JEE Mains (12^{th})

Sample Paper - III

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

Q.	Ans.
1.	(3)
2.	(2)
3.	(2)
4.	(3)
5.	(4)
6.	(3)
7.	(1)
8.	(1)
9.	(2)
10.	(2)
11.	(1)
12.	(1)
13.	(4)
14.	(3)
15.	(4)
16.	(1)
17.	(2)
18.	(2)
19.	(3)
20.	(4)
21.	(3)
22.	(1)
23.	(7)
24.	(4)
25.	(7)
26.	(3)
27.	(6)
28.	(6)
29.	(2)
30.	(5)

CHEMISTRY

Q.	Ans.
31.	(3)
32.	(4)
33.	(2)
34.	(3)
35.	(1)
36.	(1)
37.	(4)
38.	(4)
39.	(1)
40.	(1)
41.	(3)
42.	(4)
43.	(2)
44.	(3)
45.	(3)
46.	(2)
47.	(3)
48.	(4)
49.	(3)
50.	(1)
51.	(3)
52.	(0)
53.	(5)
54.	(2)
55.	(3)
56.	(5)
57.	(4)
58.	(6)
59.	(5)
60.	(2)

MATHEMATICS

Q.	Ans.
61.	(3)
62.	(3)
63.	(4)
64.	(4)
65.	(3)
66.	(3)
67.	(3)
68.	(4)
69.	(4)
70.	(1)
71.	(4)
72.	(4)
73.	(1)
74.	(1)
75.	(2)
76.	(1)
77.	(3)
78.	(3)
79.	(3)
80.	(4)
81.	(19)
82.	(6)
83.	(3)
84.	(1)
85.	(38)
86.	(8)
87.	(210)
88.	(160)
89.	(0)
90.	(4)

PHYSICS

1. **(3)**

$F_{net} = F_{dynamic} + F_{static}$

At any time t when x length is on the table then :

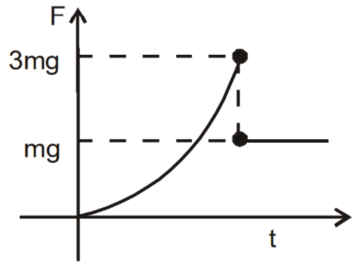
$F_{static} = x\lambda g$

$F_{dynamic} = v_{rel}\left(\frac{\lambda dx}{dt}\right) = \sqrt{2gx} \times \lambda\sqrt{2gx} = 2gx\lambda$

$F_{net} = 3\times\left(\frac{1}{2}gt^2\right)\lambda g$

$\Rightarrow F_{net} = 3\lambda g\ 1/2\ gt^2 = \frac{3}{2}\lambda g^2 t^2 = \frac{3}{2}\frac{mg^2t^2}{\ell}$

At t = $\sqrt{2\ell / g}$ all the chain will be on table so



$F_{net} = 0 + mg$ (since at this position $F_{dyn} = 0$

So the correct graph is represented by option (C

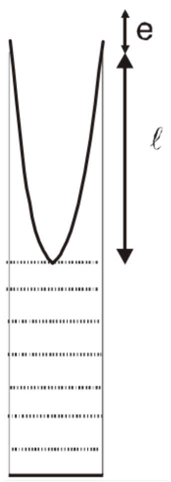
2. **(2)**

After first collision B reaches up to angular displacement α and after second collision 'B' reaches to it's initial position.

So, total time taken is

$t = \frac{T}{4} + \frac{T}{4} + \frac{T}{4} + \frac{T}{4} = T$

3. **(2)**



First resonance occurs at fundamental frequency

$f = \frac{V}{4(l+e)} \Rightarrow l + e = \frac{V}{3f}$

(where e = 0.6 × 2 = 1.2 cm)

$l + e = \frac{336}{4\times 512} = 0.164$ m

$l = 16.4 - 1.2 = 15.2$ cm

4. **(3)**

Tension in string is maximum when Torque on bob about 'O' is zero.

When bob is at A tension is maximum

$T - mg\sqrt{2} = \frac{mV^2}{\ell}$ … (i)

and $\frac{1}{2}mV^2 = mg\frac{\ell}{\sqrt{2}} - \left(1 - \frac{1}{\sqrt{2}}\right)$

$= \sqrt{2}mg\ell - mg\ell$ … (ii)

$T = mg\sqrt{2} + 2\sqrt{2}mg\ell - 2mg = mg\left[3\sqrt{2} - 2\right]$

$\therefore$ x = 3

5. **(4)**

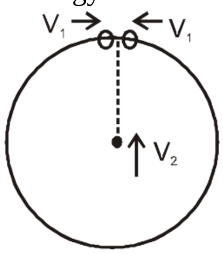
Just before the collision, situation is as shown Let the beads have velocity V_1 w.r.t. the ring and the ring has velocity V_2.

Then, by momentum

$2mv_0 = (M + 2m)V_2$

$\Rightarrow V_2 = \frac{V_0}{2}$ … (i)

By mechanical energy conservation



$\frac{1}{2}(2m)V_0^2 = \frac{1}{2}2mv^2 + \frac{1}{2}2m\left(\frac{V_0}{2}\right)^2$

where v = velocity of beads w.r.t. ground

so, $V = \frac{\sqrt{3}}{2}V_0$

6. **(3)**

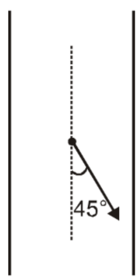
qE = mg

X = Ed

$= \left(\frac{mg}{q}\right)d$

$X = \frac{1.6\times10^{-27}\times10}{1.6\times10^{-19}}\times1\times10^{-2}$

$X = 10^{-9}$ volt



7. **(1)**

In CM frame both the masses execute SHM w

$\omega = \sqrt{\frac{k}{\mu}} = \sqrt{\frac{2k}{m}}$ SHM

Initially particles are at extreme

distance = $L_0 + (L - L_0)\cos\sqrt{\frac{2k}{m}}t$

8. **(1)**

Work done on 2 C charge = $\int q\vec{E}.d\vec{r} = q\int_1^3 E\,dr$

[$\because$ r for (1, 1, 0) = $\sqrt{2}$ & r for (3, 0, 0) = 3]

= 2 × area of E-r graph from r = $\sqrt{2}$ m to r = 3

= $2\times\left[\frac{1}{2}(3-\sqrt{2})20\right] = 20(3-\sqrt{3})J$

9. **(2)**

Rate of change of potential is increasing

$\therefore v_1 - v_2/x < v_2 - v_3/x_2$

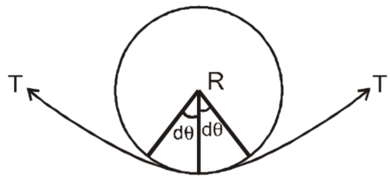
$\therefore x_1 > x_2$

10. **(2)**

$$\int_a^{3a}\frac{\mu_0 IL}{2\pi}\left[\frac{dx}{x}+\frac{dx}{4a-x}\right] = \frac{\mu_0 IL\,\ell n3}{\pi}$$

