JEE Mains (12^{th})

Sample Paper - III

DURATION : 180 Minutes **M. MARKS : 300**

General Instructions:

1. Immediately fill in the particulars on this page of the test booklet.
2. The test is of **3 hours** duration.
3. The test booklet consists of **90** questions **(75 to attempt)**. The maximum marks are **300**.
4. There are three subjects in the question paper, Subject I, II and III consisting of Section-I (**Physics),** Section-II **(Chemistry)**, Section-III **(Mathematics)**, and having **30 questions** in each part.
5. There will be a total of **20 MCQs** and **10 Numerical** Value Based Questions **(attempt any 5)**.
6. Each correct answer will give 4 marks while 1 Marks will be deducted for a wrong response.
7. No student is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. inside the examination room/hall.
8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However, the candidates are allowed to take away this Test Booklet with them.
9. **Do not fold or make any stray mark on the Answer Sheet (OMR).**

Name of the Student (In CAPITALS): ________________________________

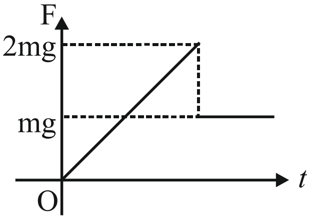
Roll Number: ________________________________

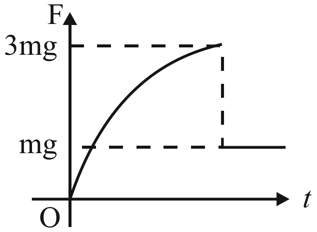
Candidate's Signature: ________________________________

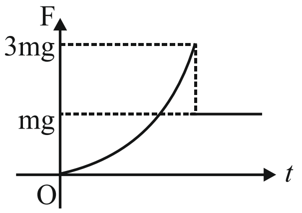
Section-I (PHYSICS)

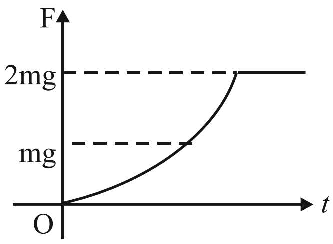
[Section – A]

1. An uniform chain of mass m and length l hangs by a thread and touches the surface of a table by its lower end. The thread is cut at time $t = 0$. Which of the following graph best represents the relation between force F exerted by table on the chain with time t. (Assume the fallen part immediately comes to rest after collision with table and do not form heap):

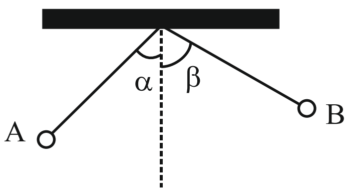
(1) 

(2) 

(3) 

(4) 

2. Two identical simple pendulums A and B have same point of suspension, having length l each. They are displaced by an angle α and β (α and β are very small and β > α) and released from rest. Find the time after which B reaches at its initial position for the first time. Assume collision to be elastic and both pendulums move in same plane



(1) $\pi\sqrt{\frac{\ell}{g}}$ (2) $2\pi\sqrt{\frac{\ell}{g}}$

(3) $\frac{\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$ (4) $\frac{2\pi\beta}{\alpha}\sqrt{\frac{\ell}{g}}$

3. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38° C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(1) 14.0 cm

(2) 15.2 cm

(3) 16.4 cm

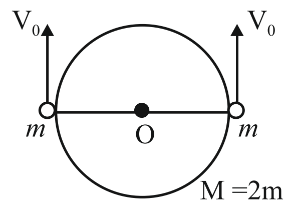
(4) 17.6 cm

4. A pendulum bob of mass m is suspended by a massless string and at rest when string is vertical. A constant horizontal force F = mg starts acting on it. The value of maximum tension in string is T = mg $\left[x\sqrt{2}-2\right]$, then the value of x is:

(1) 1 (2) 2

(3) 3 (4) 0

5. An isolated smooth ring of mass M = 2m with two small beads each of mass m is as shown in the figure. Initially both the beads are at diametrically opposite points and have velocity v_0 (for each) in same direction. The speed of the beads just before they collide for the first time is (complete system is placed on a smooth horizontal surface and assume each point of ring is touching the surface)



(1) v_0 (2) $\frac{2}{\sqrt{3}}v_0$

(3) $\frac{v_0}{2}$ (4) $\frac{\sqrt{3}}{2}v_0$

6. Two large vertical and parallel non conducting plates, have equal & opposite charge density, are at separation of 1 cm and plates are at potential difference of x volt. A proton is released at rest midway between the two plates. It is found to move, at 45° to the vertical. JUST after release. Then x is nearly

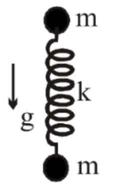
(1) 1×10^{-5} V

(2) 1×10^{-7} V

(3) 1×10^{-9} V

(4) 1×10^{-10} V

7. A mass m is hung on an ideal massless spring. Another equal mass is connected to the other end of the spring. The whole system is at rest. At $t = 0$, m is released and the system falls freely under gravity. Assume that natural length of the spring is L_0, its initial stretched length is L and the acceleration due to gravity is g. What is distance between masses as function of time



