PHYSICS **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

1. In a typical combustion engine the workdone by a gas molecule is given by $W = \alpha^2\beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be:

1) $\left[M^2LT^{-2}\right]$ **2)** $\left[M^0LT^0\right]$ **3)** $\left[M\,LT^{-2}\right]$ **4)** $\left[M\,LT^{-1}\right]$

Key: 2

Solution:

as $\left[\beta x^2\right] = [KT] = \left[M^1L^2T^{-2}\right]$

$\Rightarrow [\beta] = \left[M^1L^0T^{-2}\right]$

And as $\left[\alpha^2\beta\right] = \left[M^1L^2T^{-2}\right] \Rightarrow \left[\alpha^2\right] = \left[M^0L^2T^0\right]$

$\Rightarrow [\alpha] = \left[M^\circ L^1T^\circ\right]$

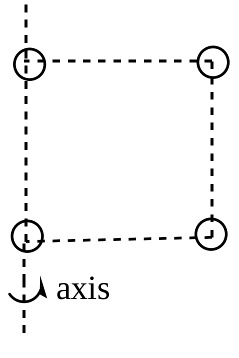
2. Four identical solid spheres each of mass ‘m’ and radius ‘a’ are placed with their centres on the four corners of a square of side ‘b’. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is:

1) $\frac{4}{5}ma^2 + 2mb^2$ **2)** $\frac{4}{5}ma^2$ **3)** $\frac{8}{5}ma^2 + 2mb^2$ **4)** $\frac{8}{5}ma^2 + mb^2$

Key: 3

Solution:

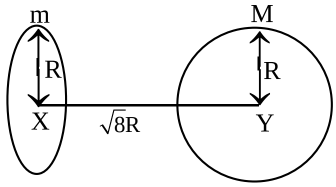
Given



then moment of inertia about the given axis will be

$$I = 4\left(\frac{2}{5}ma^2\right) + 2.\left(mb^2\right) = \frac{8}{5}ma^2 + 2mb^2$$

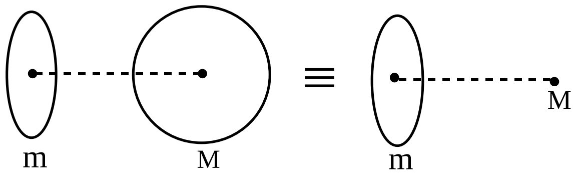
3. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass M') where both have equal radius R



1) $\frac{\sqrt{8}}{27}.\frac{GmM}{R^2}$ **2)** $\frac{1}{3\sqrt{8}}.\frac{GMm}{R^2}$ **3)** $\frac{2\sqrt{2}}{3}.\frac{GMm}{R^2}$ **4)** $\frac{\sqrt{8}}{9}.\frac{GmM}{R}$

Key: 1

Solution:



$\Rightarrow$ gravitational force between them will be

$$F = \frac{Gm}{\left(R^2 + 8R^2\right)}\frac{\sqrt{8}R}{\sqrt{R^2 + 8R^2}}M = \frac{\sqrt{8}}{27}\frac{GMm}{R^2}$$

4. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$

1) $\frac{15}{4}$ **2)** $\frac{15}{11}$ **3)** $\frac{111}{80}$ **4)** $\frac{29}{15}$

Key: No Answer

Solution:

Given $C_{Peff} = \frac{15}{4}C_{seff} \Rightarrow C_1 + C_2 = \frac{C_1C_2}{C_1 + C_2}.\frac{15}{4} \Rightarrow let\frac{C_2}{C_1} = x \Rightarrow (1+x)C_1 = \frac{15}{4}\frac{xC_1^2}{(1+x)C_1}$

$$\Rightarrow \left(1 + x^2 + 2x\right)4 = 15x \Rightarrow 4x^2 - 7x + 4 = 0 \quad \Rightarrow x = \frac{7 \pm \sqrt{49 - 64}}{2}$$

5. A large number of water drops, each of radius r, combine to have a drop of radius R. if the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be:

1) $\frac{3T}{j}\left(\frac{1}{r} - \frac{1}{R}\right)$ **2)** $\frac{3T}{rJ}$ **3)** $\frac{2T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ **4)** $\frac{2T}{rJ}$

Key: 1

Solution:

loss of surface energy = $N\left(4\pi r^2 T\right)-4\pi R^2 T$

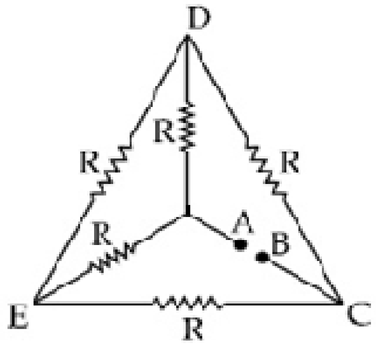
Where $R^3 = Nr^3$

$\Rightarrow$ heat produced $=\frac{4\pi T}{J}\left(Nr^2 - R^2\right)$

$\Rightarrow$ heat produced per unit volume $=\frac{4\pi T\left(Nr^2 - R^2\right)}{\frac{4}{3}\pi R^3 J}$

$=\frac{3T}{J}\left(\frac{Nr^2}{R^3}-\frac{1}{R}\right)=\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)\Rightarrow$

6. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is:

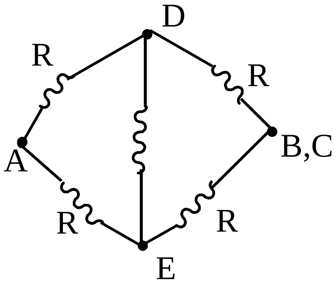


1) $\frac{R}{2}$ **2)** 2R **3)** R **4)** $\frac{3R}{2}$

Key: 3

Solution:

After redrawing



$as\,it\,is\,balanced \Rightarrow effective = R$

7. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be:

1) 1 mm **2)** 0.25 mm **3)** 0.75 mm **4)** 0.50 mm

Key: 2

Solution:

in YDSE fringe width $=\frac{D\lambda}{d}$

$=\frac{1\times 500\times 10^{-9}}{2\times 10^{-3}}=0.25\,mm$

8. A planet revolving in elliptical orbit has:

A. a constant velocity of revolution

B. has the least velocity when it is nearest to the sun.

C. its areal velocity is directly proportional to its velocity.

D. areal velocity is inversely proportional to its velocity.

E. to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below:

1) D only **2)** C only **3)** E only **4)** A only

Key: 3

Solution:

When a planet revolving in elliptical orbit its velocity is not a constant but areal velocity Remains constant

9. An alternating current is given by the equation $i - i_1 \sin \omega t + i_2 \cos \omega t$. The rms current will be:

1) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$ **2)** $\frac{1}{\sqrt{2}}(i_1 + i_2)$ **3)** $\frac{1}{2}(i_1^2 + i_2^2)^{\frac{1}{2}}$ **4)** $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{\frac{1}{2}}$

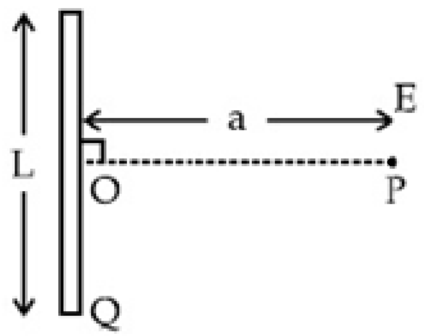
Key: 4

Solution:

$If \; = i_1 \sin \omega t + i_2 \cos \omega t$

$$\Rightarrow i_{rms} = \frac{\sqrt{i_1^{\,2} + i_2^{\,2} + 2i_1, i_2 \cos\left(\frac{\pi}{2}\right)}}{\sqrt{2}} \quad i_{rms} = \left(\frac{i_1^{\,2} + i_2^{\,2}}{2}\right)^{\frac{1}{2}}$$

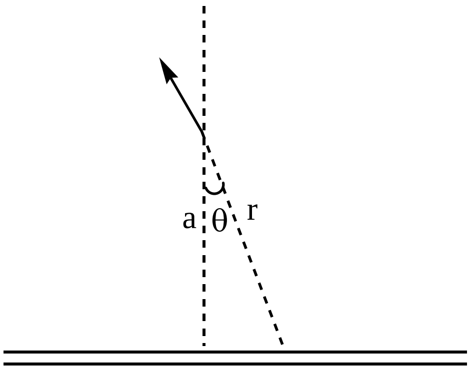
10. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L.$



1) $\frac{Q}{4\pi\varepsilon_0 L^2}$ **2)** $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$ **3)** $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$ **4)** $\frac{Q}{3\pi\varepsilon_0 L^2}$

