

JEE-Main-27-06-2022-Shift-2 (Memory Based)

Physics

Question: Which one is not showing the dimension of time-

Options:

(a) \sqrt{LC}

(b) $\frac{L}{R}$

(c) CR

(d) $\frac{C}{R}$

Answer: (d)

Solution:

$$[L] = [ML^2T^{-2}A^{-2}]$$

$$[R] = [ML^2T^{-3}A^{-2}]$$

$$[C] = [M^{-1}L^2T^4A^2]$$

$$(a) [\sqrt{LC}] = [M^0L^0T^2A^0]^{1/2} = [T]$$

$$(b) \left[\frac{L}{R}\right] = [M^0L^0T^1A^0] = [T]$$

$$(c) [CR] = [M^0L^0T^1A^0] = [T]$$

$$(d) \left[\frac{C}{R}\right] = [M^{-2}L^4T^6A^4]$$

So right answer is d.

Question: Rod of length 20 cm is moving with speed 10m/s in horizontal plane. Horizontal component of earth's magnetic field is 0.3 T. Angle of dip is 60° . Find potential of rod?

Options:

(a) 1.6 V

(b) 2.62 V

(c) 1.039 V

(d) 1.25 V

Answer: (c)

Solution:

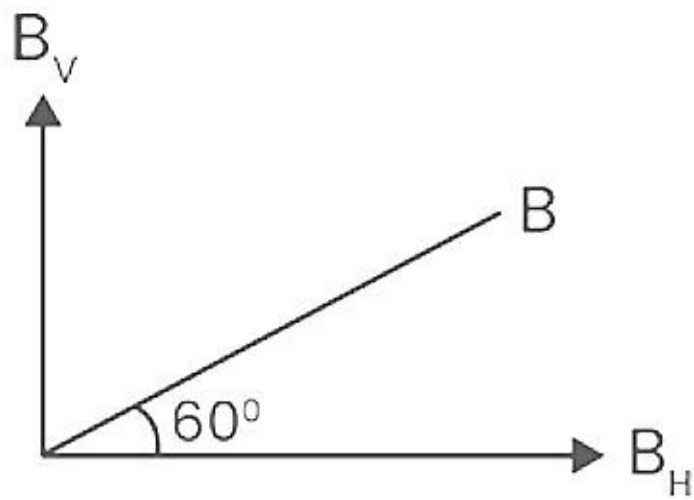
$$\tan 60^\circ = \frac{B_v}{B_H}$$

$$B_v = 0.3 \times \sqrt{3}$$

$$\varepsilon = B_v VL$$

$$\varepsilon = 0.3\sqrt{3} \times 10 \times 0.2$$

$$\varepsilon = 1.039V$$



Question: A spring of natural length L is attached with a mass M is rotated in horizontal circle with angular velocity ω . Find elongation in spring.

Options:

(a) $\frac{m\omega^2 k}{(L-m)}$

(b) $\frac{m\omega^2 L}{(k-m\omega^2)}$

(c) $\frac{m\omega^2 L}{(k-m)}$

(d) None of these

Answer: (b)

Solution:

$$m\omega^2 (L+x) = kx$$

$$m\omega^2 L = kx - m\omega^2 x$$

$$\Rightarrow x = \frac{m\omega^2 L}{(k-m\omega^2)}$$

Question: A proton and deuteron moving in circular orbit in uniform magnetic field with same velocity. Ratio of radius of deuteron to radius to proton is $x : 1$. Find value of x ?

Options:

(a) 2

(b) 5

(c) 4

(d) 7

Answer: (a)

Solution:

$$r = \frac{mv}{Bq}$$

$$r_p = \frac{m_p v}{Bq}$$

$$r_D = \frac{2m_p v}{Bq}$$

$$\frac{r_D}{r_p} = \frac{2}{1}$$

$$\Rightarrow x = 2$$

Question: If in SHM motion time period is 6 sec, what is the time taken to travel from extreme position to a distance of half the amplitude from mean position is

Options:

- (a) 4 sec
- (b) 1 sec
- (c) 0.024 sec
- (d) 0.25 sec

Answer: (b)

Solution:

Let SHM equation

$$y = A \cos \omega t$$

At $t = 0$, $y = A$

$$\text{For } y = \frac{A}{2}$$

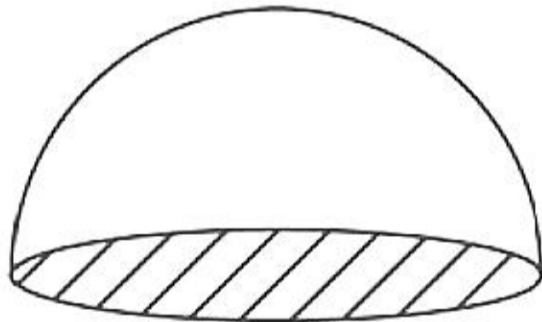
$$\frac{A}{2} = A \cos \left(\frac{2\pi}{T} \times t \right)$$

$$\cos \frac{\pi}{3} = \cos \left(\frac{2\pi}{T} \times t \right)$$

$$\frac{2\pi}{T} \times t = \frac{\pi}{3}$$

$$t = \frac{T}{6} = 1 \text{ sec}$$

Question: Find flux through shaded area



Options:

(a) $\frac{q}{2\epsilon_0}$

(b) $\frac{q\epsilon_0}{2}$

(c) $\frac{q^2\epsilon_0}{2}$

(d) None of these

Answer: (a)

Solution:

If it was a complete sphere, then flux = $\frac{q}{\epsilon_0}$

For half portion $\phi = \frac{q}{2\epsilon_0}$

Question: Transistor work like switch in

Options:

- (a) Active region
- (b) Cutoff and saturation region
- (c) Cutoff region only
- (d) Saturated region only

Answer: (b)

Solution:

Transistor acts as a switch in the saturation region and cutoff region.

Question: An electron makes a transition from lower orbit showing energy E_1 to higher orbit having energy E_2 by absorbing a photon of frequency 'f' then

Options:

- (a) $f = \frac{h}{(E_1 - E_2)}$
- (b) $f = \frac{(E_2 - E_1)}{h}$
- (c) $f = \frac{h^2}{(E_1 - E_2)}$
- (d) None of these

Answer: (b)

Solution:

Energy of photon = hf

Energy required for transition = $E_2 - E_1$

$$hf = E_2 - E_1$$

$$f = \frac{(E_2 - E_1)}{h}$$

Question: If a ray is refracted to denser medium then

Options:

- (a) Speed, wavelength and frequency remains constant
- (b) Speed and wavelength decreases, frequency constant
- (c) Speed, wavelength and frequency increases
- (d) Speed, wavelength and frequency decreases

Answer: (b)

Solution:

When a ray goes from rarer medium to denser medium then its speed and wavelength decreases but frequency always remains constant. So correct answer is (B).

Question: If a gas with gamma 1.4 expands isobarically, and work done is 400 J, then what is the heat given to the system?

Options:

- (a) 1500J
- (b) 1400J

(c) 2000J

(d) 2200J

Answer: (b)

Solution:

First law of thermodynamics is:

$$Q = p\Delta V + nC_v\Delta T$$

$$= p\Delta V + n\frac{R}{\gamma-1}\Delta T$$

$$= p\Delta V\left(1 + \frac{1}{\gamma-1}\right)$$

$$= 400\left(1 + \frac{1}{0.4}\right)$$

$$= 400 \times 3.5$$

$$= 1400J$$

Question: Dimension of the quantity which is showing Pascal- Second

Options:

(a) $ML^{-1}T^{-1}$

(b) $ML^{-2}T^{-2}$

(c) $ML^{-1}T^{-2}$

(d) MLT^{-2}

Answer: (a)

Solution: Pascal-sec are unit of Pressure – Time

$$= \frac{F}{A} \cdot t = \frac{[MLT^{-2}][T]}{[L^2]} = [ML^{-1}T^{-1}]$$

Question: A particle moving in vertical circle tied to string. Velocity at bottom is u. Magnitude of change in velocity when string becomes horizontal is $v = \sqrt{x(u^2 - gR)}$. Find value of x?

Options:

(a) 2

(b) 3

(c) 4

(d) 5

Answer: (a)

Solution: When string is horizontal, let's assume speed to be v then by conservation of energy

$$mgR + \frac{1}{2}mv^2 = \frac{1}{2}mu^2$$

$$\Rightarrow v^2 = u^2 - 2gR$$

$$\Rightarrow v = \sqrt{u^2 - 2gR}$$

Final velocity vector is $\hat{v} = \sqrt{u^2 - 2gR}\hat{j}$

Initial velocity vector is $\hat{u} = u\hat{i}$

$$|\Delta\hat{v}| = \sqrt{|\hat{v}|^2 + |\hat{u}|^2}$$

$$= \sqrt{u^2 - 2gR + u^2} = \sqrt{2u^2 - 2gR}$$

$$= \sqrt{2(u^2 - gR)}$$

Question: A stone is dropped from 4.9 m above water level. It entered with velocity v and then continue at constant velocity v . It took 4 seconds from throw to reach ground. Find depth of water.

