JEE-Main-28-06-2022-Shift-1 (Memory Based)

Physics

Question: A man of mass 60 kg comes with velocity v and jumps into a car of mass 120 kg at rest the combined velocity is $2\,ms^{-1}$ then v is

Options:

(a) 2 m/s

(b) 6 m/s

(c) 8 m/s

(d) 10 m/s

Answer: (b)

Solution:

$m_{man} = 60kg = m_1$

$v_{man} = v$

$m_{car} = 120kg = m_2$

$v_{car} = 0 = u_2$

$v_{combined} = 2m/s = v$

So by linear momenum conservation

$m_1 v + m_2 \times 0 = (m_1 + m_2) v_{combined}$

$60 \times v = (60 + 120) \times 2$

$60v = 360$

$v = 6m/s$

Question: Two waves having wavelength 4.08, 4.16 produce 40 beats in 12 s find velocity in the medium

Options:

(a) 70 m/s

(b) 50 m/s

(c) 707 m/s

(d) 305 m/s

Answer: (c)

Solution: $v_1 - v_2 = \text{Beat frequency}$

$$\frac{v}{\lambda_1} - \frac{v}{\lambda_2} = \left(\frac{40}{12}\right)$$

$$v\left(\frac{1}{4.08} - \frac{1}{4.16}\right) = \left(\frac{40}{12}\right)$$

$$v\left(\frac{4.16 - 4.08}{4.08 \times 4.16}\right) = \left(\frac{40}{12}\right)$$

$v = 707.2 \approx 707m/s$

Question: Work function of metal is $6.63 \times 10^{-19} J$ .then the maximum wavelength to remove photoelectrons in nm

Options:

(a) 100 nm

(b) 400 nm
(c) 600 nm
(d) 300 nm

Answer: (d)

Solution: $\phi = 6.63\times10^{-19}\text{g}$

$$\phi_{\text{in(ev)}} = \frac{6.63\times10^{-19}}{1.6\times10^{-19}}\text{eV}$$

$$= 4.14\text{eV}$$

We know

$$\phi = \frac{1240}{\lambda_{\text{in nm}}}$$

$$\lambda_{\text{(in nm)}} = \frac{1240}{\phi_{\text{in ev}}}$$

$$= \frac{1240}{4.14}$$

$$= 299.516 \approx 300\text{nm}$$

Question: A drop which has radius of 2 cm is broken into 64 different drops find the gain in potential energy. Surface tension of water is 0.075 N/m

Options:

(a) 1.1 milli joules
(b) 1.3 milli Joules
(c) 11.3 milli joules
(d) 1.13 milli Joules

Answer: (d)

Solution: $\frac{4}{3}\times\left(2\times10^{-2}\right)^3 = 64\times\frac{4}{3}\pi\left(\text{r}^3\right)$

$$\text{r} = \frac{2\times10^{-2}}{4}$$

$$\text{r} = 5\times10^{-3}\,\text{m}$$

Change in surface energy $= 5\times\text{A}_{\text{big}} - 5\times\text{A}_{\text{small}}$

$$= 0.075\times4\pi\left(2\times10^{-2}\right)^2 - 0.075\times4\pi\times\left(5\times10^{-3}\right)^2\times64$$

$$= 1.13\,\text{milli joules}$$

Question: Human voice lower hearing limit.
High quality music
Earth wave
Radio wave

Options:

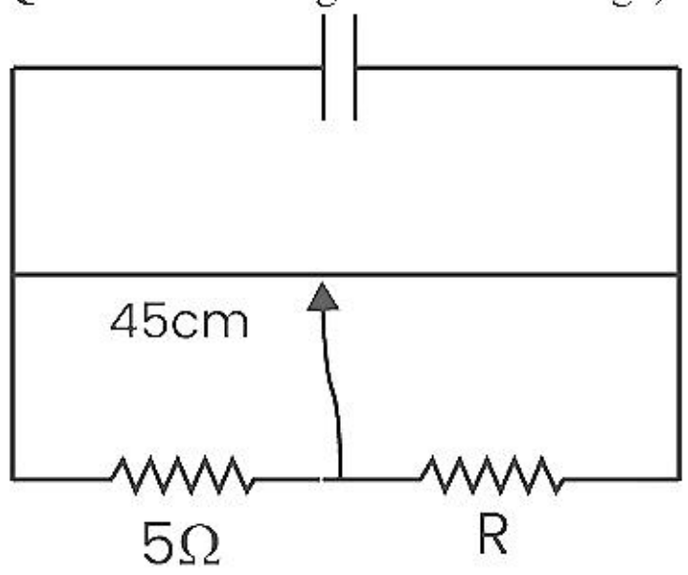
(a) 20 Hz, 440 Hz, 8 Hz, 330 kHz to 300 GHz
(b) 20 Hz, 440 Hz, 8 Hz, 130 kHz to 200 GHz
(c) 10 Hz, 40 Hz, 4 Hz, 330 kHz to 300 GHz
(d) 20 Hz, 40 Hz, 2 Hz, 330 kHz to 100 GHz

Answer: (a)

Solution: Human voice lower hearing limit $\rightarrow 20\text{Hz}$
High quality music $\rightarrow 440\text{Hz}$

Earth wave $\rightarrow$ 8Hz
Radio wave $\rightarrow$ 300kHz to 300 GHz

Question: In the given meter bridge, Find the vale of R



Options:

(a) $45/3\Omega$

(b) $55/3\Omega$

(c) $35/3\Omega$

(d) $25/3\Omega$

Answer: (b)

Solution:

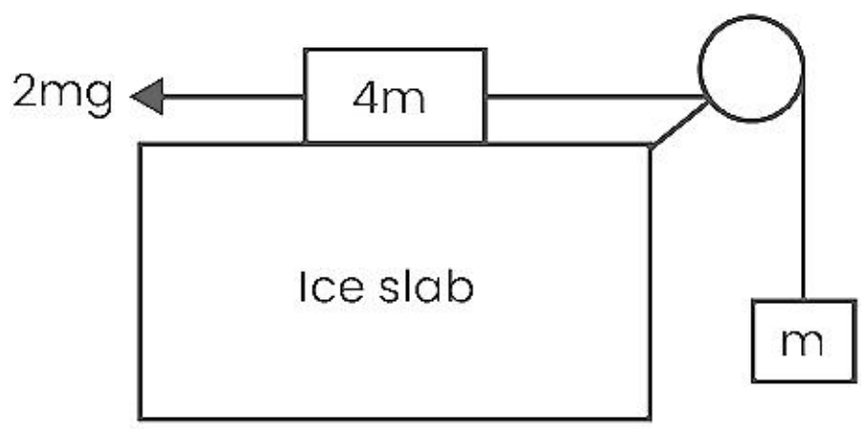
As we have obtained null direction at 45 cm. Hence

$$\frac{15}{45}=\frac{R}{(100-45)}$$

$$\frac{15}{45}=\frac{R}{55}$$

$$R=\frac{55}{3}\Omega$$

Question: If $T=\frac{x}{5}mg$ Find x = ?