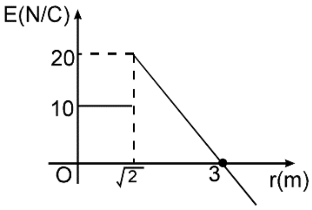
(1) $L_0 + (L - L_0)\cos\sqrt{\frac{2k}{m}}t$

(2) $L_0 + (L - L_0)\cos\sqrt{\frac{k}{m}}t$

(3) $L_0 - 2(L + L_0)\cos\sqrt{\frac{2k}{m}}t$

(4) $L_0 + (L - L_0)\sin\sqrt{\frac{2k}{m}}t$

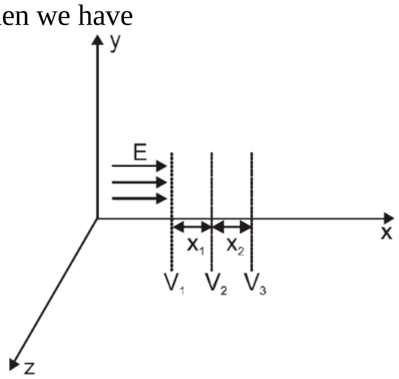
8. An electric field 'E' whose direction is radially outward varies as distance from origin 'r' as shown in the graph. E is taken as positive if its direction is away from the origin. Then the work done by electric field on a 2 C charge if it is taken from (1, 1, 0) to (3, 0, 0) is:



(1) $20(3-\sqrt{2})$ J (2) –60 J

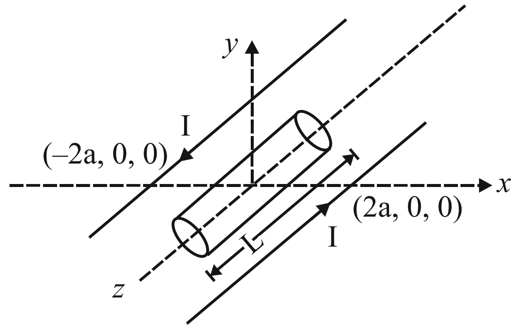
(3) 60 J (4) $20(\sqrt{2}-3)$ J

9. In an electric field shown in figure three equipotential surface are shown. If function of electric field is $E = 2x^2$ V/m, and given that $V_1 - V_2 = V_2 - V_3$ then we have



(1) $x_1 = x_2$
(2) $x_1 > x_2$
(3) $x_2 > x_1$
(4) Data insufficient

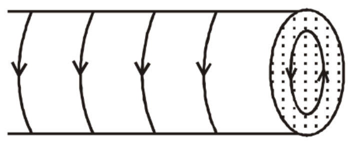
10. Two very long wires parallel to the Z-axis (in xz plane) and a distance '4a' (along x-axis) apart carry equal currents I in opposite directions as shown in the figure. A cylinder of radius a and length L has its axis on the Z-axis midway between the wires, calculate the net upward magnetic flux through half of the curved cylindrical surface above the



(1) $\frac{\mu_0 IL}{\pi}$ (2) $\frac{\mu_0 IL\ \ell n3}{\pi}$

(3) $\frac{\mu_0 IL\ \ell n2}{\pi}$ (4) $\frac{\mu_0 IL}{2}$

11. Magnetic field is uniform and has a magnitude B in the interior of a very long solenoid far from its ends. One of the ends of the solenoid is closed with a thin flat plastic cover. A single small electrical loop of radius R lies on the cover so that its center is on the axis of the solenoid. The electrical current flowing in the loop in I. Then the mechanical tension in the loop's wire is



(1) $\frac{\text{BIR}}{2}$ (2) BIR

(3) 2πBIR (4) πBIR

12. The space has electromagnetic field which varies with time whose variation is given as:

$$\vec{B} = \begin{cases} -B_0\hat{k} & \text{if} \quad 0 \le t \le \frac{\pi m}{qB_0} \\ -B_0\hat{j} & \text{if} \quad \frac{\pi m}{qB_0} \le t \le \frac{2\pi m}{qB_0} \\ 0 & \text{if} \quad \frac{2\pi m}{qB_0} \le t \le \infty \end{cases}$$

and $$\vec{E} = \begin{cases} 0 & \text{if} \quad 0 \le t \le \frac{2\pi m}{qB_0} \\ -E_0\hat{k} & \text{if} \quad \frac{2\pi m}{qB_0} \le t \le \infty \end{cases}$$

A charge particle having mass m and positive charge q is given velocity $v_0\,\hat{i}$ at origin at $t = 0$ sec. The coordinate of point on xy plane when it again passes through xy plane for the first time is:

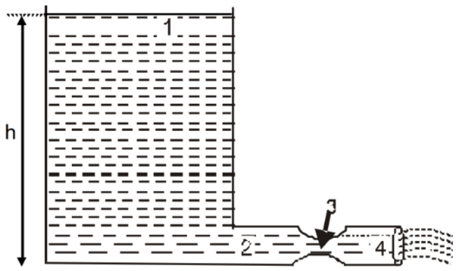
(1) $\left(\frac{2mv_0}{q}\sqrt{\frac{v_0}{E_0B_0}}, \frac{2mv_0}{qB_0}, 0\right)$

(2) $\left(\frac{2mv_0}{q}\sqrt{\frac{v_0}{E_0B_0}}, \frac{mv_0}{qB_0}, 0\right)$

