JEE-Main-28-07-2022-Shift-1 (Memory Based)

Physics

Question: A rod of mass M and length L with current I is placed on an inclined plane with angle of inclination 45 degree. A magnetic field of B tesla is applied upwards on this rod such that it is equilibrium. Find current I in terms of M,L, and B:

Options:

(a) $i=\frac{mg}{BL}\sin\theta$

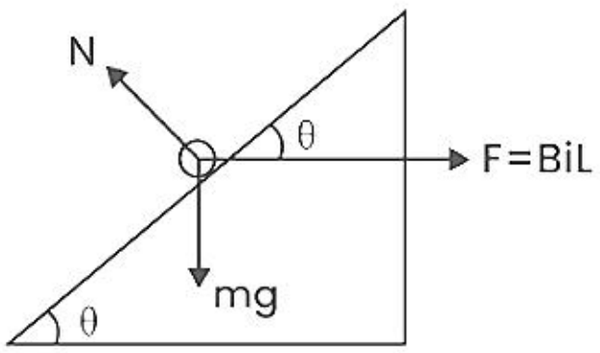
(b) $i=\frac{mg}{BL}\tan\theta$

(c) $i=\frac{BL}{mg}\tan\theta$

(d) $i=\frac{BL}{mg}\sin\theta$

Answer: (b)

Solution:



By lamrim's theorem

$$\frac{BiL}{\sin\left(90+90-\theta\right)}=\frac{mg}{\sin\left(90-\theta\right)}$$

$$\boxed{i=\frac{mg}{BL}\tan\theta}$$

Question: If the projection of vector $\vec{A}=2\hat{i}+4\hat{j}-2\hat{k}$ on $\vec{B}=\hat{i}+2\hat{j}+\alpha\hat{i}$ is equal to zero. Find the value of 'α'.

Options:

(a) $\alpha=5$

(b) $\alpha=4$

(c) $\alpha=6$

(d) $\alpha=2$

Answer: (a)

Solution:

If the projection of a vector is zero on another it means they are $\perp$ to each other.

$\Rightarrow A.B = 0$

$\Rightarrow 2+8-2\alpha = 0$

solving $\alpha = 5$

Question: Find ratio of torque if dipole moment 'p' is placed in electric field of strength 1.5 x 10^{-24} N/c and 4.5 x 10^{-24} N/c respectively.

Options:

(a) $\frac{1}{2}$

(b) $\frac{1}{6}$

(c) $\frac{1}{4}$

(d) $\frac{1}{3}$

Answer: (d)

Solution:

$\frac{1}{3} = \frac{\tau_1}{\tau_2}$

$\tau_{MAX} = PE$

$\frac{\tau_1}{\tau_2} = \frac{PE_1}{PE_2} = \frac{1.5\times10^{-24}}{4.5\times10^{-24}} = \frac{1}{3}$

Question: In case of amplitude modulation to avoid distortion, the modulation index (μ) should be

Options:

(a) $\mu \leq 1$

(b) $\mu > 1$

(c) $\mu = 2$

(d) $\mu = 0$

Answer: (a)

Solution:

Modulation index is defined as the ratio of the amplitude of the modulation wave and the amplitude of the carrier wave for avoiding distortion.

$\mu = \frac{A_m}{A_c}$

$A_c > A_m \therefore \mu \leq 1$

Question: A carnot engine has efficiency 50%. If the temperature of sink is reduced by 40°C its efficiency increases by 30% the temperature of source is?

Options:

(a) 166.67°K

(b) 466.67°K

(c) 266.67°K

(d) 366.67°K

Answer: (c)

Solution:

Initial efficiency = 0.5

$$\text{Initial efficiency } = \frac{0.5\times 30}{100} + 0.5 = 0.65$$

So A to D

$$0.5 = 1 - \frac{T_C}{T_H} \ldots (1)$$

$$0.65 = 1 - \frac{T_C - 40}{T_H}$$

$$0.65 = \left(1 - \frac{T_C}{T_H}\right) + \frac{40}{T_H}$$

$$0.15 = \frac{40}{T_H} \Rightarrow T_H = 266.67k$$

Question: What r dimensions of B^2/μ_0

Options:

(a) $\left[M^{-1}L^{1}T^{-2}\right]$

(b) $\left[M^{1}L^{-1}T^{-2}\right]$

(c) $\left[M^{1}L^{1}T^{-2}\right]$

(d) $\left[M^{-1}L^{-1}T^{2}\right]$

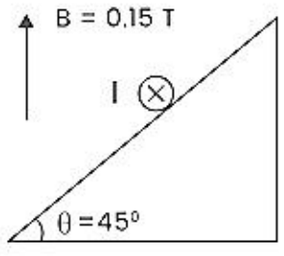
Answer: (b)

Solution:

$$\text{Dimension of } \frac{B^2}{\mu_0} = \frac{\left[MT^{-2}I^{-1}\right]^2}{\left[MLT^{-2}I^{-2}\right]}$$

$$= M^{1}L^{-1}T^{-2}$$

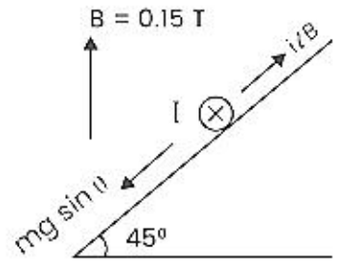
Question: A wire of linear density r = 0.45 kg m^{-1} is in equilibrium on incline as thrown



Find current in wire.

Options:

(a) 10 A
(b) 30 A
(c) 20 A
(d) 40 A
Answer: (b)
Solution:



According to diagram

$$ilB\sin\theta = d\lg\sin\theta$$

$$(\theta = 45°)$$

$$i = \frac{dg}{B}$$

$$i = \frac{0.45\times 10}{0.15}$$

$$i = 30A$$

Question: Two identical capacitors having capacity 40μF are connected in series. If dielectric of dielectric constant K is inserted in one of man the net capacity becomes 24μF. Find K.
Options:
(a) 3.5
(b) 2.5
(c) 1.5
(d) 5.5
Answer: (c)
Solution:
If capacitors are connected n series the

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$24 = \frac{40\times 40k}{(40+40k)}$$

$$960 + 960k = 1600k$$

$$(1600 - 960)k = 96°$$

$$k = \frac{960}{640}$$

k = 1.5

Question: Radioactive sample is 64 time more hazardous than permissible value. Half life of the sample is 2hrs 30 mins. After how many hours it will be safe to operation in the laboratory?

