JEE-Main-25-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: The number of bijective function $f(1,3,5,7,...,99)\rightarrow(2,4,6,8,...,100)$ if $f(3)\geq f(5)\geq....\geq f(99)$ is:

Options:

(a) ${}^{50}C_1$

(b) ${}^{50}C_2$

(c) $\frac{50!}{2}$

(d) ${}^{50}C_3\times 3!$

Answer: (a)

Solution:

Bijective function means one-one and onto.

That means for every input unique output which is non-repeating so, set $A(1,3,5,7,...,99)$ has 50 elements and set $B(2,4,6,....,100)$ has 50 elements.

Such that $f(3)\geq f(5)\geq....\geq f(99)$

This can be done in ${}^{50}C_1$ ways

Question: If $P(A)=\frac{1}{3},P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$ then $P\left(\frac{A}{B'}\right)+P\left(\frac{A'}{B}\right)=$

Options:

(a) $\frac{5}{8}$

(b) $\frac{4}{9}$

(c) $\frac{29}{24}$

(d) 3

Answer: (c)

Solution:

As $P(A)=\frac{1}{3},P(B)=\frac{1}{5}$ and $P(A\cup B)=\frac{1}{2}$

So, $P(A\cap B)=\frac{1}{30}$

Now, $P\left(\frac{A}{B'}\right)=\frac{P(A\cap B')}{P(B')}=\frac{\frac{1}{3}-\frac{1}{30}}{\frac{4}{5}}=\frac{3}{8}$

And $P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}=\frac{\frac{1}{5}-\frac{1}{30}}{\frac{1}{5}}=\frac{5}{6}$

So, $P\left(\frac{A}{B'}\right)+P\left(\frac{A'}{B}\right)=\frac{3}{8}+\frac{5}{6}=\frac{29}{24}$

Question: Let $f(x)=\left[x^2-2x\right]+|5x-7|$, and let m be minimum value of $f(x)$ and M be maximum value of $f(x)$ in $\left[\frac{5}{4},2\right]$, then:

Options:
(a) $m=-1, M=2$
(b) $m=0, M=3$
(c) $m=-1, M=4$
(d) $m=-2, M=2$

Answer: (a)

Solution:

For $x\in\left[\frac{5}{4},2\right], \left[x^2-2x\right]=-1$

So, $f(x)=-1+|5x-7|$ is lost at $x=\frac{7}{5}$ and greatest at $x=2$

$m=f\left(\frac{7}{5}\right)=-1$

And $M=f(2)=2$

Question: The tangent at the point at $A(1,3)$ & $B(1,-1)$ on the parabola $y^2-2x-2y=1$ meet at point P. Find area of ΔPAB.

Options:
(a) 4
(b) 6
(c) 7
(d) 8

Answer: (b)

Solution:

Tangent at $A(1,3)$

$3y-(x+1)-(y+3)=1$

$\Rightarrow x-2y+5=0$

Tangent at $B(1,-1)$

$-y-(x+1)-(y-1)=1$

$x+2y-1=0$

$\therefore$ P is $\left(-2,\frac{3}{2}\right)$

$$\therefore \text{ Area of } \Delta PAB=\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 1 & -1 & 1 \\ -2 & \frac{3}{2} & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[1-\left(-1-\frac{3}{2}\right)-3(1+2)+1\left(\frac{3}{2}-2\right)\right]$$

$$=\frac{1}{2}\left[-\frac{5}{2}-9-\frac{1}{2}\right]$$

$=6$

Question: If $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ & $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}, \vec{a}.\vec{b}=3,$ find projection of $\vec{b}$ on $\vec{a}-\vec{b}$

Options:

(a) $\frac{2}{\sqrt{21}}$

(b) $\frac{\sqrt{3}}{7}$

(c) $\frac{\sqrt{7}}{3}$

(d) $\frac{2}{3}$

Answer: (a)

Solution:

As $|a\times b|^2+(\vec{a}.\vec{b})^2=|a|^2|b|^2$

$(\sqrt{5})^2+(3)^2=(\sqrt{6})^2|b|^2$

Now, projection of $\vec{b}$ on $\vec{a}-\vec{b}=\dfrac{(\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

$$=\frac{\vec{a}.\vec{b}-|b|^2}{|\vec{a}-\vec{b}|}$$

Now, $\left|\vec{a}-\vec{b}\right|^2 = |a|^2 + |b|^2 - 2\vec{a}.\vec{b} = \frac{7}{3}$