Options:

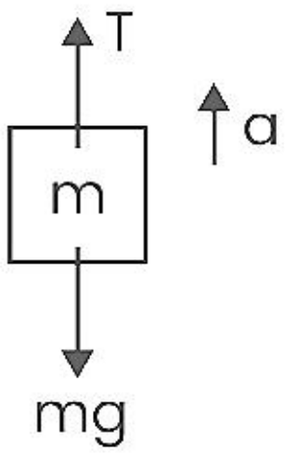
(a) 6

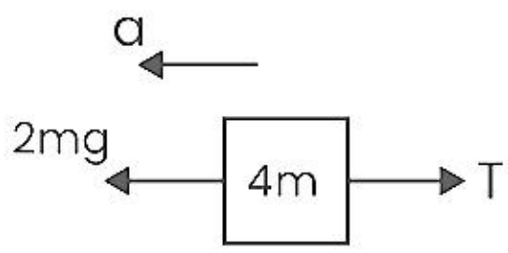
(b) 5

(c) 7

(d) 9

Answer: (a)
Solution:





2mg - T = 4ma

$T - mg = ma(1)$

$\text{from eq}(1)\text{in eq}(2)$

$2mg - T = 4(T - mg)$

$5T = 6mg$

$T = \frac{6}{5}mg$

$Hence\, x = 6$

Question: MOI of sphere about tangent
MOI of disc about disc about diameter
MOI of ring about diameter
Options:

(a) $\frac{2MR^2}{3}$

(b) $\frac{3MR^2}{2}$

(c) $\frac{MR^2}{3}$

(d) MR^2

Answer: (a)
Solution:

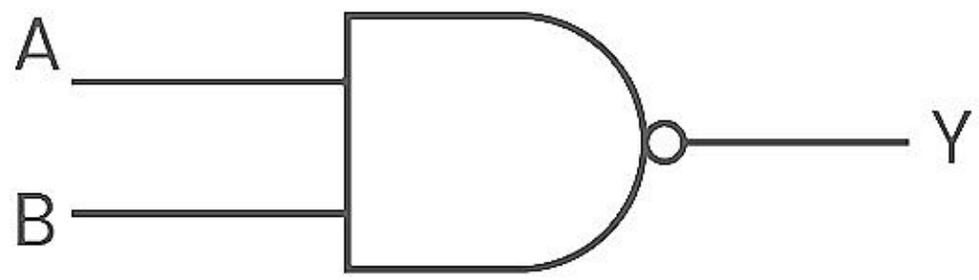
MOI of sphere about tangent $\frac{7}{5}MR^2$

MOI of disc about diameter $= \frac{MR^2}{4}$

MOI of ring about diameter $= \frac{MR^2}{2}$

MOI of hollow sphere a from axis passing through center $= \frac{2MR^2}{3}$

Question: For the following gate the output Y is given by



Options:

(a) A + B

(b) $\overline{A+B}$

(c) A. B

(d) $\overline{A.B}$

Answer: (d)

Solution:

It is a NAND Gate hence the answer is d.

Question: A particle moves in x-y plane according to rule $x = A\cos(\omega t), Y = A\sin(\omega t)$. The particle follows:

Options:

(a) An elliptical path

(b) A circular path

(c) A parabolic path

(d) A straight-line path inclined equally to x and y-axis

Answer: (b)

Solution: $x = A\sin\omega t$ or $\frac{x}{A} = \sin\omega t$ ....(i)

$y = A\cos\omega t$ or $\frac{y}{A} = \cos\omega t$ ....(ii)

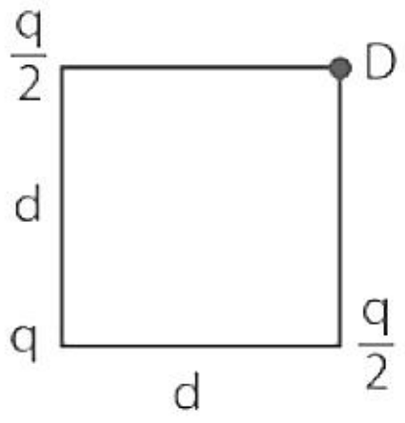
Squaring and adding, we get

$\frac{x^2}{A^2} + \frac{y^2}{A^2} = 1\left(\because \cos^2\omega t + \sin^2\omega t = 1\right)$

Or $x^2 + y^2 = A^2$

This is the equation of a circle. Hence particle follows a circular path.

Question:



Find electric field at point D?

Options:

(a) $\left(\sqrt{2}+1\right)\frac{Kq}{2d^2}$

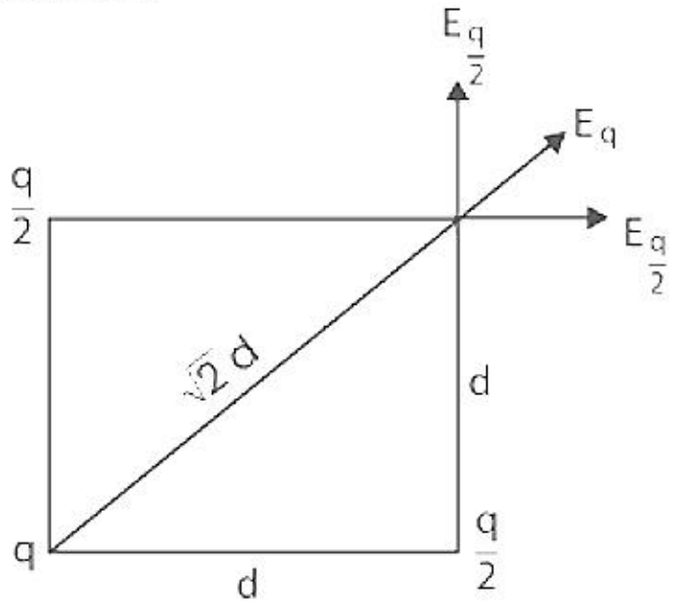
(b) $\frac{Kq}{2d^2}$

(c) $(2+1)\frac{Kq}{2d^2}$

(d) $\sqrt{2}\frac{Kq}{2d^2}$

Answer: (a)

Solution:



$$\vec{E}_{\frac{q}{2}} = \frac{k\frac{q}{2}}{d^2}\hat{i}$$

$$\vec{E}_{\frac{q}{2}} = \frac{k\frac{q}{2}}{d^2}\hat{j}$$

Resultant of electric fields due to $\frac{q}{2}$ will be in direction of field due to q.

$$\left|\vec{E}_R\right| = \sqrt{\left(\frac{kq}{2d^2}\right)^2 + \left(\frac{kq}{2d^2}\right)^2}$$

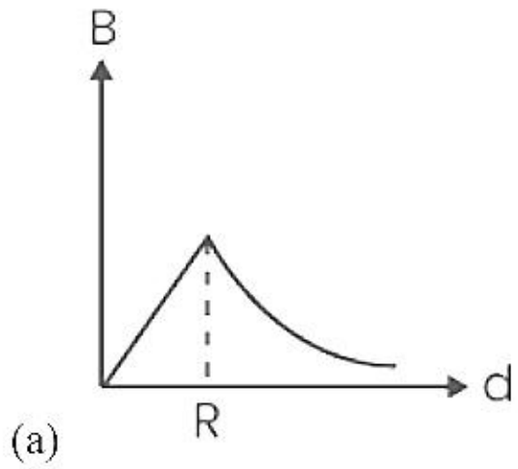
$$E_R = \sqrt{2}\frac{kq}{2d^2}$$

$$E_q = \frac{kq}{\sqrt{2}d^2} = \frac{kq}{2d^2}$$

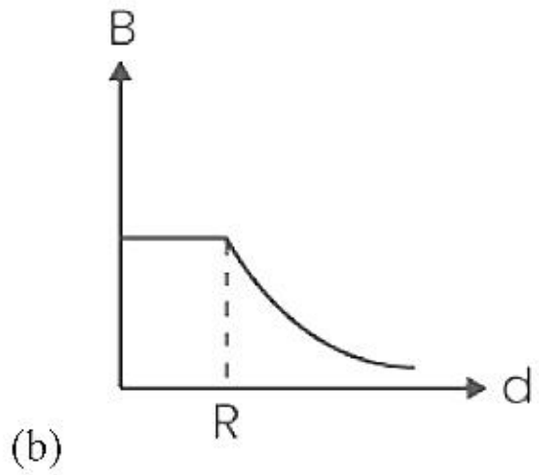
$$E_{total} = \left(\sqrt{2}+1\right)\frac{kq}{2d^2}$$

Question: A uniform current is flowing along the length of an infinite, straight, thin, hollow cylinder of radius 'R'. The magnetic field 'B' produced at a perpendicular distance 'd' from the axis of the cylinder is plotted in a graph. Which of the following figures looks like the plot?