Options:

(a) 10 hrs

(b) 15 hrs

(c) 9 hrs

(d) 7 hrs

Answer: (b)

Solution:

We know

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$N_0 = 64N_0\left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$$

$$\boxed{n = 6}$$

We know

$$n = \frac{6}{68/2} = 6$$

$$n = \frac{t}{2.5} = 6$$

$$t = 15\,\text{hr}$$

Question: In medium with relative permittivity 1 and relative permeability 4, the speed of light is

Options:

(a) 1.5 x 10^8 m/s

(b) 4.5 x 10^8 m/s

(c) 5.5 x 10^8 m/s

(d) 3.5 x 10^8 m/s

Answer: (a)

Solution:

$$\mu = \sqrt{\varepsilon_r \mu_r} = \sqrt{1\times 4} = 2$$

$$\Rightarrow v = \frac{c}{\mu} = \frac{3\times 10^8}{2}$$

$$= 1.5\times 10^8\ m/s$$

Question: Find speed of the wave which is represented by equation $y = -5\sin\frac{2\pi}{\lambda}(400t - x)$

where x and y are in meters t is seconds

Options:

(a) 800 m/s

(b) 400 m/s

(c) 80 m/s

(d) 1200 m/s

Answer: (b)

Solution:

$$y = -5\sin\left(\frac{800\pi}{\lambda}t - \frac{2\pi}{\lambda}x\right)$$

Comparing with $y = A\sin(\omega t - kx)$

$$\omega = \frac{800\pi}{\lambda}$$

$$k = \frac{2\pi}{\lambda}$$

$$V = \frac{\omega}{k} = \frac{800\pi/\lambda}{2\pi/\lambda}$$

$$V = 400 m/s$$

Question: Find percentage change in 'g'. If radius of earth shrinks by 2%,mass remains constant

Options:

(a) 2

(b) 3

(c) 4

(d) 5

Answer: (c)

Solution:

G increased by 4%

$$g = \frac{GM_e}{R_e^2}$$

$$\frac{dg}{g} = \frac{GM_e}{\frac{GM_e}{R_e^2}\frac{(x)}{R_e^3}dR_e}$$

$$\frac{dg}{g} = -2\frac{dR_e}{R_e} = -2(.02) = +0.4$$

$$\frac{dg}{g}\times 100 = 4$$

Question: In a purely inductive AC circuit, the equation of current is given as

$$i = i_0 \sin\left(\omega t - \frac{\pi}{6}\right)$$

The equation of voltage is going to be:

Options:

(a) $i_0 \omega L \sin\left(\omega t + \frac{\pi}{3}\right)$

(b) $i_0 \omega L \sin\left(\omega t - \frac{\pi}{3}\right)$

(c) $i_0 L \sin\left(\omega t + \frac{\pi}{3}\right)$

(d) $i_0 \omega L \, \mathrm{s}\cos\left(\omega t + \frac{\pi}{3}\right)$

Answer: (a)

Solution:

Voltage leads current by $\frac{\pi}{2}$

So phase of voltage $= \left(\omega t - \frac{\pi}{6} + \frac{\pi}{2}\right)$

$V_0 = i_0 Z = i_0 \omega L$

So, $V = i_0 \omega L \sin\left(\omega t + \frac{\pi}{3}\right)$

Question: Find ratio of maximum torque on dipole placed in electric field.

Given $\frac{P_1}{P_2} = 2$

$E_1 = 4.5 \times 10^{-24} \, N/C$ and $E_2 = 1.5 \times 10^{-24} \, N/C$

Options:

(a) 2

(b) 5

(c) 8

(d) 6

Answer: (d)

Solution:

$\tau_{MX} = \rho E$

$$\frac{\tau_1}{\tau_2} = \frac{P_1 E_1}{P_2 E_2} = \frac{2 \times 4.5 \times 10^{-24}}{1.5 \times 10^{-24}} = 6$$

Question: Force required to stretch a wire of cross-section area $1cm^2$ to double its length shall be: (Given Young's modulus of wire 2×10^{11} Pascal. (Assuming no significant change in area)

Options:

(a) 10M pascal

(b) 20M Pascal

(c) 30M Pascal

(d) 40M Pascal

Answer: (b)

Solution:

$$\frac{FL}{AY}=\Delta L$$

To double the length $\Delta L = L$

$$\frac{FL}{AY}=L\Rightarrow F=A.Y$$

$$F=\left(1\times10^{-4}m^2\right)\times\left(2\times10^{11}Pa\right)$$

$$=2\times10^{7}Pa$$

$$=20\text{M Pascal}$$

Question: A monkey sitting on a large tree of height 19.6m drops a mango from its hands. Below on the road soldiers are marching at a speed of 9 km/h. Find the current distance of soldier from the tree who will be able to catch the mango. (Ignore the height of soldier)

Options:

(a) 5 m

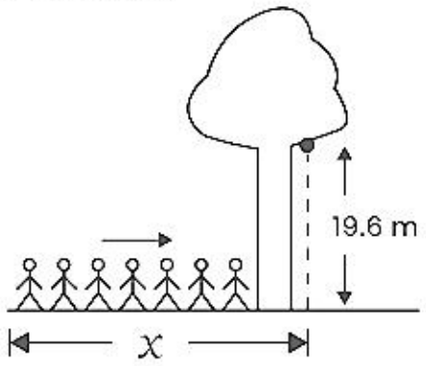
(b) 15 m

(c) 20 m

(d) 25 m

Answer: (a)

Solution:



Speed of soldiers $9\times\frac{5}{18}=2.5m/s$

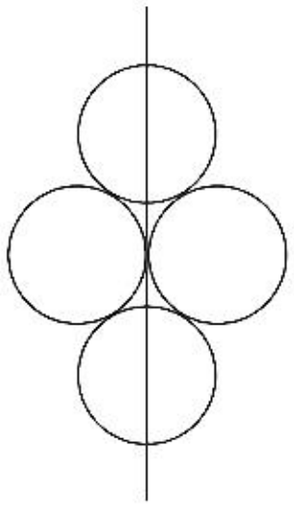
Time taken by mango to reach ground $=\sqrt{\frac{2h}{g}}$

$$t = \sqrt{\frac{2\times 19.6}{9.8}} = 28$$

Hence, distance of soldier $x = v \times t$

$x = 2.5 \times 2 = 5m$

Question: 4 Identical discs are placed as shown. If MOI is $\frac{x}{4}ma^2$. Find X (a = Diameter.)



Options:

(a) 4

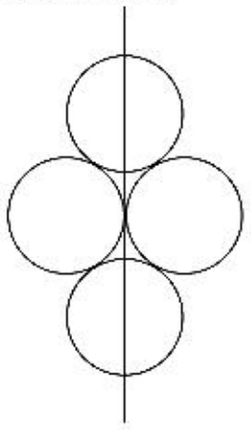
(b) 6

(c) 10

(d) 12

Answer: (d)

Solution:



Radius of each disc is a.

So net $MOI = \left(\frac{Ma^2}{4}\right)\times 2 + \left(\frac{Ma^2}{4} + Va^2\right)\times 2$

Net $MOI = \frac{Ma^2}{2} + \frac{5Ma^2}{2}$

$= \frac{6Ma^2}{2}$

$= 3Ma^2$

Hence after comparing it with $\frac{x}{4}\mu a^2$, X will be 12.

Question: Radioactive sample is 64 time more hazardous than permissible value. Half life of the sample is 30 days. After how many days it will be same to operate in the laboratory?

Options:

(a) 90 days

(b) 120 days

(c) 180 days

(d) 365 days

Answer: (c)

Solution:

We know $N = N_0\left(\frac{1}{2}\right)^n$

$N_0 = 64N_0\left(\frac{1}{2}\right)^n$

$\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^6$

$\boxed{n = 6}$

We know $n = \frac{t}{\frac{t_1}{2}} = 6$

$t = 6 \times 30\, days$

$t = 180\, days$

Question: Assertion: Average momentum of atoms depend on temperature.

Reason: If temperature is doubled and O_2 breaks into 2 'O' atoms; then RMS of 'O' atoms is doubled.

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

(c) If assertion is false, but reason is True.