Options:

- (a) 20.4 m
- (b) 29.4 m
- (c) 2.94 m
- (d) 19.4 m

Answer: (b)

Solution: Velocity after falling through a height of 4.9 m

$$v^2 - u^2 = 2as$$

$$v^2 - 0 = 2(9.8)(4.9)$$

$$v = 9.8 \text{ m/s}$$

Time taken to fall by 4.9 m

$$9.8 = 0 + 9.8t$$

$$\Rightarrow t = 1 \text{ sec}$$

Rest of 3 sec it went inside water with constant speed 9.8 m/s

$$\therefore \text{Depth of water } 9.8 \times 3 = 29.4 \text{ m}$$

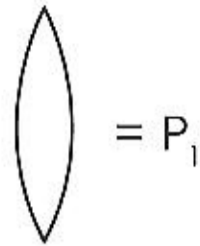
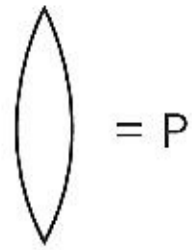
Question: A lens of power P is cut into two parts horizontally. Then one of the parts is equally divided into two vertical parts. Let's assume that P_1 is the power of the half lens and P_2, P_3 be the powers of the quarter lenses. Then choose the wrong option

Options:

- (a) $P_1 = \frac{P}{2}$
- (b) $P_2 = P_1$
- (c) $P_3 = P_2$
- (d) $P_3 = \frac{P_1}{2}$

Answer: (d)

Solution:



$$P_2 = \cup, \cup = P_3$$

$$P_1 = \frac{P}{2} \text{ and } P_3 = P_2 = P_1 = \frac{P}{2}$$

Question: SHM $x = \sin \pi \left(t + \frac{1}{3} \right)$. Find V at $t = 1$

Options:

- (a) 1.5 m/s
- (b) 15.10 m/s
- (c) 2.8 m/s
- (d) 1.57 m/s

Answer: (d)

Solution: $x = \sin \pi \left(t + \frac{1}{3} \right)$

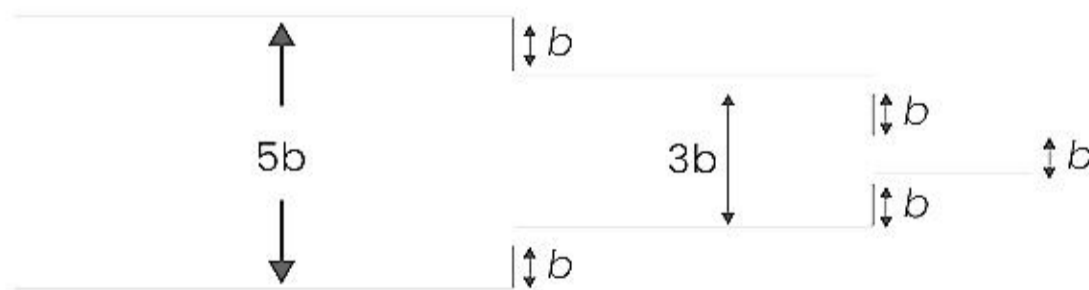
$$v = \frac{dx}{dt} = \cos \pi \left(t + \frac{1}{3} \right)$$

At $t = 1$

$$v = \pi \cos \left(\frac{4\pi}{3} \right) = -\frac{\pi}{2}$$

$$= -1.57 \text{ m/s}$$

Question: 6 capacitor plates are arranged as shown. The area of each plate is A. The capacitance of the arrangement is _____.



Options:

- (a) $\frac{15}{28} \left(\frac{\epsilon_0 A}{b} \right)$

(b) $\frac{23}{15} \left(\frac{\epsilon_0 A}{b} \right)$

(c) $\frac{15}{22} \left(\frac{\epsilon_0 A}{b} \right)$

(d) $\frac{17}{23} \left(\frac{\epsilon_0 A}{b} \right)$

Answer: (b)

Solution: All are parallel

$$C_1 = \frac{\epsilon_0 A}{5b}, C_2 = \frac{\epsilon_0 A}{3b}, C_3 = \frac{\epsilon_0 A}{b}$$

$$\text{Equivalent capacitance} = \frac{\epsilon_0 A}{b} \left(\frac{1}{5} + \frac{1}{3} + 1 \right)$$

$$= \frac{23 \epsilon_0 A}{15 b}$$

Question: 4 masses at corners of square m and a mass M at center. Find potential energy of system?

Options:

(a) $\frac{-4\sqrt{2}Gmm_1}{a} - (4 + \sqrt{2}) \frac{Gm^2}{a}$

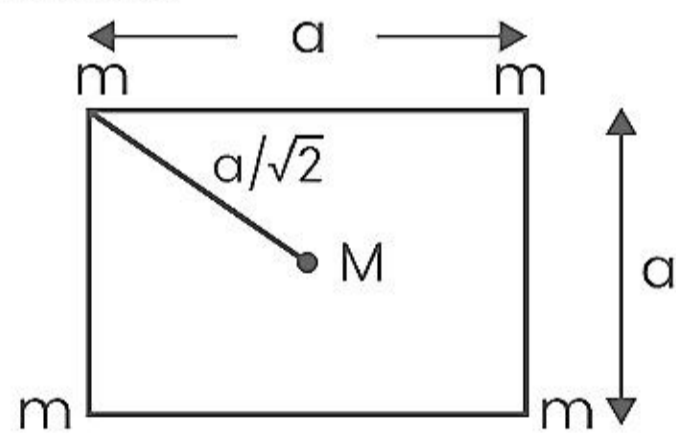
(b) $\frac{4\sqrt{2}Gmm_1}{a} + (4 + \sqrt{2}) \frac{Gm^2}{a}$

(c) $\frac{-4\sqrt{2}Gmm_1}{a} + (4 + \sqrt{2}) \frac{Gm^2}{a}$

(d) $\frac{-4\sqrt{2}Gmm_1}{a} - (4 - \sqrt{2}) \frac{Gm^2}{a}$

Answer: (a)

Solution:



$$U = - \left[4 \frac{GMm}{\left(\frac{a}{\sqrt{2}} \right)} \right] - \frac{4Gm^2}{a} - \frac{2Gm^2}{\sqrt{2}a}$$

$$= - \frac{4\sqrt{2}GMm}{a} - (4 + \sqrt{2}) \frac{Gm^2}{a}$$

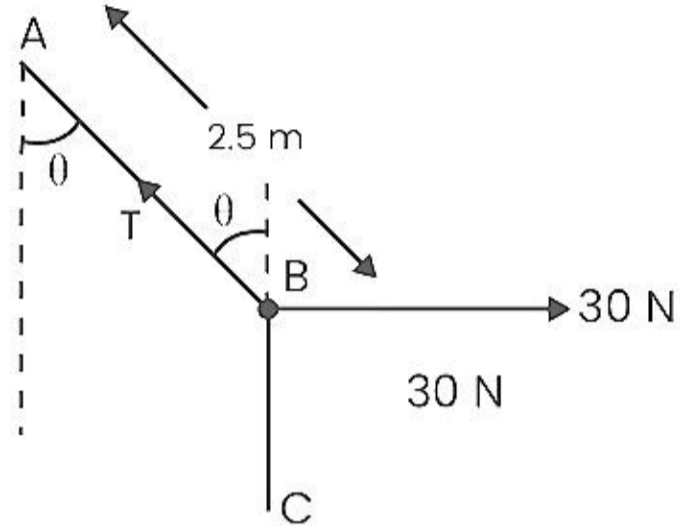
Question: A rope of mass 10 kg and length 5 m is suspended from a ceiling. If a force of 30 N is applied at the middle of the rope, then find the angle made by the rope with vertical.

Options:

- (a) $\frac{2}{5}$
- (b) $\frac{2.5}{5}$
- (c) $\frac{3}{5}$
- (d) $\frac{1}{5}$

Answer: (c)

Solution:



$AB = BC = 2.5\text{m}$ (each has mass 5 kg)

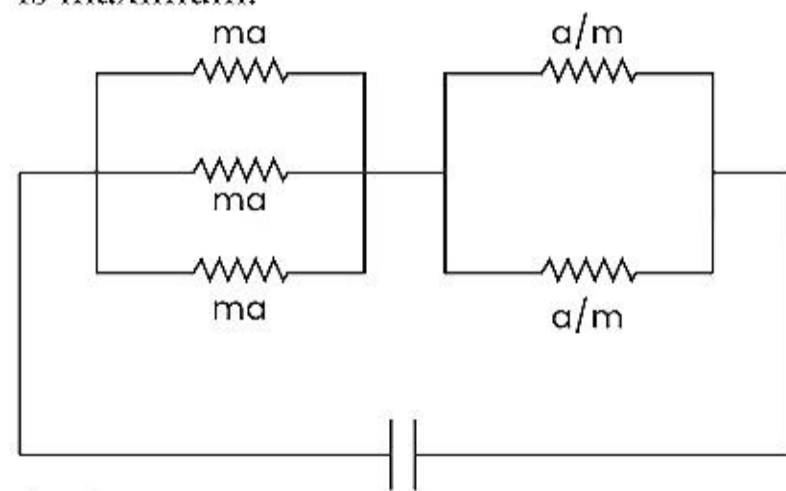
At point B

$$T \cos \theta = 5g = 50$$

$$T \sin \theta = 30$$

$$\therefore \tan \theta = \frac{3}{5}$$

Question: For the figure given, a is constant. Find value of m for which equivalent resistance is maximum.