Key: 2

Solution:



net filled will be along Los

as $dE_{Los} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dl}{r^2}\cos\theta \Rightarrow dE_{Los} = \frac{\left(Q/L\right)}{4\pi\varepsilon_0}\frac{a\sec^2\theta d\theta\cos\theta}{a^2\sec^2\theta} = \frac{Q}{4\pi\varepsilon_0 La}\cos\theta d\theta$

$\Rightarrow E = \frac{Q}{4\pi\varepsilon_0 La}\int_{-\theta_0}^{\theta_0}\cos\theta d\theta = \frac{Q}{2\pi\varepsilon_0 La}\sin\theta_0, \left(\theta_0 = \tan^{-1}\left(1/\sqrt{3}\right)\right)$

$\Rightarrow E = \frac{Q}{2\pi\varepsilon_0 L^2\sqrt{3}}$

11. The normal density of a material is ρ and its bulk modulus of elasticity is K. the magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be:

1) $\frac{PK}{\rho}$ **2)** $\frac{\rho K}{P}$ **3)** $\frac{\rho P}{K}$ **4)** $\frac{K}{\rho P}$

Key: 3

Solution:

say $m = \rho V \Rightarrow \frac{d\rho}{\rho} + \frac{dv}{V} = 0$

$\Rightarrow d\rho = -\rho\frac{dv}{v}$ but $k = \frac{dp}{(-dv/v)} = \frac{p}{(-dv/v)}$ (given dp=p)

$\Rightarrow d\rho = (-\rho)\left(\frac{-p}{k}\right) \Rightarrow d\rho = \frac{\rho P}{K}$

12. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively , then the value of $\lambda_1 : \lambda_2$ is:

1) $1:9$ **2)** $7:108$ **3)** $1:3$ **4)** $7:135$

Key: 4

Solution:

$\frac{\lambda c}{\lambda_1} = (13.6)\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$ $\frac{\lambda c}{\lambda_2} = +13.6\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$

$\frac{\lambda_1}{\lambda_2} = \frac{(16-9)/9\times16}{(16-1)/1\times16} = \frac{7}{135}$

13. Given below are two statements : one is labeled as Assertion A and the other is labelled as Reason R.

Assertion A: An electron microscope can achieve better resolving power than an optical microscope.

Reason R :The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below:

1) Both A and R are true but R is NOT the correct explanation of A

2) A is true but B is false:

3) A is false but R is true.

4) Both A and B are true and R is the correct explanation of A.

Key: 4

Solution:

The statements are correct but also correct explanation

14. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A: Body 'P' having mass M moving with speed 'u' has head-on collision elastically with another body 'Q' having mass 'm' initially at rest. If m << M, body 'Q' will have a maximum speed equal to '2u' after collision.

Reason R: During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Both A and R are correct but R is NOT the correct explanation of A.

2) A is correct but R is not correct.

3) A is not correct but R is correct.

4) Both A and B are correct and R is the correct explanation of A.

Key: 4

Solution:

$Mu + m(0) = Mv_1 + mv_2$

$\frac{1}{2}\text{Mu}^2 = \frac{1}{2}\text{Mv}_1{}^2 + \frac{1}{2}\text{mv}_2{}^2$

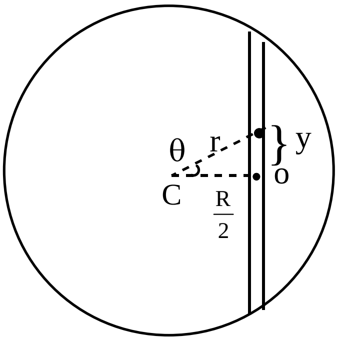
Applying $M >> m \Rightarrow V_1 = 2u$

15. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $\left(R/2\right)$ from the earth's centre, where 'R' is the radius of the Earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic notion with a time period:

1) $2\pi\sqrt{\frac{R}{g}}$ **2)** $\frac{2\pi R}{g}$ **3)** $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ **4)** $\frac{g}{2\pi R}$

Key: 1

Solution:



at a disp'y' from'O'

$$ma = -m\left(\frac{GMr}{R^3} sm\theta\right)$$

$$ma = -\frac{GMm}{R^3} y$$

$$\Rightarrow a = -\left(\frac{GM}{R^3}\right)y = -\left(\frac{g}{R}\right)y \Rightarrow T = 2\pi\sqrt{\frac{R}{g}}$$

16. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor:

1) $\frac{1}{2}, \sqrt{2}$ **2)** $\frac{1}{2}, 2\sqrt{2}$ **3)** $\frac{1}{4}, \sqrt{2}$ **4)** $\frac{1}{4}, 2\sqrt{2}$

Key: 1

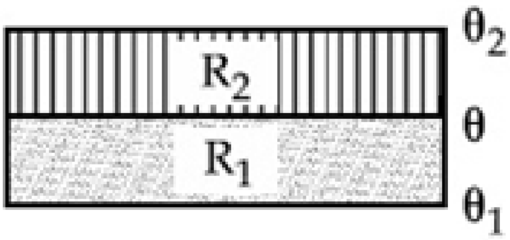
Solution:

as $\frac{1}{K_s} = \frac{1}{K_1} + \frac{1}{K_1} \Rightarrow K_s = \frac{k_1}{2}$

$$\Rightarrow T^1 = 2\pi\sqrt{\frac{m}{K_S}} = 2\pi\sqrt{\frac{m2}{k_1}} = \sqrt{2}T$$

i.e $K_s = \frac{1}{2}K_1 \,\&\, T^1 = \sqrt{2}T$

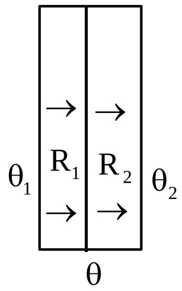
17. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by:



1) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$ **2)** $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$ **3)** $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$ **4)** $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$

Key: 3

Solution:



at steady state $\frac{\theta_1-\theta}{R_1}=\frac{\theta_1-\theta_2}{R_1+R_2}$

$\Rightarrow R_1\theta_1+R_2\theta_1-(R_1+R_2)\theta=R_1\theta_1-R_1\theta_2$

$\Rightarrow \theta=\frac{\theta_1 R_2+\theta_2 R_1}{R_1+R_2}$

18. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.
[$h=6.63\times10^{-34}$ Js and $c=3\times10^{8}ms^{-1}$]

1) 654 nm and red colour
2) 1046 nm and blue colour
3) 654 nm and orange colour
4) 1046 nm and red colour

Key: 1

Solution:

as $\frac{\lambda c}{\lambda}=E_{gap}$

$\Rightarrow \lambda=\frac{6.63\times10^{-34}\times3\times10^{8}}{1.9\times1.6\times10^{-19}}=6.54\times10^{-7}m$

$\lambda=654nm$ i.e Red

19. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by:

1) $T\alpha R^{\frac{5}{2}}$
2) $T\alpha R^{\frac{3}{2}}$
3) $T\alpha R^{2}$
4) $T\alpha R^{\frac{4}{3}}$

Key: 3

Solution:

$mR\omega^2=F=K\overline{R}^{3}$

$\Rightarrow \omega^2\alpha R^{-4}\Rightarrow T^2\alpha R^4\Rightarrow T\alpha R^2$

20. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value $|f|=40$ cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information:

1) Image is virtual, opposite side of convex mirror.
2) Image is virtual, opposite side of concave mirror.
3) Image is real, same side of concave mirror.
4) Image is real, same side of convex mirror.

Key: 1

Solution:

Given real object and the image is erect

As $\frac{1}{x_I}+\frac{1}{x_o}=\frac{1}{x_F}$ & $\frac{y_I}{y_o}=m=-\frac{x_I}{x_o}$

Given $m=+Ve$ $m=\frac{25}{100}=\frac{1}{4}=-\frac{x_I}{x_o}$

$\Rightarrow 4x_I=-x_o \Rightarrow x_I=+Ve \Rightarrow$ virtual image

$\Rightarrow \frac{1}{x_I}+\frac{1}{-4x_I}=\frac{1}{x_F} \Rightarrow x_F=\frac{4}{3}x_I=+Ve$

$f=+40cm$ convex mirror

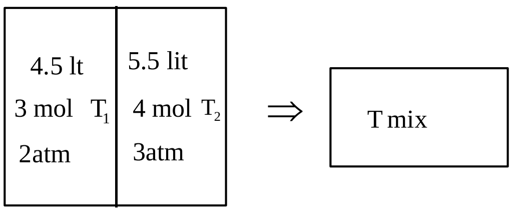
(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

21. A container is divided into two chambers by partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contain 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium pressure existing in the mixture is $x\times10^{-1}$ atm. Value of x is _________.