(d) If both the assertion and reason are false.

Answer: (c)

Solution:

Statement one is false because Average momentum is always zero. And it does not depend on any other physical parameters as well as the molecules are moving randomly.

And Statement 2 is correct because of the formula of rms speed

Question: An electron is accelerated from rest in potential ΔV then de Broglie wavelength is

Options:

(a) 13.27

(b) 13.50

(c) 16.15

(d) 12.27

Answer: (d)

Solution:

We know $\lambda = \frac{h}{P}$ or $\lambda = \frac{h}{\sqrt{2me\Delta V}}$

For e^{-1} (λ in $\mathring{A}$)

$\lambda = \frac{12.27}{\sqrt{\Delta V}} \mathring{A}$

JEE-Main-28-07-2022-Shift-1 (Memory Based)

Chemistry

Question: Which of the following has least tendency to liberate H_2 from mineral acids.
Options:
(a) Cu
(b) Mn
(c) Ni
(d) Zn
Answer: (a)
Solution: Cu cannot displace H_2 from mineral acids because it is below hydrogen in electrochemical series.

Question: Match the gases evolved in following reaction

(Column I)	(Column II)
(A) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(i) H_2
(B) $KMnO_4 + HCl \rightarrow$	(ii) N_2
(C) $Al + NaOH + H_2O \rightarrow$	(iii) O_2
(D) $NaNO_3$ + Heat$\rightarrow$	(iv) Cl_2

Options:
(a) A → (i); B → (iii); C → (ii); D → (iv)
(b) A → (iv); B → (iii); C → (i); D → (ii)
(c) A → (iii); B → (ii); C → (iv); D → (i)
(d) A → (ii); B → (iv); C → (i); D → (iii)
Answer: (d)
Solution:

$$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$$

$$2NaNO_3 \xrightarrow{\Delta} 2NaNO_2 + O_2$$

$$2KMnO_4 + 16HCl \rightarrow 2KCl + 2MnCl_2 + 5Cl_2 + 8H_2O$$

$$2Al + 2NaOH + 6H_2O \rightarrow 2Na[Al(OH)_4]^- + 3H_2$$

Question: What are the monomers of terylene?
Options:
(a) Phenol and formaldehyde
(b) Ethylene glycol and phthalic acid
(c) Adipic acid and hexamethylenediamine
(d) Ethylene glycol and terephthalic acid
Answer: (d)
Solution: Ethylene glycol and terephthalic acid are monomers which on polymerization gives terylene.

Question: Correct statements for enzyme inhibitor drugs-
I. They are competitive and non-competitive drugs

II. They bind at active and allosteric sites
III. Competitive drugs inhibits active site
IV. Non-competitive drugs inhibits allosteric sites
Options:
(a) I and II
(b) II and III
(c) III and IV
(d) All are correct
Answer: (d)
Solution: Drugs compete with the natural substrate for their attachment on the active sites of enzymes. Such drugs are called competitive inhibitors
Some drugs do not bind to the enzyme's active site. These bind to a different site of enzyme which is called allosteric site.

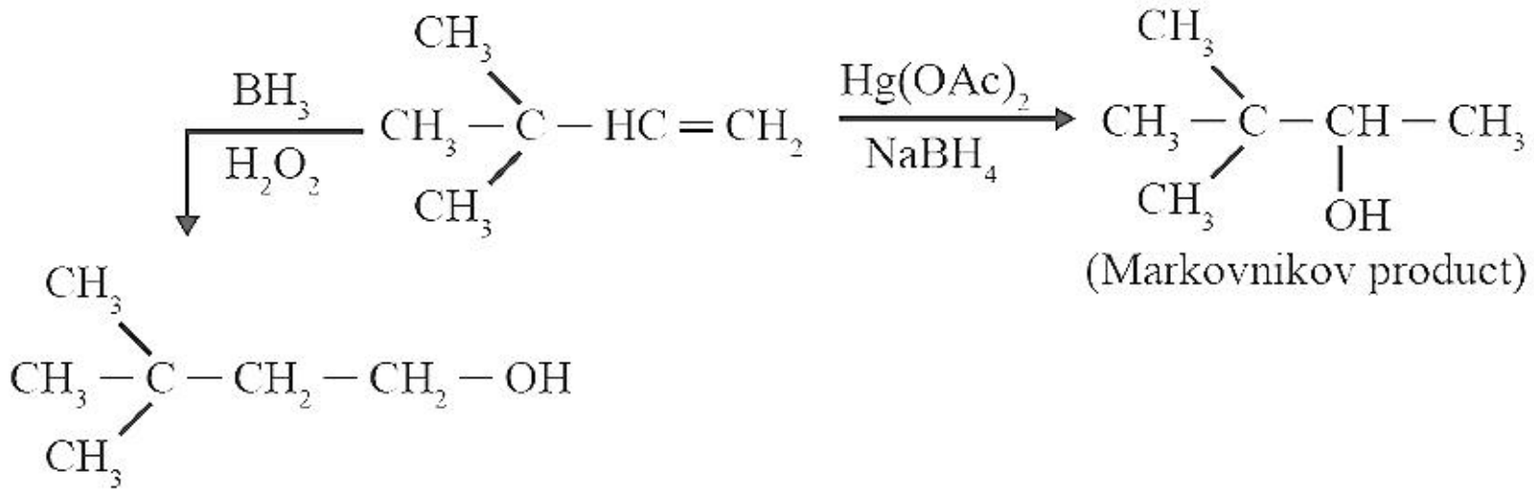
Question: Which pair has same or identical electron gain enthalpy?
Options:
(a) Rb, Cs
(b) I, At
(c) Ar, Kr
(d) Na, k
Answer: (c)
Solution: Ar & Kr have same electron gain enthalpy of 96 kJ/mol

Question: match the following.

(Column I) Reaction	(Column II) Catalyst
(A) Haber Process	(i) V_2O_5
(B) Contact process	(ii) Fe
(C) Ostwald process	(iii) Platinised asbestos

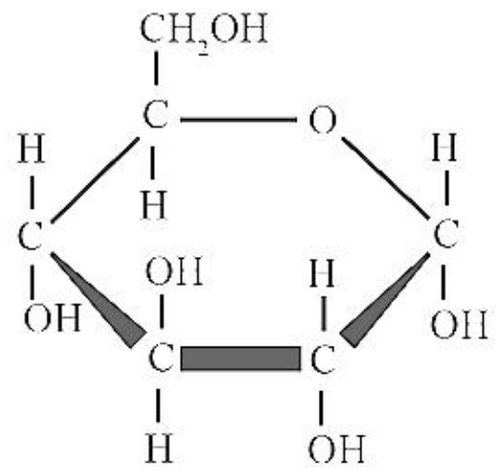
Options:
(a) A → (ii); B → (i); C → (iii)
(b) A → (ii); B → (iii); C → (i)
(c) A → (iii); B → (ii); C → (i)
(d) A → (i); B → (iii); C → (ii)
Answer: (a)
Solution: In Haber's process, catalyst used is Fe; In contact process catalyst used is V_2O_5. In Ostwald's process catalyst used is platinised asbestos