Options:

- (a) $\sqrt{\frac{1}{3}}$
- (b) $\sqrt{\frac{2}{3}}$
- (c) $\sqrt{\frac{4}{3}}$

(d) $\sqrt{\frac{3}{2}}$

Answer: (d)

Solution: Equation resistance $R = \frac{ma}{3} + \frac{a}{2m}$

$$\frac{dR}{dm} = \frac{a}{3} - \frac{a}{2m^2}$$

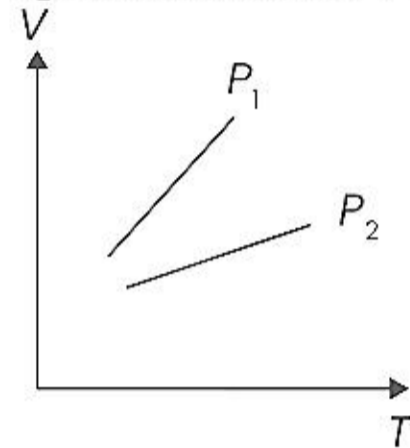
Putting equal to zero

$$\frac{a}{3} - \frac{a}{2m^2} = 0$$

$$\frac{1}{3} = \frac{1}{2m^2} \Rightarrow m^2 = \frac{3}{2}$$

$$\Rightarrow m = \sqrt{\frac{3}{2}}$$

Question: For the V-T graph we can say



Options:

(a) $P_1 < P_2$

(b) $P_1 > P_2$

(c) $P_1 = P_2$

(d) No relationship can be obtained

Answer: (a)

Solution: $PV = nRT$

$$V = \frac{nRT}{P}$$

$\frac{nR}{P}$ is the slope

More slope = less pressure

$$\therefore P_1 < P_2$$

JEE-Main-27-06-2022-Shift-2 (Memory Based)

Chemistry

Question: Assertion: Fluorine forms only one oxo acid

Reason: It is small and electronegative

Options:

- (a) Both assertion and reason are true, reason is correct explanation of assertion.
- (b) Both assertion and reason are true, but reason is not a correct explanation of assertion.
- (c) Assertion is true, but reason is false
- (d) Assertion is false, but reason is true

Answer: (a)

Solution: Due to high electronegativity and small size, fluorine forms only one oxoacid, HOF known as fluoric (I) acid or hypofluorous acid.

Question: The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is

Options:

- (a) N_2O
- (b) NH_3
- (c) N_2
- (d) Cl_2

Answer: (c)

Solution: $NH_4Cl(aq) + NaNO_2(aq) \rightarrow N_2(g) + 2H_2O(l) + NaCl(aq)$

Question: Which of the following ions have half & completely filled f-orbital respectively in lanthanides ions?

[Given Atomic No.: Eu-63, Sm-62, Tm-69, Tb-65, Yb-70, Dy-66]

Options:

- (a) Eu^{2+} , Tm^{2+}
- (b) Tb^{4+} , Yb^{2+}

(c) Dy^{3+} , Yb^{3+}

(d) Sm^{2+} , Tm^{3+}

Answer: (b)

Solution: Electronic configuration of $\text{Tb}^{4+} = [\text{Xe}] 4f^7$ and for $\text{Yb}^{2+} = [\text{Xe}] 4f^{14}$

Question: In 3d series, the metal having the highest negative M^{2+}/M standard electrode potential is

Options:

(a) Cu

(b) Fe

(c) Zn

(d) Cr

Answer: (d)

Solution: The element Cr has the highest negative M^{2+}/M standard electrode potential (– 0.90 V).

Question: Correct order of increasing ionic radii of Na^+ , F^- , Mg^{2+} , O^{2-} , N^{3-}

Options:

(a) $\text{O}^{2-} < \text{Na}^+ < \text{F}^- < \text{Mg}^{2+} < \text{N}^{3-}$

(b) $\text{N}^{3-} < \text{Na}^+ < \text{O}^{2-} < \text{Mg}^{2+} < \text{F}^-$

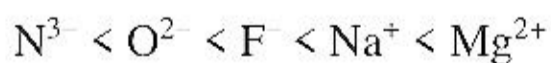
(c) $\text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-} < \text{N}^{3-}$

(d) $\text{Mg}^{2+} < \text{N}^{3-} < \text{O}^{2-} < \text{Na}^+ < \text{F}^-$

Answer: (c)

Solution: The ionic radii of isoelectronic species increases with a decrease in the magnitudes of nuclear charge.

The arrangement of the given species in order of their increasing nuclear charge is as follows:



Therefore, the arrangement of the given species in order of their increasing ionic radii is as follows: $\text{Mg}^{2+} < \text{Na}^+ < \text{F}^- < \text{O}^{2-} < \text{N}^{3-}$

Question: Match the following.

Column-I	Column-II
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A) Antipyretic	i) Reduces pain
B) Analgesic	ii) Reduces stress
C) Tranquilizer	iii) Reduces fever
D) Antacid	iv) Reduces acidity (Smooth)

Options:

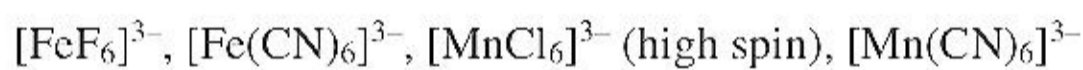
- (a) A → i; B → iv; C → ii; D → iii
 (b) A → iii; B → iv; C → ii; D → i
 (c) A → i; B → ii; C → iii; D → iv
 (d) A → iii; B → i; C → ii; D → iv

Answer: (d)

Solution:

- A) Antipyretic ⇒ Reduces fever
 B) Analgesic ⇒ Reduces pain
 C) Tranquilizer ⇒ Reduces stress
 D) Antacid ⇒ Reduces acidity (Smooth)

Question: Increasing order of magnetic moment

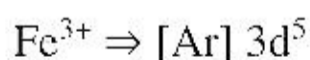
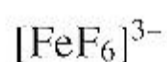


Options:

- (a) $[\text{Fe}(\text{CN})_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-} < [\text{MnCl}_6]^{3-} < [\text{FeF}_6]^{3-}$
 (b) $[\text{Fe}(\text{CN})_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-} < [\text{FeF}_6]^{3-} < [\text{MnCl}_6]^{3-}$
 (c) $[\text{MnCl}_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-} < [\text{FeF}_6]^{3-} < [\text{Fe}(\text{CN})_6]^{3-}$
 (d) $[\text{FeF}_6]^{3-} < [\text{Mn}(\text{CN})_6]^{3-} < [\text{Fe}(\text{CN})_6]^{3-} < [\text{MnCl}_6]^{3-}$

Answer: (a)

Solution:

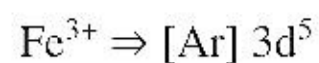


No. of unpaired electrons = 5

As F^- is a weak field ligand

$$\mu = \sqrt{n(n+2)} = \sqrt{5(5+2)} = 5.92 \text{ BM} \dots (1)$$

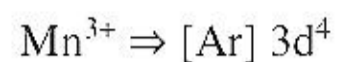
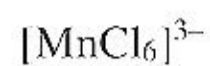




No. of unpaired electrons = 1

As CN^- is a strong field ligand

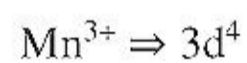
$$\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = 1.73 \text{ BM}$$



No. of unpaired electrons = 4

As Cl^- is a weak field ligand

$$\mu = \sqrt{n(n+2)} = \sqrt{4(4+2)} = 4.89 \text{ BM}$$

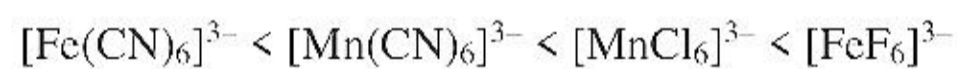


No. of unpaired electrons = 2

As CN^- is a strong field ligand

$$\mu = \sqrt{n(n+2)} = \sqrt{2(2+2)} = 2.87 \text{ BM}$$

Increasing order of magnetic moment



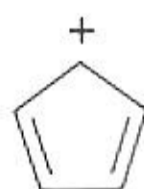
Question: Which of the following species is most stable?

