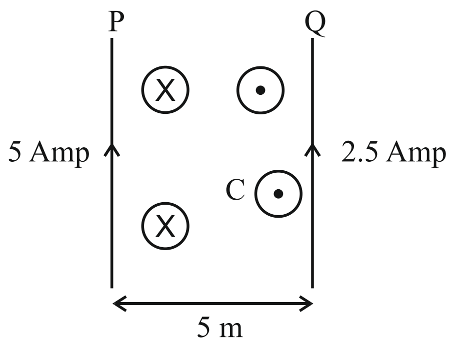
So, $\frac{3}{4} = \frac{a^2 - y^2}{a^2}$ or $y^2 = \frac{a^2}{4}$ or $y = \frac{a}{2}$.

7. **(4)**

When current flow in both wire in same direction then magnetic field at half way due to P wire.

$\vec{B}_P - = \frac{\mu_0 I_1}{2\pi \frac{5}{2}} = \frac{\mu_0 I_1}{\pi.5} = \frac{\mu_0}{\pi}$ (where I_1 = 5 Amp)

The direction of B_P is downward ⊗



Magnetic field at half way due to Q wire

$\vec{B}_Q = \frac{\mu_0 I_2}{2\pi \frac{5}{2}} = \frac{\mu_0}{2\pi}$ [upward ⊙]

[where I_2 = 2.5 Amp.]

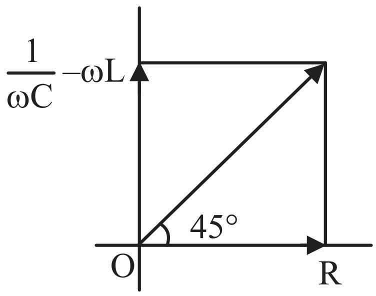
Net magnetic field at half way:

$$\vec{B} = \vec{B}_P + \vec{B}_Q = \frac{\mu_0}{\pi} \otimes + \frac{\mu_0}{2\pi} \odot = \frac{\mu_0}{2\pi} \otimes$$

Hence net magnetic field at midpoint = $\frac{\mu_0}{2\pi}$

8. **(4)**

From figure,



$$\tan 45° = \frac{\frac{1}{\omega C} - \omega L}{R}$$

$$\Rightarrow \frac{1}{\omega C} - \omega L = R \Rightarrow \frac{1}{\omega C} = R + \omega L$$

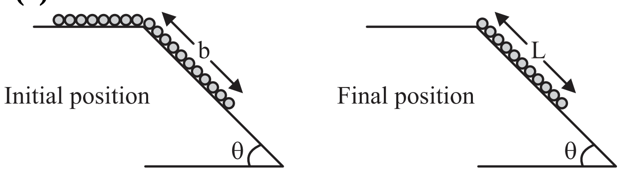
$$C = \frac{1}{\omega(R+\omega)} = \frac{1}{2\pi f(R + 2\pi f L)}$$

9. **(1)**

$[X] = [(\text{velocity})^2 \text{ Density}]$

$= [(L^2T^{-2})ML^{-3}] = [ML^{-1}T^{-2}]$

10. **(3)**



Loss in PE = gain in KE

$$\Rightarrow mg\left(\frac{L}{2}\sin\theta\right) - \left(\frac{m}{L}b\right)g\left(\frac{b}{2}\sin\theta\right)$$

$$= \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{g\sin\theta}{L}(L^2 - b^2)}$$

11. **(1)**

$$\ell_{cm} = \frac{\sum_{i=1}^{n} m_i \ell_i}{\sum_{i=1}^{n} m_i} = \frac{m\ell\sum_{i=1}^{n} n^2}{m\Sigma n}$$

$$= \frac{n(n+1)(2n+1)}{6 \times \frac{n(n+1)}{2}} = \left(\frac{2n+1}{3}\right)\ell$$

12. **(3)**

For conservation of vertical momentum, the second part must have a vertical downward velocity of 50 m/s. For conservation of horizontal momentum, the second part must have a horizontal velocity of 120 m/s. $v = \sqrt{50^2 + 120^2}$ = 130 m/s.

13. **(3)**

$$v_{rms} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3 \times (24 \times 10^5 \times 10^{-1}) \times (10 \times 10^{-3})}{(20 \times 10^{-3})}}$$

= 600 m/s.

14. **(2)**

Heat liberated by water to attain 0°C = cosθ = 10 × 1 × 40 = 400 cal

Amount of ice melted = $\frac{400}{800}$ = 5g

∴ Total amount of water = 10 + 5 = 15 gm

15. **(3)**

As amplitudes are A and 2A, so intensities would be in the ratio 1 : 4, let us say I and 4I.

$$I_{max} = I_0 = I + 4I + 2\sqrt{4I^2} = 9I \Rightarrow I = \frac{I_0}{9}$$

Intensity at any point, $I' = I + 4I + 2\sqrt{4I^2}\cos\phi$; I'

$$= 5I + 4I\cos\phi = \frac{I_0}{9}(5 + 4\cos\phi)$$

16. **(4)**

$hv = \phi + E_1$ ….. (i)