Question: $(CH_3)_3C - CH = CH_2$ was reacted with $Hg(OAC)_2$ and on one side BH_3/H_2O_2 which of the following product will form?
Options:
(a) $Hg(OAC)_2$ – Markovnikov product; BH_3/H_2O_2 – anti Markovnikov product
(b) $Hg(OAC)_2$ – anti Markovnikov product; BH_3/H_2O_2 – Markovnikov product
(c) Both will give Markovnikov product
(d) Both will give anti Markovnikov product
Answer: (a)
Solution:

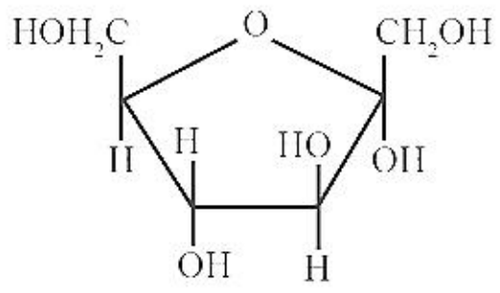


Question: Statement 1: BOD level in polluted water is more than clean water
Statement 2: Eutrophication leads to decrease in oxygen
Options:
(a) Both statement 1 and 2 are correct
(b) Statement 1 is correct but statement 2 is incorrect
(c) Statement 1 is incorrect but statement 2 is correct
(d) Both statement 1 and 2 are incorrect.
Answer: (a)
Solution: Both statements are correct.
Statement-1: BOD is directly proportional to the amount of pollution. So, High BOD indicates highly polluted water.
Statement 2: Oxygen depletion, or hypoxia, is a common consequence of eutrophication, both in fresh water and seawater.

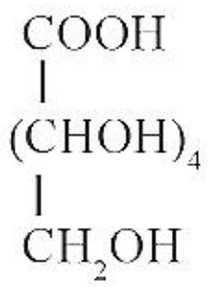
Question: Structure of pyranose is
Options:
(a)



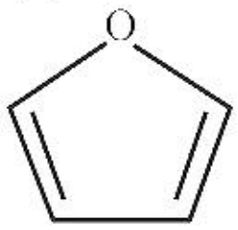
(b)



(c)



(d)



Answer: (a)

Solution: The six membered cyclic structure of glucose is called pyranose structure (α– or β–), in analogy with pyran.

Question: Which reaction is involved in leaching?

Options:

(a) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na[Al(OH)_4]$

(b) $Al_2O_3 \cdot xH_2O \rightarrow Al_2O_3 + xH_2O$

(c) $Al_2O_3 + Mg \rightarrow MgO + Al$

(d) Both (a) and (b)

Answer: (d)

Solution:

$Al_2O_3(s) + 2NaOH(aq) + 3H_2O(l) \rightarrow 2Na[Al(OH)_4](aq)$

$2Na[Al(OH)_4](aq) + 2CO_2(g) \rightarrow Al_2O_3 \cdot xH_2O(s) + 2NaHCO_3(aq)$

$Al_2O_3 \cdot xH_2O(s) \xrightarrow{1470K} Al_2O_3(s) + xH_2O(g)$

Question: Phenol + $Na_2Cr_2O_7 \rightarrow$ Product

Options:

(a) Benzoquinone

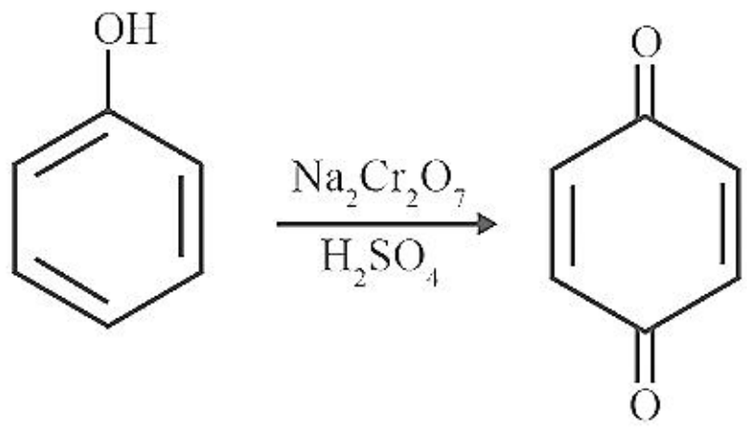
(b) Picric acid

(c) Benzaldehyde

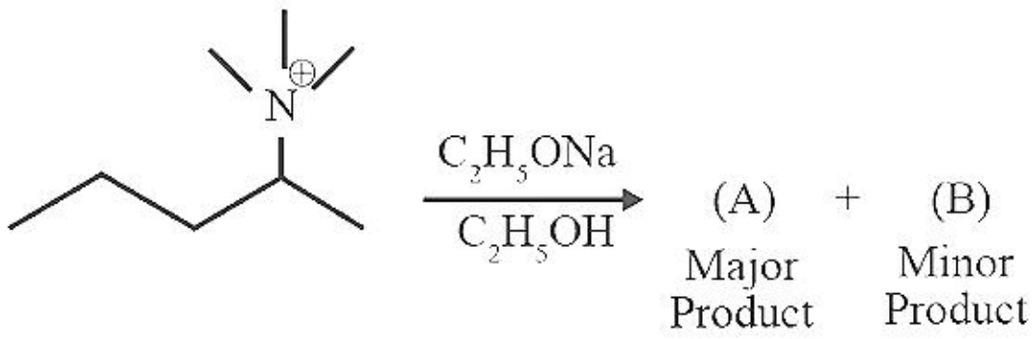
(d) None of these

Answer: (a)

Solution:



Question:



Options:

(a)

(A) , (B) Hofmann product

(b)

(B) , (A) Hofmann product

(c)

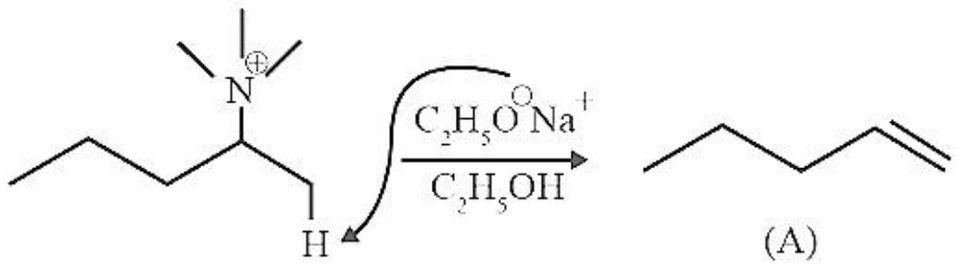
(A) , (B) Saytzeff product

(d)

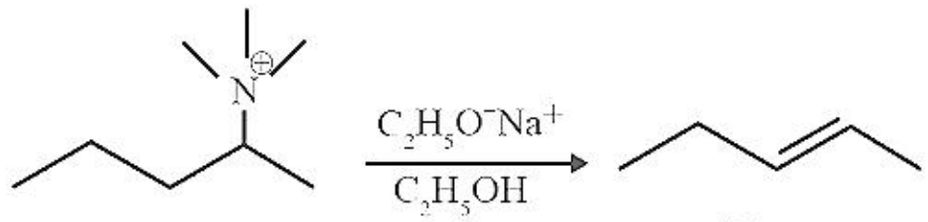
(B) , (A) Saytzeff product

Answer: (b)

Solution:



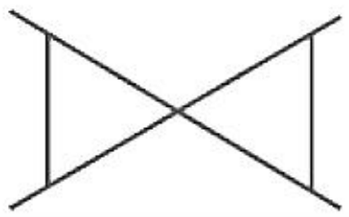
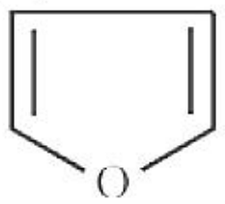
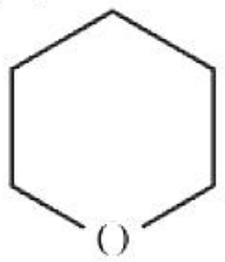
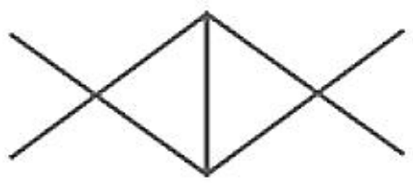
(A)
(Hofmann eliminations product)



(B)
(Minor)

Question: Match the following.