11. **(1)**

B at end = $\frac{1}{2}$ B at interior = $\frac{1}{2}$B



$$IdL\left(\frac{B}{2}\right) = 2T\sin d\theta$$

dL = R(2dθ)

IR.2dθ $\frac{B}{2}$ = 2T dθ

T = $\frac{BIR}{2}$

12. **(1)**

$$\frac{2mv_0}{qB_0} = \frac{1}{2}\left(\frac{qE_0}{m}\right)t^2$$

$$\frac{4m^2v_0}{q^2E_0B_0} = t^2$$

x = v_0 × t ⇒ $v_0\ \frac{2m}{q}\sqrt{\frac{v_0}{E_0B_0}}$

13. **(4)**

Velocity of efflux at section (4) is v = $\sqrt{2gh}$

Applying Bernoulli's equation between section (3) and (4)

$$P_3 + \frac{1}{2}\rho v_3^2 = P_4 + \frac{1}{2}\rho v_4^2$$

$$\Rightarrow P_3 + \frac{1}{2}\rho\left(2\sqrt{2gh}\right)^2 = P_0 + \frac{1}{2}\rho\left(\sqrt{2gh}\right)^2$$

⇒ $P_3 = P_0 - 3\sigma gh$

14. **(3)**

$$a = \left[\frac{F-(m+M)g\sin\theta}{(m+M)}\right]$$

So, f = ma cos θ

$$= \left[\frac{F-(m+M)g\sin\theta}{(m+M)}\right]m\cos\theta$$

15. **(4)**

$$\delta_a = \left(\frac{3}{2}-1\right)\times A = \frac{A}{2}$$

$$\delta_w = \left(\frac{3/2}{4/3}-1\right)A = \frac{A}{8}$$

$$\frac{\delta_{air}}{\delta_{water}} = \frac{4}{1}$$

16. **(1)**

$$\frac{1}{40}-\frac{1}{-60} = \frac{1}{f} \quad \ldots (1)$$

$$\frac{1}{v}-\frac{1}{-60} = \frac{1}{2f} \quad \ldots (2)$$

$$\frac{1}{v}+\frac{1}{60} = \frac{1}{2}\left(\frac{1}{40}+\frac{1}{60}\right)$$

$$\Rightarrow \frac{1}{v}+\frac{1}{60} = \frac{1}{2}\ \frac{100}{40\times 60}$$

$$\Rightarrow -\frac{1}{v} = \frac{1}{48} - \frac{1}{60}$$

⇒ v = 240 cm

17. **(2)**

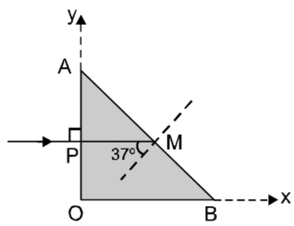
Clearly, PM = $\frac{3}{2}$ cm

$$37° > \sin^{-1}\frac{1}{n_0 + a(3/2)}$$

$$\frac{5}{3} > \frac{1}{n_0 + \frac{3a}{2}}$$

$$3n_0 + \frac{9a}{2} > 5$$

$$\frac{9a}{2} > 1 \quad ; a > \frac{2}{9}$$



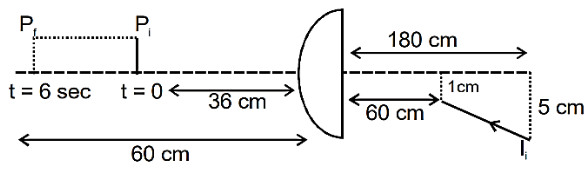
18. **(2)**

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{15}\right)$$

f = 30 cm

$$\frac{1}{v}+\frac{1}{36} = \frac{1}{30} \qquad \frac{1}{v} = \frac{1}{30}-\frac{1}{36} = \frac{1}{180}$$

v = 180 cm



19. **(3)**

Let the focal length of each piece be

Then

$$\frac{1}{f_1}=\frac{1}{f}+\frac{1}{f}$$

$$\frac{1}{f_2}=\frac{1}{f}+\frac{1}{f}$$

$\Rightarrow f_1 = f_2$

For the third arrangement the liquid forms a concave lens which has a diverging effect. So $f_3 > f_1 = f_2$

20. **(4)**

$$\vec{E}=E_x\hat{i}+E_y\hat{j},\quad \Delta V=-E_x\,\Delta x-E_y\,\Delta y$$

for A and B

$16 - 4 = -E_x(-2-2) - E_y(2-2)$

$E_x = 3$ V/m

For B and C

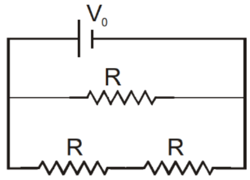
$12 - 16 = -E_x\{2-(-2)\} - E_y(4-2)$

$E_y = -4$ V/m

$\therefore \vec{E}=(3\hat{i}-4\hat{j})$ V/m

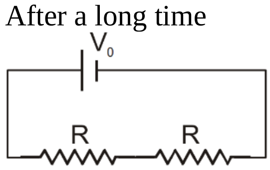
21. **(3)**

At $t = 0$



$$P_{consume}=\frac{V_0^2}{2R/3}=\frac{3}{2}P_0$$

After a long time



$$P_{consume}=\frac{V_0^2}{2R}=\frac{P_0}{2}$$

Since current in B_1 decreases with time so its brightness decreases

Initially brightness of B_2 is less than B_1 but later on B_2 will be brighter.

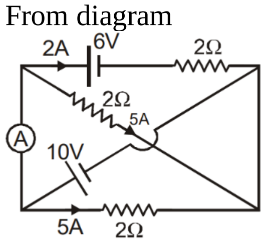
22. **(1)**

$$W_{NA}+W_{NG}+W_G=\frac{1}{2}m_BV^2-0$$

$$W_{NA}+0+0=\frac{1}{2}(2)(1)^2-0=1$$

23. **(7)**

From diagram



Current through ammeter

= (2 + 5) = 7A

24. **(4)**

$$\frac{q_1}{x-4}-\frac{q^2}{4}0 \qquad \frac{q_1}{x+7}-\frac{q_2}{7}=0$$

$$\frac{q_1}{q_2}=\frac{x-4}{4} \qquad \frac{q_1}{q_2}=\frac{x+7}{7}$$

$$\frac{x-4}{4}=\frac{x+7}{7}$$

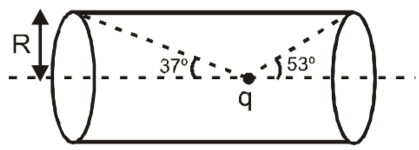
$7x - 28 = 4x + 28$

$3x = 56$

$$x=\frac{56}{3}\ \Rightarrow \frac{q_1}{q_2}=\frac{\frac{56}{3}+7}{7}=\frac{11}{3}$$

$$|q_2|=+\frac{12}{11}\mu c \qquad \Rightarrow q_1=\frac{12}{11}\times\frac{11}{3}=4\mu c$$