$2hv = \phi + E_2$ …. (ii)

1 × 2 then equation

$2\phi + 2E_1 = \phi + E_2$

Hence $E_2 > 2E_1$

17. **(3)**

When x > 0, the particle will be moving towards origin if

$$\frac{dx}{dt} < 0 \Rightarrow x\frac{dx}{dt} < 0$$

When x < 0, the particle will be moving towards origin if $\frac{dx}{dt} > 0 \Rightarrow x\frac{dx}{dt} < 0$

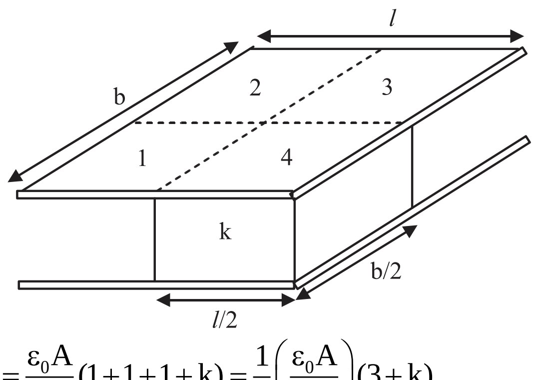
Hence in both cases $x\frac{dx}{dt} < 0$

18. **(2)**

The given capacitor can be broken down as four capacitors C_1 , C_2 , C_3 and C_4 in parallel as shown.

$C_{eq} = C_1 + C_2 + C_3 + C_4$

$$= \frac{\varepsilon_0(A/4)}{d} + \frac{\varepsilon_0(A/4)}{d} + \frac{\varepsilon_0(A/4)}{d} + \frac{k\varepsilon_0(A/4)}{d}$$



$$= \frac{\varepsilon_0 A}{4d}(1+1+1+k) = \frac{1}{4}\left(\frac{\varepsilon_0 A}{d}\right)(3+k)$$

$$C_{eq} = \frac{C}{4}(k+3)$$

19. **(2)**
Velocity components tangential to the direction of impact remains unchanged before and after the collision.

20. **(4)**

$$V = 2.10 \times 10^6 \text{ m/sec} \approx \frac{C}{137}, \frac{V}{C} = \frac{1}{137}$$

21. **(52.8)**
This is example of uniform circular motion.

$$\omega = \frac{2\pi}{T} = 2\pi n = 2\pi \times \frac{7}{100} = 0.44 \text{ rad/sec.}$$

V = Rω = 0.44 × 12 × 10 = 52.8 cm/sec.

22. **(36)**
Equivalent resistance in series = 6R
Equivalent resistance in parallel = R/6

23. **(20)**

$$E = \frac{-dV}{dx} V = 5x^2 + 10x - 9,$$

$$E = \frac{-d}{dx}(5x^2 + 10x - 9) = -(10x + 10)$$

On putting value of x in it we get,

$$E = -(10 \times 1 + 10) = -20\frac{V}{m}$$

24. **(15)**
$Process_{1\to 2}$ and $Process_{3\to 4}$ are isochoric processes.
$W_{12} = 0$, $W_{34} = 0$,
$W_{23} = nR(T_3 - T_2) = 3R\,(16000 - 400) = 3600\text{ R}$
$W_{41} = nR(T_1 - T_4) = 3R\,(200 - 800) = -1800\text{ R}$
W = (3600 – 1800) R = 1800 R = 15 KJ

25. **(246)**
Since beat frequency is 6 Hz, therefore, frequency of second fork will be either 262 Hz or 250 Hz. Since on loading with wax beat frequency decreases, frequency of second fork should be 250 Hz. So after waxing, the frequency of first tuning fork should be either 254 or 246 Hz. Since after removing a portion of wax from first fork beat frequency becomes zero. Therefore frequency of first fork after waxing should be 246 Hz.

26. **(33.33)**
According to displacement method

$$f = \frac{a^2 - d^2}{4a} = \frac{(1.5)^2 - (0.5)^2}{4 \times 1.5} = \frac{2}{6} = 33.3 \text{ cm}$$

27. **(1)**

$$\frac{E}{t} = \frac{lv}{t} = VI$$

P × 0.5% = VI × 0.5%

$$= 20 \times 100 \times 10 \times 10^{-3} \times \frac{0.5}{100}$$

= 1 W

28. **(25)**
$\phi = 7t^2 - 4t$

$$\Rightarrow \text{Induced emf} : |e| = \frac{d\phi}{dt} = 14t - 4$$

$$\Rightarrow \text{Induced current} : i = \frac{|e|}{R} = \frac{|14t - 4|}{20}$$

$$= \frac{|14 \times 0.25 - 4|}{20} (\text{at } t = 0.25 \text{ s})$$

$$= \frac{0.5}{20} = 2.5 \times 10^{-2} \text{ A}$$

29. **(3)**
$24 \times 10 = 6 \times 12 + 8v_3$
$v_3 = 21$ m/s

30. **(5)**

$$A_p = \frac{A_v^2 R_{in}}{R_0} = 50 \times 50 \times \frac{100}{200} = 1250$$

CHEMISTRY

31. **(1)**
Coloured metaborates are formed is borax bead test.