(3) $\left(\frac{mv_0}{q}\sqrt{\frac{v_0}{E_0B_0}}, \frac{2mv_0}{qB_0}, 0\right)$

(4) $\left(\frac{mv_0}{q}\sqrt{\frac{v_0}{E_0B_0}}, \frac{mv_0}{qB_0}, 0\right)$

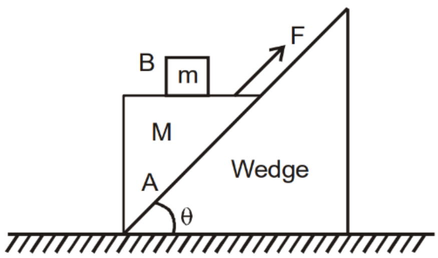
13. A large tank is filled with water (density ρ), upto height h. Water is coming out from section (4). Fluid pressure at section (3) at the instant shown is: (Given: Area $A_2 = A_4 = 2A_3$ and A_2, A_3, A_4 are very small as compared to A_1; atmospheric pressure = P_0). Assume water to be non-viscous and incompressible.



(1) $P_0 + \rho gh$ (2) $P_0 - \rho gh$

(3) $P_0 - 2\rho gh$ (4) $P_0 - 3\rho gh$

14. Wedge is fixed on horizontal surface. Triangular block A of mass M is pulled upward by applying a constant force F parallel to incline of the wedge as shown in the figure and there is no friction between the wedge and the block A, while coefficient of friction between A and block B of mass m is μ. If there is no relative motion between A and B then frictional force developed between A and B is



(1) $\left[\frac{F+(m-M)g\sin\theta}{(m+M)}\right] m\cos\theta$

(2) μmg

(3) $\left[\frac{F-(m+M)g\sin\theta}{(m+M)}\right] m\cos\theta$

(4) μmg/2

15. A thin prism of glass is placed in air and water respectively. If $n_g = \frac{3}{2}$ and $n_w = \frac{4}{3}$, then the ratio of deviation produced by the prism for a small angle of incidence when placed in air and water separately is:

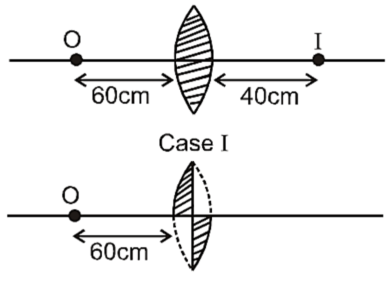
(1) 9 : 8

(2) 4 : 3

(3) 3 : 4

(4) 4 : 1

16. A converging equiconvex thin lens forms real image of a particle as shown in case I. If now lens is cut as shown in case II then select the correct alternative/alternatives:



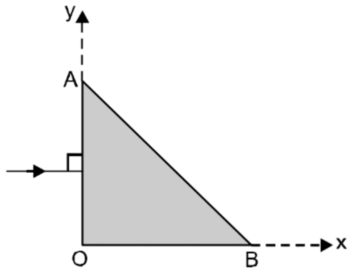
(1) Image in case II will be at 240 cm from lens.

(2) Images is erect

(3) Image in case II will be at the same location of case I.

(4) There will be two distinguished image

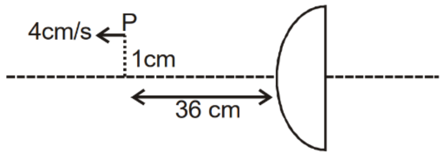
17. A triangular medium has varying refracting index $n = n_0 + ax$, where x is the distance (in cm) along x-axis from origin and $n_0 = \frac{4}{3}$. A ray is incident normally on face OA at the mid-point of OA. The range of a so that light does not escape through face AB when it falls first time on the face AB (OA = 4 cm, OB = 3 cm and AB = 5 cm): (Surrounding medium is air



(1) $a > \frac{1}{9}$ (2) $a > \frac{2}{9}$

(3) $a > \frac{1}{3}$ (4) None of these

18. A very thin plano convex lens of refractive index μ = 3/2 and radius of curvature 15 cm kept fixed as shown:



A point object P starts moving with constant speed 4 cm/s parallel to optical axis from the shown position at t = 0. The average speed of image of object from t = 0 to t = 6 sec is

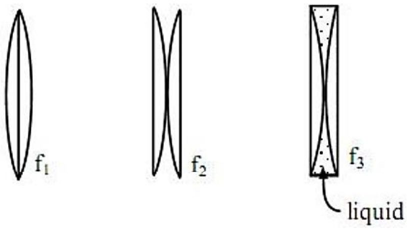
(1) 20 cm/s

(2) greater than 20 cm/s

(3) less than 20 cm/s

(4) can't say

19. The following figure shows different arrangements of two identical pieces of thin plano-convex lenses. The refractive index of the liquid used is less than that of the glass. The effective focal lengths in the three cases are related as:



(1) $f_1 = f_2 > f_3$ (2) $f_1 \neq f_2 \neq f_3$

(3) $f_1 = f_2 < f_3$ (4) $f_1 = f_2 = f_3$

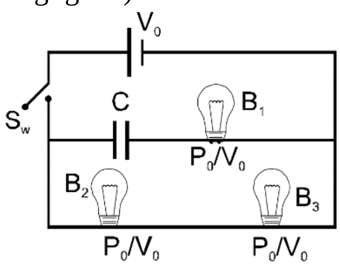
20. A uniform electric field exists in xy plane. The potential of points A (2, 2), B(–2, 2) and C(2, 4) are 4V, 16V, 12V, respectively. The electric field is