25. **(7)**

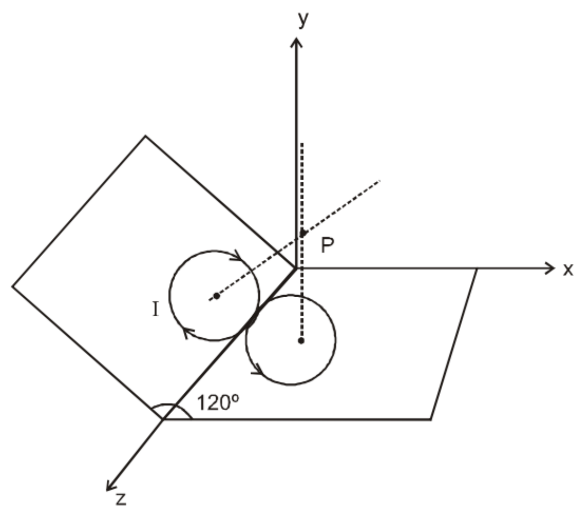


$$\phi=\frac{q}{\epsilon_0}\left(\frac{2\pi(1-\cos 37°)}{4\pi}\right)-\frac{q}{\epsilon_0}\left(\frac{2\pi(1-\cos 53°)}{4\pi}\right)$$

$$\phi=\frac{q}{\epsilon_0}\left[1-\frac{1}{2}\left(1-\frac{4}{5}\right)-\frac{1}{2}\left(1-\frac{3}{5}\right)\right]$$

$$\frac{q}{\epsilon_0}\left[1-\frac{1}{10}-\frac{2}{20}\right]=\frac{7q}{10\epsilon_0}$$

26. **(3)**



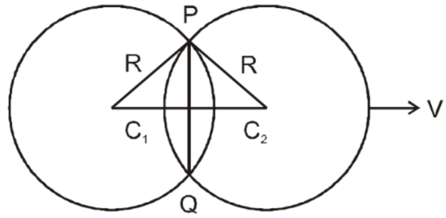
$$B_1=\frac{\mu_0 I}{2}\frac{R^2}{(R^2+3R^2)^{3/2}}=\frac{\mu_0 I}{16R}$$

$$B^2=B_1^2+B_1^2+2B_1B_1\cos 120$$

$$\Rightarrow B=B_1=\frac{\mu_0 I}{16R}=\frac{3\mu_0 I}{48R}$$

27. **(6)**

$\varepsilon = \left|(\vec{V} \times \vec{B}).\vec{\ell}\right|$



$\varepsilon = VB(PQ)$

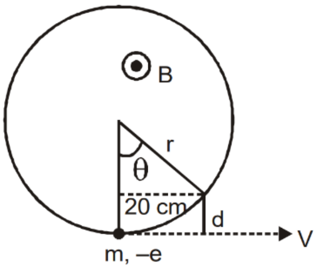
$= VB\ 2\sqrt{R^2 - \left(\frac{vt}{2}\right)^2} = VB\sqrt{4R^2 - V^2t^2}$

$= 4 \times 0.25\sqrt{4 \times 25 - 16 \times 4}$

= 6 volt $\quad (V_Q > V_P)$

28. **(6)**

Method I



$r = \frac{mV}{eB} \cong 3.4m$

$r\sin\theta = 20cm$

$\theta \cong \frac{1}{17}$ radian

$r - r\cos\theta = d$

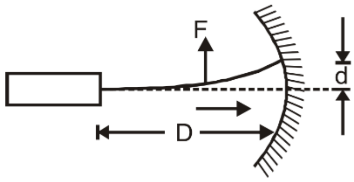
$d \cong \frac{r\theta^2}{2} \cong 6mm$

Method II

The maximum magnetic force is perpendicular to the velocity and has a magnitude

F = e *v* B

This force produces an acceleration, a, perpendicular to the line from the electron gun to the screen. The force remains in this direction since d is assumed to be small.



Thus, a = $\frac{F}{m} = \frac{e\nu B}{m}$ and d = $\frac{1}{2}at^2$

Where t = $\frac{D}{\nu}$ is the time it takes for the electrons to reach the screen.

Hence, d = $\frac{1}{2}\frac{e\nu B}{m}\left(\frac{D}{\nu}\right)^2$ or d = $\frac{eBD^2}{2m\nu}$

Substituting the numerical values:

d = $\frac{1.6 \times 10^{-19} \times 5 \times 10^{-5} \times (0.2)^2}{2 \times 9.1 \times 10^{-31} \times 3 \times 10^7}$

$\therefore$ d = 5.8 × 10^{-3} m

i.e., d = 6 m

29. **(2)**

The minima will be heard at P when a crest from S_1 and a trough from S_2 reach there at the same time. This will happen if $L_1 - L_2$ is λ/2 or λ + (λ/2) or 2λ + (λ/2) and so on. Hence, the increase in L1 between consecutive minima is λ1 and from the data we see that λ = 0.40 m. Then from λ = v/f ⇒ f = 340/0.40 = 850 Hz.

30. **(5)**

$\frac{P_0^2}{2\rho V} = I = \frac{Power}{4\pi r^2}$

Put the values to get the answer

CHEMISTRY

31. **(3)**

$E = n.h.\frac{c}{\lambda}$

$n = \frac{E.\lambda.}{h.c} = \frac{20 \times 600 \times 10^{-9}}{\frac{20}{3} \times 10^{-34} \times 3 \times 10^8} = 6 \times 10^{19}$

$= \frac{6 \times 10^{19}}{N_A} N_A = \frac{6 \times 10^{19}}{6 \times 10^{23}} \times N_A = 10^{-4} N_A$

32. **(4)**

$R_H = \frac{m_e.e^4}{8\varepsilon_0^2 h^3.c} \qquad \frac{R_H}{R_x} = \frac{1}{\left(\frac{2}{3}\right)^4}$

$R_X = \frac{m_e.\left(\frac{2}{3}e\right)^4}{8\varepsilon_0^2 h^3 c} \quad R_x = R_{4x}\left(\frac{2}{3}\right)^4 = \frac{16R_H}{81}$

33. **(2)**

Let mass be w:

$(K.E)_{N_2} = \frac{w}{28} \cdot \frac{3}{2} RT_{N_2}$

$(K.E)_{A_r} = \frac{w}{40} \cdot \frac{3}{2} RT_{A_r}$

$\frac{5}{1} = \frac{(K.E)_{N_2}}{(K.E.)_{A_r}} = \frac{40}{28} \times \frac{T_{N_2}}{T_{Ar}}$

$\frac{5}{1} = \frac{10}{7} \times \frac{T_{N_2}}{T_{A_r}} = \frac{T_{N_2}}{T_{A_r}} = \frac{7}{2}$

34. **(3)**

$-w_{rev} = -nRT = 5 \times 2 \times 373 cal = -3.73\ Kcal$

$\Delta E = q + w \qquad q = 5 \times 9.72 = 48.6$

$\Delta E = 48.6 + (-3.73) = 44.87\ Kcal / mole$

35. **(1)**

$pK_{b(CN^-)} = 4.7$, $pK_a + pK_b = pK_w$

$pK_a = 14 - 4.7 = 9.3$

For acid buffer $pH = pK_a + \log\frac{2.5}{2.5}$

$pH = pK_a = 9.3$

36. **(1)**

$$\overset{+5}{XO_3^-} \longrightarrow \overset{+7}{XO_4^-} \quad \Delta G^o = -2F(-0.36) = 0.72F$$

$$\overset{+5}{XO_3^-} \longrightarrow \overset{+3}{XO_2^-} \quad \Delta G^o - 2F(-0.342) = 0.684F$$

$$2XO_3^- \rightleftharpoons 2XO_4^- + XO_2^- \quad \Delta G^o = 1.404F$$

$\Delta G^o = -2.303RT\log K$

$1.404F = -2.303RT\log k$

37. **(4)**

$a = 2(r_+ + r_-) \Rightarrow V_{unit\ cell} = 8(r_+ + r_-)^3$

Number of $Na^+ = 4$ $V_{Na^+} = 4\times\frac{4}{3}\pi(r_+)^3$

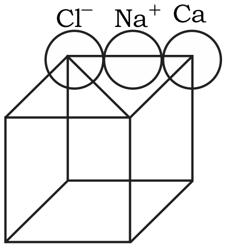
Number of $Cl^- = 6\times\frac{1}{8} + 5\times\frac{1}{2} = \frac{13}{4}$

$V_{Cl^-} = \frac{13}{4}\times\frac{4}{3}\pi(r_-)^3$

Total volume occupied $= \frac{13}{4}\times\frac{4}{3}\pi(r_-)^3 + \frac{16}{3}\pi(r_+)^3$

$= \frac{13}{3}\pi(r_-)^3 + \frac{16}{3}\pi(r_+)^3$

Packing fraction $= \dfrac{\frac{13}{3}\pi(r_-)^3 + \frac{16}{3}\pi(r_+)^3}{8(r_+ + r^-)^3}$



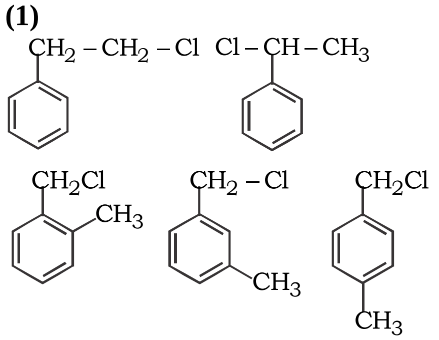
38. **(4)**

I.E for H-atom $= E_\infty - E_1 = E_0Z^2 = x$ kJ

Thus; $E_3 - E_2 = E_0.Z^2\left(\frac{1}{4} - \frac{1}{9}\right)$

$= \frac{E_0.Z^2\times 5}{36} = \frac{5x}{36}$ kJ

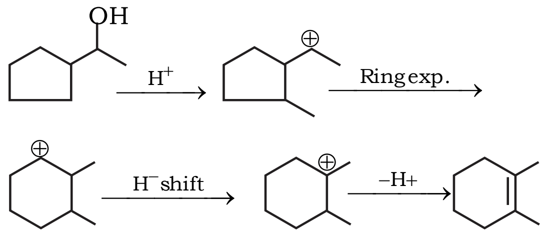
39. **(1)**



40 **(1)**

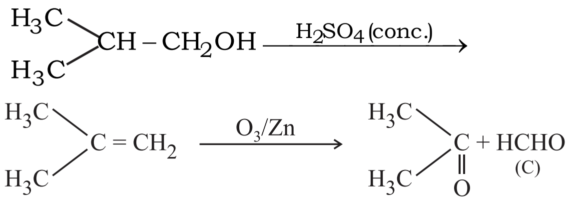
S_N1 process.

41. **(3)**

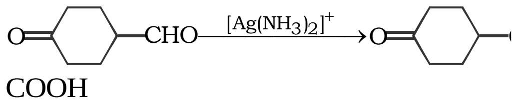


42. **(4)**

$\frac{0.37}{M} = \frac{112}{22400}$ $M = 74$



43. **(2)**



44. **(3)**

Theoretical

45. **(3)**

$C_{(g)} + O_2(g) \longrightarrow CO_2(g)\ \Delta S = 0$

46. **(2)**

$[Co(NH_3)_5NO_3]SO_4 \xrightarrow{\text{cation exchanges}} H_2SO_4$

Number of equivalent of H_2SO_4 = No. of equivalent of NaOH

$0.001\times 2 = 20\times 0.1\times 10^{-3}$

47. **(3)**

48. **(4)**

$Pb^{+2} + 2HCl \longrightarrow PbCl_2 \xrightarrow{H_2S} Pbs$

Consider IInd group in qualitative analysis.

49. **(3)**

Fact.

50. **(1)**

Protective power $\propto \frac{1}{\text{Gold number}}$ Gold number of A is least, therefore, it has the highest protective power.