$|a-b| = \frac{\sqrt{7}}{3}$

$\therefore$ Projection is $\frac{3-\frac{7}{3}}{\frac{\sqrt{7}}{3}} = \frac{2}{\sqrt{21}}$

Question: Shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = 7$ and $\frac{7-x}{2} = y-2 = z-6$ is

Options:

(a) $2\sqrt{29}$

(b) 1

(c) $\frac{\sqrt{37}}{29}$

(d) $\frac{\sqrt{29}}{22}$

Answer: (a)

Solution:

$L_1 : \frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1}$

$L_2 = \frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1}$

$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|b_1 \times b_2|}\right|$

Here, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 6\hat{i} + 4\hat{j} + 8\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 14\hat{i} - 4\hat{j} + 6\hat{k}$

$\therefore d = \left|\frac{(14\hat{i} - 4\hat{j} + 6\hat{k})\cdot(6\hat{i} + 4\hat{j} + 8\hat{k})}{\sqrt{36+16+64}}\right| = 2\sqrt{29}$

Question: $\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right) = ?$

Answer: $\frac{1}{32}$

Solution:

$$\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$$

$$\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$$

$$\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

$$\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)=\frac{1}{2^5}=\frac{1}{32}$$

Question: $\sum_{n=1}^{21}\frac{3}{(4n-3)(4n+1)}=?$

Answer: $\frac{63}{85}$

Solution:

$$T_n=\frac{3}{(4n-3)(4n+1)}$$

$$T_n=\frac{3}{4}\left(\frac{4}{(4n-3)(4n+1)}\right)$$

$$=\frac{3}{4}\left[\frac{(4n+1)-(4n-3)}{(4n-3)(4n+1)}\right]$$

$$T_n=\frac{3}{4}\left[\frac{1}{4n-3}-\frac{1}{4n+1}\right]$$

$$\Rightarrow T_1=\frac{3}{4}\left(\frac{1}{1}-\frac{1}{5}\right)$$

$$\Rightarrow T_2=\frac{3}{4}\left(\frac{1}{5}-\frac{1}{9}\right)$$

$$\vdots$$

$$T_{21}=\frac{3}{4}\left(\frac{1}{81}-\frac{1}{85}\right)$$

$$S_{21}=\frac{3}{4}\left(1-\frac{1}{85}\right)$$

$$=\frac{3}{4}\left(\frac{84}{85}\right)=\frac{63}{85}$$

Question: Find remainder when $(11)^{1011}+(1011)^{11}$ is divided by 9.

Answer: 8.00

Solution:

Given, $(11)^{1011}+(1011)^{11}$

$\Rightarrow (9+2)^{1011}+(1008+3)^{11}$

$\Rightarrow 9 \text{ Integer}+2^{1011}+9 \text{ Integer}+311$

$\Rightarrow (2^3)^{337}+3(3^2)^5$

$\Rightarrow (9-1)^{337}$

$\Rightarrow 9 \text{ Integer}-1$

$\Rightarrow 9 \text{ Integer}-1-8+8$

$\therefore$ Remainder will be 8.

Question: $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{2\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$

Answer: 14.00

Solution:

$$\lim_{x\to\frac{\pi}{4}}\frac{2\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$$

Since its $\dfrac{0}{0}$ form, lets apply L-Hospital rule

$$\lim_{x\to\frac{\pi}{4}} 0-\frac{7(\cos x+\sin x)^6(-\sin x+\cos x)}{0-\sqrt{2}\cos 2x.(2)}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{7}{2\sqrt{2}}\frac{(\cos x+\sin x)^5(\cos^2 x-\sin^2 x)}{\cos^2 x}$$

$$\frac{7}{2\sqrt{2}}\left(\sqrt{2}\right)^5=\frac{7\left(2\times 2\sqrt{2}\right)}{2\sqrt{2}}=14$$

Question: If $x^2+px^2+qx+1=0\ (p<q)$ has only one root α, then α belongs to:

Answer:

Solution:

$f(0)=1$

& $f(-1)=-1+p-q+1=p-q<0$

$\therefore f(0)>0$ & $f(-1)<0$

$\therefore f(x)$ must have root between $(-1,0)$