Options:

(a)



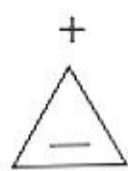
(b)



(c)



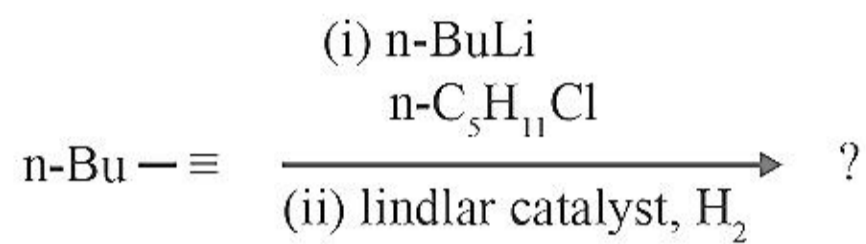
(d)



Answer: (d)

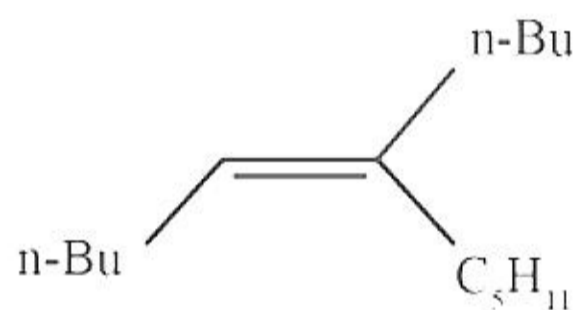
Solution: Due to aromaticity, most stable species is (d).

Question: Major product is

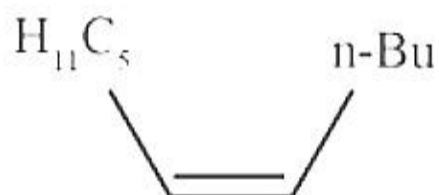


Options:

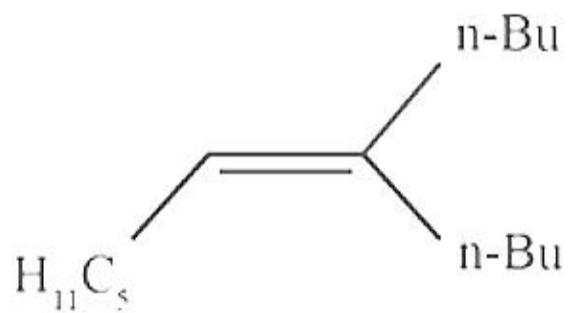
(a)



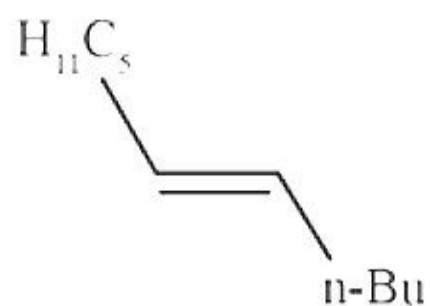
(b)



(c)

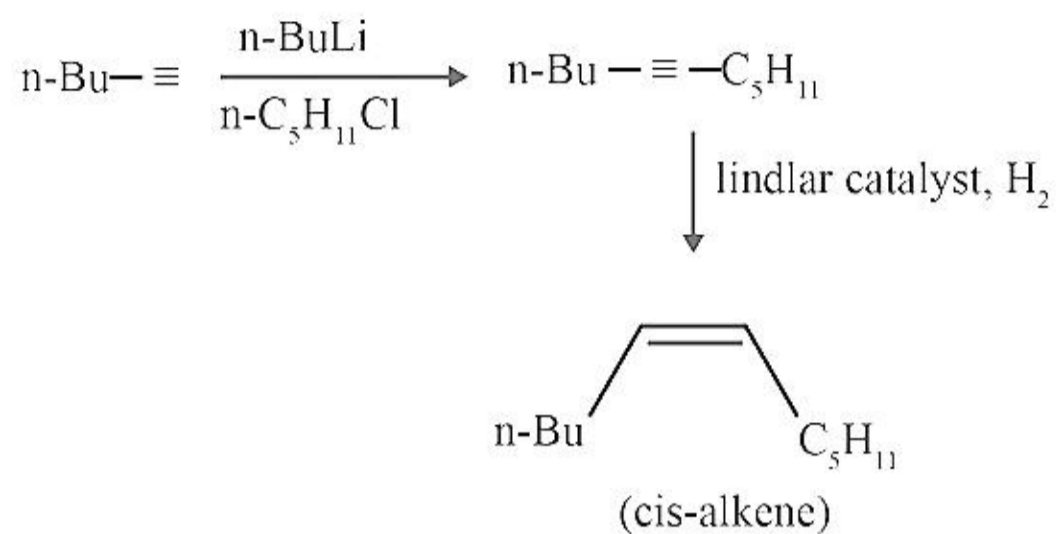


(d)



Answer: (b)

Solution:



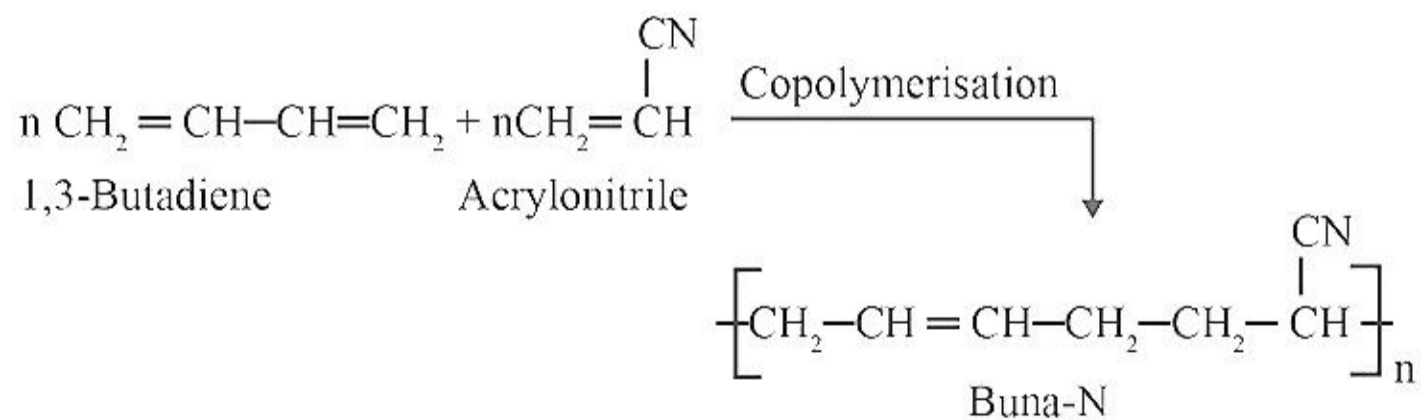
Question: What is the monomer of Buna-N?

Options:

- (a) 1,3-Butadiene and Acrylonitrile
- (b) 2-Chloro-1,3-butadiene
- (c) 1,3-Butadiene and Styrene
- (d) Phenol and formaldehyde

Answer: (a)

Solution:



Question: pH of 10^{-3} M NaOH is

Options:

- (a) 11

(b) 9

(c) 7

(d) 12

Answer: (a)

Solution:

$$[\text{OH}^-] = 10^{-3} \text{ M}$$

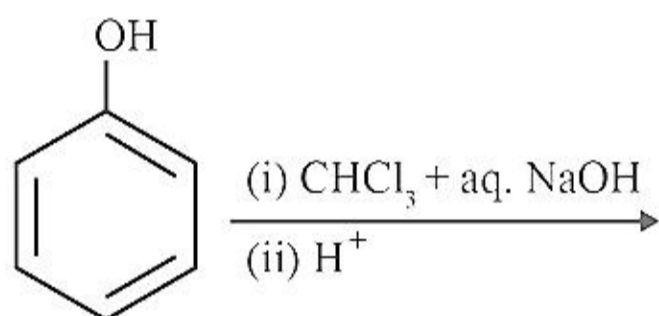
$$\text{pOH} = -\log_{10}[\text{OH}^-]$$

$$\text{pOH} = -\log_{10}10^{-3} = -(-3\log_{10}10) = 3$$

$$\text{pH} + \text{pOH} = 14 \text{ at } 298 \text{ K}$$

$$\text{pH} = 14 - 3 = 11$$

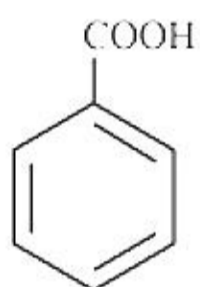
Question: Consider the following reaction,



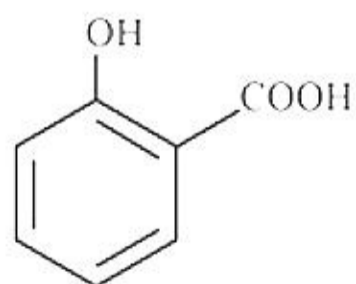
The major product formed in the above reaction is:

Options:

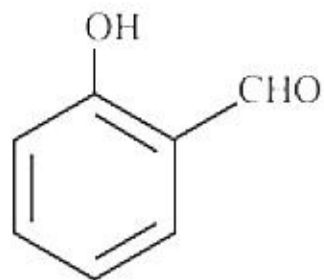
(a)



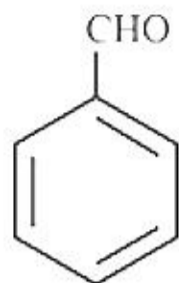
(b)



(c)