51. **(3)**

$$Ph-N_2^{\oplus}Cl \longrightarrow Ph-Cl+N_2(g)$$

$V_\infty \ \alpha \ a_0$

$V_t \ \alpha \ x$

$$K=\frac{2.303}{t}\log\frac{V_\infty}{(V_\infty-V_t)}$$

From graph slope =

$$-\frac{K}{2.303}=-\frac{(2-8)}{(30-0)} \Rightarrow K=\frac{1}{5}\times 2.303$$

For 75% reaction; $x=a_0\times\frac{3}{4}$

$t=3\,mins$

52. **(0)**

At constant temp. change in internal energy is zero

53. **(5)**

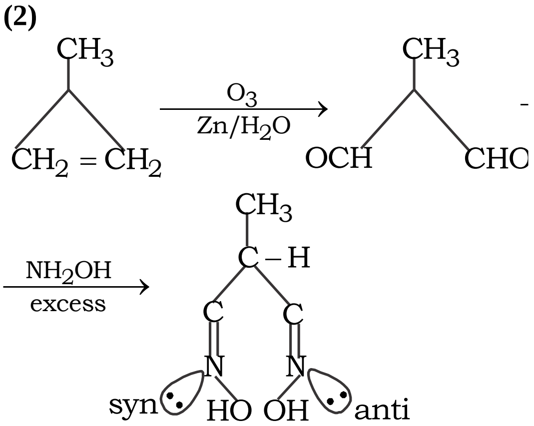
Initial mole $=5\times1=5$

Final mole $0.5\times5=2.5$

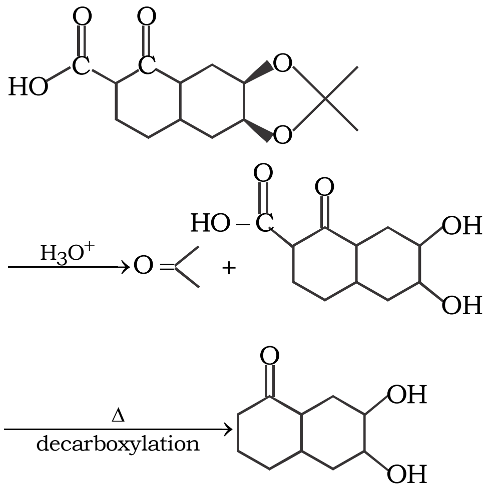
Moles adsorbed $5-2.5=2.5$

Mole adsorbed per gram of charcoal $=\frac{2.5}{0.5}=5$

54. **(2)**

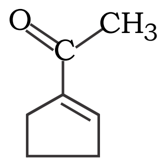


55. **(3)**



56. **(5)**

Intramolecular aldol of condensation



57. **(4)**

Fact.

58. **(6)**

Oxidation number of cobalt is +3.

59. **(5)**

$Na_2Fe_3^{II}Fe_2^{III}[S:_8\ O_{22}](OH)_2\ 3+2=5$

60. **(2)**

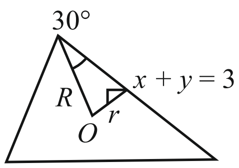
Complex I$\rightarrow$ $3d^3$ Hence ratio of CFSE=2

Complex II$\rightarrow 3d^6$

MATHEMATICS

61. **(3)**

$$r=\left|\frac{0+0-3}{\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$



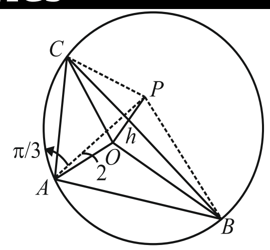
Again $\frac{r}{R}$ = sin30° = ½

$R = 2r$

$\therefore\ R + r = 3r = \frac{9}{\sqrt{2}}$

62. **(3)**

$\frac{h}{2}=\tan\frac{\pi}{3}=\sqrt{3} \Rightarrow h=2\sqrt{3}$



63. **(4)**

$PAP^{-1} = B$

P is non-singular

now $PAP^{-1} = A$ If $P = I$

$\therefore$ We can say ARA. $\Rightarrow$ Reflexive

$PAP^{-1} = B$

$\therefore\ P^{-1}PAP^{-1}P = P^{-1}BP$

$\Rightarrow A = P^{-1}BP$

Now PAP^{-1}

$\Rightarrow PP^{-1}BPP^{-1} = B$

$\therefore$ $ARB \Rightarrow BRA$

$\therefore$ R is symmetric

$ARB \Rightarrow PAP^{-1} = B$ ...(I)

$BRC \Rightarrow PBP^{-1} = C$ ...(II)

$\Rightarrow P(PAP^{-1})P^{-1} = C$

$\Rightarrow (P)^2 A(P^2)^{-1} = C$

$\Rightarrow ARC$.

$\therefore$ R is transitive.

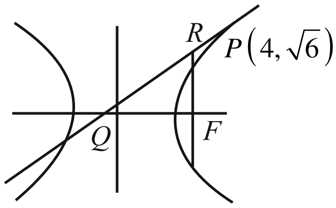
$\therefore$ R is an equivalence Relation.

64. **(4)**

$$\frac{x^2}{4} - \frac{y^2}{2} = 1$$

$$e = \sqrt{1 + \frac{2}{4}} = \frac{\sqrt{6}}{2}$$

$$F\left(\sqrt{6}, 0\right)$$



Equation of tangent at P is $x - \dfrac{\sqrt{6}y}{2} = 1$

$\therefore$ It cuts x axis at $Q(1, 0)$, latus rectum at F, $x = \sqrt{6}$ cuts the tangent at.

$$\Rightarrow \sqrt{6} - \frac{\sqrt{6}y}{2} = 1$$

$$y = \frac{2\left(\sqrt{6} - 1\right)}{\sqrt{6}}$$

Area of $\Delta QFR = \dfrac{1}{2}\left(\sqrt{6} - 1\right)\left(\dfrac{\sqrt{6} - 1}{\sqrt{6}}\right)$

$$= \frac{7 - 2\sqrt{6}}{\sqrt{6}} = \frac{7}{\sqrt{6}} - 2$$

65. **(3)**

$f(x) = \dfrac{x-2}{x-3}, x \neq 3$ $\quad$ $g(x) = 2x - 3$

$y = \dfrac{x-2}{x-3}$ $\quad$ $y = 2x - 3$

$$\Rightarrow x = \frac{3y-2}{y-1} = f^{-1}(y)$$

$$\Rightarrow x = \frac{y+3}{2} = g^{-1}(y)$$

$\therefore$ $f^{-1}(x) + g^{-1}(x) = \dfrac{13}{2}$

$$\Rightarrow \frac{3x-2}{x-1} + \frac{x+3}{2} = \frac{13}{2}$$

$\Rightarrow x^2 - 5x + 6 = 0 \Rightarrow x = 2, 3$

$\therefore$ Sum of possible values of x is 5.

66. **(3)**

Mean = $\bar{x} = \dfrac{na - na}{2n} = 0$

Variance = $\dfrac{1}{2n}\sum_{i=1}^{2n} x_i^2 - \bar{x}^2 = \dfrac{2na^2}{2n} = a^2$

If b added to all of value, then mean = 0 + b = b = 5

$(S.D.)^2 \Rightarrow a^2 = (2\ 0)^2$

$\therefore$ $a^2 + b^2 = 400 + 25 = 425$

67. **(3)**

$|\vec{a}| = |\vec{b}|$ $\quad$ $\vec{a}\cdot\vec{b} = 0$

$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}| = |\vec{a}|$

$\Rightarrow |\vec{b}| = 1 = |\vec{a}|$

Let $\vec{c} = \vec{a} + \vec{b} + \vec{a}\times\vec{b}$ -

$|\vec{c}|^2 = |\vec{a}+\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 + 2\left(\vec{a}+\vec{b}\right)\cdot(\vec{a}\times\vec{b})$

$= |\vec{a}|^2 + |\vec{b}|^2 + |\vec{a}|^2$

$= 2|a|^2 + |\vec{b}|^2 = 3$

$\therefore$ $\vec{c} = \sqrt{3}$.