(Column I)	(Column II)
(A)	(i) Aromatic compound

	
(B) 	(ii) Spiro compound
(C) 	(iii) Bicyclic compound
(D) 	(iv) Non-planar heterocyclic compound

Options:

(a) A → (i); B → (iii); C → (ii); D → (iv)

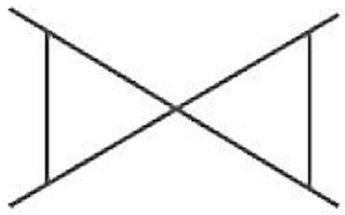
(b) A → (iv); B → (iii); C → (i); D → (ii)

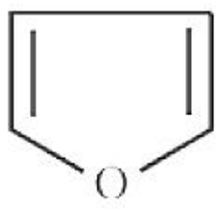
(c) A → (iii); B → (ii); C → (iv); D → (i)

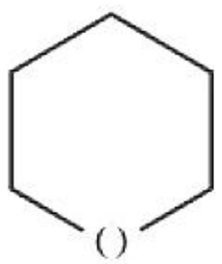
(d) A → (ii); B → (i); C → (iv); D → (iii)

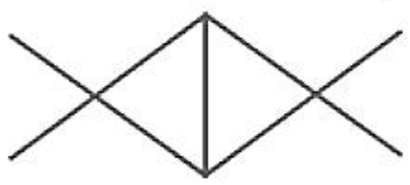
Answer: (d)

Solution:

 ⇒ Spiro compound

 ⇒ Aromatic compound

 ⇒ Non-planar heterocyclic compound

 ⇒ Bicyclic compound

Question: Batteries reaction matching

(Column I)	(Column II)
(A) Primary battery	(i) $Pb(s) + PbO_2(s) + 2H_2SO_4 \downarrow 2PbSO_4(s) + 2H_2O$
(B) Secondary battery	(ii) $Zn(s) + 2NH_4^+ + 2MnO_2 \downarrow [Zn(NH_3)_2]^{2+} + Mn_2O_3 + H_2O$
(C) Mercury cell	(iii) $Zn(Hg) + HgO(s) \downarrow ZnO(s) + Hg(l)$

Options:

(a) A → (iii); B → (ii); C → (i)

(b) A → (ii); B → (i); C → (iii)

(c) A → (i); B → (ii); C → (iii)

(d) A → (ii); B → (iii); C → (i)

Answer: (b)

Solution:

(A) Primary battery ⇒ **(ii)** $Zn(s) + 2NH_4^+ + 2MnO_2 \downarrow [Zn(NH_3)_2]^{2+} + Mn_2O_3 + H_2O$

(B) Secondary battery ⇒ **(i)** $Pb(s) + PbO_2(s) + 2H_2SO_4 \downarrow 2PbSO_4(s) + 2H_2O$

(C) Mercury cell ⇒ **(iii)** $Zn(Hg) + HgO(s) \downarrow ZnO(s) + Hg(l)$

Question: Consider the following reaction:

$$\underset{1\,mol}{X} + \underset{1\,mol}{Y} + \underset{0.05\,mol}{3Z} \rightarrow XYZ_3$$

Calculate the mass of XYZ_3 formed at the end of the reaction (in g).

[Given: molar masses of X, Y and Z are 10, 20, 30 g/mol respectively.]

Answer: 2.00

Solution:

$$\underset{1\,mol}{X} + \underset{1\,mol}{Y} + \underset{0.05\,mol}{3Z} \rightarrow XYZ_3$$

$$\underset{1-\left(\frac{0.05}{3}\right)}{} \quad \underset{\left(\frac{1-0.05}{3}\right)}{} \quad \underset{0}{} \quad \underset{\left(\frac{0.05}{3}\right)}{}$$

(Z is limiting reagent)

Molar mass of $XYZ_3 = 10 + 20 + (3 \times 30) = 120$ g/mol

$$\text{Mass of } XYZ_3 = \frac{0.05}{3} \times 120 = 2 \text{ g}$$

Question: Half life of a reaction is 2 hr 30 min. Calculate the time (in min) required to reduce reactant concentration to $\frac{1}{64}$ (Round off to nearest integer)

Answer: 899.00

Solution:

$t_{1/2} = 2$ hr 30 min

$$[A_{final}] = \frac{1}{64}[A_{initial}]$$

$$t = \frac{2.303}{k}\log\frac{[A_{initial}]}{[A_{final}]}$$

$$t = \frac{2.303}{0.693} \times (t_{1/2}) \log\left[\frac{64}{1}\right]\left[\because t_{1/2} = \frac{0.693}{k}\right]$$

$t = 3.32 \times (150) \log(64)$

$= 498 \times 1.8060$

$t = 899.38$ min

Question: How many of the following are paramagnetic?

O_2^{2-} , He_2^-, C_2 , B_2, Li_2, O_2^+, O_2^-

Answer: 5.00

Solution: He_2^+, C_2^-, B_2, O_2^- and O_2^-

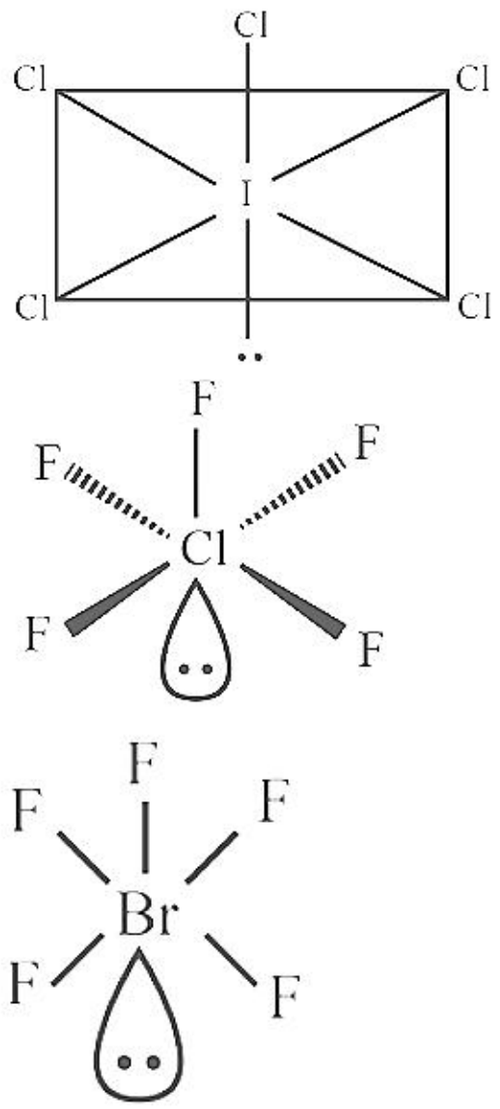
These are paramagnetic species due to presence to unpaired electron