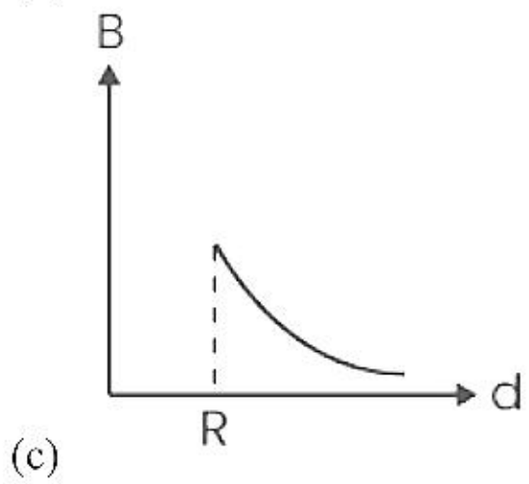
Options:



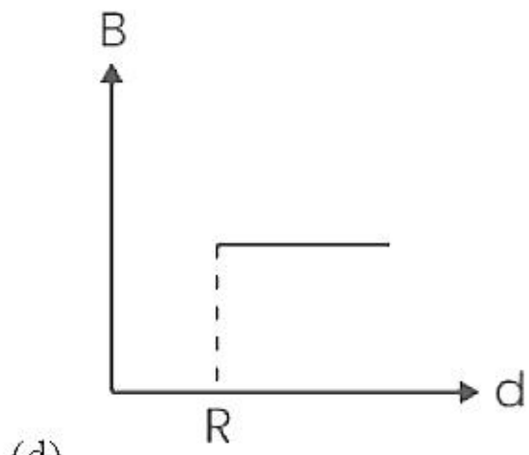
(a)



(b)



(c)



(d)

Answer: (c)

Solution: For

$d < R$

$B \times 2\pi d = \mu_0 I_{in} = 0$

As B=0

For $d > R$

$B \times 2\pi d = \mu_0 I$

$$B = \frac{\mu_0 I}{2\pi d}$$

Question: Read the assertion and reason carefully to mark the correct option out of the options given below.

Assertion: Dimensions of pressure × time is same as of coefficient of viscosity

Reason: Coefficient of viscosity $= \frac{\text{Force}}{\text{Velocity gradient}}$

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both the assertion and reason are false.

Answer: (c)

Solution: Coefficient of viscosity, $\eta = \frac{F}{\frac{A dv}{dx}}$

$[F] = [\text{Force}] = MLT^{-2}$

$[A] = [\text{Area}] = L^2$

$\left[\frac{dv}{dx}\right] = [\text{Velocity gradient}] = \frac{LT^{-1}}{L} = T^{-1}$

$\therefore [\eta] = \frac{MLT^{-2}}{L^2T^{-1}} = \left[ML^{-1}T^{-1}\right]$

$[P] = \left[ML^{-1}T^{-2}\right]$

$[P] \times [T] = \left[ML^{-1}T^{-1}\right] = [\eta]$

So, Assertion is true but reason is false.

Question: A pendulum of length 250 cm is released from rest when string makes angle of 60° with vertical. Find its maximum velocity

Options:

(a) 2 m/s

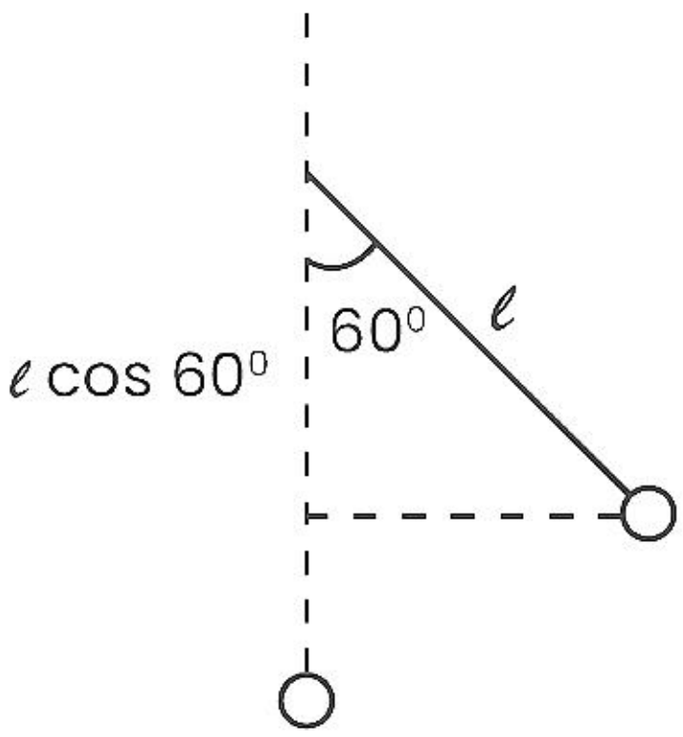
(b) 3 m/s

(c) 5 m/s

(d) 6 m/s

Answer: (c)

Solution:



$$mg\left(\ell - \ell\cos 60^\circ\right) = \frac{1}{2}mv^2$$

$$20(2.50 - 1.25) = v^2$$

$$v = \sqrt{25}$$

$$v = 5m/s$$

Question: If $\vec{r} = \left(3\hat{i} + \hat{j}\right)m, \vec{v} = \left(3\hat{j} - \hat{k}\right)m/s^{-1}$ $m = 1kg$, $\left|\vec{L}\right| = \sqrt{x}Nm - s$. Find the value of x.

Options:

(a) 50

(b) 61

(c) 110

(d) 91

Answer: (d)

Solution: $v = 5m/s$

$$\vec{L} = M\left(\vec{r} \times \vec{v}\right)$$

$$\vec{L} = 1\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 0 \\ 0 & 3 & -1 \end{vmatrix}$$

$$= \hat{i}(-1-0) - \hat{j}(-3-0) + \hat{k}(9-0)$$

$$\vec{L} = -\hat{i} + 3\hat{j} + 9\hat{k}$$

$$\left|\vec{L}\right| = \sqrt{1^2 + 3^2 + 9^2} = \sqrt{91}$$

$$x = 91$$

Question: The period of two planets A and B is $T_A = 2T_B$. Find the ratio of radius of their revolution.

Options:

(a) $2^{\frac{1}{3}}$

(b) $4^{\frac{1}{3}}$

(c) $3^{\frac{1}{3}}$

(d) $6^{\frac{1}{3}}$

Answer: (b)

Solution:

$T^2 \propto r^3$

$T_A^2 \propto r_A^3$

$T_B^2 \propto r_B^3$

$$\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3$$

$$\frac{r_A}{r_B} = 4^{\frac{1}{3}}$$

Question: Read the assertion and reason carefully to mark the correct option out of the options given below.