32. **(3)**
$Zn^{+2}(d^{10})$ unpaired $e^- = 0$
$Sc^{+3}(d^{0})$ unpaired $e^- = 0$

33. **(3)**
$NH_3 + Cl_2(\text{excess}) \rightarrow NCl_3$

34. **(3)**
N_2^+

$$1s^2(\sigma)1s^2(\sigma^*)2s^2(\sigma)2s^2\sigma^*\pi\, 2p_x^2\ 2p_y^2\ \sigma 2p_z^1$$

$4e^-$ in antibonding M.O.

35. **(1)**
I.P. B < C < O < N

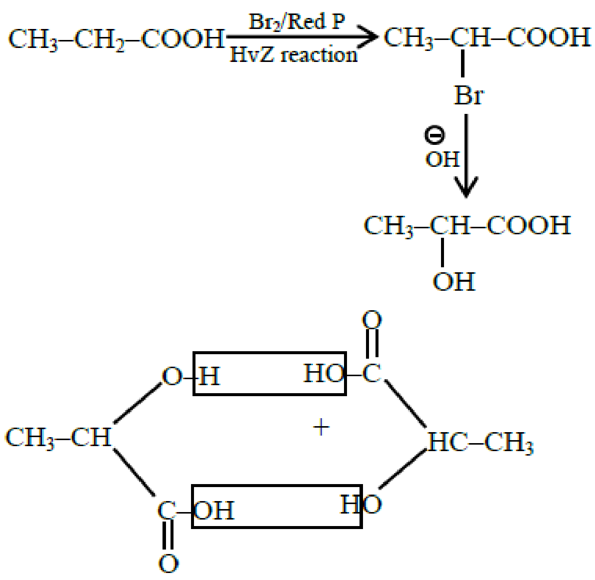
36. **(3)**

37. **(3)**

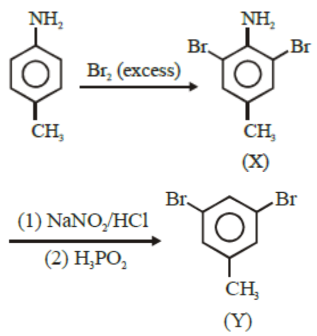
38. **(2)**

Due to presence of EWG $\left[R-\overset{O}{\overset{||}{C}}-\right]$ Rate of SN^2 reaction is higher than SN^1 reaction.

39. **(4)**



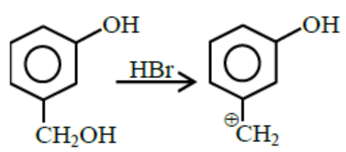
40. **(2)**

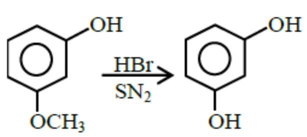


41. **(2)**

Morphine

42. **(2)**





43. **(4)**

$$d = \frac{m \times M_w}{N_A \times a^3}$$

$$d = \frac{4 \times 195}{6 \times 10^{23} \times (0.392 \times 10^{-7})^3} = 21.5 \text{ gm/cm}^3$$

44. **(2)**

45. **(3)**

$A \longrightarrow$ Products

t = 0 1 M

t = 1hr 0.125 M

We know

$$Kt = 2.303 \log \frac{a_0}{a_t}$$

$$K = \frac{2.303}{60 \times 60} \log \frac{1}{0.125} \quad [t = 1 \text{ hr} = 60 \times 60 \text{ sec}]$$

$$= \frac{2.303}{3600} \log 8$$

$$= \frac{2.303}{3600} \times 3 \log 2 [\log 8 = \log 2^3 = 3 \log 2]$$

K = 0.00058 sec^{-1}

$$\therefore t_{1/2} = \frac{0.693}{0.00058} \text{sec} \quad \left[t_{1/2} = \frac{0.693}{K}\right] = 1200 \text{ sec.}$$

46. **(2)**

Combination of weak base (NH_4OH) and its salt with strong acid (NH_4Cl) act as basic buffer solution

$$\underset{\text{(weak acid)}}{NH_4OH} + \underset{\text{(Strong acid)}}{HCl} \rightleftharpoons NH_4Cl + H_2O$$

Initial	2 mole	1 mole	0	0
After reaction	(2 – 1)	(1 – 1) = 0	1 mole	1 mole
	= 1 mole			

1 mole NH_4OH and 1 mole NH_4Cl are present in mixture after reaction. Hence it will also act as basic buffer.