Question: Number of structure having square pyramidal shape is/are

ICl_2, ICl_5, BrF_5, ClF_5

Answer: 3.00

Solution:



Question: MnO_4^{2-} disproportionates in acidic medium to form two compounds of manganese A and B. Oxidation state of Mn is less in B than in A. Spin only magnetic moment of B (in B.M.) is: (Round off to nearest integer)

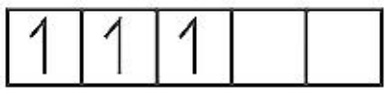
Answer: 4.00

Solution:

$$3\overset{(+6)}{\underset{(A)}{Mn}}O_4^{2-} + 4H^+ \rightarrow 3\overset{(+7)}{\underset{(A)}{Mn}}O_4^- + \overset{(+4)}{\underset{(B)}{Mn}}O_2 + 2H_2O$$

MnO_2 – Mn is present in +4 oxidation state

Mn^{+4} – $[Ar]3d^3$



n = 3

$$\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}$$

$= 3.89 \text{ BM} \approx 4 \text{ BM}$

Question: K_a for C_3H_7COOH is 2×10^{-5}. Find pH of 0.2 M solution

Answer: 9.00

Solution:

$$\underset{(0.2-x)}{C_3H_7COO^-} + H_2O \rightleftharpoons \underset{x}{C_3H_7COOH} + \underset{x}{OH^-}$$

$$K_h = \frac{x^2}{0.2-x} = \frac{x^2}{0.2}$$

$$K_h = \frac{K_w}{K_a}$$

$$\frac{10^{-14}}{2\times10^{-5}} = \frac{x^2}{0.2}$$

$x = 10^{-5}$ mol/L

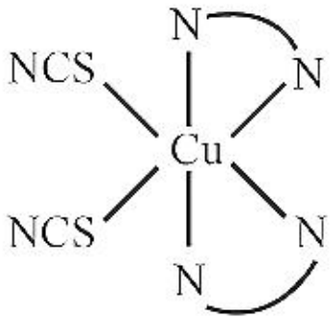
$[OH^-] = 10^{-5}$

$H^+ = 10^{-14}/10^{-5} = 10^{-9} \therefore pH = 9$

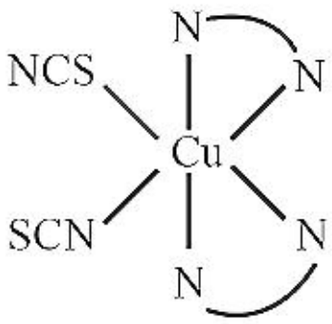
Question: Number of possible isomers of $[Cu(en)_2(SCN)_2]$

Answer: 6.00

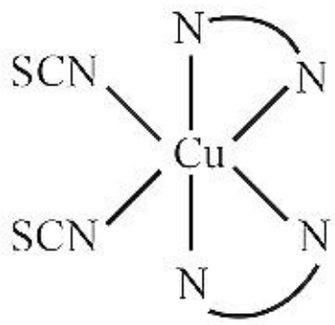
Solution: $[Cu(en)_2(SCN)_2]$



(I)



(II)



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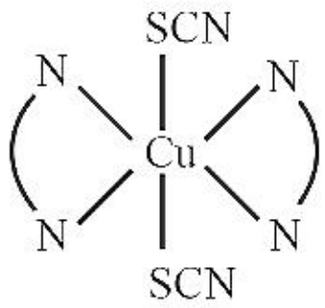

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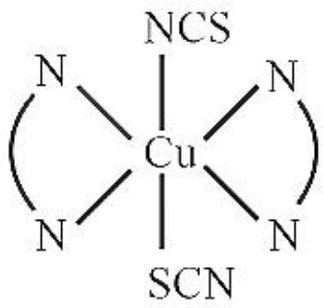
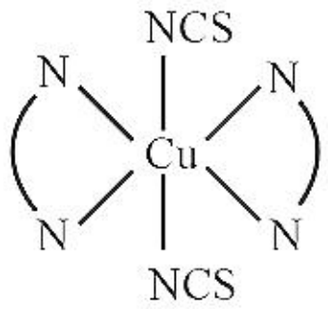


(V)



(VI)

JEE-Main-28-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: Let $A=\begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 1 & 0 & 0\end{bmatrix}$ & $B_0=A^{49}+2A^{98}$. If $B_n=adj\left(B_{n-1}\right)\forall n\geq 1$ then $\left|B_4\right|$

Options:

(a) 3^{28}

(b) 3^{30}

(c) 3^{32}

(d) 3^{36}

Answer: (c)

Solution:

$$A^2=\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$$

$$\therefore B_0=I+2A=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 2 & 0\\ 2 & 0 & 0\\ 0 & 0 & 2\end{bmatrix}=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 0\\ 0 & 0 & 3\end{bmatrix}$$

As $B_n=adj\left(B_{n-1}\right)$

$$\therefore\ B_4=adj\left(B_3\right)=adj\left(adj\ B_2\right)=adj\left(adj\left(adj\ B_1\right)\right)$$

$$=adj\left(adj\left(adj\left(adj\ B_0\right)\right)\right)$$

$$\therefore \left|B_4\right|=\left|B_0\right|^{2^4}=\left|B_0\right|^{16}$$

$$\left|B_0\right|=\begin{bmatrix}1 & 2 & 0\\ 2 & 1 & 0\\ 0 & 0 & 3\end{bmatrix}=3(-3)$$

$$\therefore \left|B_0\right|^{16}=9^{16}=3^{32}$$

Question: Considering the principal value of ITF, the sum of all solutions of $\cos^{-1}x-2\sin^{-1}x=\cos^{-1}2x$ is

Options:

(a) 0

(b) 1

(c) $\frac{1}{2}$

(d) $-\frac{1}{2}$

Answer: (a)

Solution:

$\cos^{-1}x-2\sin^{-1}x=\cos^{-1}2x$

$\cos^{-1}x-\cos^{-1}2x=2\sin^{-1}x$

$\cos\left(\cos^{-1}x-\cos^{-1}2x\right)=\cos\left(2\sin^{-1}x\right)$

$x(2x)+\sqrt{1-x^2}\sqrt{1-4x^2}=1-2x^2$

$4x^2-1=\sqrt{1-x^2}\sqrt{1-4x^2}$

$\left(4x^2-1\right)^2=\left(1-4x^2-x^2+4x^4\right)$

$16x^4-8x^2+1=1-5x^2+4x^4$

$\Rightarrow 12x^4-3x^2=0$

$\Rightarrow x^2\left(4x^2-1\right)=0$

$\Rightarrow x=0,\frac{1}{2},-\frac{1}{2}$

Sum = 0

Question: If minimum value of $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5},x>0$ is 14 then $\alpha=?$

Options:

(a) 32

(b) 64

(c) 128

(d) 256

Answer: (c)

Solution:

$f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5}$

$f'(x)=5x-\frac{5\alpha}{x^6}=0$

$\Rightarrow x^7=\alpha$

$\Rightarrow x=\alpha^{\frac{1}{7}}$

$f\left(\alpha^{\frac{1}{7}}\right)=14$

$\frac{5\alpha^{\frac{2}{7}}}{2}+\frac{\alpha}{\alpha^{\frac{5}{7}}}=14$

$\frac{7\alpha^{\frac{2}{7}}}{2}=14$

$\alpha^{\frac{2}{7}}=4$

$$\alpha = 4^{\frac{7}{2}} = 2^7 = 128$$

Question: $x\,dy = \left(\sqrt{x^2 + y^2} + y\right)dx$ curve passes through $(1,0)$. Find $y(2) = ?$

Answer: $\dfrac{3}{2}$

Solution:

$$x\,dy = \left(\sqrt{x^2 + y^2} + y\right)dx$$

$$\frac{dy}{dx} = \sqrt{1 + \frac{y^2}{x^2}} + \frac{y}{x}$$

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\Rightarrow \frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

$$= \ln\left|v + \sqrt{1 + v^2}\right| = \ln|x| + \ln c$$

$$\Rightarrow v + \sqrt{1 + v^2} = cx$$

$$\Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx$$

$$x = 1,\ y = 0$$

$$\Rightarrow 0 + 1 = c$$

$$\Rightarrow c = 1$$

$$\Rightarrow \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = x$$

$$\Rightarrow \frac{y}{2} + \sqrt{1 + \frac{y^2}{4}} = 2$$

$$\Rightarrow \sqrt{1 + \frac{y^2}{4}} = 2 - \frac{y}{2}$$

$$\Rightarrow 1 + \frac{y^2}{4} = 4 + \frac{y^2}{4} - 2y$$

$$\Rightarrow 2y = 3$$

$$\Rightarrow y = \frac{3}{2}$$

Question: Find remainder when $7^{2022} + 3^{2022}$ is divided by 5

Answer: 3.00

Solution:

Given, $7^{2022}+3^{2022}$

$7^{2022}+3^{2022}=(49)^{1011}+9^{1011}$

$=(50-1)^{1011}+(10-1)^{1011}$

$=(5k-1)+(5\lambda-1)$

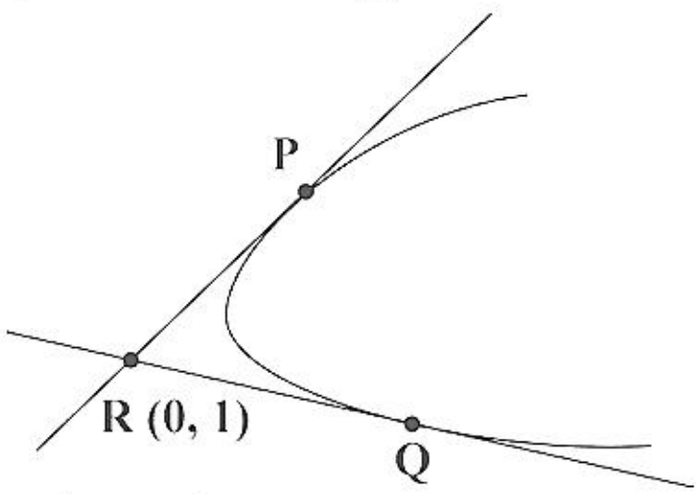
Remainder $=5-2=3$

Question: $y^2=2x+3$. For $A(1,0)$, two tangents are drawn which meets parabola at P & Q. Find orthocentre of APQ.

Answer: (2, -1)

Solution:

$y^2=2x-3 \qquad ....(1)$

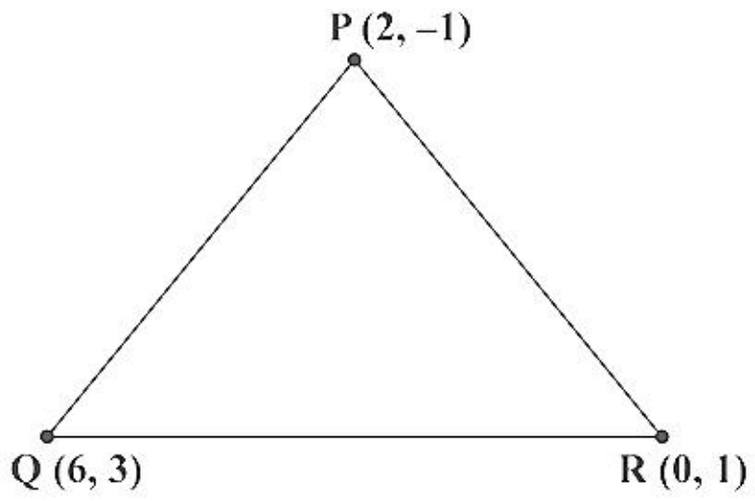


$PQ=T=0$

$y=x-3 \qquad ....(2)$

Solving (1) & (2) we get

$(2,-1)$ and $(6,3)$



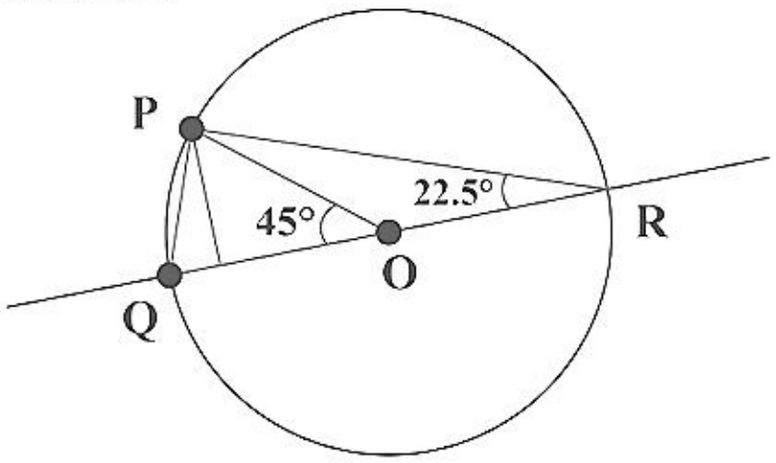
$m_{QR}=-1,\ m_{PQ}=1$

Orthocentre $\equiv(6,3)$

Question: A line passing through centre "O" of circle with radius 4 units, cuts the circle at Q & R. P is a point on the circle such that OP makes $45°$ with QR. Find area of ΔPQR.

Answer: $\frac{16}{\sqrt{2}}$

Solution:



$QR = 8$

$PR = QR\cos 22.5° = 8\cos 22.5°$

$PQ = QR\sin 22.5° = 8\sin 22.5°$

$$\text{Area} = \frac{1}{2}PR \cdot PQ$$

$$= \frac{1}{2}8\cos 22.5° \cdot 8\sin 22.5°$$

$$= 16\sin 22.5°$$

$$= \frac{16}{\sqrt{2}}$$

Question: A 6-8 digit password is to be made using A, B, C, D, E, 1, 2, 3, 4, 5, with repetition allowed. If number of such passwords containing atleast one digit is $x \times 5^6$ then $x = ?$

Answer: 7073.00

Solution:

6 digit $\_\ \_\ \_\ \_\ \_\ \_ : 10^6 - 5^6$

7 digit $\_\ \_\ \_\ \_\ \_\ \_\ \_ : 10^7 - 5^7$

8 digit $\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_ : 10^8 - 5^8$

Total: $10^6 - 5^6 + 10^7 - 5^7 + 10^8 - 5^8$

$= 7073\left(5^6\right)$

$\therefore x = 7073$

Question: Out of total candidate 60% were female, 40% were male. 60% of total passed. Female passed was twice of males passed. A candidate who passes was chosen. Find probability that it was a female.