Assertion: Work done by n mole of gas in adiabatic process is $w = \frac{nR(T_2 - T_1)}{1-\gamma}$

Reason: If work is done on a gas, then temperature will increase.

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

(c) If assertion is true, but reason is false.

(d) If both the assertion and reason are false.

Answer: (c)

Solution: Assertion is true because this is the correct expression for work done in adiabatic process.

Reason is false because this is not true for many cases like for isothermal process temperature remains constant.

Question: Resolving power of a telescope for the aperture 24.4 cm for the wavelength $\lambda = 2440\,\text{Å}$ is

Options:

(a) 1.2×10^5

(b) 2.2×10^5

(c) 8.2×10^5

(d) 6.2×10^5

Answer: (c)

Solution: $\frac{1}{R.P} = \frac{1.22\lambda}{a}$

$$R.P = \frac{24.4\times10^{-2}}{1.22\times24\times2440\times10^{-10}} = 0.082\times10^7$$

$R.P = 8.2\times10^5$

Question: Centripetal acceleration is given by $= k^2rt^2$ if radius is constant then find the power delivered by the force

Options:

(a) mk^2r^2t

(b) m^2kr^2t

(c) mkr^2t

(d) $mkrt$

Answer: (a)

Solution: Centripetal acceleration, $a_c = k^2rt^2$

Where, $a_c = \frac{v^2}{r}$

$\Rightarrow \frac{v^2}{r} = k^2rt^2$

$\Rightarrow v = krt \ldots.(1)$

Tangential acceleration, $a_t = \frac{dv}{dt} = kr \ldots.(2)$

Tangential force acting on the particle, $F = ma_t = mkr$

Power delivered, $P = \bar{F}.\bar{v} = Fv\cos\theta$

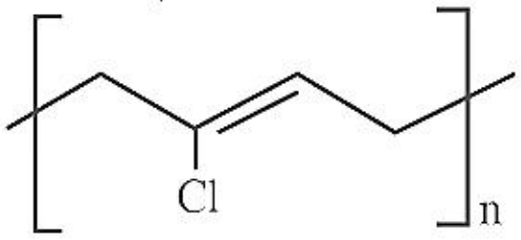
$\therefore P = Fv = (mkr) \times krt (\because \theta = 0°)$

$\Rightarrow P = mk^2r^2t$

JEE-Main-28-06-2022-Shift-1 (Memory Based)

Chemistry

Question: Which of the following is not copolymer?
Options:
(a) Buna S
(b) Neoprene
(c) PHBV
(d) Styrene butadiene
Answer: (b)
Solution: Neoprene is a homopolymer and the monomer from which it is obtained is 2-chloro-1,3-butadiene



Neoprene

Question: Most stable Lanthanide in divalent form:
Options:
(a) Eu^{+2}
(b) Sm^{+2}
(c) Yb^{+2}
(d) Ce^{+2}
Answer: (a)
Solution: Most stable lanthanide in divalent form is Eu^{+2}
Electronic configuration of Europium is $4f^7 6s^2$.
Hence, +2 state is most stable for Eu.

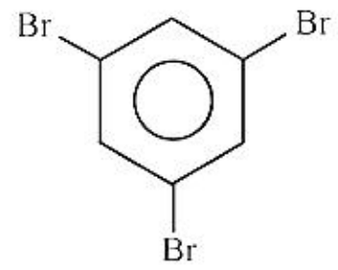
Question: Write the product formed when primary amine reacts with nitrous acid at room temperature?
Options:
(a) Alcohol
(b) Diazonium salt
(c) Amide
(d) Acid halide
Answer: (a)
Solution: Primary aliphatic amines react with nitrous acid to form aliphatic diazonium salts which being unstable, liberate nitrogen gas quantitatively and alcohols.

$$R-NH_2 + HNO_2 \xrightarrow{NaNO_2+HCl} \left[R-\overset{+}{N}_2\,C\overset{-}{l}\right] \xrightarrow{H_2O} ROH + N_2 + HCl$$

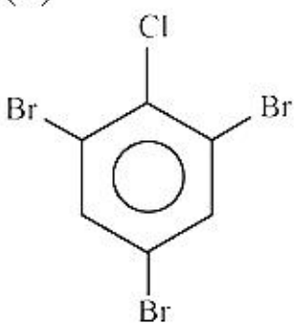
Question: Aniline on reaction with $Br_2 + H_2O$ then $NaNO_2$ and H_3PO_2 gives

Options:

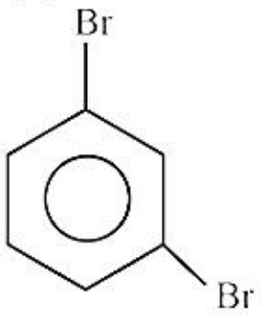
(a)



(b)



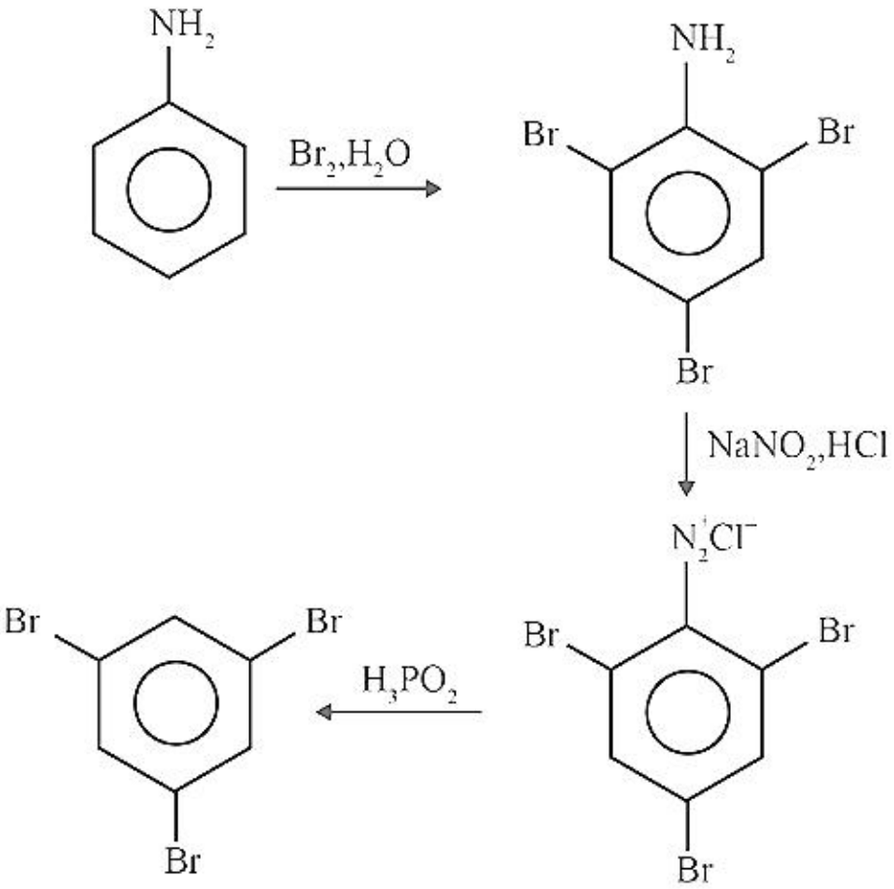
(c)



(d) None of these

Answer: (a)

Solution:



Question: The Zeta potential is property of colloidal particles for

Options:

(a) Colour

(b) Brownian movement

(c) Charge on surface of colloidal particle

(d) Tyndall effect
Answer: (c)
Solution: The potential difference between the fixed layer and the diffused layer of opposite charges is called the zeta potential or electro kinetic potential. It is used to define or explain the process of preferential adsorption of ions from solution in the electrical charge on colloidal particles.