Key: 25.5

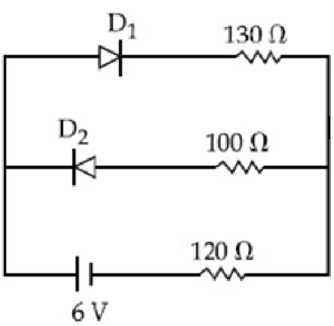
Solution:



As $\Delta Q_{net}=0 \Rightarrow \frac{f}{2}(2\times4.5)+\frac{f}{2}(3\times5.5)=\frac{f}{2}P(5.5+4.5)$

$\Rightarrow 9+16.5=P(10) \Rightarrow P=2.55\,atm$ $\qquad \Rightarrow P=25.5\times10^{-1}\,atm$

22. The circuit contains two diodes each with a forward resistance of 50 Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is ________mA.



Key: 20.

Solution:

D_2: Reverse biased acts as ∞ resistance

$\Rightarrow$current in 120Ω, I=$\frac{6}{120+130+50}=\frac{6}{300}A$ $\quad\quad I=20\,mA$

23. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is $x\times10^{-2}$. The value of x is _______.

Key: 33.33

Solution:

$$\frac{E\max}{E\min}=\frac{16}{8}\Rightarrow\frac{E_c(1+m)}{E_c(1-m)}=\frac{16}{8}\quad\Rightarrow 1+m=2-2m\Rightarrow 3m=1\Rightarrow m=\frac{1}{3}$$

$$\Rightarrow m=\frac{1}{3}=0.3333=x\times10^{-2}\Rightarrow x=33.33$$

24. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The workdone by the battery is _____________ J.

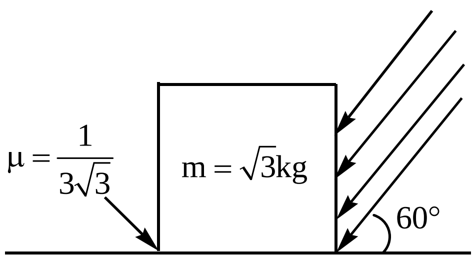
Key: 300

Solution:

Work done by the battery = Eq

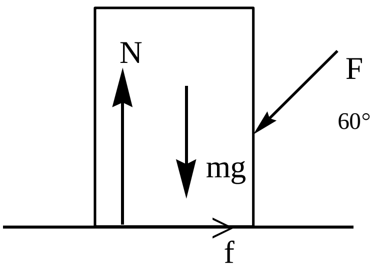
$=15\times20=300\,J$

25. As shown in the figure , a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle $60°$ with horizontal such that it does not move, will be $3x$. The value of x will be ________.$\left[g=10m/S^2;\sin60°=\frac{\sqrt{3}}{2};\cos60°=\frac{1}{2}\right]$



Key: 0.33

Solution:



$$N - mg - F\sin 60° = 0$$

$$N = mg + \frac{\sqrt{3}}{2}F \quad \text{And } F\cos 60° = f \le \mu N \quad \Rightarrow \frac{F}{2} \le \mu\left(mg + \frac{\sqrt{3}}{2}F\right)$$

$$\Rightarrow \frac{F}{2} \le \frac{1}{3\sqrt{3}}\left(10\sqrt{3} + \frac{\sqrt{3}}{2}F\right) \quad \Rightarrow 3\sqrt{3} \le 20\sqrt{3} + \sqrt{3}F$$

$$\Rightarrow 2\sqrt{3}F \le 20\sqrt{3} \Rightarrow F \le 10N \quad 3x \le 10 \Rightarrow x = \frac{10}{3} = 0.33$$

26. In a series LCR resonant circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be_____.

Key: $200\sqrt{2}$ **(282.84)**

Solution:

$$Q = \frac{L\omega}{R} \text{ where } \omega = \frac{1}{\sqrt{LC}} \text{ if f= also fixed then } Q^1 = 400 \Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}} = 100 \text{ (given)}$$

$$\Rightarrow Q' = \frac{1}{R/2}\sqrt{\frac{2L}{C}} = 2\sqrt{2}\frac{1}{R}\sqrt{\frac{L}{C}} = 200\sqrt{2} = 282.84$$

27. The mass per unit length of a uniform wire is 0.135g/cm. A transverse wave of the form $y = -0.21\sin(x + 30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is________(Round-off to the nearest integer)

Key: 1215

Solution:

Wave velocity $c = 30$ m/s

And as $c=\sqrt{\frac{T}{\mu}} \Rightarrow T=\mu C^2=0.135\times10^{-1}\times900 \quad \Rightarrow T=12.15 \Rightarrow x=1215$

28. A person standing on a spring balance inside a stationary lift measures 60 kg. the weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be _______N. $\left[g=10m/s^2\right]$

Key: 492

Solution:

Given mg=60g

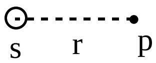
Now in an accelerating lift $\mathrm{w}^1=\mathrm{m}(\mathrm{g}-\mathrm{a})$

$\mathrm{w}^1=60(10-1.8)\,\mathrm{N}=6\times8.2\,\mathrm{kgwt} \qquad \mathrm{w}^1=49.2\,\mathrm{kgwt} \qquad =492\,\mathrm{N}$

29. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x\times10^{-1}V/m$. Value of x is _______. (Round-off to the nearest interger) [Take $\varepsilon_0=8.85\times10^{-12}C^2N^{-1}m^{-2}, c=3\times10^8ms^{-1}$]

Key: 137

Solution:



as $\mathrm{I}=\frac{\mathrm{P}}{4\pi \mathrm{r}^2}=\frac{1}{2}\varepsilon_o \mathrm{E}_0{}^2\mathrm{C}$

$$\Rightarrow \frac{\frac{1.25}{100}\times1000}{4\pi\times4}=\frac{1}{2}8.85\times10^{-12}\times\mathrm{E}_0{}^2\times3\times10^8$$

$\Rightarrow \mathrm{E}_0{}^2=187.2538 \Rightarrow \mathrm{E}_0=13.68\,\mathrm{v/m} \qquad \Rightarrow \mathrm{x}=137$

30. A boy pushes a box of mass 2 kg with force $\vec{F}=\left(20\hat{i}+10\hat{j}\right)N$ on a frictionless surface. If the box was Initially at rest, then_______ m is displacement along the x-axis after 10 s.

Key: 500

Solution:

$2\mathrm{a}_\mathrm{x}=20 \Rightarrow \mathrm{a}_\mathrm{x}=10\mathrm{m/s}^2$

In 10^{sec} $\Delta\mathrm{x}=0+\frac{1}{2}10\times10^2=500\mathrm{m}$

CHEMISTRY **Max Marks: 100**

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

31. Find A, B and C in the following reactions:

$$NH_3 + A + CO_2 \rightarrow (NH_4)_2 CO_3$$

$$(NH_4)_2 CO_3 + H_2O + B \rightarrow NH_4HCO_3$$

$$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$$

1) $A - O_2; B - CO_2; C - Na_2CO_3$ **2)** $A - H_2O; B - O_2; C - NaHCO_3$

3) $A - H_2O; B - CO_2; C - NaHCO_3$ **4)** $A - H_2O; B - O_2; C - Na_2CO_3$

Key: 3

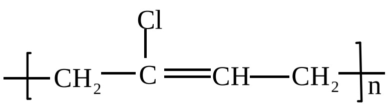
Solution:

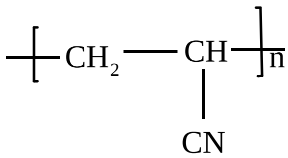
$$2\mathrm{NH_3} + \underset{(\mathrm{A})}{\mathrm{H_2O}} + \mathrm{CO_2} \rightarrow (\mathrm{NH_4})_2\mathrm{CO_3}$$

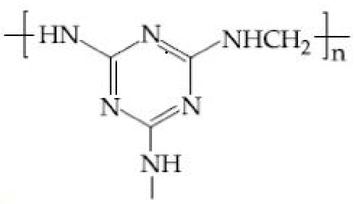
$$(\mathrm{NH_4})_2\mathrm{CO_3} + \mathrm{H_2O} + \underset{(\mathrm{B})}{\mathrm{CO_2}} \rightarrow 2\mathrm{NH_4HCO_3}$$