47. **(3)**

$$C_{(s)} + 2H_{2(g)} + \frac{1}{2}O_{2(g)} \rightarrow CH_3OH_{(l)}$$

$$\Delta H = (\Delta H_f)_{CH_3OH} = 2$$

This required reaction ≡ +(2)nd reaction + 2 × (3)rd reaction –(1)st reaction

= – 393 + 2×(–286) – (–726)

= – 393 – 572 + 726 = – 239 KJ mol^{-1}

48. **(3)**

$$\underset{\text{(Red)}}{HIn} \rightleftharpoons H^+ + \underset{\text{yellow}}{In^-}$$

$$pH = pKIn + \log 10 \frac{(In^-)}{(HIn)}$$

At 1^{st} condition;

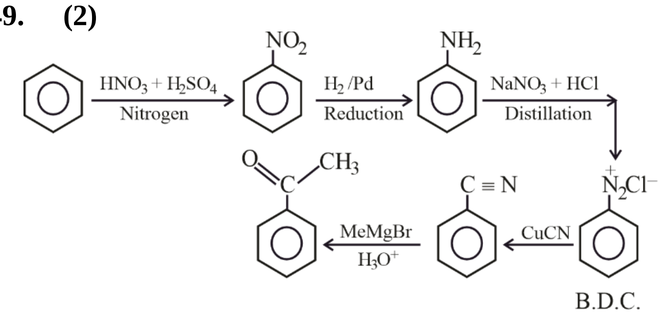
$$(pH)_1 = pKIn + \log_{10}\frac{10}{90} = pKIn - \log_{10} 9 \quad \ldots(1)$$

At 2^{nd} condition;

$$(pH)_2 = pKIn + \log_{10}\frac{90}{10} = pKIn + \log_{10} 9 \quad \ldots(2)$$

change in pH = 2 $\log_{10}9$ = 2 $\log_{10}32$ = 4 $\log_{10}3$
= 4 × 0.48 = 1.92

49. **(2)**



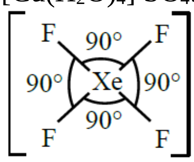
50. **(2)**

$$\underset{(b)}{H_2N} - C - \underset{(c)}{NH} - CH_3$$
$$\quad \| $$
$$\quad \underset{(a)}{NH}$$

Due to +M effect of both (2) and (3).
(1) is most basic.

51. **(13)**

$[Cu(H_2O)_4]\ SO_4.H_2O \Rightarrow x = 4$



$\Rightarrow$ 90° angles = y = 4
$[Pt(NH_3)_6]^{+y}$, $4Cl^-$
(Number of ions = z = 5)
x + y + z
4 + 4 + 5
= 13

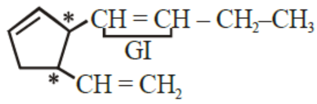
52. **(13)**

EAN = atomic number – O.S. + 2(CN)
$Ni(CO)_x \Rightarrow 36 = 28 - 0 + 2(x)$
$8 = 2x \Rightarrow x = 4$
$Fe(CO)_y \Rightarrow 36 = 26 - 0 + 2y$
$2y = 10 \Rightarrow y = 5$
$K_z[Fe(CN)_6] \Rightarrow 36 = 26 - O.S. + 2 \times 6$
$\Rightarrow$ O.S. = +2
$K^{+1}[Fe(CN)6]^{-z}$
$+2 + 6(-1) = -z$
$-4 = -z \Rightarrow z = 4$
$x + y + z \Rightarrow 4 + 5 + 4 = 13$

53. **(6)**

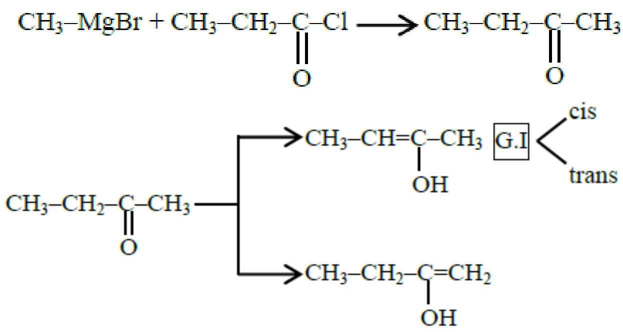
$PCl_5 \Rightarrow sp^3d$
$SF_6 \Rightarrow sp^3d^2$
$IF_7 \Rightarrow sp^3d^3$
$CCl_4 \Rightarrow sp^3$
1 + 2 + 3 = 6

54. **(8)**



n = 3
no of S.I. = 23 = 8

55. **(3)**



56. **(7)**

SBH can't reduced ester group.

57. **(1)**

$$Y_A = \frac{P_A^\circ x_A}{P_A^\circ x_A + P_B^\circ x_B}$$

$$= \frac{1 \times \frac{1}{4}}{1 \times \frac{1}{4} + 3 \times \frac{3}{4}} = \frac{\frac{1}{4}}{\frac{10}{4}} = 0.1$$