(1) $(4\hat{i} + 5\hat{j})$ V/m (2) $(3\hat{i} + 4\hat{j})$ V/m

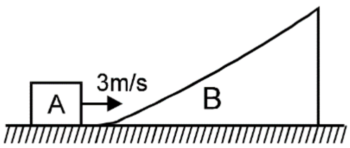
(3) $-(3\hat{i} + 4\hat{j})$ V/m (4) $(3\hat{i} - 4\hat{j})$ V/m

[Section – B]

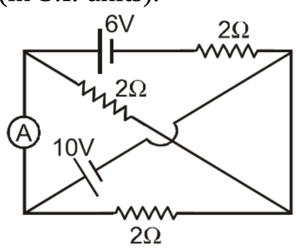
21. A uncharged capacitor is connected in circuit as shown in figure. Power ratings of bulbs are given in diagram. At $t = 0$ switch is closed then calculate ratio of power consumption in the circuit $t = 0$ and $t = \infty$. (Internal resistance of battery and connecting wires is negligible)



22. In the figure shown *A* is of mass 1 kg and B of mass 2 kg. A moves with velocity 3 m/s and rises on *B* which is initially at rest. All the surfaces are smooth. By the time *A* reaches the highest point on *B*, find the work done by *A* on *B* in joule

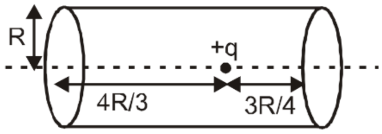


23. An ideal ammeter is connected in a circuit as shown in circuit diagram. What will be the reading of ammeter (in S.I. units).



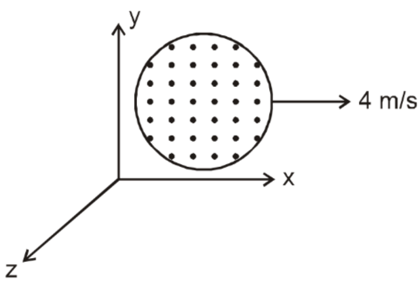
24. A positive charge $+q_1$ is located to the left of a negative charge $-q_2$. On a line passing through the two charges, there are two places where the total potential is zero. The reference is assumed to be at infinity. The first place is between the charges and is 4.00 cm to the left of the negative charge. The second place is 7.00 cm to the right of the negative charge. If $q_2 = -12/11$ μC, what is the value of charge q_1 in μC

25. A point charge $+q$ is placed on the axis of a closed cylinder of radius R and heigh $\frac{25R}{12}$ as shown. If electric flux coming out from the curved surface of cylinder $\frac{xq}{10\in_0}$, then calculate x.

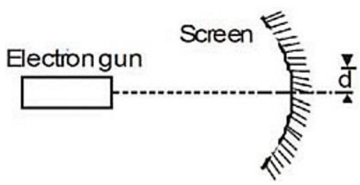


26. A thin conducting wire in the shape of a 'figure of eight' is situated with its circular loops in two planes making an angle of 120° with each other if the current in the loop is I and the radius is R, the magnetic induction at a point of intersection P of the two axes of the loops $\frac{N\mu_0 i}{48R}$. Find N.

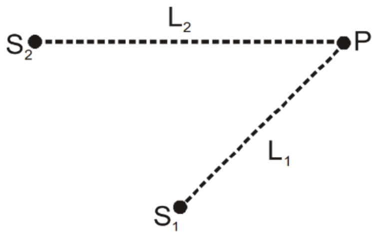
27. A uniform magnetic field $\vec{B} = 0.25\hat{k}T$ exists in a circular region of radius R = 5 m. A loop of radius R = 5m lying in x-y plane encloses the magnetic field at t = 0 and then pulled at uniform velocity $\vec{v} = 4\hat{i}$ m/s. Find the emf induced (in volts) is the loop at t = 2 sec.



28. Estimate the approximate maximum deflection d of the electron beam (in mm) near the centre of a TV screen due to the Earth's magnetic field of 5.0×10^{-5} T. Assume that the screen is 20 cm from the electron gun which produces electrons of speed $v = 3.0 \times 10^7$ ms^{-1} as shown in figure. You may assume that d is so small that the magnetic force always acts in the same direction



29. Two sound sources shown in the figure vibrate in phase. By moving S_1 along PS_1 consecutive minima are heard when $L_1 - L_2$ has values, 20 cm, 60 cm and 100 cm. If the frequency of sound source is $\frac{1700}{n}$ Hz. Then find the value of n: [Speed of sound is 340 m/s]



30. A point source of power 50π watts is producing sound waves. The velocity of sound is 330 m/s, density of air is 1.0 kgm^{-3}. Then (using $\pi = 22/7$) at $r = \sqrt{330}$ m from the point source then the pressure amplitude is x Nm^{-2}. Find the value of x.