Again $\vec{c}\cdot\vec{a} = |\vec{a}|^2 + \left(\vec{a}\times\vec{b}\right)\vec{a}$ -

$= 1 + 0 = 1$

$\therefore$ angle between $\vec{c}$ and $\vec{a}$

$\cos\theta = \dfrac{\vec{c}.\vec{a}}{|\vec{c}||\vec{a}|} = \dfrac{1}{\sqrt{3}}$

$\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$

68. **(4)**

$15 \sin^4\alpha + 10\cos^4\alpha = 6, \alpha \in R$

$\Rightarrow 15\tan^4\alpha + 10 = 6\sec^4\alpha$

$= 6(1 + \tan^2\alpha)^2$

$= 6(1 + 2\tan^2\alpha + \tan^4\alpha)$

$\Rightarrow 9\tan^4\alpha - 12\tan^2\alpha + 4 = 0$

$\Rightarrow (3\tan^2\alpha - 2)^2 = 0$

$\Rightarrow \tan^2\alpha = \dfrac{2}{3} \Rightarrow \sin^2\alpha = \dfrac{2}{5}, \cos^2\alpha = \dfrac{3}{5}$

$\therefore$ $27\sec^6\alpha + 8\cos^6\alpha = 27\left(\dfrac{5}{3}\right)^3 + 8\left(\dfrac{5}{2}\right)^3$

125 + 125 = 250

69. **(4)**

P	**Q**	**$P \Rightarrow Q$**	**~Q**	**$(P \Rightarrow Q) \wedge$ ~Q**	**~P**	**$((P \Rightarrow Q) \wedge$ ~Q$) \Rightarrow$ ~P**
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

70. **(1)**

$$\frac{dy}{dx}=(y+1)\left[(y+1)e^{x^2/2}-x\right]\ldots\ldots\text{(i)}$$

$$\Rightarrow \frac{1}{(y+1)^2}\frac{dy}{dx}+\frac{x}{(y+1)}=e^{x^2/2}$$

Let $\frac{1}{y+1}=t \Rightarrow -\frac{1}{(y+1)^2}\frac{dy}{dx}=\frac{dt}{dx}$

(I) $\Rightarrow \frac{dt}{dx}-xt=-e^{x^2/2}$

Solution

I.F. = $e^{-\int x\,dx}=e^{-\frac{x^2}{2}}$

$$\frac{e^{-x^2/2}}{y+1}=-x+c$$

$y(2) = 0 \Rightarrow e^{-2} = c - 2 \Rightarrow c = 2 + e^{-2}$

$$\frac{e^{-x^2/2}}{y+1}=2+e^{-2}-x$$

$x = 1 \Rightarrow y + 1 = \frac{e^{3/2}}{1+e^2}$

(I)

$$\therefore \frac{dy}{dx}=\frac{e^3}{(1+e^2)^2}\times e^{1/2}-\frac{e^{3/2}}{1+e^2}=-\frac{e^{3/2}}{(1+e^2)^2}$$

71. **(4)**

$S_1 = n[a + (2n - 1)d]$ ...(i)

$S_2 = 2n[a + (4n - 1)d]$ ...(ii)

$S_2 - S_1 = 2n[a + (4n - 1)d] - n[a + (2n - 1)d]$

$= n[a + (6n - 1)d] = 1000$

$\therefore S_{6n} = 3n[a + (6n - 1)d] = 3000$

72. **(4)**

LHL at $x = 0$

$$\lim_{x\to 0}\frac{2\sin\left\{\left(\frac{a+3}{2}\right)x\right\}\cos\left(\frac{a-1}{2}\right)x}{\left\{\left(\frac{a+3}{2}\right)x\right\}}\times\frac{\left(\frac{a+3}{2}\right)x}{2x}=b$$

$\Rightarrow \frac{a+3}{2}=b \Rightarrow a - 2b + 3 = 0$ ...(i)

RHL at $x = 0$

$$\lim_{x\to 0}\frac{\sqrt{x+bx^3}-\sqrt{x}}{bx^{5/2}}=b$$

$$\lim_{x\to 0}\frac{bx^3}{bx^{5/2}\left[\sqrt{x+bx^3}+\sqrt{x}\right]}=b$$

$$\lim_{x\to 0}\frac{1}{\sqrt{1+bx^2+1}}=b$$

$\Rightarrow b=\frac{1}{2}$

$\Rightarrow a = 2b - 3 = -2$

$a + b = -2 + \frac{1}{2} = -\frac{3}{2}$

73. **(1)**

Equation of tangent to $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta,\sin\theta\right)$

$\Rightarrow \frac{x}{3\sqrt{3}}\cos\theta+\frac{y}{1}\sin\theta=1 \quad \theta\in\left(0,\frac{\pi}{2}\right)$

$\therefore$ Sum of intercepts on coordinate axes

$z = 3\sqrt{3}\sec\theta + \text{cosec}\,\theta$

$\therefore \frac{dz}{d\theta} = 3\sqrt{3}\sec\theta\tan\theta - \text{cosec}\,\theta\cot\theta$

for z min; $\frac{dz}{d\theta}=0 \Rightarrow 3\sqrt{3}\sec\theta\tan\theta = \text{cosec}\,\theta\cot\theta$

$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\therefore \theta = \frac{\pi}{6}$

$\theta<\frac{\pi}{6},\frac{dz}{d\theta}>0$ and $\theta<\frac{\pi}{6}\Rightarrow\frac{dz}{d\theta}<0$

$\therefore z$ in minimum for $\theta = \frac{\pi}{6}$.

74. **(1)**

System of equation has not trivial solutions

$$\therefore \Delta = \begin{vmatrix}4 & \lambda & 2\\ 2 & -1 & 1\\ \mu & 2 & 3\end{vmatrix}=0$$

$R_1 - R_2$

$$\Rightarrow \begin{vmatrix}0 & \lambda+2 & 0\\ 2 & -1 & 1\\ \mu & 2 & 3\end{vmatrix}=0$$

$\Rightarrow (\lambda + 2)[\mu - 6] = 0$

$\therefore \mu = 6$ and $\lambda \in R$

75. **(2)**

$\frac{1}{3}\le f(t)\le 1 \qquad \forall t \in [0, 1]$

$0\, f(t) \le \frac{1}{2} \qquad \forall t \in (1, 3]$

$g(3) = \int_0^1 f(t)\,dt+\int_1^3 f(t)\,dt$ ...(1)