Answer: $\frac{2}{3}$

Solution:

Given,

Total = 100

Female = 60

Male = 40

Passed = 60

Female passed = 40, Male passes = 20

Probability $= \frac{40}{60} = \frac{2}{3}$

Question: If $f(x) = \int_0^x e^{x-t} f'(t)\,dt + e^x\left(x^2 - x + 1\right)$, then find $f(x)\big|_{\min}$.

Answer: ()

Solution:

$$f(x) = \int_0^x e^{x-t} f'(t)\,dt + e^x\left(x^2 - x + 1\right)$$

$$f(x) = e^x \int_0^x e^{-t} f'(t)\,dt + e^x\left(x^2 - x + 1\right)$$

$$f'(x) = e^x\left[\int_0^x e^{-t} f'(t)\,dt + e^{-x} f'(x)\right] + e^x\left(x^2 - x + 1 + 2x - 1\right)$$

$$f'(x) = e^x\left[\int_0^x e^{-t} f'(x)\,dt\right] + f'(x) + e^x\left(x^2 + x\right)$$

$$\Rightarrow 0 = e^x\left[\int_0^x e^{-t} f'(t)\,dt + x^2 + x\right]$$

$$\Rightarrow 0 = \int_0^x e^{-t} f'(x)\,dt + x^2 + x$$

$$\Rightarrow f'(x) = -e^{-x}(2x + 1)$$

Put $f'(x) = 0 \Rightarrow x = \frac{-1}{2}$

$$f'(x) = -e^x(2x + 2 - 1)$$

$$f'(x) = -e^x(2x + 2) + e^x$$

$$\Rightarrow f(x) = -e^x(2x) + e^x + C$$

$$f(0) = 1$$

$$\Rightarrow C = 0$$

$$f(x) = -e^x(2x) + e^x$$

$$f\left(\frac{-1}{2}\right) = -e^{\frac{-1}{2}}\left(2\left(\frac{-1}{2}\right)\right) + e^{\frac{-1}{2}} = 0$$

Question: Find principal range of $\cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$

Answer: ()

Solution:

$$y=\frac{x^2-4x+2}{x^2+3}$$

$$\Rightarrow x^2y+3y=x^2-4x+2$$

$$\Rightarrow x^2(y-1)+4x+3y-2=0$$

$$D\geq 0$$

$$16-4(y-1)(3y-2)\geq 0$$

$$4-(3y^2-2y-3y+2)\geq 0$$

$$\Rightarrow -3y^2+5y+2\geq 0$$

$$\Rightarrow 3y^2-5y-2\leq 0$$

$$\Rightarrow y\in\left[\frac{-1}{3},2\right]$$

$$\text{Range}\in\left[0,\cos^{-1}\left(\frac{-1}{3}\right)\right]$$

Question: Eccentricity of $x^2-y^2=1$ is reciprocal of eccentricity of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. $y=\left(\frac{\sqrt{5}}{2}\right)x+k$ is common tangent. Find $4\left(a^2+b^2\right)$.

Answer: $\frac{6}{7}$

Solution:

$$x^2-y^2=1$$

$$e=\sqrt{2}$$

$$b^2=a^2\left(1-\frac{1}{2}\right)$$

$$b^2=\frac{a^2}{2}$$

$$y=\frac{\sqrt{5}}{2}x+R$$

$$R^2=\frac{1\times 5}{4}-1=\frac{1}{4}$$

$$R^2=a^2m^2+b^2$$

$$\frac{1}{4}=\frac{a^2 5}{4}+b^2$$

$$1 = 5a^2 + 4b^2$$

$$1 = 5a^2 + 2a^2$$

$$a^2 = \frac{1}{7},\ b^2 = \frac{1}{14}$$

$$4\left(a^2 + b^2\right) = 4\left(\frac{1}{7} + \frac{1}{14}\right)$$

$$= 4\left(\frac{3}{14}\right)$$

$$= \frac{6}{7}$$

Question: If $a_{n+2} = \frac{2}{a_{n+1}} + a_n, a_1 = 1, a_2 = 2$ & $\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right)\left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \times \ldots \times \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha} \times {}^{61}C_{31}$

then $\alpha = ?$

Answer: ()

Solution:

$\because a_{n+2} \cdot a_{n+1} - a_{n+1} \cdot a_n = 2$ where $a_1 = 1, a_2 = 2$ and $a_3 = 2$

Let $T_r = a_{r+1} \cdot a_r$

So, T_r is A.P. with common difference 2 and first term 2.

Clearly $T_r = 2r$

Now $\prod_{i=1}^{30}\left(\frac{a_i + \frac{1}{a_{i+1}}}{a_{i+2}}\right) = \prod_{i=1}^{30}\left(\frac{T_r + 1}{T_{r+1}}\right)$

$$= \prod_{i=1}^{30}\left(\frac{2r+1}{2r+2}\right) = \frac{3 \cdot 5 \cdot 7 \ldots . 61}{4 \cdot 6 \cdot 8 \ldots 62} = \frac{62!}{2\left(4 \cdot 6 \cdot 8 \ldots . 62\right)^2}$$

$$= \frac{62!}{2^{61} \cdot (31!)^2}$$

$$= 2^{\ 61} \cdot {}^{62}C_{31}$$

$$= 2^{-60} \cdot {}^{61}C_{30}$$

$$2^{-60} \cdot {}^{61}C_{31}$$

Question: Let $z_1 \in C, \left|z_1 - 3\right| = \frac{1}{2}, z_2 \in C, \left|z_2 + \left|z_2 - 1\right|\right| = \left|z_2 - \left|z_2 + 1\right|\right|$, then least value of $\left|z_1 - z_2\right|$ is:

Answer: $\frac{3}{2}$

Solution:

$$\left|z_2+\left|z_2-1\right|\right|^2=\left|z_2-\left|z_2+1\right|\right|^2$$

$$\Rightarrow\left(z_2+\left|z_2-1\right|\right)\left(\overline{z}_2+\left|z_2-1\right|\right)=\left(z_2-\left|z_2+1\right|\right)\left(\overline{z}_2-\left|z_2+1\right|\right)$$

$$\Rightarrow z_2\left(\left|z_2-1\right|+\left|z_2+1\right|\right)+\overline{z}_2\left(\left|z_2-1\right|+\left|z_2+1\right|\right)=\left|z_2+1\right|^2-\left|z_2-1\right|^2$$

$$\Rightarrow\left(z_2+\overline{z}_2\right)\left(\left|z_2+1\right|+\left|z_2-1\right|\right)=2\left(z_2+\overline{z}_2\right)$$

Either $z_2+\overline{z}_2=0$ or $\left|z_2+1\right|+\left|z_2-1\right|=2$

So, z_2 lies on imaginary axis or on real axis with in $[-1,1]$. Also $\left|z_1-3\right|=\frac{1}{2}$, lies on the circle having centre 3 and radius $\frac{1}{2}$.

Clearly $\left|z_1-z_2\right|_{\min}=\frac{3}{2}$