Section-II (CHEMISTRY)

[Section - A]

31. Consider a 20 watt light source that emits monochromatic light of wavelength 600 nm. The number of photons ejected per second in form of Avogadro's constant N_A is approximately:

(1) N_A (2) 10^{-2} N_A
(3) 10^{-4} N_A (4) 10^{-6} N_A

32. The charge on the electron and proton are reduced to $\frac{2}{3}$rd of their original values. Let the present value of Rydberg constant of H-atom be R. What will be the new value of Rydberg constant?

(1) $\frac{2R}{3}$ (2) $\frac{4R}{9}$
(3) $\frac{8R}{27}$ (4) $\frac{16R}{81}$

33. Equal mass of N_2 and Ar have their total translational kinetic energy in the ratio 5:1. What would be the ratio of temperature of N_2 to that of Ar?

(1) 10:7 (2) 7:2
(3) 7:4 (4) 9:4

34. A reversible isothermal evaporation 90 g of water is carries out at 100^0C. Heat of evaporation of water is 9.72 kcal/mol. Assuming water vapour to behave like an ideal gas. What is the change in internal energy of the system?

(1) 48.6 kcal (2) 52.33 kcal
(3) 44.87 kcal (4) 56.06 kcal

35. The pK_b of cyanide ion is 4.7 the pH of solution prepared by mixing 2.5 moles of KCN and 2.5 moles of HCN in water and making total volume upto 500 ml is:

(1) 9.30 (2) 7.30
(3) 10.30 (4) 8.30

36. If $E^o_{XO_3^-/XO_4^-} = -0.36V$ and $E^o_{XO_3^-/XO_2^-} = -0.342V$ at 27^0C, the equilibrium constant is:

$$2XO_3^{-1} \rightleftharpoons XO_2^{\Theta} + XO_4^-$$

$$\left[\frac{2.303RT}{F} = 0.06 \text{ and } \log 5 = 0.7\right]$$

(1) logK= -23.4 (2) logK= -2.2
(3) logK= -20.2 (4) logK= -10.6

37. The correct expression for the packing fraction of sodium chloride if the ions on one face diagonal are removed is (r_+ = radius of Na^+ & r_- = radius of Cl^-)

(1) $\dfrac{\frac{4}{3}\pi r_-^3 + \frac{13}{3}\pi r_+^3}{8(r_+ + r_-)^3}$

(2) $\dfrac{\frac{16}{3}\pi r_-^3 + \frac{13}{3}\pi r_+^3}{8(r_+ + r_-)^3}$

(3) $\dfrac{\frac{13}{3}\pi r_-^3 + \frac{4}{3}\pi r_+^3}{8(r_+ + r_-)^3}$

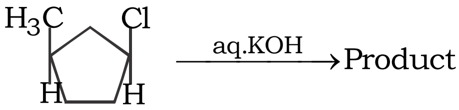
(4) $\dfrac{\frac{13}{3}\pi r_-^3 + \frac{16}{3}\pi r_+^3}{8(r_+ + r_-)^3}$

38. The ionization energy of hydrogen atom (in the ground state) is x kJ. The energy required for an electron to jump from 2^{nd} orbit to the 3^{rd} orbit will be

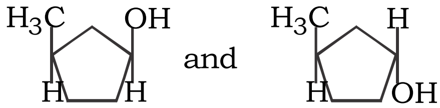
(1) x/6 (2) 5x
(3) 7.2x (4) 5x/36

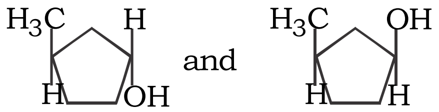
39. How many aromatic isomers are possible for compound with molecular formula C_8H_9Cl which do not contain Cl atom in aromatic nucleus?

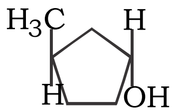
(1) 5 (2) 7
(3) 4 (4) 3

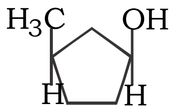
40. 

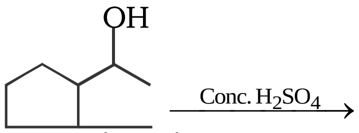
Major and minor product obtained respectively are:

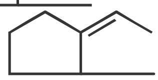
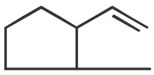
(1) 

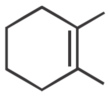
(2) 

(3)  (only)

(4)  (only)

41.  major product is:

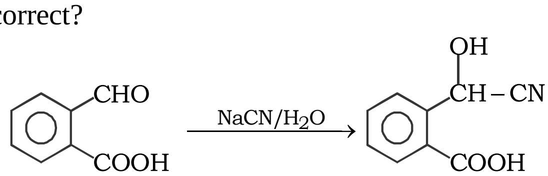
(1)  (2) 

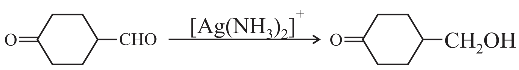
(3)  (4) 

42. 0.37 g of ROH is added to CH_3MgI and the gas evolved measured 112 cc at STP. On dehydration ROH gives an alkene which on ozonolysis give acetone and product (C). ROH on oxidation easily gives an acid (D) containing the same number of carbon atoms. Select incorrect statement:

(1) Molecular weight of ROH = 74
(2) $(H_3C)_2CH-CH_2OH$
(3) C = HCHO
(4) D = $CH_3CH_2CH_2COOH$