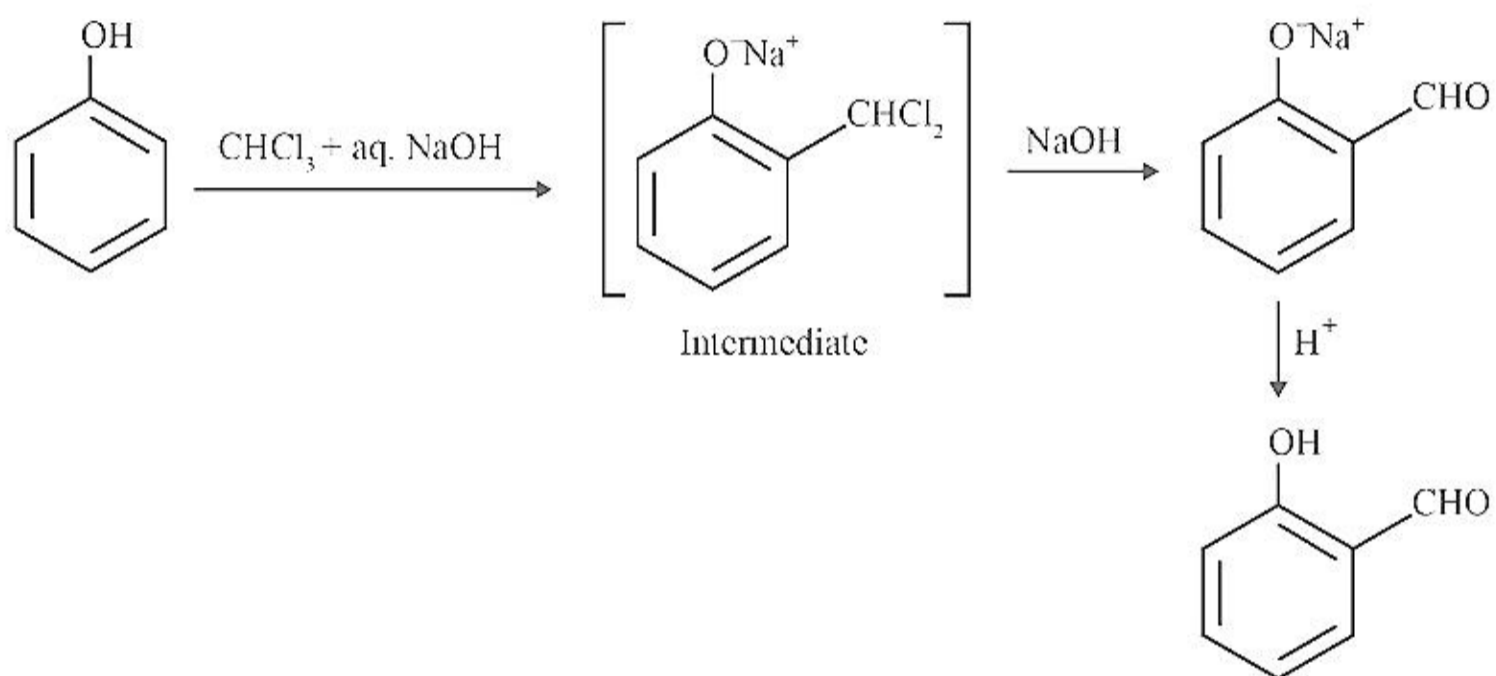


(d)



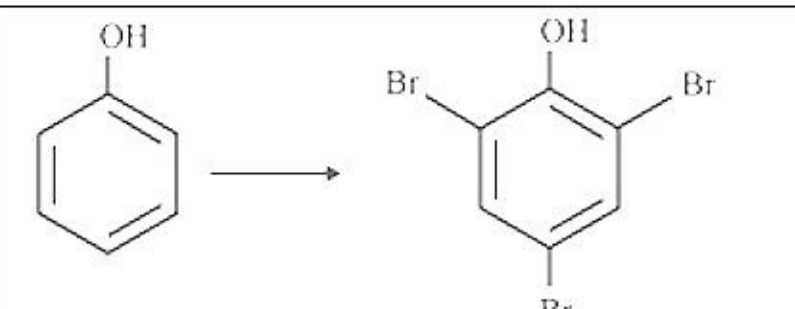
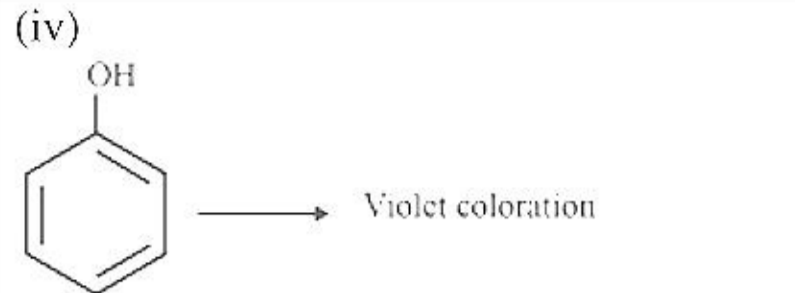
Answer: (c)

Solution:



Question: Match the reagent with organic conversions.

Column-I	Column-II
(i)	(P) $\text{Na}_2\text{Cr}_2\text{O}_7/\text{H}^+$
(ii)	(Q) Zn
(iii)	(R) FeCl_3

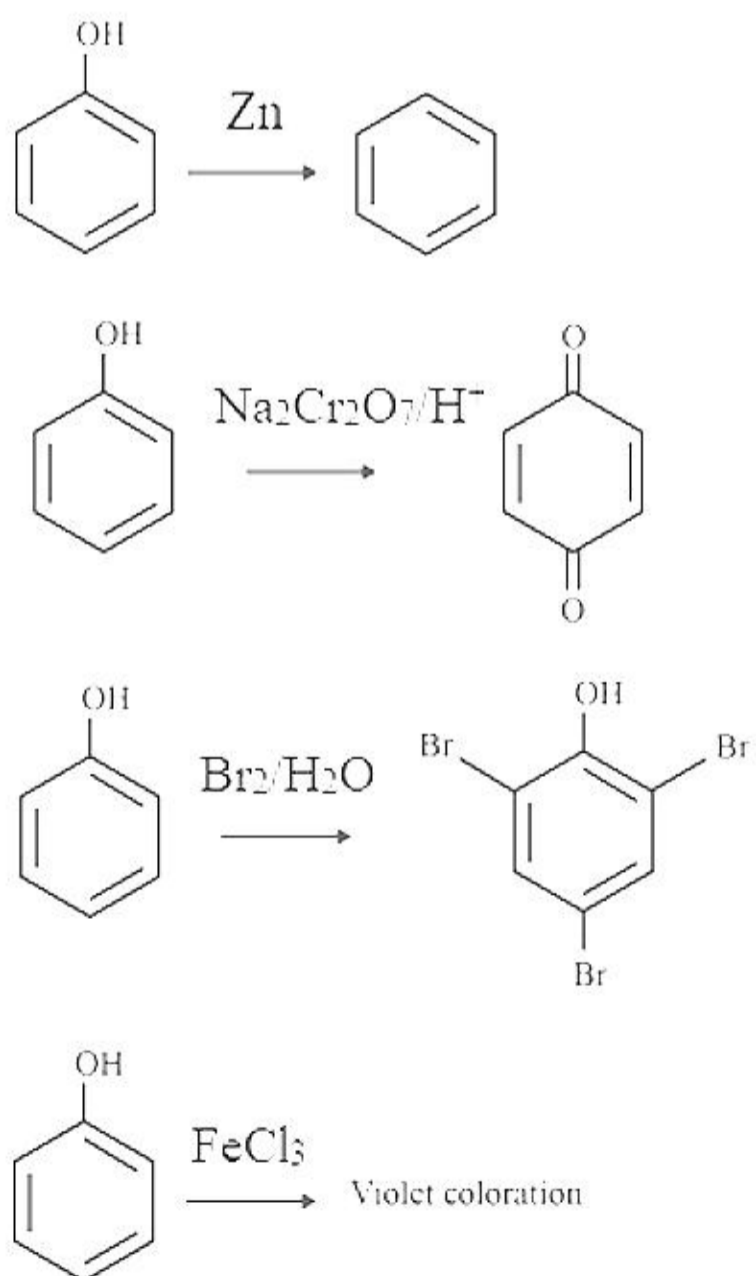
	
<p>(iv)</p> 	(S) Br ₂ /H ₂ O

Options:

- (a) i → Q; ii → P; iii → S; iv → R
 (b) i → P; ii → Q; iii → R; iv → S
 (c) i → S; ii → R; iii → Q; iv → P
 (d) i → R; ii → P; iii → S; iv → Q

Answer: (a)

Solution:



Question: Match the acid radicals present in column I with their characteristic observation in column II

Column-I	Column-II
i) CO_3^{2-}	(P) Brisk Effervescence
ii) NO_3^-	(Q) White precipitate
iii) SO_4^{2-}	(R) Brown ring
iv) S^{2-}	(S) Rotten egg smell

Options:

- (a) i \rightarrow S; ii \rightarrow R; iii \rightarrow Q; iv \rightarrow P
- (b) i \rightarrow P; ii \rightarrow Q; iii \rightarrow R; iv \rightarrow S
- (c) i \rightarrow P; ii \rightarrow R; iii \rightarrow Q; iv \rightarrow S
- (d) i \rightarrow P; ii \rightarrow R; iii \rightarrow S; iv \rightarrow Q

Answer: (c)

Solution:

i) $\text{CO}_3^{2-} \Rightarrow$ Brisk Effervescence

ii) $\text{NO}_3^- \Rightarrow$ Brown ring

iii) $\text{SO}_4^{2-} \Rightarrow$ White precipitate

iv) $\text{S}^{2-} \Rightarrow$ Rotten egg smell

Question: Statement 1: In extraction of gold, the oxidation state of gold in the cyanide complex formed is +3.