Question: The stability of α-Helix structure of protein is due to
Options:
(a) Hydrogen Bonding
(b) Vander Waal's
(c) Sulphide Linkage
(d) None of these
Answer: (a)
Solution: Hydrogen bonding is responsible for the stability of alpha-helical structure of proteins.

Question: Hybridization of P in PF_5 is sp^xd^y. What is y?
Options:
(a) 3
(b) 2
(c) 4
(d) 1
Answer: (d)
Solution: Hybridization of P in PF_5 is sp^3d.
Therefore y = 1

Question: Which of the following has minimum synergic bond?
Options:
(a) $[Mn(CO)_5]$
(b) $[Mn_2(CO)_{10}]$
(c) $[Cr(CO)_6]$
(d) $[Fe(CO)_5]$
Answer: (c)
Solution: More the number of d-electrons, higher is the synergic bonding
a) d^7, b) d^7, c) d^5, d) d^8
So, (c) has minimum synergic bonding

Question: The purple colour after lassaigne test of sulphur is due to the formation of
Options:
(a) $Na_4[Fe(CN)_6S]$
(b) $Na_4[Fe(CN)_5NCS]$
(c) $Na_4[Fe(CN)_5NOS]$
(d) $Na_2[Fe(CN)_5NCS]$
Answer: (c)
Solution: $Na_2S + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NOS]$

Sodium sulphide reacts with sodium nitroprusside to form a violet colour compound, which confirms the presence of sulphur.

Question: If work function of a metal is 6.63×10^{-19} J. Find minimum wavelength of light required to emit photoelectron.

Options:

(a) 100 nm

(b) 200 nm

(c) 300 nm

(d) 400 nm

Answer: (c)

Solution:

$$K_{max} = 0,\ \phi = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\phi} = \frac{6.63\times10^{-34}\times3\times10^{8}}{6.63\times10^{-19}} = 300\,nm$$

Question: Which of the following gives N_2 on thermal decomposition?

Options:

(a) $B_2(NO)_3$

(b) $Ba(N_3)_2$

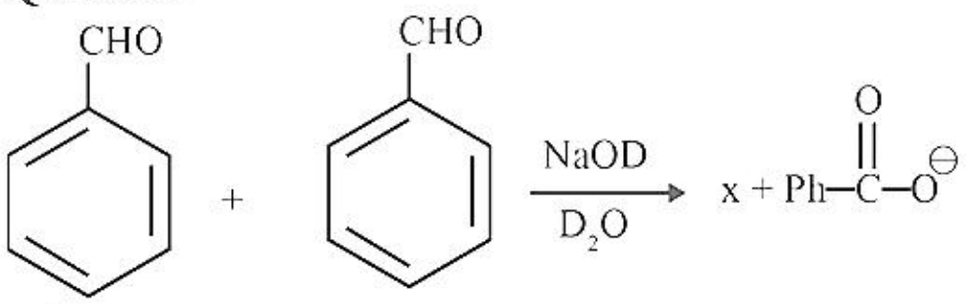
(c) $NaNO_3$

(d) HNO_3

Answer: (b)

Solution: $Ba(N_3)_2 \rightarrow Ba + 3N_2$

Question:



x is

Options:

(a) Ph–CH_2–OD

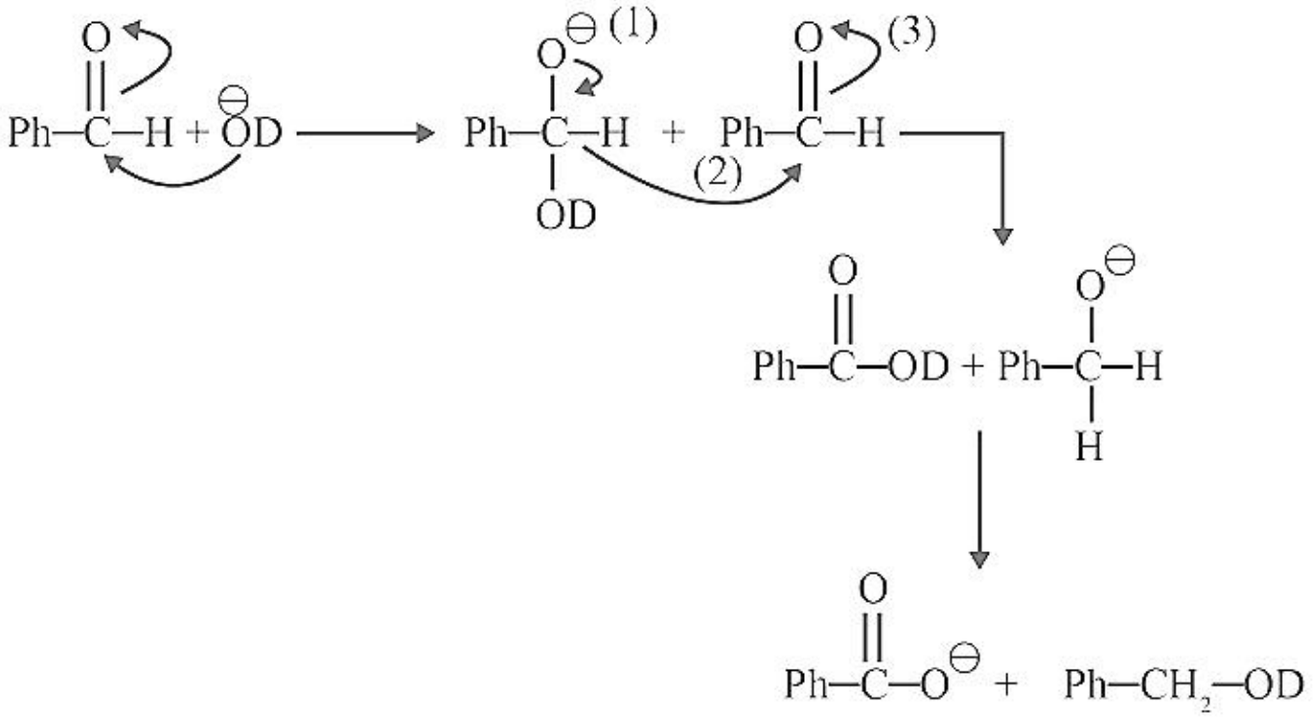
(b) Ph–COOD

(c) Ph–CHO

(d) None

Answer: (a)

Solution:



Question: $H_2 + CuO \rightarrow$ Product
Options:
(a) Cu
(b) CuO_2
(c) Cu_2O
(d) None
Answer: (a)
Solution: $H_2 + CuO \rightarrow Cu + H_2O$

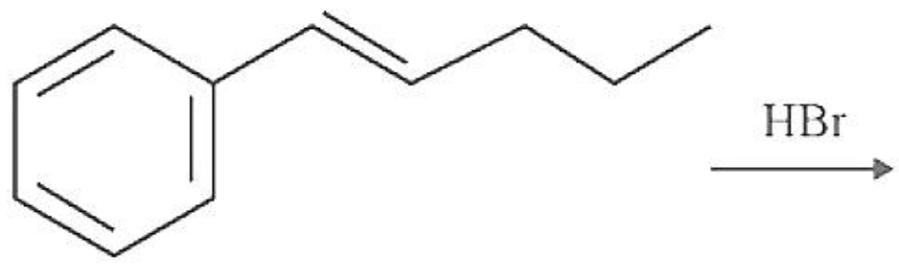
Question: Find the valence shell electronic configuration of the element present above of element E of group 16 and Period –4
Options:
(a) $2s^2 2p^6$
(b) $2s^2 2p^5$
(c) $3s^2 3p^4$
(d) $3s^2 3p^3$
Answer: (c)
Solution: E selenium (Se)
Element present above Se is sulphur. Valence shell electronic configuration of sulphur is $[Ne]3s^2 3p^4$