43. In which of the following, the given product is incorrect?

(1) 

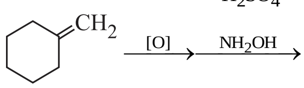
(2) 

(3) $C_6H_5CHO + CH_3CH_2CHO \xrightarrow{dil\ OH^-} C_6H_5-CH=C(CH_3)-CHO$

(4) $C_6H_5CHO + \text{Semicarbazine} \rightarrow C_6H_5-CH=N-NHCONH_2$

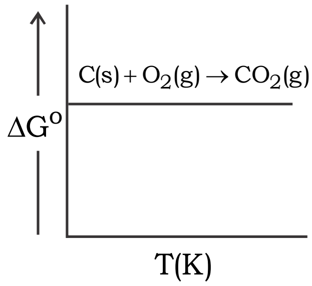
44. Identify the incorrect products in the following reaction sequences:

(1) $(CH_3)_2CO + HCN \xrightarrow[H_2SO_4]{CH_3OH} \xrightarrow{Peroxide}$ PMMA

(2)  $\xrightarrow[(2)\ Polymerisation]{(1) H_2SO_4}$ Nylon – 6

(3) $CH \equiv CH + HCN \xrightarrow{Ba^{2+}} \xrightarrow{Polymerisation}$ Perlon

(4) $2RCl + Si \xrightarrow{Cu} \xrightarrow{H_2O} \xrightarrow{Polymerisation}$ Silicone

45. 

Ellinghum diagram for the formation of CO_2 is straight line in the given graph. This is due to:

(1) Increase in entropy during CO_2 formation
(2) Decrease in entropy during CO_2 formation
(3) Entropy remains constant during CO_2 formation
(4) Cannot be predicted

46. 0.001 moles of $[Co(NH_3)_5(NO_3)(SO_4)$ was passed through a cation exchanger and the acid coming out of it required 20 ml of 0.1 M NaOH for neutralization. Hence, the complex is:

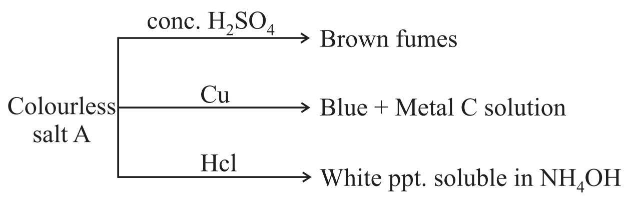
(1) $[Co(NH_3)_5SO_4]NO_3$
(2) $[Co(NH_3)_5NO_3]SO_4$
(3) $[Co(NH_3)_5]NO_3SO_4$
(4) None of these

47. The salt used for performing bead test in qualitative inorganic analysis is:

(1) $K_2SO_4.Al_2(SO_4)_3.24H_2O$
(2) $FeSO_4(NH_4)_2SO_4.6H_2O$
(3) $Na(NH_4)HPO_4.4H_2O$
(4) $CaSO_4.2H_2O$

48. An aqueous solution of a substance gives a white precipitate on treatment with dil. HCl. Which dissolved on heating, On passing H_2S in hot acidic solution a black precipitate is formed. The substance is

(1) Hg_2^{2+} salt (2) Cu^{2+} salt
(3) Ag^+ salt (4) Pb^{2+} salt

49. 

The salt A can be:

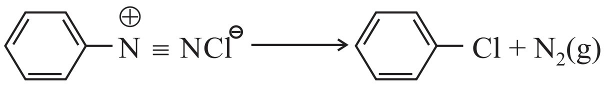
(1) $Cu(NO_3)_2$ (2) $NaNO_3$
(3) $AgNO_3$ (4) $Pb(NO_3)_2$

50. If gold number of a, b, c and d are 0.005, 0.05, 0.5 and 5 respectively, then which the following will have the highest protective power:

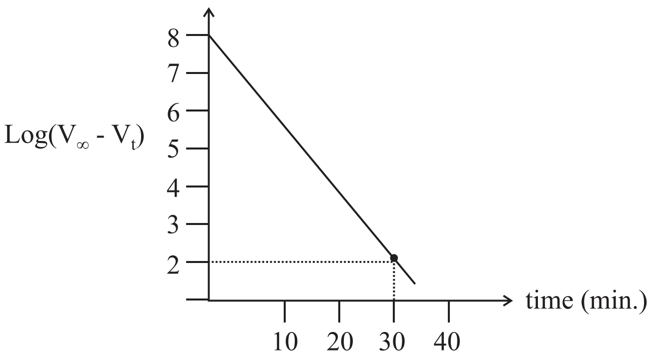
(1) a
(2) b
(3) c
(4) d

[Section - B]

51. For the first order reaction



following observation is made :



where V_t (in cc) is volume of N_2 collected at time $t \,\&\, V_\infty$ (in cc) is volume of N_2 collected after a long time. What is the time taken in minute for 75% reaction?

52. Change in entropy in expansion is given by:

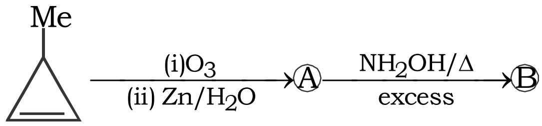
(I) $\Delta s\left(\dfrac{ds}{dv}\right)_u$ (II) $dv + \left(\dfrac{ds}{du}\right)_v du$

At constant temperature then in how many cases entropy change is decide exactly by only factor l at 298 K?