58. **(3)**

$KC = 10^{30}$

$$E_{cell} = E°_{cell} - \frac{0.059}{2} \log K_C$$

$E°_{cell} = 0.01 \log \times 10^{30}$
$E°_{cell} = + 0.30\ V$

59. **(18)**

60. **(1)**

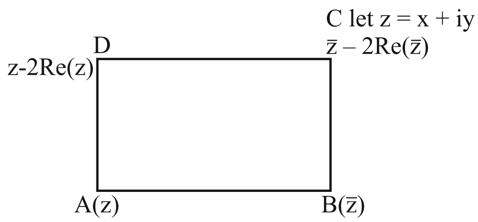
$$k_h = \frac{k_w}{k_a} = \frac{10^{-14}}{10^{-5}} = 10^{-9}$$

$$h = \sqrt{\frac{k_h}{C}} = \sqrt{\frac{10^{-9}}{0.1}} = 10^{-4}$$

h (in %) = $10^{-4} \times 100$
= 0.01%

MATHEMATICS

61. **(1)**



$\therefore$ length of side = 4

Then $|z-\bar{z}|=4|$

$|2iy|=4 \Rightarrow |y|=2$

Also $|z-(z-2\text{Re}(z))|=4$

$|2x|=4 \Rightarrow |x|=2$

$|z|=\sqrt{x^2+y^2}=2\sqrt{2}$

62. **(4)**

Negation of $x \leftrightarrow \sim y$

$\equiv \sim(x \leftrightarrow \sim y)$

$\equiv x \leftrightarrow \sim(\sim(y)$

$\equiv x \leftrightarrow y$

$\equiv (x \wedge y) \vee (\sim x \wedge \sim y)$

63. **(4)**

$S' = 2^{10} + 2^9.3 + 2^8.3^2 + \ldots..+2.3^9 + 3^{10}$

G.P $\rightarrow a = 2^{10}$, $r = \frac{3}{2}$, $n = 11$

$$S' = 2^{10}.\frac{\left(\left(\frac{3}{2}\right)^{11}-1\right)}{\frac{3}{2}-1} = 2^{11}\left(\frac{3^{11}}{2^{11}}-1\right)$$

$= 3^{11} - 2^{11}$

64. **(2)**

Here $D = \begin{vmatrix} 2 & -4 & \lambda \\ 1 & -6 & 1 \\ \lambda & -10 & 4 \end{vmatrix} = (\lambda - 3)(3\lambda + 2)$

$D = 0 \Rightarrow \lambda = 3, -\frac{2}{3}$

$D_1 = \begin{vmatrix} 1 & -4 & \lambda \\ 2 & -6 & 1 \\ 3 & -1 & 4 \end{vmatrix} = 2(3-\lambda)$

For $\lambda = -\frac{2}{3}, D_1 \neq 0$

65. **(2)**

$$I=\int\left(e^{2x}+2e^x-e^{-x}-1\right)e^{\left(e^x+e^{-x}\right)}dx$$

$$I=\int\left(e^{2x}+e^x-1\right)e^{\left(e^x+e^{-x}\right)}dx+\int\left(e^x-e^{-x}\right)e^{e^x+e^{-x}}dx$$

$$I=\int\left(e^x+1-e^{-x}\right)e^{e^x+e^{-x}}dx+e^{x^e+e^{-x}}$$

$e^x + e^{-x} + x = du$

$(e^x - e^{-x} + 1)dx = du$

$I = e^{e^x+e^{-x}} + e^{e^x+e^{-x}} = e^{e^x+e^{-x}}\left(e^x+1\right)$

then $g(x) = e^x + 1$

$g(0) = 2$

66. **(3)**

a, b, c are in AP then

$2b = a + c$

$28 = 3^{2\sin2\theta-1} + 3^{4-2\sin\theta}$

Put $3^{2\sin2\theta-1} = x$

$28 = \frac{x}{3}+\frac{81}{x} \Rightarrow x^2 - 84x + 243 = 0$

$(x-3)(x-81) = 0$

$2\sin2\theta = 1$ or 4

$\sin2\theta = \frac{1}{2}$

Terms are 1, 14, 27,………then

$T_6 = 1 + 5(13)$

67. **(4)**

$n(C) = 73$, $n(T) = 65$, $n(C \cap T) = x$

$n(C \cup T) \leq 100$

$\Rightarrow n(C) + n(T) - n(C \cap T) \leq 100$

$\Rightarrow x > 38$

$n(C \cap T) \leq \min(n(C), n(T)) \Rightarrow x \leq 65$

$\Rightarrow 38 \leq x \leq 65$

68. **(2)**

$y^2 = 4x$ and $x^2 = 4y$

Any tangent of $y^2 = 4x$ is $y = mx+\frac{1}{m}$

It also tangent for $x^2 = 4y$

$\therefore \frac{1}{m} = -m^2 \Rightarrow m = -1$

$\therefore$ common tangent is $y = -x - 1$, it also touches $x^2 + y^2 = c^2$

$\therefore 1 = c^2\cdot(1 + 1) \Rightarrow c^2 = \frac{1}{2}$

69. (4)

$\therefore x^2 = |x|^2 = t$ let

$9t2 - 18t + 5 = 0$

$(3t - 1)(3t - 5) = 0$

$$|x| = \frac{1}{3}, \frac{5}{3}$$

Product of roots = $\frac{1}{3}\left(-\frac{1}{3}\right)\left(\frac{5}{3}\right)\left(\frac{-5}{3}\right) = \frac{25}{81}$