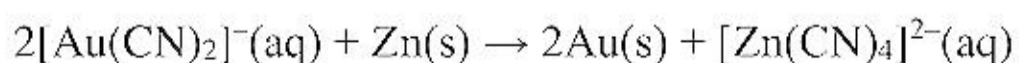
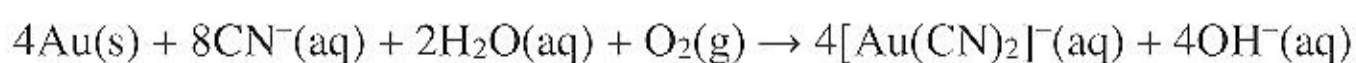
Statement 2: When the cyanide complex is treated with zinc, Zn gets oxidised to +2 state.

Options:

- (a) Statement 1 and statement 2 both are correct.
- (b) Statement 1 is correct but statement 2 is wrong.
- (c) Statement 1 is wrong but statement 2 is correct.
- (d) Statement 1 and statement 2 both are wrong.

Answer: (c)

Solution:

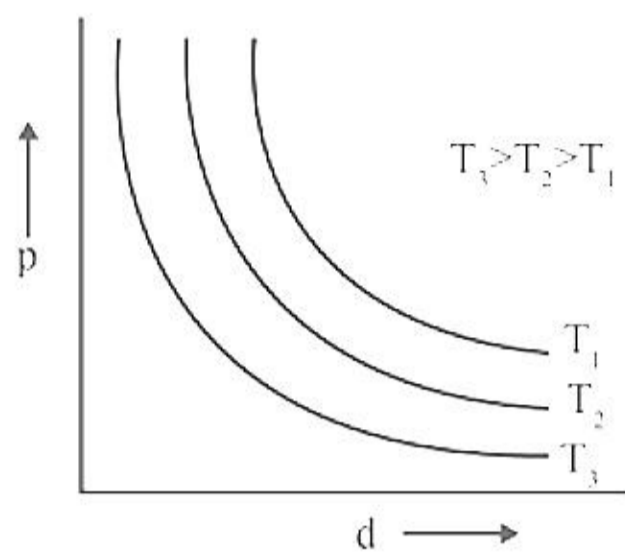


Oxidation state of Gold in complex $[\text{Au}(\text{CN})_2]^-$ is +1 and oxidation state of Zn in Zn complex is +2.

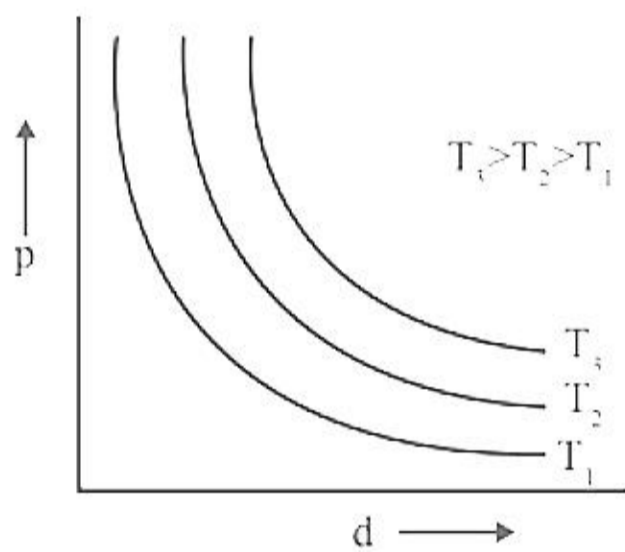
Question: Which of the following curve is correct for an ideal gas?

Options:

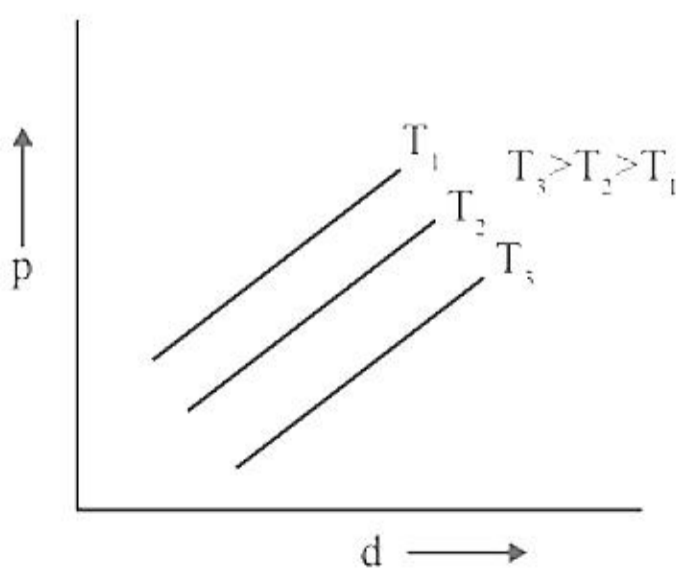
(a)



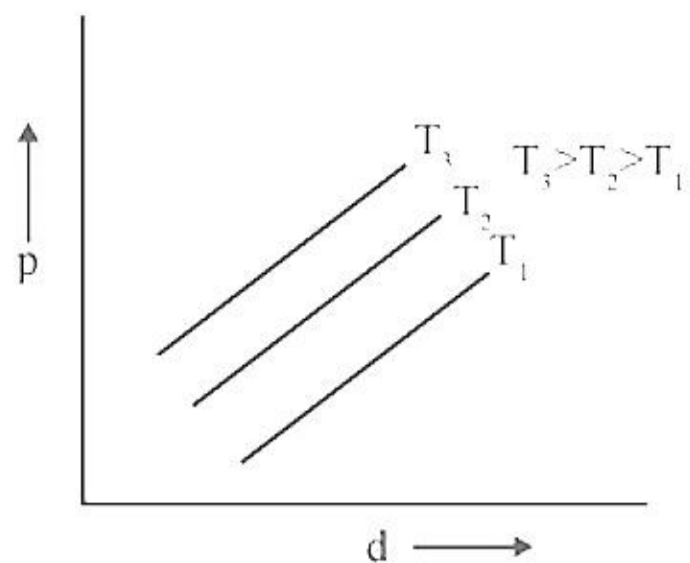
(b)



(c)



(d)



Answer: (d)

Solution:

$$PM = dRT \therefore P \propto d \cdot T$$

So, curve is linear and $T_3 > T_2 > T_1$

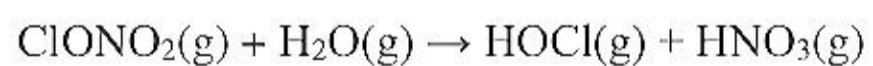
Question: In the stratospheric clouds, the hydrolysis of chlorine nitrate gives product A and B and also when it reacts with HCl it gives product B and C. What is Product A, B and C?

Options:

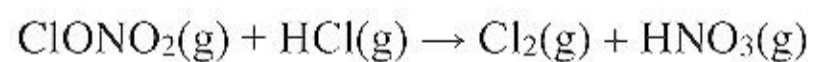
- (a) HOCl, HNO₃, Cl₂
- (b) Cl₂, HNO₃, HOCl
- (c) HClO₂, HNO₂, HOCl
- (d) HOCl, HNO₂, Cl₂O

Answer: (a)

Solution:



(A)



(C) (B)

JEE-Main-27-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: Shortest distance between lines

$$\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$$

$$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

Options:

(a) $\frac{18}{\sqrt{5}}$

(b) $6\sqrt{3}$

(c) $\frac{46}{3\sqrt{5}}$

(d) $\frac{22}{3\sqrt{5}}$

Answer: (a)

Solution:

Given, $L_1 = \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$$L_2 = \frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$$

$$\therefore \text{Shortest distance } d = \frac{|(\bar{a}_2 - \bar{a}_1) \cdot (\bar{b}_1 \times \bar{b}_2)|}{|\bar{b}_1 \times \bar{b}_2|}$$

$$\bar{a}_2 - \bar{a}_1 = (-3\hat{i} + 6\hat{j} + 5\hat{k}) - (3\hat{i} + 2\hat{j} + \hat{k})$$

$$= -6\hat{i} + 4\hat{j} + 4\hat{k}$$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

$$= \hat{i}(10) - \hat{j}(8) + \hat{k}(-4)$$

$$\Rightarrow d = \frac{|-60 - 32 - 16|}{\sqrt{100 + 64 + 16}}$$

$$= \frac{|108|}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

Question: $\alpha = \sin 36^\circ$ is a root of which of the following?