Question: Calculate the amount of electricity needed to convert $Cr_2O_7^{2-}$ to Cr^{3+}
Options:
(a) 289500 C
(b) 579000 C
(c) 5790 C
(d) 28950 C
Answer: (b)
Solution: $Cr_2O_7^{2-} + 14H^+ + 6e^- \rightarrow 2Cr^{3+} + 8H_2O$
From balanced equation, we get n = 6
Charge required = nF = 6 × 96500 C = 579000 C

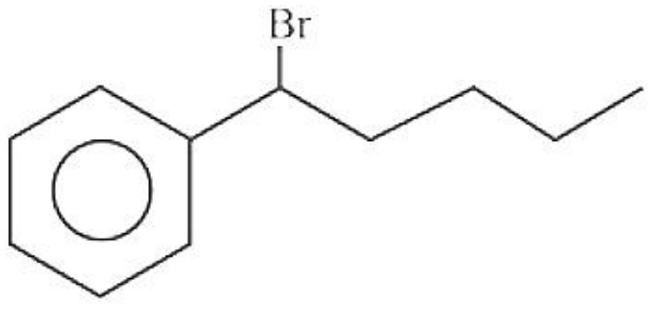
Question: Find heat absorbed for an isothermal process in vacuum
[Given: $V_1 = 4L$, $V_2 = 10$ L]
Options:
(a) 0
(b) 1
(c) 2
(d) 3
Answer: (a)
Solution: For an isothermal process in vacuum
$dU = 0$
$dW = 0$
$dU = dq + dW$
$\therefore dq = 0$

Question: Assertion: For Group 15 elements, the acidity of pentavalent oxide of is greater than trivalent oxide.
Reason: Down the group acidity Decreases.
Options:
(a) Both assertion and reason are true, reason is correct explanation of assertion.
(b) Both assertion and reason are true, but reason is not a correct explanation of assertion.
(c) Assertion is true, but reason is false
(d) Assertion is false, but reason is true
Answer: (b)
Solution: The oxide in the higher oxidation state of the element is more acidic than that of lower oxidation state. Their acidic character decreases down the group. The oxides of the type E_2O_3 of nitrogen and phosphorus are purely acidic, that of arsenic and antimony amphoteric and those of bismuth predominantly basic.

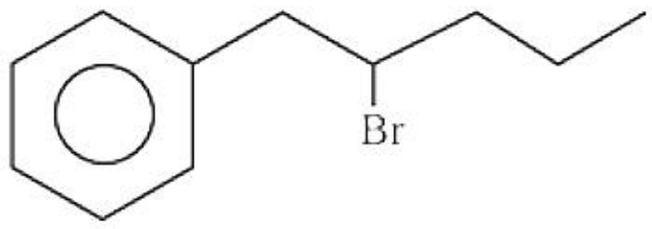
Question:



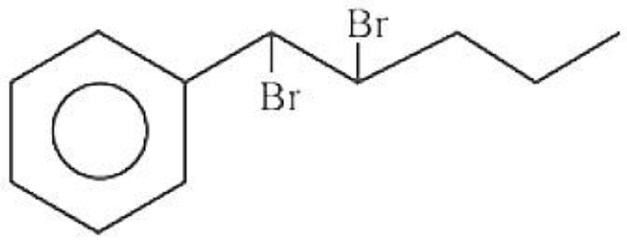
Options:
(a)



(b)



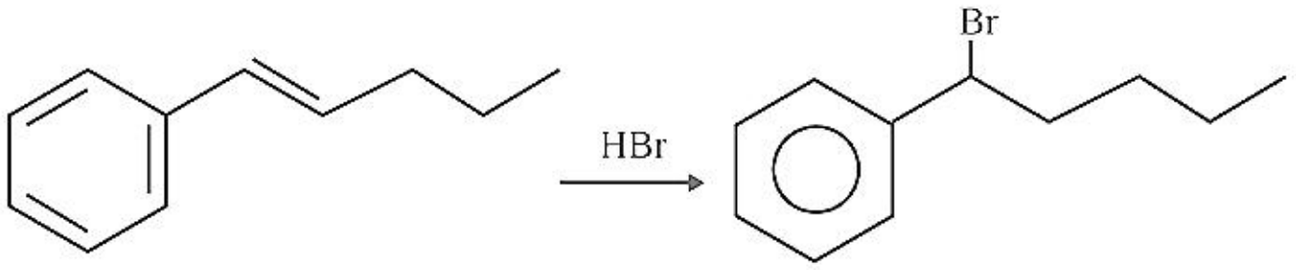
(c)



(d) None

Answer: (a)

Solution:



Question: Statement-1: $[Ni(CN)_6]^{4-}$ is square planar d^2sp^3 and diamagnetic ,whereas $Ni(CO)_4$ is square planar and sp^3 Hybridised and paramagnetic.

Statement-2: $NiCl_4^{2-}$ and $NiCO_4$ has the same d configuration, geometry and hybridization.

Options:

(a) Statement 1 and statement 2 both are correct.

(b) Statement 1 is correct but statement 2 is wrong.

(c) Statement 1 is wrong but statement 2 is correct.

(d) Statement 1 and statement 2 both are wrong.

Answer: (d)

Solution: $[Ni(CN)_6]^{4-}$ is paramagnetic and sp^3d^2, octahedral

$Ni(CO)_4$ is tetrahedral, dsp^2 hybridized and diamagnetic

$NiCl_4^{2-}$ is square planar, sp^3 hybridised and paramagnetic

Therefore, both the statements 1 and 2 are false

Question: 0.5 g of bromine organic compound gives 0.40g of AgBr, what is percent of bromine present in organic compound?

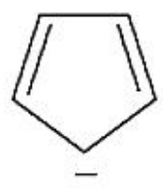
Answer: 34.00

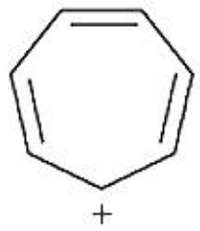
Solution:

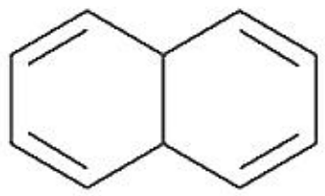
$$\%\text{of Br} = \frac{80}{188} \times \frac{\text{Mass of AgBr} \times 100}{\text{Mass of compound}}$$

$$= \frac{80}{188} \times \frac{0.4 \times 100}{0.5} = 34\%$$

Question: How many are aromatic compounds?