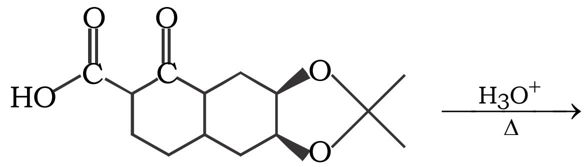
(1) He	(2) HCl
(3) CH_4	(4) SO_4
(5) NO_2	(6) H_2
(7) Xe	(8) O_2
(9) N_2	

53. 5 L of 1 M dibasic acid is shaken with 0.5 g charcoal. The final concentration of the solution after adsorption is 0.5 M. what is the amount of dibasic acid adsorbed per gram of charcoal?

54. Total number of optically active stereoisomers of B are:



55. Total number of oxygen atom present in the final product bearing maximum carbon atoms of the given reaction is:



56. $CH_3-\underset{\underset{O}{\|}}{C}-CH_2-CH_2-CH_2-CH_2-\overset{\overset{O}{\|}}{C}-H \xrightarrow[\text{dil.}]{\text{NaOH}}$

The number 'C' atom in the ring of major compound formed in above reaction are?

57. What is the net negative charge on adenosine triphosphate (ATP)?

58. How many d-electrons are present in $[Co(NH_3)_5CO_3]ClO_4$?

59. A given silicate is amphibole $Na_2\overset{ii}{Fe}x\,\overset{iii}{Fe}y[Si_8O_{22}](OH)_2$. Then value of $x + y$ is:

60. $CrCl_3 + \underset{(6\,\text{molal})}{NH_3} \longrightarrow \underset{(1)}{\text{Complex}}$

$CoCl_3 + \underset{(6\,\text{mole})}{NH_3} \longrightarrow \underset{(2)}{\text{Complex}}$

Ratio of CFSE in complex 2 to the CFSE in complex 1 (consider Δq in both cases same)

Section-III (MATHEMATICS)

[Section – A]

61. Let the centroid of an equilateral triangle *ABC* be at the origin. Let one of the sides of the equilateral triangle be along the straight line $x + y = 3$. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then $(R + r)$is equal to :

(1) $7\sqrt{2}$	(2) $2\sqrt{2}$
(3) $\dfrac{9}{\sqrt{2}}$	(4) $3\sqrt{2}$

62. A pole stands vertically inside a triangular park *ABC*. Let the angel of elevation of the top of the pole from each corner of the park be $\dfrac{\pi}{3}$. If the radius of the circumcircle of ΔABC is 2, then the height of the pole is equal to :

(1) $\dfrac{1}{\sqrt{3}}$	(2) $\sqrt{3}$
(3) $2\sqrt{3}$	(4) $\dfrac{2\sqrt{3}}{3}$

63. Define a relation R over a class of $n \times n$ real matrices A and B as "ARB iff there exists a non singular matrix P such that $PAP^{-1} = B$". Then which of the following is true?

(1) R is reflexive, symmetric but not transitive
(2) R is symmetric, transitive but no reflexive
(3) R is reflexive, transitive but not symmetric
(4) R is an equivalence relation

64. Consider a hyperbola $H: x^2 - 2y^2 = 4$. Let the tangent at a point $P(4, \sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1), x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to:

(1) $4\sqrt{6} - 1$
(2) $4\sqrt{6}$
(3) $\sqrt{6} - 1$
(4) $\frac{7}{\sqrt{6}} - 2$

65. Let $f: R - \{3\} \to R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g: R \to R$ be given as $g(x) = 2x - 3$. Then the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to:

(1) 2 (2) 7
(3) 5 (4) 3

66. Let in a series of $2n$ observations, half of them are equal to a and remaining half are equal to $-a$. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 650 (2) 925
(3) 425 (4) 250

67. Let $\vec{a}$ and $\vec{b}$ are two non-zero vectors perpendicular to each other and $|\vec{a}| = |\vec{b}|$. If $|\vec{a} \times \vec{b}| = |\vec{a}|$, then the angle between the vectors $(\vec{a} + \vec{b} + (\vec{a} \times \vec{b}))$ and $\vec{a}$ is equal to:

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$
(3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

68. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\text{cos}ec^6\alpha$ is equal to:

(1) 500 (2) 350
(3) 400 (4) 250

69. If P and Q are two statements, then which of the following compound statement is a tautology?

(1) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow Q$
(2) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow (P \wedge Q)$
(3) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow P$
(4) $((P \Rightarrow Q) \wedge \sim Q) \Rightarrow \sim P$

70. Let $y = y(x)$ be the solution of the differential equatlion $\frac{dy}{dx} = (y+1)\left((y+1)e^{x^2/2} - x\right), 0 < x < 2.1,$ with $y(2) = 0$. Then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to:

(1) $\frac{-e^{3/2}}{\left(e^2+1\right)^2}$ (3) $-\frac{2e^2}{\left(1+e^2\right)^2}$
(3) $\frac{e^{5/2}}{\left(1+e^2\right)^2}$ (4) $\frac{5e^{1/2}}{\left(e^2+1\right)^2}$