Options:

(a) $16x^4 - 10x^2 + 5 = 0$

(b) $16x^4 + 20x^2 - 5 = 0$

(c) $16x^4 - 20x^2 + 5 = 0$

(d) $16x^4 - 10x^2 - 5 = 0$

Answer: (c)

Solution:

Given, $16x^4 - 20x^2 + 5 = 0$

Let $x^2 = t$

$$\Rightarrow 16t^2 - 20t + 5 = 0$$

$$\Rightarrow t = \frac{20 \pm \sqrt{20^2 - 4 \times 16 \times 5}}{2 \times 16}$$

$$\Rightarrow t = \frac{20 \pm \sqrt{80}}{32}$$

$$\Rightarrow t = \frac{5 \pm \sqrt{5}}{8}$$

$$x^2 = \frac{5 \pm \sqrt{5}}{8} \Rightarrow x = \frac{\sqrt{10 \pm 2\sqrt{5}}}{4}$$

$$x = \frac{\sqrt{10 - 2\sqrt{5}}}{4} = \sin 36^\circ$$

Question: If equation of parabola whose vertex is at $(5, 4)$ and directrix is $3x + y - 29 = 0$ is

$x^2 + ay^2 + bxy + cx + dy + k = 0$ then $a + b + c + d + k$ is

Options:

(a) 576

(b) 575

(c) -575

(d) -576

Answer: (d)

Solution:

Given, vertex $(5, 4)$ & directrix at $3x + y - 29 = 0$

Thus, foot of perpendicular from vertex to directrix

Thus, by calculating, focus will be (2, 3)

Now, equation of parabola

$$\sqrt{(x-2)^2 + (y-3)^2} = \left| \frac{3x+y-29}{\sqrt{9+1}} \right|$$

$$10[x^2 + 4 - 4x + y^2 + 9 - 6y] = 9x^2 + y^2 + 841 + 6xy - 174x - 58y$$

$$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$$

Comparing with

$$x^2 + ay^2 + bxy + cx + dy + k = 0$$

$$a=9, b=-6, c=134, d=-2, k=-711$$

$$\therefore a+b+c+d+k = -576$$

Question: Let foot of perpendicular from point $(1, a, h)$ on line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P .

Distance of P from the plane $3x + 4y + 12z + 23 = 0$

Options:

(a) $\frac{63}{13}$

(b) 5

(c) $\frac{50}{13}$

(d) 4

Answer: (b)

Solution:

$$\text{Given, } \frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = k$$

\therefore Any point on line is

$$P(4k-2, 2k+1, 3k-1)$$

Let $(1, 2, 4)$

$$\text{D.R. of } PQ = 4k-3, 2k-1, 3k-5$$

Now, PQ is perpendicular to line

$$\therefore 4(4k-3) + 2(2k-1) + 3(3k-5) = 0$$

$$16k - 12 + 4k - 2 + 9k - 15 = 0$$

$$29k - 29 = 0$$

$$k = 1$$

$$\therefore P(2, 3, 2)$$

Now, distance of P from $3x + 4y + 12z + 23 = 0$ is

$$= \frac{|6+12+24+23|}{\sqrt{9+16+144}}$$

$$= \frac{65}{13} = 5$$

Question: If m, n are number of local maximum and local minimum points of function

$$f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2te^t} dt. \text{ Then ordered pair } (m, n) \text{ is}$$

Options:

(a) (3, 4)

(b) (2, 3)

(c) (3, 2)

(d) (2, 2)

Answer: (b)

Solution:

$$\text{Given, } f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2te^t} dt$$

$$f'(x) = \frac{x^4 - 5x^2 + 4}{2x^2 e^{x^2}} (2x) - 0$$

$$= \frac{(x^2 - 4)(x^2 - 1)x}{x^2 e^{x^2}}$$

$$= \frac{(x-2)(x+2)(x-1)(x+1)x}{x^2 e^{x^2}}$$

\therefore Number of local maximum (m) = 2

Number of local minimum (n) = 3

$\therefore (m, n) = (2, 3)$

Question: Let \vec{a} and \vec{b} be two vectors along diagonals of parallelogram having area $2\sqrt{2}$. Let angle between \vec{a} and \vec{b} be acute. $|\vec{a}| = 1$, $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}(\vec{a} \times \vec{b}) - 2\vec{b}$ then acute angle between \vec{b} and \vec{c} is

Options:

(a) $\frac{-\pi}{4}$

$$(b) \frac{5\pi}{6}$$

$$(c) \frac{\pi}{4}$$

$$(d) \frac{3\pi}{4}$$

Answer: (d)

Solution:

$$\frac{1}{2}|\bar{a} \times \bar{b}| = 2\sqrt{2}$$

$$\Rightarrow |\bar{a} \times \bar{b}| = 4\sqrt{2}$$

$$|a| = 1, |a \cdot b| = |a \times b|$$

$$|a||b|\cos\theta = |a||b|\sin\theta$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

$$\text{Now, } |a||b|\sin\theta = 4\sqrt{2}$$

$$\Rightarrow |b| = 8$$

$$\bar{c} = 2\sqrt{2}(\bar{a} \times \bar{b}) - 2\bar{b}$$

$$\bar{c} \cdot \bar{b} = 2\sqrt{2}(a \times b) \cdot b - 2|b|^2$$

$$c \cdot b = -2|b|^2 = -64$$

$$|c| \cdot |b| \cdot \cos\alpha = -64$$

$$|c| \cos\alpha = -8$$

$$|c|^2 = |2\sqrt{2}a \times b - 2\bar{b}|^2$$

$$= 4|\sqrt{2}a \times b - b| \cdot |\sqrt{2}a \times b - b|$$

$$= 4|4(a \times b)^2 - 2\sqrt{2}(a \times b) \cdot b + |b|^2|$$

$$= 4|4 \times 32 - 0 + 64|$$

$$= 4 \times 64 \times 2$$

$$|c| = 16\sqrt{2}$$

$$\Rightarrow 16\sqrt{2} \cos\alpha = -8$$

$$\Rightarrow \cos\alpha = \frac{-1}{\sqrt{2}}$$

$$\Rightarrow \alpha = \frac{3\pi}{4}$$

Question: Let A and B be two 3×3 matrices such that $AB = I$, $|A| = \frac{1}{8}$. Then $|\text{adj}(B \text{adj}(2A))|$

Options:

(a) 32

(b) 128

(c) 64

(d) 16

Answer: (c)

Solution:

Given, $AB = I$, $|A| = \frac{1}{8}$

$$|\text{adj } B(\text{adj } 2A)|$$

$$= |B \text{adj}(2A)|^2$$

$$= |B|^2 |\text{adj}(2A)|^2$$

$$= |B|^2 (|2A|)^2$$

$$= |B|^2 ((2^3 |A|)^2)^2$$

$$= |B|^2 (2^{12}) |A|^4$$

$$= 2^{12} |B|^2 |A|^4$$

$$= 2^{12} |A|^2$$

$$= \frac{2^{12}}{2^6} = 2^6 = 64$$

Question: Let f be a differential function is $\left(0, \frac{\pi}{2}\right)$. If $\int_{\cos x}^1 t^2 f(t) dt = \sin^3 x + \cos x$. Then

$\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to

Options:

(a) $\frac{9}{\sqrt{2}} - 6$

(b) $6 - \frac{9}{\sqrt{2}}$

(c) $6 - 9\sqrt{2}$

(d) $\frac{9}{2} - 6\sqrt{2}$

Answer: (b)

Solution:

$$\text{Given, } \int_{\cos x}^1 t^2 f(t) dt = \sin^3 x + \cos x$$

Differentiating w.r.t. x

$$\cos^2 x f(\cos x) \cdot \sin x = 3 \sin^2 x \cdot \cos x - \sin x$$

$$f(\cos x) = \frac{3 \sin x \cos x}{\cos^2 x} - \frac{1}{\cos^2 x}$$

$$f(\cos x) = 3 \tan x - \sec^2 x$$

Again differentiating

$$-f'(\cos x)(\sin x) = 3 \sec^2 x - 2 \sec x \cdot \sec x \tan x$$

$$f'(\cos x) = \frac{2 \sec^2 x \cdot \tan x}{\sin x} - \frac{3 \sec^2 x}{\sin x}$$

$$f'(\cos x) = \frac{2}{\cos^2 x} - \frac{3}{\cos^2 x \cdot \sin x}$$

$$\text{Now, put } \cos x = \frac{1}{\sqrt{3}}, \sin x = \frac{\sqrt{2}}{\sqrt{3}}$$

$$f'\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\left(\frac{1}{\sqrt{3}}\right)^3} - \frac{3}{\left(\frac{1}{\sqrt{3}}\right)^2 \left(\frac{\sqrt{2}}{\sqrt{3}}\right)}$$

$$\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right) = 6 - \frac{9}{\sqrt{2}}$$

Question: Find the number of complex number z , show that $|z - (4 + 3i)| = 2$ &

$$|z| + |z - 4| = 6$$

Options:

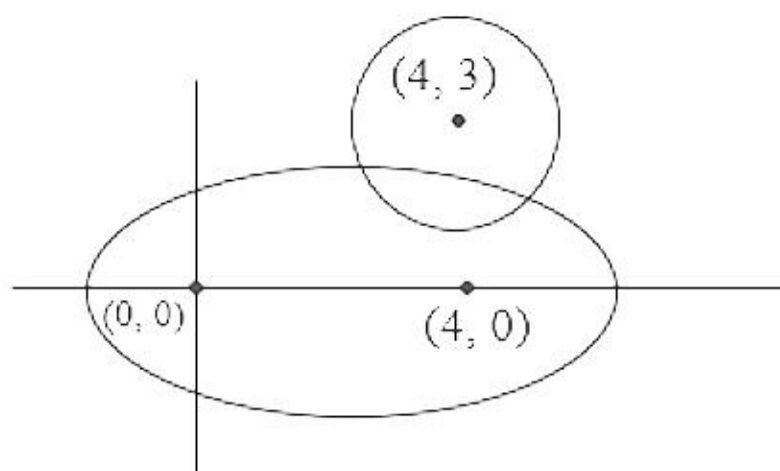
- (a) 1
- (b) 2
- (c) 3
- (d) 4

Answer: (b)

Solution:

$$|z - (4 + 3i)| = 2 \text{ and } |z| + |z - 4| = 6$$

$$c \equiv (4, 3), r = 2$$



Circle and ellipse intersect at 2 points

Question: Which of the following is a tautology?