Now $\frac{1}{3}\le\int_0^1 f(t)\,dt\le 1$ ...(2)

and $0(3-1) \le \int_1^3 f(t)\le\frac{1}{2}(3-1)$

$\Rightarrow 0 \le \int_0^1 f(t)\,dt\le 1$ ...(3)

$\Rightarrow$ (2) + (3)

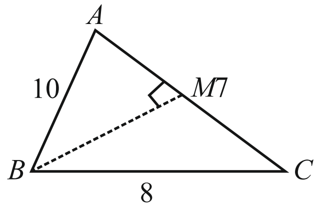
$\Rightarrow \frac{1}{3}\le\int_0^3 f(t)\,dt\le 2$

$\Rightarrow g(3) \in \left[1/3, 2\right]$

76. **(1)**

$|\overrightarrow{BC}| = 8, |\overrightarrow{CA}| = 7, |\overrightarrow{AB}| = 10$

$A = 8, b = 7, c = 10$



$\cos A = \dfrac{49 + 100 - 64}{2(7)(10)} = \dfrac{17}{28}$

Projection of AB along $AC = AB \cos A$

$= 10 \times \dfrac{17}{28} = \dfrac{85}{14}$

77. **(3)**

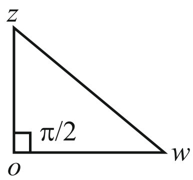
$\omega = 1 - \sqrt{3}i$, $\boldsymbol{z}$ is such that $|\boldsymbol{z}\omega| = 1$

$|\omega| = 2 \qquad \Rightarrow \quad |\boldsymbol{z}|\,|\omega| = 1$

$\Rightarrow |\boldsymbol{z}| = \dfrac{1}{2}$

$\therefore$ Area of triangle

$= \dfrac{1}{2} |\omega|\,|\boldsymbol{z}| = ½$



78. **(3)**

$4y^2 = x^2(4 - x)(x - 2) \ge 0$

x [2, 4]

$y = \pm\dfrac{1}{2} x\sqrt{(4-x)(x-2)}$

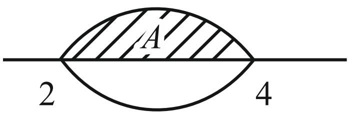
$A \quad = \dfrac{1}{2}\displaystyle\int_2^4 x\sqrt{(4-x)(x-2)}\,dx$

$= \dfrac{1}{2}\displaystyle\int_2^4 x\sqrt{6x - x^2 - 8}\,dx$

$= \dfrac{1}{2}\displaystyle\int_2^4 (x-3+3)\sqrt{1-(x-3)^2}\,dx$

$= \dfrac{1}{2}\displaystyle\int_2^4 (x-3)\sqrt{1-(x-3)^2}\,dx$

$+ \dfrac{3}{2}\displaystyle\int_2^4 \sqrt{1-(x-3)^2}\,dx$



put $x - 3 = \sin\theta$

$\Rightarrow dx = \cos\theta\, d\theta$

$A \quad = \dfrac{1}{2}\displaystyle\int_{-\pi/2}^{\pi/2} \sin\theta\cos^2\theta\, d\theta + \dfrac{3}{2}\int_{-\pi/2}^{\pi/2}\cos^2\theta\, d\theta$

$= \dfrac{3}{2}\displaystyle\int_0^{\pi/2}(1 + \cos 2\theta)\, d\theta$

$A \quad = 0 + \dfrac{3}{2}\left(\theta + \dfrac{\sin 2\theta}{2}\right)_0^{\frac{\pi}{2}} = \dfrac{3\pi}{4}$

$\therefore$ Required total area $= 2A = \dfrac{3\pi}{2}$.

79. **(3)**

p = probability of getting success in a single trial

q = probability of getting failure in a single trial

$p + q = 1$

given, ${}^5C_1\, p^1\, q^4 = 0.4096$ ...(i)

and ${}^5C_2\, p^2\, q^3 = 0.2048$ ...(ii)

$\dfrac{{}^5C_1 pq^4}{{}^5C_2 p^2 q^3} = \dfrac{0.4096}{0.2048} = 2$

$\Rightarrow \dfrac{q}{p} = 4 \quad \Rightarrow \quad p = \dfrac{1}{5} \quad q = \dfrac{4}{5}$

$\therefore\ p(x = 3) = 5c_3\, p^3\, q^2 = 10 \times \dfrac{1}{125} \times \dfrac{16}{25} = \dfrac{32}{625}$

80. **(4)**

Let centre of variable circle is at $C(h, k)$ and radius $= r$

$CC_1 = 3 - r$

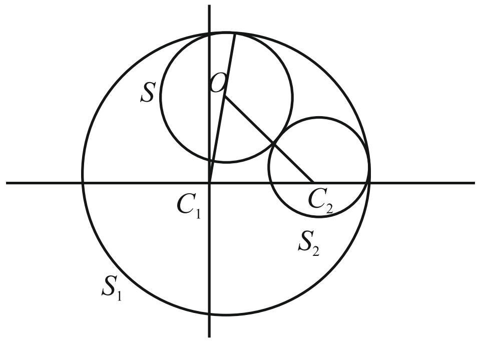
$CC_2 = 1 + r$

$CC_1 + CC_2 = 4$ constant.

$\therefore$ Locus of C is an ellipse.

$\sqrt{(h-2)^2 + k^2} = 4 - \sqrt{h^2 + k^2}$

or, point $\left(2, \pm\dfrac{3}{2}\right)$ satisfying the expression



81. **(19)**

$\displaystyle\sum_{k=0}^{10}(2^2 + 3k)\,{}^nC_k = 4\sum_{k=0}^{10}{}^{10}C_k + 3\sum_{k=0}^{10} k\,{}^{10}C_k$

$n = 10$ (assumption)

$= 4.2^{10} + 3\displaystyle\sum_{k=0}^{10} k \times \frac{10}{k}\,{}^9C_{k-1}$

$= 4.2^{10} + 30 \times 2^9$

$= 2^{10}(4 + 15) = 19 \times 2^{10} = \alpha \times 3^{10} + \beta \times 2^{10}$

Comparing $\alpha = 0 \qquad \beta = 19$

$\therefore\ \alpha + \beta = 19$

82. **(6)**

$P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$

Its characteristic equation

$\begin{bmatrix} 2-\lambda & -1 \\ 5 & -3-\lambda \end{bmatrix} = 0 \Rightarrow \lambda^2 + \lambda - 1 = 0$

$\therefore P^2 = I - P$ ...(1)

$P^3 = P - P^2 = P - (I - P) = 2P - I$

$P^4 = 2P^2 - P = 2(I - P) - P = 2I - 3P$.

$P^5 = 2P - 3P^2 = 2P - 3(I - P) = 5P - 3I$

$P^6 = 5P^2 - 3P = 5(I - P) - 3P = 5I - 8P$

$\therefore N = 6$

83. **(3)**

$f(x + y) = f(x) . f(y)$

$\Rightarrow f(x) = \lambda^x$.