1) 

2) 

3) 

4) 

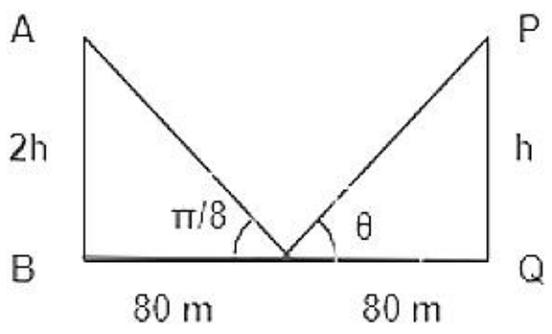
Answer: 2.00

Solution: Both 1 & 2 are cyclic, planar and follows Huckel's rule

JEE-Main-28-06-2022-Shift-1 (Memory Based)

MATHEMATICS

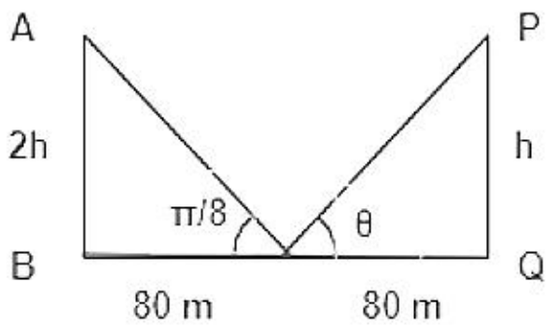
Question:



Find $\tan^2\theta$.

Answer: $4\left(3-2\sqrt{2}\right)$

Solution:



$$\tan\frac{\pi}{8}=\frac{h}{80},$$

$$\tan\theta=\frac{2h}{80}$$

$$\Rightarrow \tan\theta=2\tan\frac{\pi}{8}$$

$$\Rightarrow \tan^2\theta=4\tan^2\frac{\pi}{8}$$

$$=4\left(\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}\right)$$

$$=4\left(\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}\right)$$

$$=\frac{4\left(\sqrt{2}-1\right)}{\sqrt{2}+1}$$

$$=4\left(\sqrt{2}-1\right)^2$$

$$= 4\left(3 - 2\sqrt{2}\right)$$

Question: Find probability that 3 digit number has atleast two odd numbers.

Answer: $\frac{19}{36}$

Solution:

Total 3 digit numbers $= 9 \times 10 \times 10 = 900$

Numbers with 3 odd digit $= 5 \times 5 \times 5 = 100$

Number with 2 odd, one even digit $= 4 \times 5 \times 5 + 2 \times 5 \times 5 \times 5$

$= 350$

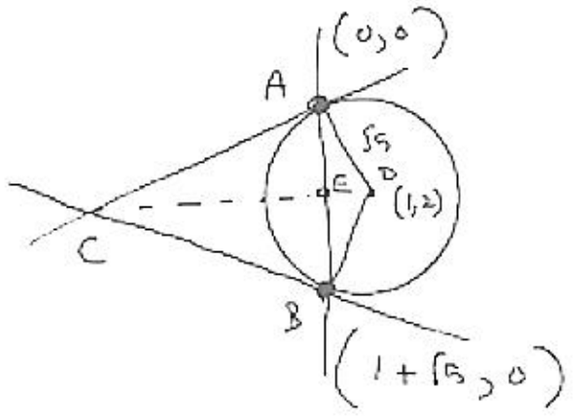
Favourable number of cases $= 125 + 350 = 475$

Probability $= \frac{475}{900} = \frac{19}{36}$

Question: Equation of circle $(x-1)^2 + (y-2)^2 = 5$ drawing 2 tangents at $(0,0)$ & $\left(1+\sqrt{5}, 0\right)$ they intersect. Find area of that triangle.

Answer: $\frac{1+\sqrt{5}}{4}$

Solution:



Equation of $AB \equiv y = 0$

$(x-1)^2 + (y-2)^2 = 5$

$\Rightarrow x^2 + y^2 - 2x - 4y = 0$

Let C be (α, β)

Equation of $AB = \alpha x + \beta y - (x + \alpha) - 2(y + \beta) = 0$

$\Rightarrow x(\alpha - 1) + y(\beta - 2) - \alpha - 2\beta = 0$

$\Rightarrow \alpha = 1, -\alpha - 2\beta = 0$

$\beta = \frac{-1}{2}$

$$CE = \frac{1}{2},\ AB = 1+\sqrt{5}$$

$$\text{Area} = \frac{1}{2}\left(\frac{1}{2}\right)\left(1+\sqrt{5}\right) = \frac{1+\sqrt{5}}{4}$$

Question: $|A| = 2$. Find $\left||A|\,adj\left(5\,adj\ A^3\right)\right|$.

Answer: $2^{15}\times 5^6$

Solution:

$$\left||A|\,adj\left(5\,adj\ A^3\right)\right|$$

$$= |A|^3\left|adj\left(5\,adj\ A^3\right)\right|$$

$$= |A|^3\left|5\,adj\ A^3\right|^2$$

$$= |A|^3\,5^6\left|adj\ A^3\right|^2$$

$$= |A|^3\,5^6\left(\left|A^3\right|^2\right)^2$$

$$= |A|^3\,5^6\,|A|^{12}$$

$$= 2^{15}\cdot 5^6$$

Question: $\overline{a} = 2\hat{i}+\hat{j}+3\hat{k},\ \overline{b} = 3\hat{i}+3\hat{j}+\hat{k}$ & $\overline{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$, $\overline{a},\overline{b},\overline{c}$ are coplanar, $\overline{a}\cdot\overline{c} = 5$ & $\overline{b}$ is perpendicular to $\overline{c}$. Find $122\left(c_1+c_2+c_3\right)$.

Answer: 150.00

Solution:

$$\overline{a} = 2\hat{i}+\hat{j}+3\hat{k},\ \overline{b} = 3\hat{i}+3\hat{j}+\hat{k}\ \&\ \overline{c} = c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$$

$$\overline{a}\cdot\overline{c} = 5,\ \overline{b}\perp\overline{c},\ 122\left(c_1+c_2+c_3\right) = ?$$

$$a\cdot c = 2c_1+c_2+3c_3 = 5 \qquad ....(1)$$

$$b\cdot c = 3c_1+3c_2+c_3 = 0 \qquad ....(2)$$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{vmatrix} = 0$$

$$c_1(1-9)-c_2(2-9)+c_3(6-3) = 0$$

$$-8c_1+7c_2+3c_3 = 0 \qquad ....(3)$$

From (1), (2) and (3)

$$c_1 = \frac{10}{122},\ c_2 = \frac{-85}{122},\ c_3 = \frac{225}{122}$$

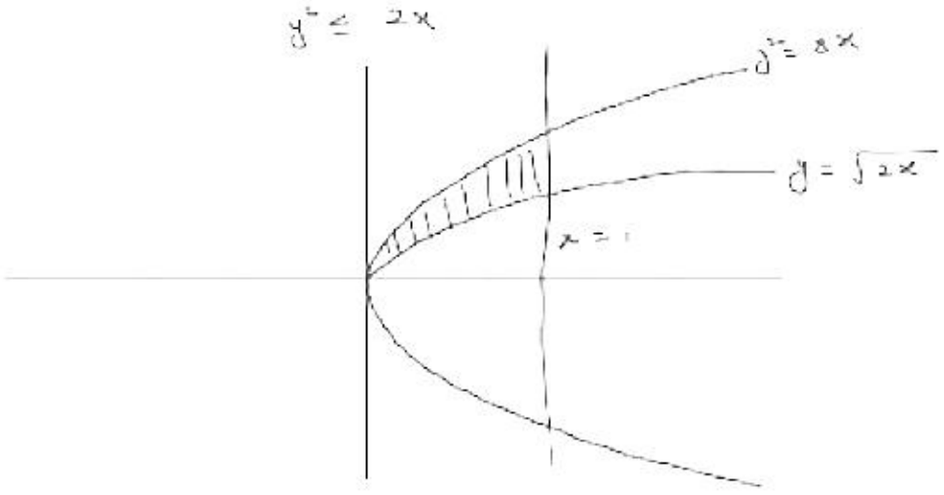
$$122\left(c_1+c_2+c_3\right)=150$$

Question: $y^2 \le 8x,\ y \ge \sqrt{2x}, x \le 1$. Find area.