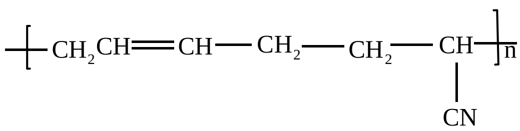
$$\mathrm{NH_4HCO_3} + \mathrm{NaCl} \rightarrow \underset{(\mathrm{C})}{\mathrm{NaHCO_3}} + \mathrm{NH_4Cl}$$

32. The structure of Neoprene is:

1) 

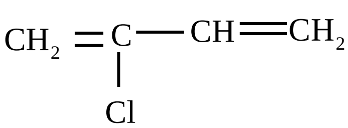
2) 

3) 

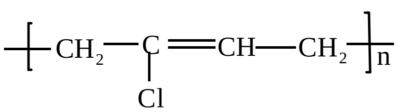
4) 

Key: 1

Solution:

Chloroprene is 

Neoprene is an addition polymer of chloroprene



33. Statements about heavy water are given below:

A. Heavy water is used in exchange reactions for the study of reaction mechanisms.

B. Heavy water is prepared by exhaustive electrolysis of water.

C. Heavy water has higher boiling point than ordinary water.

D. Viscosity of H_2O is greater than D_2O.

Choose the most appropriate answer from the options given below:

1) A only C only **2)** A and B only

3) A, B and C only **4)** A and D only

Key: 3

Solution:

A, B, and C only

Viscosity of D_2O is greater than that of H_2O

34. The presence of ozone in troposphere:

1) generates photochemical smog

2) protects us from the UV radiation

3) protects us from the X-ray radiation

4) protects us from greenhouse effect

Key: 1

Solution:

In troposphere ozone causes photochemical smog

35. The orbital having two radial as well as two angular node is:

1) 4f **2)** 5d **3)** 3p **4)** 4d

Key: 2

Solution:

For 5d orbital, no. of redial nodes =2, ($n-l-1$)

$\therefore$ of angular nodes =2, (l-value)

36. Given below are two statements:

Statement I: A mixture of chloroform and aniline can be separated by simple distiliation

Statement II: When separating aniline from a mixture of aniline and water by steam Distillation aniline boils below its boiling point.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Statement I is true but Statement II is false

2) Statement I is false but Statement II is true

3) Both Statement I and Statement II are false

4) Both statement I and Statement II are true

Key: 4

Solution:

S_1 Chloroform and aniline differ much in their boiling points.

$(61.2°c)$ $(184.1°c)$

$\therefore$ They can be separated by single distiliation

S_2 In the steam distiliation of aniline it boils at a temperature much below its normal boiling point. It boils off at a lamp below $100°c$

37. Math list-I with List-II.

List-I	**List-II**
Electronics Configuration of elements	$\Delta_i H$ in $kJ\,mol^{-1}$
(a) $1s^2 2s^2$	(i) 801
(b) $1s^2 2s^2 2p^4$	(ii) 899
(c) $1s^2 2s^2 2p^3$	(iii) 1314
(d) $1s^2 2s^2 2p^1$	(iv) 1402

Choose the most appropriate answer from the options given below:

1) $(a)\to(i)$, $(b)\to(iv)$, $(c)\to(iii)$, $(d)\to(ii)$

2) $(a)\to(ii)$, $(b)\to(iii)$, $(c)\to(iv)$, $(d)\to(i)$

3) $(a)\to(i)$, $(b)\to(iii)$, $(c)\to(iv)$, $(d)\to(ii)$

4) $(a)\to(iv)$, $(b)\to(i)$, $(c)\to(ii)$, $(d)\to(iii)$

Key: 2

Solution:

Order of I.E values of Be, B, N and ‘O’ is $N > O > Be > B$

38. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal]

1) Nd **2)** Dy **3)** Yb **4)** Pr

Key: 3

Solution:

Configuration of Yb is $6s^2 4f^{14}$ Its main oxidation state is +2 only.

39. Given below are two statements:

Statement I: $o-$Nitrophenol is steam volatile due to intramolecular hydrogen bonding.

Statement II: $o-$Nitrophenol has high melting due to hydrogen bonding.

In the light of the above statements, choose the most appropriate answer from the options given below:

1) Both Statement I and Statement II are false

2) Statement I is true but Statement II is false

3) Both Statement I and statement II are true

4) statement I is false but Statement II is true

Key: 2

Solution:

$o-$Nitrophenol - intramolecular hydrogen bonding-more volatile (i.e) lower B.P

p-nitrophenol-intramolecular hydrogen bonding – less volatile-(i.e) higher B.P

p-nitrophenol has higher m.p because if its symmetric structure which may lead to close packing in solid state

40. Which of the following is ‘a’ FALSE statement?

1) Carius method is used for the estimation of nitrogen in an organic compound

2) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture.

3) Carius tube is used in the estimation of sulphur in an organic compound.

4) Kjeldahl’s method is used for the estimation of nitrogen in an organic compound.

Key: 1

Solution:

Estimation of 'S' and 'X' carius method

Estimation of 'N'-Kjeldahl method and dumes method

41) $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\text{ K}]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with Hydroxy1 amine but does not give Tollen's test. Identify A and B

1) 1,1-Dichlorobutane and 2-Butanone

2) 1,1-Dichlorobutane and Butanal

3) 2,2-Dichlorobutane and Butan-2-one

4) 2,2-Dichlorobutane and Butanal

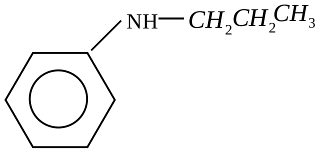
Key: 3

Solution:

$$\underset{(C_4H_8Cl_2)}{CH_3-CH_2-CCl_2-CH_3} \xrightarrow[\text{Hydrolysis}]{\text{aq KOH}} \underset{(C_4H_8O)}{CH_3-CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3}$$

Gem halides give the corresponding carbonyl compounds on alkaline hydrolysis

42) An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is:

1) 

2) $CH_3CH_2CH_2\overset{H}{N}-CH_2CH_3$

3) $CH_3CH_2CH_2NHCH_3$

4) $CH_3CH_2NH_2$

Key: 2

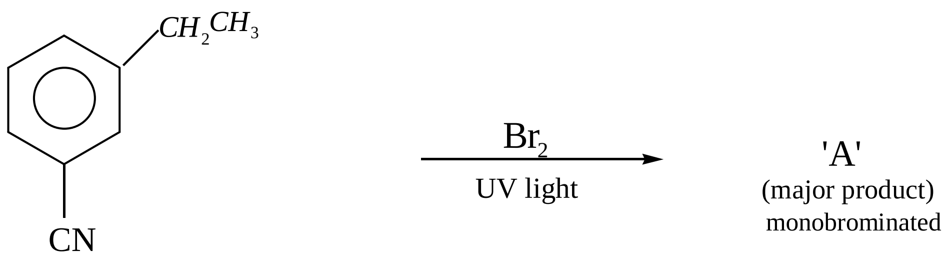
Solution:

A 2° amine gives a product insoluble in alkaline solutions when treated with $C_6H_5SO_2Cl$

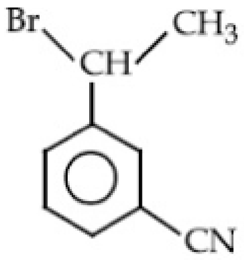
But the 2° amine is obtained by ammonolysis of ethyl chloride .

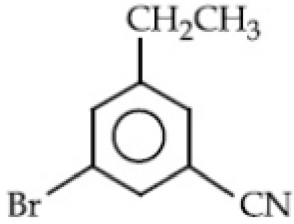
$\therefore$ The 2° amine has to be $CH_3-CH_2-CH_2-NH-CH_2CH_3$

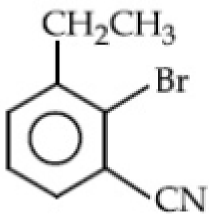
43) For the given reaction:

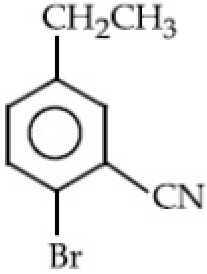


What is 'A'?