71. Let S_1 be the sum of first $2n$ terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1,)$ is 1000, then the sum of the first $6n$ terms of the arithmetic progression is equal to:

(1) 5000
(2) 1000
(3) 7000
(4) 3000

72. Let $f: R \to R$ be function defined as

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin 2x}{2x}, & if\ x < 0 \\ b & ,if\ x = 0 \\ \frac{\sqrt{x+bx^3} - \sqrt{x}}{bx^{5/2}}, & if\ x > 0 \end{cases}$$

If f is continuous at $x = 0$, then the value of $a + b$ is equal to :

(1) -2 (2) -3
(3) $-\frac{5}{2}$ (4) $-\frac{3}{2}$

73. Let a tangent be drawn to the ellipse $\frac{x^2}{27} + y^2 = 1$ at $\left(3\sqrt{3}\cos\theta, \sin\theta\right)$ where $\theta \in \left(0, \frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to:

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$
(3) $\frac{\pi}{8}$ (4) $\frac{\pi}{3}$

74. Let the system of linear equation
$4x+\lambda y+2z=0$
$2x-y+z=0$
$\mu x+2y+3z=0, \lambda, \in R$
has a non-trivial solution. Then which of the following is true?
(1) $\mu=6, \lambda \in R$ (2) $\mu=-6, \lambda \in R$
(3) $\lambda=2, \mu \in R$ (4) $\lambda=3, \mu \in R$

75. Let $g(x)=\int_0^x f(t)dt$ where f is continuous function in [0, 3] such that $\frac{1}{3} \le f(t) \le 1$ for all $t \in [0,1]$ and $0 \le f(t) \le \frac{1}{2}$ for all $t \in (1,3]$. The largest possible interval in which $g(3)$ lies is:
(1) [1, 3] (2) $\left[\frac{1}{3},2\right]$
(3) $\left[-1,-\frac{1}{2}\right]$ (4) $\left[-\frac{3}{2},-1\right]$

76. In a triangle ABC, If $|\overrightarrow{BC}|=8, |\overrightarrow{CA}|=7, |\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to
(1) $\frac{85}{14}$ (2) $\frac{25}{4}$
(3) $\frac{127}{20}$ (4) $\frac{115}{16}$

77. Let a complex number be $\omega=1-\sqrt{3}i$. Let another complex number z be such that $|z\omega|=1$ and arg(z) – arg(w) = $\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and ω is equal to
(1) 4 (2) $\frac{1}{4}$
(3) $\frac{1}{2}$ (4) 2

78. The area bounded by the curve $4y^2=x^2(4-x)(x-2)$ is equal to:
(1) $\frac{\pi}{16}$ (2) $\frac{\pi}{8}$
(4) $\frac{3\pi}{2}$ (4) $\frac{3\pi}{8}$

79. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to:
(1) $\frac{40}{243}$ (2) $\frac{80}{243}$
(3) $\frac{32}{625}$ (4) $\frac{128}{625}$

80. Let $S_1: x^2+y^2=9$ and $S_2:(x-2)^2+y^2=1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points:
(1) $\left(\frac{1}{2}, \pm\frac{\sqrt{5}}{2}\right)$ (2) $\left(0, \pm\sqrt{3}\right)$
(3) $(1, \pm 2)$ (4) $\left(2, \pm\frac{3}{2}\right)$

[Section – B]

81. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1+x)^n$. If $\sum_{k=0}^{10}\left(2^2+3k\right) {}^nC_k=\alpha.3^{10}+\beta.2^{10}, \alpha, \beta \in R,$ then $\alpha+\beta$ is equal to….

82. Let I be an idetitiy matrix of order 2×2 and $P=\begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n=5I-8P$ is equal to……..

83. Let $f: R \to R$ satisfy the equation $f(x+y)=f(x).f(y)$ for all $x, y \in R$ and $f(x) \ne 0$ for any $x \in R$. If the function f is differentiable at x = 0 and $f'(0)=3$, then $\lim_{h\to 0}\frac{1}{h}\left(f(h)-1\right)$ is equal to

84. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i}-j+k\right)-b=0$ be (–3, 5, 2). Then, the value of $|a+b|$ is equal to

85. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point (1, –1, α) lies on the plane P the value of $|5\alpha|$ is equal to

86. Let $P(x)$ be a real polynomial of degree 3 which vanishes at x = – 3. Let $P(x)$ have local minima at x = 1, local maxima at $x=-1$ and $\int_{-1}^{1}P(x)dx=18$, then the sum of all the coefficients of the polynomial $P(x)$ is equal to..

87. The term independent of x expansion of $\left[\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right]^{10}, x \ne 1$ is equal to

88. If $\sum_{r=1}^{10} r!\left(r^3+6r^2+2r+5\right)=\alpha(11!)$, then the value of α is equal to….

89. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by x^2+x+1, then $P(1)$ is equal to

90. Let $y = y(x)$ be the solution of the differential equation $xdy - ydx = \sqrt{\left(x^2-y^2\right)}dx, x \geq 1.$ with $y(1)=0.$ If the area bounded by the line $x=1, x=e^{\pi}$, $y=0$ and $y=y(x)$ is $\alpha e^{2\pi}+\beta$, then the value of $10(\alpha+\beta)$ is equal to…