Options:

(a) $(\neg p \wedge q) \vee (p \vee \neg p)$

(b) $(p \rightarrow q) \vee q$

(c) $(p \leftrightarrow q) \vee (p \wedge q)$

(d) $p \wedge (p \leftrightarrow q)$

Answer: (a)

Solution:

$$(\neg p \wedge q) \vee (p \vee \neg p)$$

$$\equiv (\neg p \wedge q) \vee T$$

$$\equiv T$$

Hence $(\neg p \wedge q) \vee (p \vee \neg p)$ is a tautology

Question: For some real numbers α and β , $a = \alpha - i\beta$. If system of equations

$$hx + (1+i)y = 0 \text{ and } 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0 \text{ has more than one solution, then } \frac{\alpha}{\beta} \text{ is}$$

Answer: $2 - \sqrt{3}$

Solution:

For more than one solution

$$\begin{vmatrix} 4i & (1+i) \\ 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) & \bar{a} \end{vmatrix} = 0$$

$$\Rightarrow 4i\bar{a} - (1+i)8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 0$$

$$\Rightarrow 4i\bar{a} - 4(1+i)(-1 + \sqrt{3}i) = 0$$

$$\Rightarrow i\bar{a} - (-1 + \sqrt{3}i - i - \sqrt{3}) = 0$$

$$\Rightarrow i\bar{a} - (-1 - \sqrt{3} + \sqrt{3}i - i) = 0$$

$$\Rightarrow -\bar{a} - (-i - \sqrt{3}i - \sqrt{3} + 1) = 0$$

$$\Rightarrow \bar{a} = \sqrt{3} - 1 + i(1 + \sqrt{3})$$

$$\alpha = \sqrt{3} - 1, \beta = -1 - \sqrt{3}$$

$$\frac{\alpha}{\beta} = \frac{\sqrt{3} - 1}{-1 - \sqrt{3}} = \frac{(\sqrt{3} - 1)(1 - \sqrt{3})}{-(1 + \sqrt{3})(1 - \sqrt{3})}$$

$$= \frac{-(3 + 1 - 2\sqrt{3})}{-(1 + \sqrt{3})}$$

$$= 2 - \sqrt{3}$$

Question: If $y(x) = (x^x)^x, x > 0, \frac{d^2x}{dy^2} + 20$ at $x = 1$ is ____

Answer: 24.00

Solution:

Given, $y = x^{x^2}$

$$\log y = x^2 \log x$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (2x \log x + x)$$

$$\frac{dy}{dx} = y(2x \log x + x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx}(2x \log x + x) + y(2 + 2 \log x + 1)$$

Now, at $x = 1, \frac{dy}{dx} = 1$

$$\frac{d^2y}{dx^2} = 1 + 1(3) = 4$$

$$\therefore \frac{d^2y}{dx^2} + 20 = 4 + 20 = 24$$

Question: $S = 2 + \frac{6}{7} + \frac{12}{49} + \frac{20}{343} + \dots$ Find $4S$.

Answer: $\frac{343}{27}$

Solution:

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \dots$$

$$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \dots$$

$$\frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \dots$$

$$\frac{6S}{49} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \dots$$

$$\left(\frac{6}{7} - \frac{6}{49}\right)S = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \dots$$

$$\frac{36}{49}S = 2\left(1 + \frac{1}{7} + \frac{1}{7^2} + \dots\right)$$

$$\Rightarrow S = 2 \cdot \frac{1}{1 - \frac{1}{7}} \cdot \frac{49}{36}$$

$$\Rightarrow 4S = \left(\frac{7}{3}\right)^3$$

Question: $\cot\left(\sum_{x=1}^{50} \tan^{-1}\left(\frac{1}{1+x+x^2}\right)\right)$

Answer: $\frac{26}{25}$

Solution:

$$\tan^{-1}\left(\frac{1}{1+x+x^2}\right) = \tan^{-1}\left(\frac{(x+1)-x}{1+x(x+1)}\right)$$

$$= \tan^{-1}(x+1) - \tan^{-1}(x)$$

$$\therefore \sum_{x=1}^{50} \tan^{-1}\left(\frac{1}{1+x+x^2}\right) = \tan^{-1}2 - \tan^{-1}(1) + \tan^{-1}3 - \tan^{-1}(2) + \dots + \tan^{-1}(51) - \tan^{-1}(50)$$

$$= \tan^{-1}(51) - \tan^{-1}(1)$$

$$= \tan^{-1}\left(\frac{50}{52}\right)$$

$$= \cot^{-1}\left(\frac{26}{25}\right)$$

$$\therefore \cot \left(\sum_{x=1}^{50} \tan^{-1} \left(\frac{1}{1+x+x^2} \right) \right) = \frac{26}{25}$$

Question: If a_1, a_2, \dots & b_1, b_2, \dots are two AP's such that $a_1 = 2$, $a_{10} = 3$ & $a_1 b_1 = 1 = a_{10} b_{10}$, then find $a_4 b_4$.

Answer: $\frac{28}{27}$

Solution:

$$a_1, a_2, \dots \Rightarrow \text{A.P.}$$

$$b_1, b_2, \dots \Rightarrow \text{A.P.}$$

$$a = 2,$$

$$a_{10} = 3 = a + 9d_1$$

$$\Rightarrow 3 = 2 + 9d_1$$

$$\Rightarrow d_1 = \frac{1}{9}$$

$$a_1 b_1 = 1$$

$$\Rightarrow b_1 = \frac{1}{2}$$

$$b_{10} = \frac{1}{3} = \frac{1}{2} + 9d_2$$

$$\Rightarrow \frac{-1}{6} = 9d_2$$

$$\Rightarrow d_2 = \frac{-1}{54}$$

$$a_4 b_4 = (a + 3d_1)(b + 3d_2)$$

$$= \left(2 + 3 \times \frac{1}{9} \right) \left(\frac{1}{2} + 3 \times \left(\frac{-1}{54} \right) \right)$$

$$= \left(2 + \frac{1}{3} \right) \left(\frac{1}{2} - \frac{1}{18} \right)$$

$$= \left(\frac{7}{3} \right) \left(\frac{16}{2 \times 18} \right)$$

$$= \frac{7}{3} \times \frac{4}{9} = \frac{28}{27}$$

Question: A is 2×2 matrix each element of A is picked from the set $\{0, 1, 2, 3, 4, 5\}$. How many matrices A are possible if sum of its elements is a prime member p , $2 < p < 8$

Answer: 180.00

Solution:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$a+b+c+d=3$$

$$0 \ 0 \ 0 \ 3 \rightarrow 4$$

$$0 \ 0 \ 1 \ 2 \rightarrow 12$$

$$0 \ 1 \ 1 \ 1 \rightarrow 4$$

total = 20

Or $a+b+c+d=5$

$$0 \ 0 \ 0 \ 5 \rightarrow 4$$

$$0 \ 0 \ 1 \ 4 \rightarrow 12$$

$$0 \ 0 \ 2 \ 3 \rightarrow 12$$

$$0 \ 2 \ 2 \ 1 \rightarrow 12$$

$$0 \ 3 \ 1 \ 1 \rightarrow 12$$

$$1 \ 1 \ 1 \ 2 \rightarrow 4$$

total = 56

Or $a+b+c+d=7$

$$0 \ 0 \ 1 \ 6 \rightarrow 12$$

$$0 \ 0 \ 3 \ 4 \rightarrow 12$$

$$0 \ 1 \ 1 \ 5 \rightarrow 12$$

$$0 \ 1 \ 2 \ 4 \rightarrow 24$$

$$0 \ 1 \ 3 \ 3 \rightarrow 12$$

$$0 \ 2 \ 2 \ 3 \rightarrow 12$$

$$1 \ 1 \ 1 \ 4 \rightarrow 4$$

$$1 \ 1 \ 2 \ 3 \rightarrow 12$$

$$1 \ 2 \ 2 \ 2 \rightarrow 4$$

total = 104

$$\text{Answer} = 104 + 56 + 20 = 180$$