1) 

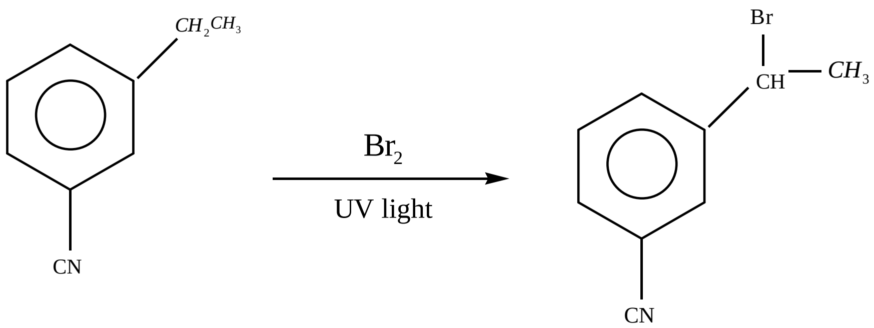
2) 

3) 

4) 

Key: 1

Solution:



as kOH

monobominated

(Radical Substitution)

44) Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead Storage batteries. Compound A is:

1) $PbSO_4$ **2)** PbO_2 **3)** PbO **4)** Pb_3O_4

Key: 2

Solution:

PbO_2 strong oxidant.Its amphoteric nature is reflected in its reaction with alkali forming Plumbate.

Used in lead strong battery.

45. Which of the following vitamin is helpful in delaying the blood clotting?

1) Vitamin K **2)** Vitamin B **3)** Vitamin E **4)** Vitamin C

Key: 1

Solution:

Vitamin K

Helpful in delaying blood clotting (NCERT)

46. Match List-I with List-II.

List-I	**List-II**
(Ore)	(Element Present)
(a) Kernite	(i) Tin
(b) Cassiterite	(ii) Boron
(c) Calamine	(iii) Fluorine
(d) Cryolite	(iv) Zinc

Choose the most appropriate answer from the options given below:

1) $(a)\rightarrow(ii),(b)\rightarrow(i),(c)\rightarrow(iv),(d)\rightarrow(iii)$

2) $(a)\rightarrow(i),(b)\rightarrow(iii),(c)\rightarrow(iv),(d)\rightarrow(ii)$

3) $(a)\rightarrow(ii),(b)\rightarrow(iv),(c)\rightarrow(i),(d)\rightarrow(iii)$

4) $(a)\rightarrow(iii),(b)\rightarrow(i),(c)\rightarrow(ii),(d)\rightarrow(iv)$

Key: 1

Solution:

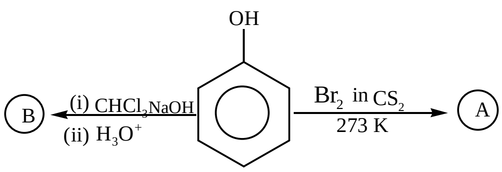
Kernite : $Na_2B_4O_7 4H_2O$

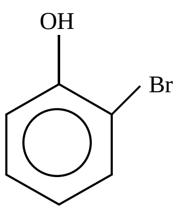
Cassiterite : SnO_2

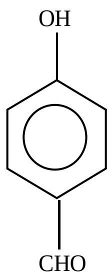
Calamine : $ZnCO_3$

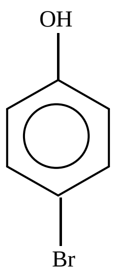
Cryolite : $3NaFAlF_3$

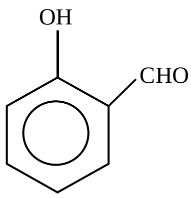
47. Identify the major products A and B respectively in the following reactions of phenol:

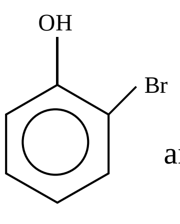


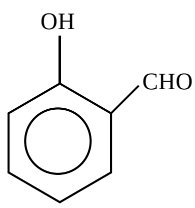
1) 

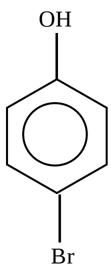
and 

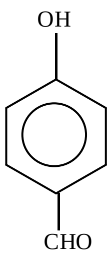
2) 

and 

3) 

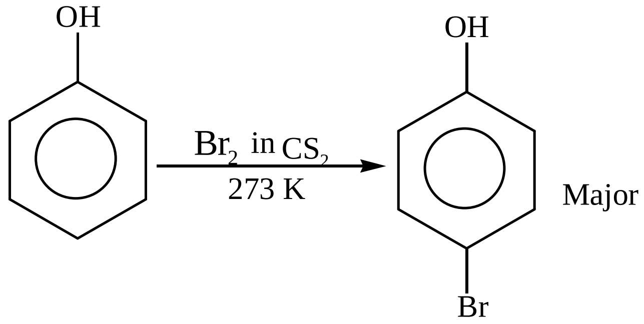
and 

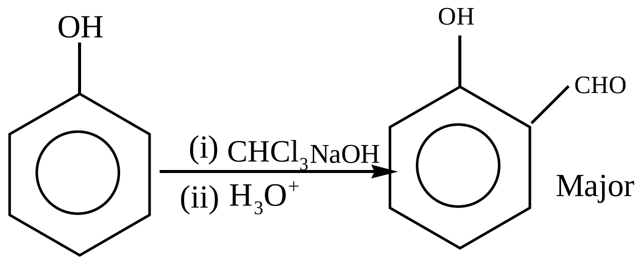
4) 

and 

Key: 2

Solution:





48. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y, X and Y respectively are:

1) $X = SO_2,\ Y = Cr_2O_3$ **2)** $X = SO_3, Y = Cr_2O_3$

3) $X = SO_3,\ Y = Cr_2(SO_4)_3$ **4)** $X = SO_2,\ Y = Cr_2(SO_4)_3$

Key: 4

Solution:

$$\text{Sulphite} \xrightarrow[\text{warm}]{\text{dil } H_2SO_4} SO_{2(g)}\uparrow$$

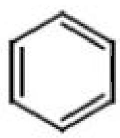
$SO_{2(g)}$ reduces acidified dichronite to chronic sulphate, $Cr_2(SO_4)_3$ which is

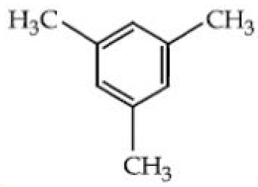
is green in colour

49. For the given reaction:

$$\underset{\displaystyle CH_3}{\overset{|}{CH}}=CHBr \xrightarrow[\text{2. Red hot iron tube, 873 K}]{\text{1. } NaNH_2} \underset{\text{(major product)}}{\text{(A)}}$$

What is 'A'?

1) 

2) 

3) $CH_3CH_2CH_2NH_2$

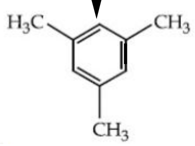
4) $\underset{\displaystyle CH_3}{\overset{|}{CH}}=CH-NH_2$

Key: 2

Solution:

$$CH_3-CH=CHBr \xrightarrow[-HBr]{NaNH_2} CH_3-C\equiv CH$$

$\downarrow$ red hot iron tube



50. Given below are two statements: one is labelled as Assertion A and the other is labelled As Reason R.

Assertion A: Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.

Reason R: Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.

In the light of above statements, choose the most appropriate answer from the options given below:

1) Both A and R are true and R is the correct explanation of A

2) Both A and R are true but R is NOT the correct explanation of A

3) A is true but R is false

4) A is false but R is true

Key: 4

Solution:

S-1 : False

S-2: True

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

51. For a chemical reaction $A+B=C+D$

$\left(\Delta_r H^\theta = 80 \text{kJ mol}^{-1}\right)$ the entropy change $\Delta_r S^\theta$ depends on the temperature T(in K) as

$\Delta_r S^\theta = 2T$ (J $K^{-1} mol^{-1}$).

Minimum temperature at which it will become spontaneous is_______ K. (Integer)

Key: 201

Solution:

For $A + B \to C + D$,

$\Delta H^\circ = 80 \times 10^3 \, J \, mol^{-1}$

$\Delta S^\circ = 2T \, J \, K^{-1} \, mol^{-1}$

From $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$ for minimum temperature at which it becomes Spontaneous

$\Delta H^\circ = T\Delta S^\circ = T(2T) = 4T^2$

$\therefore T^2 = \frac{80 \times 10^3}{4} = 20 \times 10^3$

Or $T = 200K$

52. 224 mL of $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution.The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour Pressure of solution (assuming the solution is dilute) ($P^\circ_{(H_2O)} = 24$ mm of Hg) is $x \times 10^{-2}$ mm of Hg, the value of x is ________ (Interger answer)

Key: $Po - Ps = 18. \times 10^{-2}$

Solution:

no. of moles of SO_2 $n = \frac{PV}{RT}$ $= \frac{1 \times 224 \times 10^{-3}}{0.0821 \times 298}$

$= 9 \times 10^{-3}$

= 9 milli moles

mill moles $NaOH = 0.1 \times 100 = 10$

$SO_{2_{(ClO)}} + NaOH_{(ag)} \to Na_2SO_3$

9	10	0
9-5	10-10	5

No. of moles of $H_2O = \frac{36}{18} = 2$

$\frac{Po - Ps}{Po} = \frac{3 \times 5 \times 10^{-3}}{2} \times 2$ $Po - Ps = \frac{15 \times 24 \times 10^{-3}}{2}$