70. (2)

$$I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx \cdot I = \int_{-\pi/2}^{\pi/2} \frac{1}{1+e^{\sin x}}dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^{\sin x}}{e^{\sin x}}dx \left\{ \begin{matrix} \text{Replace} \\ x \to (a+b+x) \end{matrix} \right\}$$

$$\int_a^b (f(x))dx = \int_c^b f(a+b+x)dx$$

$$2I = \int_{-\pi/2}^{\pi/2} 1dx \Rightarrow I = \frac{1}{2}\int_{-\pi/2}^{\pi/2} dx$$

$$I = \frac{1}{2}[x]_{-\pi/2}^{\pi/2} \Rightarrow I = \frac{\pi}{2}$$

71. (1)

$S = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + ...$ upto 10 terms

$$S = \tan^{-1}\left(\frac{2-1}{1+1\cdot 2}\right) + \tan^{-1}\left(\frac{3-2}{1+2\cdot 3}\right) +$$

$$+\tan^{-1}\left(\frac{4-3}{1+3\cdot 4}\right) + ... + \tan^{-1}\left(\frac{11-10}{1+11\cdot 10}\right)$$

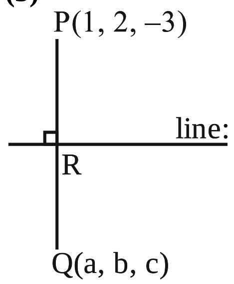
$S = (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + ... + (\tan^{-1}11 - \tan^{-1}10)$

$S = \tan^{-1}11 - \tan^{-1}1$

$S = \tan^{-1}(11) - \frac{\pi}{4}$

$\tan(S) = \frac{5}{6}$

72. (3)



line: $\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1} = r$

$R(-1 + 2r, 3 - 2r, -r)$

dr's of PR are $(2 - 2r, -1 + 2r, -3 + r)$

Then $2(2 - 2r) + 2(1 - 2r) + 1(3 - r) = 0$

$9 - 9r = 0 \Rightarrow r = 1$

$R(1, 1, -1)$

Then $a + 1 = 2$, $b + 2 = 2$, $c - 3 = -2$

$a = 1$, $b = 0$, $c = 1$

$\therefore a + b + c = 2$

73. (3)

Given $\frac{dy}{2+y} = \frac{-e^x dx}{5+e^x}$

$ln(2 + y) = -ln(5 + e^x) + lnC$

$y = \frac{C}{5+e^x} - 2 \Rightarrow y(0) = 1$

$y = \frac{18}{5+e^x} - 2$

$\therefore y = (\log_e 13) = -1$

74. (3)

$P(x) = 0 \Rightarrow x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1 \Rightarrow \therefore \alpha = 2$

Now, $\lim_{x \to 2^+} \frac{\sqrt{1-\cos(x^2-x-2)}}{x-2}$

$$\Rightarrow \lim_{x\to 2^+} \frac{\sqrt{2\sin^2\left(\frac{x^2-x-2}{2}\right)}}{x-2}$$

$$\Rightarrow \lim_{x\to 2^+} \frac{\left|\sin\left(\frac{x^2-x-2}{x-2}\right)\right|}{x-2}$$

$\Rightarrow$ for $x \to 2^+$, $\frac{x^2-x-2}{2} \to 0^+$

$$\Rightarrow \lim_{x\to 2^+} \frac{\sqrt{2}\sin\frac{(x^2-x-2)}{2}}{\left(\frac{x^2-x-2}{2}\right)} \cdot \frac{\frac{x^2-x-2}{2}}{(x-2)}$$

$$\Rightarrow \lim_{x\to 2^+} \frac{1}{\sqrt{2}} \cdot \frac{(x-2)(x+1)}{(x-2)} = \frac{3}{\sqrt{2}}$$

75. (1)

For ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, $a = 4$, $b = 3$,

$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

A and B are foci then $PA + PB = 2a$

$= 2(4) = 8$

76. (4)

$f(x)$ is differentiable then will also continuous then

$f(\pi) = -1$, $f(\pi^+) = -k_2$

$k_2 = 1$

Now $f'(x) = \begin{cases} 2k_1(x-\pi) & ; x \le \pi \\ -k_2 \sin x & ; x > \pi \end{cases}$

then $f'(\pi^-) = f'(\pi^+) = 0$

$f''(x) = \begin{cases} 2k_1 & ; x \le \pi \\ -k_2 \cos x & ; x > \pi \end{cases}$ then $2k_1 = k_2$

$k_1 = \frac{1}{2}$

77. **(4)**

$C_3 \to C_2 - C_1$

$$f(\theta) = \begin{vmatrix} -\sin^2\theta & -1 & 1 \\ -\cos^2\theta & -1 & 1 \\ 12 & -2 & -2 \end{vmatrix} = 4(\cos^2\theta - \sin^2\theta)$$

$= 4(\cos 2\theta),\ \theta \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