Answer: $\dfrac{2\sqrt{2}}{3}$

Solution:

Given, $y^2 \le 8x,\ y \ge \sqrt{2x}, x \le 1$.



$$\text{Area} = \int_0^1 \left(\sqrt{8x}-\sqrt{2x}\right)dx$$

$$= \int_0^1 \left(2\sqrt{2}x-\sqrt{2}\sqrt{x}\right)dx$$

$$= \int_0^1 \sqrt{2}\sqrt{x}\,dx$$

$$= \sqrt{2}\left.\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right|_0^1$$

$$= \frac{2\sqrt{2}}{3}$$

Question: Find number of numbers divisible by 6, formed using $3,6,1,7,2,5$.

Answer: 72.00

Solution:

Given $3,6,1,7,2,5$

$3+6+1+7+2+5=24$

Case I: $3,1,7,2,5$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1} = 24$

Case II: $6,1,7,2,5$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{2} = 48$

Total $= 24 + 48 = 72$

Question: $\sum_{k=1}^{31} {}^{31}C_k \times {}^{31}C_{k-1} + \sum_{k=1}^{30} {}^{30}C_k \times {}^{30}C_{k-1} = \frac{\alpha \times 60!}{30! \times 30!}$. $16\alpha = ?$

Answer: $\frac{2371}{8}$

Solution:

$$\sum_{k=1}^{31} {}^{31}C_k \times {}^{31}C_{k-1} + \sum_{k=1}^{30} {}^{30}C_k \times {}^{30}C_{k-1}$$

$$= \sum_{k=1}^{31} {}^{31}C_{31-k} \, {}^{31}C_{k-1} + \sum_{k=1}^{30} {}^{30}C_k \, {}^{30}C_{31-k}$$

$$= {}^{62}C_{30} + \left({}^{60}C_{31}\right)$$

$$= \frac{62!}{32!30!} + \frac{60!}{31!29!}$$

$$= \frac{60!}{30!}\left(\frac{61 \times 62}{32!} + \frac{1}{31 \times 29!}\right)$$

$$= \frac{60!}{30!30!}\left(\frac{61 \times 62}{31 \times 32} + \frac{30}{31}\right)$$

$$= \frac{60!}{30!30!}\left(\frac{2371}{496}\right)$$

$$\alpha = \frac{2371}{496}$$

$$16\alpha = \frac{2371}{31}$$

Question: $x\left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = x + y\left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right)$. Let $y = y(x)$ satisfy above differential equation such that $y(1) = 0$. If $y(2\alpha) = \alpha$, find α.

Answer: ()

Solution:

$$x\left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right)\frac{dy}{dx} = x + y\left(\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right)$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\left[\frac{x}{\sqrt{x^2-v^2x^2}}+e^v\right]\left(v+x\frac{dv}{dx}\right)=v\left[\frac{x}{\sqrt{x^2-v^2x^2}}+e^v\right]+1$$

$$\left[\frac{1}{\sqrt{1-v^2}}+e^v\right]\left(v+x\frac{dv}{dx}\right)=v\left[\frac{1}{\sqrt{1-v^2}}+e^v\right]+1$$

$$=\frac{v}{\sqrt{1-v^2}}+ve^v+\frac{x}{\sqrt{1-v^2}}\frac{dv}{dx}+xe^v\frac{dv}{dx}=\frac{v}{\sqrt{1-v^2}}+ve^v+1$$

$$\Rightarrow x\frac{dv}{dx}\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)=1$$

$$\Rightarrow\int\left(\frac{1}{\sqrt{1-v^2}}+e^v\right)dv=\int\frac{dx}{x}$$

$$\Rightarrow\sin^{-1}\left(\frac{y}{x}\right)+e^{\frac{y}{x}}=\ln x+c$$

$$\because\ y(1)=0\Rightarrow 0+1=0+c\Rightarrow c=1$$

$$\therefore\ y(2\alpha)=\alpha\Rightarrow\sin^{-1}\left(\frac{1}{2}\right)+e^{\frac{1}{2}}=\sin 2\alpha+1$$

$$\frac{\pi}{6}+e^{\frac{1}{2}}-1=\ln 2\alpha$$

$$\Rightarrow 2\alpha=e^{\frac{\pi}{6}+\sqrt{e}-1}$$

$$\Rightarrow\alpha=\frac{1}{2}\cdot e^{\frac{\pi}{6}+\sqrt{e}-1}$$

Question: Find the number of real solution of $4x^7+3x^3+5x+1=0$

Answer: 1.00

Solution:

Let $f(x)=4x^7+3x^3+5x+1$

$f'(x)=28x^6+9x^2+5>0$

$f(x)$ is increasing

No. of roots = 1

Question: $(a_n)\to GP$, $a_1\times a_3\times a_5\times a_7=\frac{1}{1296}$, $a_2+a_4=\frac{7}{36}$, then $a_6+a_8+a_{10}$

Answer: 43.00

Solution:

$$a\cdot ar^2\cdot ar^4\cdot ar^6=\frac{1}{1296}$$

$$a^4\cdot r^{12}=\frac{1}{64}$$

$$ar^3 = \frac{1}{6}$$

$$a_2 + a_4 = \frac{7}{36}$$

$$ar + ar^3 = \frac{7}{36}$$

$$ar = \frac{7}{36} - \frac{1}{6} = \frac{1}{36}$$

$$\Rightarrow r^2 = 6$$

$$a_6 + a_8 + a^{10}$$

$$ar^5 + ar^7 + ar^9$$

$$= \frac{1}{36} \times 6^2 + \frac{1}{36} \times 6^3 + \frac{1}{36} \times 6^4$$

$$= 1 + 6 + 36 = 43$$

Question: Mean & variance of 15 observations is 8 & 3 respectively. One observation is wrongly taken as 5 in place of 20. Find correct variance.

Answer: 11.00

Solution:

$$\text{Variance} = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2$$

$$3 = \frac{\sum x^2}{15} - 8^2$$

$$\frac{\sum x^2}{15} = 67$$

$$\sum x^2 = 1005$$

$$\text{New } \sum x^2 = 1005 - 5^2 + 20^2$$

$$= 1380$$

$$\text{New } \sum x = 8 \times 15 - 5 \times 20 = 135$$

$$\text{Variance} = \frac{1380}{15} - \left(\frac{135}{15}\right)^2$$

$$= 92 - 81 = 11$$

Question: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $e = \sqrt{\frac{5}{2}}$ & $\text{LR} = 6\sqrt{2}$. If $y = 2x + c$ is tangent, find c.

Answer: ± 2

Solution:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\ e=\sqrt{\frac{5}{2}}$$

$$\frac{2b^2}{a}=6\sqrt{2}$$

$$b^2=a^2\left(\frac{5}{2}-1\right)$$

$$b^2=\frac{3}{2}a^2$$

$$\Rightarrow\frac{2\times\frac{3}{2}a^2}{a}=6\sqrt{2}$$

$$\Rightarrow a=2\sqrt{2}$$

$$\Rightarrow a^2=8$$

$$\Rightarrow b^2=12$$

$$y=2x+c$$

$$c^2=a^2m^2-b^2$$

$$=8\times2-12=4$$

$$\Rightarrow c=\pm2$$