$Po - Ps = 18. \times 10^{-2}$

53. 3.12 g of oxygen is adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is_______.
[R=0.0821 L atm $K^{-1} mol^{-1}$]

Key: 2

Solution:

Amount of oxygen absorbed per gram of platinum}$= \frac{3.12}{1.2}$ gm

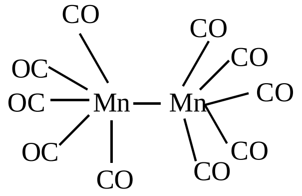
Volume of oxygen $= \frac{w}{m} \frac{KT}{P}$

$= \frac{3.12}{1.2} \times \frac{1}{32} \times \frac{0.0821 \times 300}{1}$ $= 2.0011 \approx 2$

54. Number of bridging CO ligands in $\left[Mn_2(CO)_{10}\right]$ is __________.

Key: 0

Solution:



Number of bridging 'CO' ligands –Zero

55. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}, If energy change ΔE during the reaction is $-20\,kJ$, then the activation energy for the reverse reaction in kJ is ________ (Interger answer)

Key:

Solution:

$\Delta E = Ea_{forward} - Ea_{backward}$

$-20\,kJ = 30\,kJ - Ea_{(backward)}$

$Ea_{(backward)} = 30+20 = 50$ kJ

56. The number of significant figures in 50000.020×10^{-3} is ________.

Key: 8

57. Consider the following reaction

$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{+2} + 4H_2O, E^\circ = 1.51V$

The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is______ (Interger answer)

Key: 25

Solution:

1 mol of $MnO_4^- \equiv 5$mole e^-

$\equiv 5F$

5 mol of $MnO_4^- \equiv 25F$

58. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at $27°C$. The intial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_p is $x \times 10^{-2}$. The value of x is ______ (Interger answer)

$[R = 0.08206\ dm^3 atm\ K^{-1}\ mol^{-1}]$

Key: 100

Solution:

$$AB_{2(9)} \rightleftharpoons A_{(g)} + 2B_{(9)}$$

$$\begin{array}{ccc} 1 & - & - \\ 1-x & x & 2x \end{array}$$

Intial P= $\frac{nRT}{V} = \frac{1\times1}{12}\times\frac{300}{25}$

= 1

$Kp = \frac{P_A \times P_B{}^2}{P_{AB_2}}$ total no. of moles= $1+\alpha$

$$Kp = \frac{0.45\times0.9\times0.9}{0.55} = 66.2\times10^{-2}$$

Intial pressure $P = \frac{nRT}{V}$

$= \frac{1\times1}{12}\times\frac{300}{25}$ =1

$$AB_2 \rightleftharpoons A_{(g)} + 2B_{(9)}$$

$1-x$ x $2x$

Total P= $1+2x$

$1+2x = 1.9$ $x = 0.45$

59. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is_____ (Interger answer) (Z : compressibility factor)

Key: 1

Solution:

$$P(V_m - b) = KT$$

$$PV_m = KT + Pb$$

$$Z = 1 + \frac{Pb}{RT} \qquad \left[\frac{\partial Z}{\partial P}\right]_T = \frac{b}{RT}$$

$$x = 1$$

60. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is ______. Given—

Key:

Solution:

In alkaline medium $Cr_2O_7^{2-}$ changes to CrO_4^{2}

There is no change in oxidation of ‘Cr’

In the product oxidation state of ‘Cr’ is +6

(SINGLE CORRECT ANSWER TYPE)

This section contains 20 multiple choice questions. Each question has 4 options (1), (2), (3) and (4) for its answer, out of which ONLY ONE option can be correct.

Marking scheme: +4 for correct answer, 0 if not attempted and –1 in all other cases.

61. The maximum slope of the curve $y=\frac{1}{2}x^4-5x^3+18x^2-19x$ occurs at the point:

1) $\left(3,\frac{21}{2}\right)$ **2)** $(2,2)$ **3)** $(2,9)$ **4)** $(0,0)$

Key : 2

Solution:

$$y=\frac{1}{2}x^4-5x^3+18x^2-19x$$

$$m=\frac{dy}{dx}=2x^3-15x^2+36x$$

$$\frac{dm}{dx}=6x^2-30x+36=0$$

$$x^2-5x+6=0$$

$$(x-2)(x-3)=0$$

$$\frac{d^2m}{dx^2}=6(2x-5)<0 \text{ for } x=2 \qquad \therefore \max.\text{at } x=2$$

$$y=8-40+72-38=2$$

62. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1,2 and 3 only is :

1) 77 **2)** 35 **3)** 82 **4)** 42

Key : 1

Solution:

The required numbers contain

$1,1,1,1,1,2,3 \rightarrow \frac{7!}{5!}=42$

$1,1,1,1,2,2,2, \rightarrow \frac{7!}{4!3!}=35$

Total wags $=77$

63. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t=0$. The number of bacteria is increased by 20% in 2 hours. If population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is equal to :

1) 2 **2)** 8 **3)** 16 **4)** 4

Key : 4

Solution:

Let x be the number of bacteria present at t

Then $\frac{dx}{dt} = kx$

$\Rightarrow \ln(x) = kt + c,$ At t = 0 , $x = 1000$

$\Rightarrow c = \ln 1000$

At t = 2, $x = 1000 + 1000 \times \frac{20}{100} = 1200$

Log 1200 $= 2k_1 + \log 1000$

$\Rightarrow 2k_1 = \log\left(\frac{1200}{1000}\right) = \log\left(\frac{6}{5}\right)$ $\Rightarrow k_1 = \frac{1}{2}\log\left(\frac{6}{5}\right)$

$\therefore \log x = \frac{1}{2}\log\left(\frac{6}{5}\right)t + \log(1000)$

Given $\log 2000 = \frac{1}{2}\log\left(\frac{6}{5}\right).k + \log(1000)\log\left(\frac{6}{5}\right)$ $\Rightarrow k = 2\log 2$

$\therefore \left(\frac{k}{\log e^2}\right) = (2)^2 = 4$

64. If $\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c}; 0 < x < 1,$ then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is :

1) $\frac{1-y^2}{y\sqrt{y}}$ **2)** $1-y^2$ **3)** $\frac{1-y^2}{2y}$ **4)** $\frac{1-y^2}{1+y^2}$

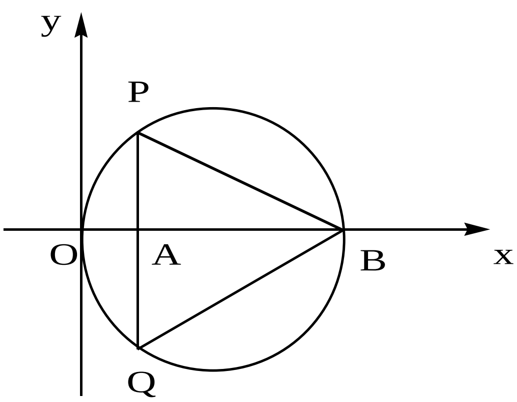
Key : 4

Solution:

$$\frac{\sin^{-1} x}{a} = \frac{\cos^{-1} x}{b} = \frac{\tan^{-1} y}{c} = \frac{\sin^{-1} x + \cot^{-1} x}{a+b} = \frac{\pi}{2(a+b)}$$

$$\tan^{-1} y = \frac{\pi c}{2(a+b)} \qquad \cos\left(\frac{\pi c}{a+b}\right) = \cos\left(2\tan^{-1} y\right) = \frac{1-y^2}{1+y^2}$$

65. In the circle given below, let OA = 1 unit, OB = 13 unit and $PQ \perp OB$. Then , the area of the triangle PQB (in square units) is :



1) $26\sqrt{3}$ **2)** $24\sqrt{2}$ **3)** $24\sqrt{3}$ **4)** $26\sqrt{2}$

Key : 3

Solution:

OA = 1 , OB = 13 $\Rightarrow$ AB = 12

$AP.AQ = OA.AB$

$AP^2 = 1 \times 12 (\because AP = AQ)$

$AP = 2\sqrt{3} \Rightarrow PQ = 4\sqrt{3}$

Area of = $\Delta PQB = \frac{1}{2} PQ.AB$

$= \frac{1}{2} \times 4\sqrt{3} \times 12 = 24\sqrt{3}$

66. The intersection of three lines $x - y = 0, x + 2y = 3$ and $2x + y = 6$ is a :

1) Right angled triangle **2)** Equilateral triangle

3) Isosceles triangle **4)** None of the above

Key : 3

Solution:

Given line $x - y = 0$-------(1)

$x + 2y = 3$--------(2)

$2x + 4 = 6$-------(3)

If A is angle between (1) ,(2)

$\cos A = \frac{|1-2|}{\sqrt{10}} = \frac{1}{\sqrt{10}}$

If B is angle between (1),(3)

Cos B = $\frac{2-1}{\sqrt{10}} = \frac{1}{\sqrt{10}}$

Cos C = $\frac{4}{5}$

$\therefore$ The triangle is isosceles

67. The sum of the infinite series $1 + \frac{2}{3} + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^4} + \frac{22}{3^5} + ...$ is equal to :

1) $\frac{9}{4}$ **2)** $\frac{11}{4}$ **3)** $\frac{15}{4}$ **4)** $\frac{13}{4}$

Key : 4

Solution:

$1 + 2\left(\frac{1}{3}\right) + \frac{7}{3^2} + \frac{12}{3^3} + \frac{17}{3^3} + ...$

$= (1+3) - 3 + (-3+5)\left(\frac{1}{3}\right) + ...$

$= 4 + \frac{(-3)}{1 - \frac{1}{3}} + \frac{5\left(\frac{1}{3}\right)}{\left(1 - \frac{1}{3}\right)^2} \quad = 4 - \frac{9}{2} + \frac{15}{4} = \frac{13}{4}$

68. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4[th], 6[th] and 8[th] terms is equal to :

1) 35 **2)** 26 **3)** 32 **4)** 30

Key : 1

Solution:

Given $ar+ar^5=\frac{25}{2}$

$(ar^2)(ar^4)=25$

$(ar^3)=25$

$ar^3=5$

$\frac{5}{r^3}\times r+\frac{5}{r^3}\times r^5=\frac{25}{2}$

$\frac{1}{r^2}+r^2=\frac{5}{2}=\frac{1}{2}+2\Rightarrow r^2=2$

$T_6+T_6+T_8=ar^3(1+r^2+r^4)$ $5(1+2+4)=35$

69. Consider the three planes

$P_1:3x+15y+21z=9$

$P_2:x-3y-z=5$ and $P_3:2x+10y+14z=5$

Then , which one of the following is true ?

1) P_1 and P_2 are parallel **2)** P_1 and P_3 are parallel

3) P_2 and P_3 are parallel **4)** P_1 , P_2 and P_3 all are parallel

Key : 2

Solution:

D.R's of normal of $P_1=(3,15,21)$ or $(1,5,7)$

,, ,, $P_2=(1,-3,-1)$

,, ,, $P_3=(2,10,14)$ or $(1,5,7)$

$\therefore$ P_1 is parallel to P_3

70. Let A be a symmetric matrix of order 2 with integer entries . If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is :

1) 6 **2)** 1 **3)** 12 **4)** 4

Key : 4

Solution:

Let $A=\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$A^2=\begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$=\begin{bmatrix} a^2+b^2 & ab+bc \\ ab+bc & b^2+c^2 \end{bmatrix}$

Given $a^2+2b^2+c^2=1$, a,b,c are integers

$a=\pm1, b=c=0$ (or) $a=b=0$, $c=\pm1$

$\therefore$ Total number of matrices $=4$

71. The value of $\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx$ is :

1) 2π **2)** 4π **3)** $\frac{\pi}{4}$ **4)** $\frac{\pi}{2}$

Key : 3

Solution:

$$I=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^x}dx=\int_{-\pi/2}^{\pi/2}\frac{\cos^2 x}{1+3^{-x}}dx=\int_{-\pi/2}^{\pi/2}\frac{3^x\cos^2 x}{3^x+1}dx$$

$$\therefore 2I=\int_{-\pi/2}^{\pi/2}\cos^2 x\,dx=2\int_{0}^{\pi/2}\cos^2 x\,dx$$

$$=2\times\frac{1}{2}\times\frac{\pi}{2} \qquad \therefore I=\frac{\pi}{4}$$

72. Let $R=\{(P,Q)|$P and Q are at the same distance from the origin$\}$ be a relation , then the equivalence class of $(1,-1)$ is the set :

1) $S=\{(x,y)|x^2+y^2=\sqrt{2}\}$ **2)** $S=\{(x,y)|x^2+y^2=2\}$

3) $S=\{(x,y)|x^2+y^2=1\}$ **4)** $S=\{(x,y)|x^2+y^2=4\}$

Key : 2

Solution:

P ,Q lie on the circles $x^2+y^2=r^2$ $(\because OP=OQ)$ $(1,-1)$ lies on the circle $x^2+y^2=r^2\Rightarrow r^2=2$

$\therefore$ Equialance class of $(1,-1)$ is $S=\{(x,y)/(x^2+y^2=2)\}$

73. The maximum value of the term independent of 't' in the expansion of $\left(tx^{\frac{1}{5}}+\frac{(1-x)^{\frac{1}{10}}}{t}\right)^{10}$ where $x\in(0,1)$ is :

1) $\frac{10!}{3(5!)^2}$ **2)** $\frac{2.10!}{3\sqrt{3}(5!)^2}$ **3)** $\frac{10!}{\sqrt{3}(5!)^2}$ **4)** $\frac{2.10!}{3(5!)^2}$

Key : 2

Solution:

The term independent of t is T_6 ($\because$ middle term)

$T_6=10_{c_5}\ x(1-x)^{1/2}$, is maximum when $\frac{x}{2}=1-x$ $\Rightarrow x=\frac{2}{3}$

Maximum of $T_6=\frac{10!}{(5!)^2}\times\frac{2}{3}\times\frac{1}{\sqrt{3}}$

74. The value of $\lim\limits_{h\to 0}\left\{\dfrac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$ is :

1) $\dfrac{2}{\sqrt{3}}$ **2)** $\dfrac{4}{3}$ **3)** $\dfrac{2}{3}$ **4)** $\dfrac{3}{4}$

Key : 2

Solution:

$$\lim_{h\to 0}2\left\{\frac{\sqrt{3}\lim\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$$

$$\lim_{h\to 0}\frac{2}{\sqrt{3}\times\sqrt{3}}\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\pi/6+h\right)}{h}\right\}=\frac{2}{3}\operatorname*{Let}_{h\to 0}\frac{\sqrt{3}\cos\left(\frac{\pi}{6}+h\right)+\sin\left(\frac{\pi}{6}+h\right)}{1}$$

$$=\frac{2}{3}\times\left(\frac{3}{2}+\frac{1}{2}\right)=\frac{4}{3}$$

75. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is :

1) $\dfrac{15}{2^{14}}$ **2)** $\dfrac{15}{2^{8}}$ **3)** $\dfrac{15}{2^{13}}$ **4)** $\dfrac{15}{2^{12}}$

Key : 3

Solution:

$$P(x=7)=P(x=9)\quad\Rightarrow n_{c_7}=n_{c_9}\Rightarrow n=16$$

$$P(x=2)=\frac{16_{c_2}}{2^{16}}=\frac{15}{2^{13}}$$

76. If $\vec{a}$ and $\vec{b}$ are perpendicular , then $\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\left(\vec{a}\times\vec{b}\right)\right)\right)$ is equal to :

1) $\dfrac{1}{2}\left|\vec{a}\right|^4\vec{b}$ **2)** $\left|\vec{a}\right|^4\vec{b}$ **3)** $\vec{0}$ **4)** $\vec{a}\times\vec{b}$

Key :2

Solution:

$$\bar{a}.\bar{b}=0$$

$$\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right)\right)=\bar{a}\times\left(\bar{a}\times\left(\bar{0}-|\bar{a}|^2\,\bar{b}\right)\right)$$

$$=-\left|\bar{a}\right|^2\bar{a}\times\left(\bar{a}\times\bar{b}\right)$$

$$=\left|\bar{a}\right|^4\bar{b}$$

77. The value of $\sum\limits_{n=1}^{100}\int\limits_{n-1}^{n}e^{x-[x]}dx$, where $[x]$ is the greatest integer $\le x$, is :

1) $100(1-e)$ **2)** $100(1+e)$ **3)** $100e$ **4)** $100(e-1)$

Key : 4

Solution:

$$\sum_{n=1}^{100}:\int_{n-1}^{n} e^{x-(x)}dx = \sum_{n=1}^{100}\int_{0}^{1} e^{x}dx \ (\because x-(x) \text{ is periodic with period 1}) \qquad = 100(e-1)$$