$f(\theta)\text{max} = M = 0$

$f(\theta)\text{min} = m = -4$

78. **(1)**

$$\bar{x} = \frac{2+4+10+12+14+x+y}{7} = 8$$

$\Rightarrow 42 + x + y = 56$

$\Rightarrow x + y = 14$

$$\sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$$

$$16 = \frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2$$

$$\Rightarrow 16 + 64 = \frac{460 + x^2 + y^2}{7}$$

$\Rightarrow 560 = 460 + x^2 + y^2$

$\Rightarrow x^2 + y^2 = 100$ ….(2)

$\Rightarrow xy = 48$

$(x-y)^2 = (x+y)^2 - 4xy = 4|$

$|x - y| = 2$

79. **(3)**

Equation $\frac{x^2}{5} + \frac{y^2}{4} = 1$ then $P\left(\sqrt{5}\cos\theta, 2\sin\theta\right)$

$(PQ)^2 = 5\cos^2\theta + 4(\sin\theta + 2)^2$

$= \cos^2\theta + 16\sin\theta + 20$

$= -\sin^2\theta + 16\sin\theta + 21$

$= 85 - (\sin\theta - 8)^2$

$= (PQ)^2_{\max} = 85 - 49 = 36$,

$\because (\sin\theta - 8)^2 \in [49, 81]$

80. **(3)**

Volume of parallelepiped $v = \left|\left[\vec{a}\ \vec{b}\ \vec{c}\right]\right|$

$$v = \begin{vmatrix} 1 & 1 & n \\ 2 & 4 & -n \\ 1 & n & 3 \end{vmatrix} = \pm 158$$

$1(12 + n^2) - 1(6 + n) + n(2n - 4) = \pm 158$

$3n^2 - 5n - 152 = 0$ or $3n^2 - 5n + 164 = 0$

D < 0 (no real roots)

$n = 8, -\frac{19}{3} \Rightarrow n = 8$

then, $\vec{b}\cdot\vec{c} = 2 + 4n - 3n = 10$

$\vec{a}\cdot\vec{c} = 1 + n + 3n = 33$

81. **(13)**

$T_{r+1} = {}^{22}C_r (x^m)^{22-r}\, x^{-2r}$

$T_{r+1} = {}^{22}C_r\, x^{m(22-r)-2r}$

$22m - mr - 2r = 1$

$22m - 1 = r(m + 2)$

$$r = \frac{22m - 1}{m + 2}$$

$$r = \frac{22m + 44 - 45}{m + 2}$$

$$r = 22 - \frac{3\cdot 3\cdot 5}{m+2}$$

So possible value of m = 1, 3, 7, 13, 43

But ${}^{20}C_r = 1540$

Only possible condition is $m = 13$

82. **(8)**

$-5 < \frac{x}{2} < 5$

$\Rightarrow \left[\frac{x}{2}\right] = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4$

Hence, function is discontinues at = –4, –3, –2, –1, 1, 2, 3, 4 number of values is 8.

83. **(30)**

$2x - y + 3 = 0$ ….(i)

$4x - 2y + \alpha = 0 \Rightarrow 2x - y + \frac{\alpha}{2} = 0$ …(ii)

$6x - 3y + \beta \Rightarrow 2x - y + \frac{\beta}{2} = 0$ …(iii)

$$d_1 = \frac{\left|\frac{\alpha}{2} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{1}{\sqrt{5}} \Rightarrow |\alpha - 6| = 2$$

$\Rightarrow \alpha - 6 = 2, -2 \Rightarrow \alpha = 8, 4$

$$d_2 = \frac{\left|\frac{\beta}{3} - 3\right|}{\sqrt{2^2 + 1^2}} = \frac{2}{\sqrt{5}} \Rightarrow |\beta - 9| = 6$$

$\Rightarrow \beta - 9 = 6, -6 \Rightarrow \beta = 15, 3$

Sum of all value of α and β = 30.

84. **(11)**

P(at least 2 show 3 or 5)

$$= {}^4C_2 \cdot \left(\frac{2}{6}\right)^2\left(\frac{4}{6}\right)^2 + {}^4C_3\left(\frac{2}{6}\right)^3\left(\frac{4}{6}\right) + {}^4C_4\left(\frac{2}{6}\right)^4$$

$$= \frac{384 + 128 + 16}{6^4} = \frac{11}{27}$$

$n = 27$

$\therefore$ expectation of number of times = np

$$= 27.\frac{11}{27} = 11$$

85. **(240)**

Syllabus

S–2, L–2, A, B, Y, U

Required = ${}^2C_1.{}^5C_2.\frac{4!}{2!} = 2 \cdot 10 \cdot \frac{24}{2} = 240$

86. **(3)**

$x_k \cdot y_k = (\sec\theta)^{2^{\frac{1}{k-1}}} - (\tan\theta)^{2^{\frac{1}{k-1}}}$

$\Rightarrow x_k \cdot y_k = y_{k-1}$

Now, $y_n \prod_{k=0}^{n} x_k = y_n \cdot \prod_{k=0}^{n} \frac{y_{k-1}}{y_k}$

$= y_n \times \frac{y_{-1}}{y_0} \times \frac{y_0}{y_1} \times \ldots\ldots \times \frac{y_{n-1}}{y_n} = y_{-1}$

$= (\sec\theta)^2 - (\tan\theta)^2 = 1$

87. **(3)**

Given, $f(x) = \left(\pi\sqrt{2} + \cos^{-1}\alpha\right)x^2 + 2(\cos^{-1}\beta)x + \pi\sqrt{2} - \cos^{-1}\alpha$

Clearly, graph of $f(x)$ is parabola opening upward.