$f'(x) = \lambda^x \ \ell n\lambda$

and $f'(0) = \ell n\lambda = 3$ and $\lambda = e^3$.

$\therefore f(x) = e^{3x}$.

$\therefore \lim_{x\to 0}\left(\frac{e^{3x}-1}{3x}\right)\times 3 = 3$

84. **(1)**

Plane $2x - y + z - b = 0$ ...(1)

$P(1, 3, a)$

$Q(-3, 5, 2)$

$\overrightarrow{PQ}$ is collinear with $2\hat{i} - \hat{j} + k$

and $\frac{-4}{2} = \frac{2}{-1} = \frac{2-a}{1} \Rightarrow a = 4$

Mid point of P and Q is $M(-1, 4, 3)$ lies on

(I) $\Rightarrow -2 -4 + 3 - b = 0$

$\Rightarrow b = -3$

85. **(38)**

Equation of plane is

$a(x - 1) + b(y + 6) + c(z + 5)\ 0$

$\therefore 4a - 3b + 7c = 0$ ...(1)

and $3a + 4b + 2c = 0$ ...(2)

$\Rightarrow \frac{a}{-34} = \frac{b}{13} = \frac{c}{25}$

$\therefore$ plane $-34(x - 1) + 13(y + 6) + 25(z + 5) = 0$

Now $(1, -1, \alpha)$ lies on it

$\therefore 65 + 25(\alpha + 5) = 0$

$25\alpha = -5(25 + 13)$

$5\alpha = -38$

$|5\alpha| = 38$

86. **(8)**

$P'(x) = a(x - 1)(x + 1) = a(x^2 - 1)$

$\therefore P(x) = a\left(\frac{x^3}{3} - x\right) + b$

$P(-3) = 0$

$\Rightarrow 0 = a(-9 + 3) + b$

$\Rightarrow b = 6a$

$\int_{-1}^{1} P(x)\,dx = 18$

$\Rightarrow \int_{-1}^{1}\left[a\left(\frac{x^3}{3} - x\right) + b\right]dx = 18$

$\Rightarrow 2b = 18 \Rightarrow b = 9$

$\therefore a = 3/2$

$\therefore P(x) = \frac{1}{2}x^3 - \frac{3}{2}x + 9$

$\therefore$ Sum of all coefficients

$= \frac{1}{2} - \frac{3}{2} + 9 = 8$

87. **(210)**

$\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{\sqrt{x}\left(\sqrt{x}-1\right)}\right]^{10}$

$\Rightarrow \left[\left(x^{1/3}+1\right) - \frac{\left(\sqrt{x}+1\right)}{\sqrt{x}}\right]^{10}$

$= \left[x^{1/3} + 1 - x^{-1/2}\right]^{10}$

general term $= {}^{10}C_r x^{\frac{10-r}{3}} \times x^{-\frac{r}{2}} = {}^{10}C_r x^{\frac{20-5r}{6}}$

For independent of $\frac{20-5r}{6} = 0 \Rightarrow r = 4$

$\therefore$ Coefficient $= {}^{10}C_4 = 210$

88. **(160)**

$\sum_{r=1}^{10} r!(r^3 + 6r^2 + 2r + 5)$

$\left\{(r+1)(r+2)(r+3) = r^3 + 6r^2 + 11r + 6\right\}$

$= \sum_{r=1}^{10} r!\left[(r+1)(r+2)(r+3) - 9r - 1\right]$

$= \sum_{r=1}^{10} (r+3)! - 9r!(r+1-1)r!$

$= \sum_{r=1}^{10} (r+3)! - 9(r+1)! + 9r! - r!$

$= \sum_{r=1}^{10}\left[(r+3)! - r!\right] - 9\sum_{r=1}^{10}\left[(r+1)! - r!\right]$

$= (11! + 12! + 13! - 1! - 2! - 3!) - 9(11! - 1!)$

$= 13! + 12! + 13! - 9! - 9!\ 11! + 9!$

$= 13! + 12! + 11! - 9(11!)$

$= 13! + 4(11!) = 11!(12 \times 13 + 4)$

$= 160(11!)$

89. **(0)**

$P(x) = f(x^3) + xg(x^3)$ ...(1)

$x^2 + x + 1 = (x - w)(x - w^2)$ is a factor of (1)

$\therefore P(w) = f(1) + wg(1) = 0$ ...(2)

$P(w^2) = f(1) + w^2 g(1) = 0$ ...(3)

$(2) \times w - 3 \Rightarrow$

$f(1)w - f(1) = 0$ and $f(1)(w - 1) = 0$

$\Rightarrow f(1) = 0$

Again (2) + (3)

and $2f(1) - g(1) = 0 \Rightarrow g(1) = 2f(1)$

$\therefore P(1) = f(1) + g(1) = 3f(1) = 3 \times 0 = 0$

90. **(4)**

$xdy - ydx = \sqrt{x^2 - y^2}\ x \geq 1.$

$d\left(\frac{y}{x}\right) = \frac{1}{x}\sqrt{1-\left(\frac{y}{x}\right)^2}$

$\Rightarrow \int \frac{d(y/x)}{\sqrt{1-\left(\frac{y}{x}\right)^2}} = \int \frac{dx}{x} \Rightarrow \sin^{-1}\left(\frac{y}{x}\right)$

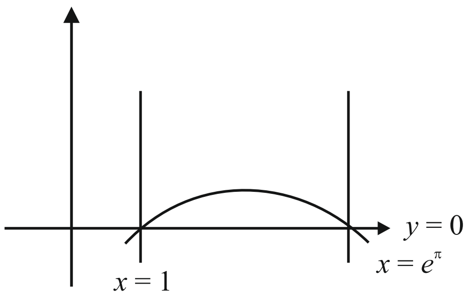
$= ln\, x + c$

$\sin^{-1}(0) = 0 + c \Rightarrow c = 0$

$y = x \sin(\ell n x)$

$x = 1,\qquad y = 0$

$x = e^{\pi}\qquad y = 0$



Required area $A = \int_1^{e^{\pi}} x \sin(\ell nx)\, dx.$

Let $lnx = t \Rightarrow \frac{1}{x}dx = dt.$

x	1	e^{π}
t	0	π

$A = \int_0^{\pi} e^{2t} \sin t\, dt = \left[\frac{e^{2t}(2\sin t - \cos t)}{5}\right]_0^{\pi}$

$= \frac{e^{2\pi}(1+1)}{5} = \frac{2e^{2\pi}}{5} = \alpha e^{2\pi} + \beta$

$\therefore \quad \alpha = \frac{2}{5}, \beta = 0$

$10(\alpha + \beta) = 10\left(\frac{2}{5} + 0\right) = 4$