78. The value of $\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\(a+2)(a+3) & a+3 & 1\\(a+3)(a+4) & a+34 & 1\end{vmatrix}$ is :

1) $(a+2)(a+3)(a+4)$ **2)** $(a+1)(a+2)(a+3)$

3) 0 **4)** -2

Key : 4

Solution:

$R_2 \to R_2 \to R_1, R_3 \to R_3 - R_2$

$$\begin{vmatrix}(a+1)(a+2) & a+2 & 1\\2(a+2) & 1 & 0\\2(a+3) & 1 & 0\end{vmatrix} = -2$$

79. Let f be any function defined on R and let it satisfy the condition :

$|f(x)-f(y)\le|(x-y)^2|, \forall(x,y)\in R$ If $f(0)=1,$ then :

1) $f(x)>0, \forall x\in R$ **2)** $f(x)<0, \forall x\in R$

3) $f(x)=0, \forall x\in R$ **4)** $f(x)$ can take any value in R

Key : 1

Solution:

$$\left|\lim_{x\to y}\frac{f(x)-f(y)}{x-y}\right| \le \lim_{x\to y}|x-y| \qquad |f^1(y)|\le 0, \forall y \qquad \Rightarrow f^1(x)=0, \forall x$$

$$\Rightarrow f(x)=1, \forall x(\because f(0)=1) \qquad \Rightarrow f(x)>0, \forall x$$

80. If $(1,5,35),(7,5,5),(1,\lambda,7)$ and $(2\lambda,1,2)$ are coplanar , then the sum of all possible values of λ is :

1) $\frac{44}{5}$ **2)** $-\frac{44}{5}$ **3)** $-\frac{39}{5}$ **4)** $\frac{39}{5}$

Key : 1

Solution:

$$\left[\overline{AB}\ \overline{BC}\ \overline{CD}\right]=0 \quad \Rightarrow \begin{vmatrix}6 & 0 & -30\\-6 & \lambda-5 & 2\\2\lambda-1 & 1-\lambda & -5\end{vmatrix}=0$$

$$6(25-5\lambda+2\lambda-2)-30\left[6\lambda-6-2\lambda^2+11\lambda-5\right]=0$$

$$10\lambda^2-88\lambda+78=0$$

Sum of the values of $\lambda=\frac{88}{10}=\frac{44}{5}$

(NUMERICAL VALUE TYPE)

This section contains 10 questions. Each question is numerical value type. For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to second decimal place. (e.g. 6.25, 7.00, 0.33, 30, 30.27, 127.30). Attempt any five questions out of 10.
Marking scheme: +4 for correct answer, 0 if not attempted and 0 in all other cases.

81. Let $m, n \in N$ and gcd $(2, n) = 1$. If $30\binom{30}{0} + 29\binom{30}{1} + ... + 2\binom{30}{28} + 1\binom{30}{29} = n.2^m$, then n+m is equal to _____

Key : 45

Solution:

g.c.d $(2, \lambda) = 1$ $\Rightarrow$ n is odd

$$30\left(30_{c_0}\right) + 29\left(30_{c_1}\right) + ... + 1\left(30_{c_{29}}\right) = n.2^m$$

$$1.30_{c_1} + 2.30_{c_2} + ... + 30.30_{c_{30}} = n.2^m$$

$$30.2^{29} = n.2^m$$

$$\Rightarrow 15 \times 2^{30} = n.2^m \left(\because n \text{ is odd}\right)$$

$$\therefore n = 15, m = 30$$

$$\Rightarrow n + m = 45$$

82. The number of solutions of the equation $\log_4(x-1) = \log_2(x-3)$ is _____

Key : 1

Solution:

$$\log_4(x-1) = \log_{2^2}(x-3)^2$$

$$(x-3)^2 = x-1, x > 3$$

$$x^2 - 7x + 10 = 0$$

$x = 5$ only $\Rightarrow$ one solution

83. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0$ is _____

Key : 2

Solution:

$y^2 = a\left(x + \frac{\sqrt{a}}{2}\right), a > 0 \Rightarrow$ order = 1

$$2y\frac{dy}{dx} = a$$

$$\therefore y^2 = 2y\frac{dy}{dx}(x) + \frac{1}{2}\left(2y\frac{dy}{dx}\right)^{3/2}$$

$$2\left(y^2 - 2xy\frac{dy}{dx}\right) = \left(2y\frac{dy}{dx}\right)^{3/2}$$

$4\left(y^2 - 2xy\frac{dy}{dx}\right)^2 = y^3\left(\frac{dy}{dx}\right)^3 \Rightarrow$ degree = 3

Difference $= 3 - 1 = 2$

84. Let $(\lambda,2,1)$ be a point on the plane which passes through the point $(4,-2,2)$. If the plane is perpendicular to the line joining the points $(-2,-21,29)$ and $(-1,-16,23)$ then $\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4$ is equal to ________

Key : 8

Solution:

D.R's of the normal to the plane

$=(-1+2,-16+21,23-29)$

$=(1,5,-6)$

Eg of plane $1(x-4)+5(y+2)-6(z-2)=0$

$\lambda-4+5(2+2)-6(1-2)=0$

$\lambda=-22$

$\left(\frac{\lambda}{11}\right)^2-\frac{4\lambda}{11}-4=4+8-4=8$

85. The value of the integral $\int_0^{\pi}|\sin 2x|\,dx$ is _____

Key : 2

Solution:

$\int_0^{\pi}|\sin 2x|dx=\frac{1}{2}\int_0^{2\pi}|\sin t|dt$

$=\frac{1}{\cancel{2}}\times\cancel{2}\int_0^{\pi}\sin t\ dt=2$

86. The sum of 162^{th} power of the roots of the equation $x^3-2x^2+2x-1=0$ is _____

Key: 3

Solution:

Given equation $x^2-2x^2+2x-1=0$

$(x-1)(x^2-x+1)=0$

$x=1, x=-w,-w^2$

$\alpha^{162}+\beta^{162}+\gamma^{162}=1+\left(-w^3\right)^{54}+\left(w^6\right)^{54}$

$=1+1+1=3$

87. The number of integral values of 'k' for which the equation $3\sin x+4\cos x=k+1$ has a solution , $k\in R$ is___

Key : 11

Solution:

$3\sin x+4\cos x=k+1$ has a solution

$\Rightarrow -\sqrt{9+16}\le k+1\le\sqrt{9+16}$

$-6\le k\le 4$

Number of integer of k = 11

88. If $y=y(x)$ is the solution of the equation $e^{\sin y}\cos y\dfrac{dy}{dx}+e^{\sin y}\cos x=\cos x, y(0)=0$; then

$1+y\left(\dfrac{\pi}{6}\right)+\dfrac{\sqrt{3}}{2}y\left(\dfrac{\pi}{3}\right)+\dfrac{1}{\sqrt{2}}y\left(\dfrac{\pi}{4}\right)$ is equal to __

Key : 1

Solution:

$$e^{\sin y}\cos y\frac{dy}{dx}+e^{\sin y}\cos x=\cos x, y(0)=0$$

$$e^{\sin y}=t$$

$$\Rightarrow e^{\sin y}\cos y\frac{dy}{dx}=\frac{dt}{dx}$$

$$\frac{dt}{dx}+t(\cos x)=\cos x$$

$$I.F=e^{\int\cos x dx}=e^{\sin x}$$

Solution is $te^{\sin x}=\int e^{\sin x}.\cos x dx$

$$=e^{\sin x}+c$$

$$e^{\sin y}=1+ce^{-\sin x}, y(0)=0\Rightarrow c=0$$

$$\Rightarrow y=0,\forall x$$

Given expression $1+y\left(\dfrac{\pi}{6}\right)+\dfrac{\sqrt{3}}{2}y\left(\dfrac{\pi}{3}\right)............=1+0=1$

89. If $\sqrt{3}(\cos^2 x)=(\sqrt{3}-1)\cos x+1$, the number of solution of the given equation when $x\in\left[0,\dfrac{\pi}{2}\right]$ is _____

Key : 1

Solution:

$$\sqrt{3}\cos^2 x=(\sqrt{3}-1)\cos x+1$$

$$\sqrt{3}\cos x(\cos x-1)+(\cos x-1)=0$$

$$(\sqrt{3}\cos x+1)(\cos x-1)=0, x\in\left[0,{}^{\pi}\!/_{2}\right]$$

$n=0$ Only, No. of solutions=1

90. The area bounded by the lines $y=|\,|x-1|-2\,|$ is______

Key: No solution (Question wrong)

Solution:

Given equation should be as $|y|=\big||x-1|-2\big|$