As range of $f(x)$ is $[0, \infty)$ so discriminant = 0

$\Rightarrow b^2 - 4ac = 0$

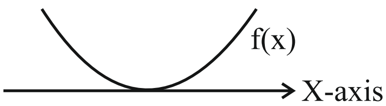
$\Rightarrow 4\left(\cos^{-1}\beta\right)^2 - 4\left(\pi\sqrt{2} + \cos^{-1}\alpha\right)(\pi\sqrt{2} - \cos^{-1}\alpha) = 0$

$\Rightarrow 4\left(\cos^{-1}\beta\right)^2 - 4\left(2\pi^2 - \left(\cos^{-1}\alpha\right)^2\right) = 0$

$\Rightarrow \left(\cos^{-1}\alpha\right)^2 + \left(\cos^{-1}\beta\right)^2 = 2\pi^2$

$\Rightarrow \cos^{-1}\alpha = \cos^{-1}\beta = \pi$

$\Rightarrow \alpha = \beta = -1$



$\therefore |\alpha - \beta| + 2\alpha\beta + 1 = 0 + 2 + 1 = 3$

88. **(3)**

$\frac{\tan 46^\circ + \tan 14^\circ}{1 - \tan 46^\circ \tan 14^\circ} = \tan(46^\circ + 14^\circ) = \sqrt{3}$ ……..(i)

$\frac{\tan 74^\circ - \tan 14^\circ}{1 + \tan 74^\circ \tan 14^\circ} = \tan(74^\circ - 14^\circ) = \sqrt{3}$ ……..(ii)

$\frac{\tan 74^\circ + \tan 46^\circ}{1 - \tan 74^\circ \tan 46^\circ} = \tan(74^\circ + 46^\circ) = -\sqrt{3}$ …..(iii)

From Equ. (i), (ii) and (iii)

$\tan 46^\circ \tan 14^\circ = 1 - \frac{\tan 46^\circ + \tan 14^\circ}{\sqrt{3}}$

$\tan 74^\circ \tan 14^\circ = \frac{\tan 74^\circ - \tan 14^\circ}{\sqrt{3}} - 1$

$\tan 74^\circ \tan 46^\circ = \frac{\tan 74^\circ + \tan 46^\circ}{-\sqrt{3}}$

$\therefore$ $\tan 46^\circ \tan 14^\circ - \tan 74^\circ \tan 14^\circ + \tan 74^\circ \tan 46^\circ = 3$

89. **(1)**

Given $r_1 + r_2 + r = r_2 \Rightarrow r_1 + r_3 = r_2 - r$

$\Rightarrow \frac{\Delta}{s-a} + \frac{\Delta}{s-c} = \frac{4}{s-b} - \frac{\Delta}{s}$

$\Rightarrow \frac{s-c+s-a}{(s-a)(s-c)} = \frac{s-s+b}{s(s-b)}$

$\Rightarrow (s-a)(s-c) = s(s-b)$

$\Rightarrow s^2 - s(a+c) + ac = s^2 - sb$

$\Rightarrow s(a+c-b) - ac = 0$

$\Rightarrow (a+c)^2 - b^2 - 2ac = 0$

$\Rightarrow a^2 + c^2 + 2ac - b^2 = 2ac = 0$

$\Rightarrow \angle B = \frac{\pi}{2}$ and $\angle A + \angle C = 90^\circ$

$\therefore \sec^2 A + \cos^2 B - \cot^2 C$

$= \sec^2\left(90^\circ - C\right) + \cos^2 90 - \cot^2 C$

$= \operatorname{cosec}^2 C + 0 - \cot^2 C = 1$

90. **(5)**

We have, $z_1\left(z_1^2 - 3z_2^2\right) = 2$ ……(i)

And $z_2\left(3z_1^2 - z_2^2\right) = 11$ …..(ii)

Multiplying Eq. (ii) by $i\left(-\sqrt{1}\right)$ and then adding in Eq. (i), we get

$z_1^3 - 3z_1z_2^2 + i\left(3z_1^2z_2 - z_2^3\right) = 2 + 11i$

$\Rightarrow (z_1 + iz_2)^3 = 2 + 11i$ ……(iii)

Again, multiplying Eq. (ii) by ($-i$) and then adding in Eq. (i) we get,

$z_1^3 - 3z_1z_2^2 - i\left(3z_1^2z_2 - z_2^3\right) = 2 - 11i$

$\Rightarrow (z_1 - iz_2)^3 = 2 - 11i$ …..(iv)

Now, on multiplying Eq. (iii) and (iv), we get

$\left(z_1^2 + z_2^2\right)^3 = 4 + 121 = 125 = 5^3$

$\therefore$ $z_1^2 + z_2^2 = 5$