SECTION –A

1. Consider three observations a, b and c such that b = a+c. If the standard deviation of a+2, b+2, c+2 is d, then which of the following is true?

(1) $b^2 = a^2 + c^2 + 3d^2$

(2) $b^2 = 3\left(a^2 + c^2\right) - 9d^2$

(3) $b^2 = 3\left(a^2 + c^2\right) + 9d^2$

(4) $b^2 = 3\left(a^2 + c^2 + d^2\right)$

Ans. **(2)**

Sol. for a, b, c

mean = $\bar{x} = \dfrac{a+b+c}{3}$

$\bar{x} = \dfrac{2b}{3}$

S.D. of a, b, c = d

$d^2 = \dfrac{a^2 + b^2 + c^2}{3} - \dfrac{4b^2}{9}$

$b^2 = 3a^2 + 3c^2 - 9d^2$

2. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counter clockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and (0, 0) is equal to :

(1) 1

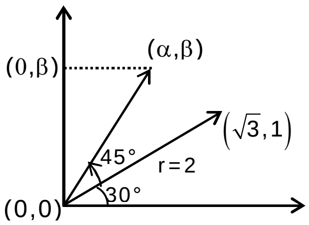
(2) $\dfrac{1}{2}$

(3) $\dfrac{1}{\sqrt{2}}$

(4) $2\sqrt{2}$

Ans. **(2)**

Sol.



$\left(\alpha, \beta\right) \equiv (2\cos 75°, 2\sin 75°)$

Area = $\dfrac{1}{2}$ (2 cos75°)(2 sin 75°)

= sin(150°) = $\dfrac{1}{2}$ square unit

3. If for a>0, the feet of perpendiculars from the points A(a, –2a, 3) and B(0, 4, 5) on the plane lx + my + nz = 0 are points C(0, –a, –1) and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{41}$

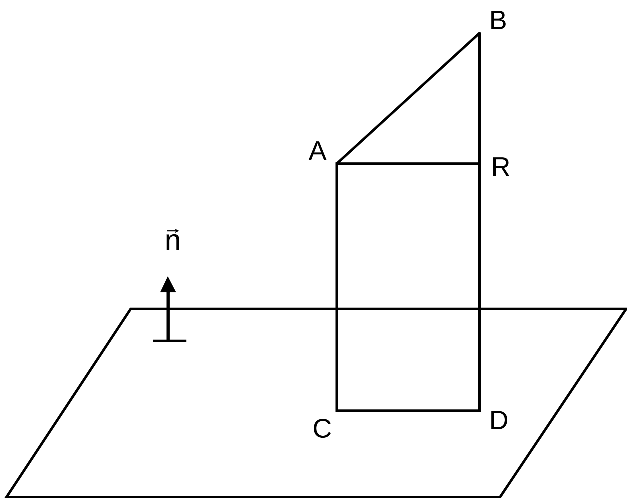
(2) $\sqrt{55}$

(3) $\sqrt{31}$

(4) $\sqrt{66}$

Ans. **(4)**

Sol.



$CD = AR = |AB|\sin\phi$

$CD = |AB|\sqrt{1-\cos^2\phi}$

$|AB|\sqrt{1-\left(\frac{\overrightarrow{AB}.\vec{n}}{|\overrightarrow{AB}|}\right)^2}$

$= \sqrt{(AB)^2-(\overrightarrow{AB}\cdot\vec{n})^2}$

$\text{Cos}\ \phi = \frac{\overrightarrow{AB}\cdot\vec{n}}{|\vec{n}||\overrightarrow{AB}|}$

$|\overrightarrow{AB}| = a\hat{i}-(2a+4)\hat{j}-2\hat{k}$

$\overrightarrow{AB}\cdot\vec{n} = \ell a - (2a+4) - 2n$

C on plane

$0\ell - am - n = 0$ .... (1)

$\overrightarrow{AC} \parallel \vec{n}$

$\frac{a}{\ell}=\frac{-a}{m}=\frac{4}{n}$

$m = -\ell$ & $an + 4m = 0$ ... (2)

From (1) and (2)

$a^2m + an = 0$

$\underline{4m + an = 0}$

$(a^2 - 4)m = 0 \Rightarrow \boxed{a = 2}$.

$2m + n = 0$ ... (1)

$m + \ell = 0$

$\ell^2 + m^2 + n^2 = 1$

$m^2 + m^2 + 4m^2 = 1$

$m^2 = \frac{1}{6}$

$m = \frac{1}{\sqrt{6}}$

$n = \frac{-2}{\sqrt{6}}$

$\ell = \frac{-1}{\sqrt{6}}$

Now $\overrightarrow{AB}.\vec{n} = 2\left(\frac{-1}{\sqrt{6}}\right) - 8\left(\frac{1}{\sqrt{6}}\right) - 2\left(\frac{-2}{\sqrt{6}}\right)$

$= \frac{-2-8+4}{\sqrt{6}} = -\sqrt{6}$

$|\overrightarrow{AB}| = \sqrt{4+64+4} = \sqrt{72}$

$CD = \sqrt{72-6}$

$CD = \sqrt{66}$

4. The range of $a \in R$ for which the function

$f(x) = (4a-3)(x + \log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$, $x \neq 2n\pi, n \in N$ has critical points, is :

(1) $\left[-\frac{4}{3}, 2\right]$

(2) $[1, \infty)$

(3) $\left(-\infty,-1\right]$

(4) (–3, 1)

Ans. **(1)**

Sol. $f(x) = (4a-3)(x+\ln 5) + 2(a-7)\left(\dfrac{\cos\frac{x}{2}}{\sin\frac{x}{2}}\cdot\sin^2\frac{x}{2}\right)$

$f(x) = (4a-3)(x+\ln 5) + (a-7)\sin x$

$f'(x) = (4a-3) + (a-7)\cos x = 0$

$\cos x = \dfrac{-(4a-3)}{a-7}$

$-1 \le -\dfrac{4a-3}{a-7} \le 1$

$-1 \le \dfrac{4a-3}{a-7} \le 1$

$\dfrac{4a-3}{a-7} - 1 \le 0$ and $\dfrac{4a-3}{a-7} + 1 \ge 0$

$\Rightarrow \dfrac{-4}{3} \le a \le 2$

5. Let the functions f: R→R and g: R →R be defined as :

$f(x) = \begin{cases} x+2, & x<0 \\ x^2, & x \ge 0 \end{cases}$ and $g(x) = \begin{cases} x^3, & x<1 \\ 3x-2, & x \ge 1 \end{cases}$

Then, the number of points in R where (fog)(x) is NOT differentiable is equal to :

(1) 1

(2) 2

(3) 3

(4) 0

Ans. **(1)**

Sol. $fog(x) = \begin{cases} x^3+2, & x \le 0 \\ x^6, & 0 \le x \le 1 \\ (3x-2)^2, & x \ge 1 \end{cases}$

$\because$ fog(x) is discontinuous at x = 0 then non-differentiable at x = 0

Now,

at x = 1

$\text{RHD} = \lim\limits_{h\to 0}\dfrac{f(1+h)-f(1)}{h} = \lim\limits_{h\to 0}\dfrac{(3(1+h)-2)^2-1}{h} = 6$

$\text{LHD} = \lim\limits_{h\to 0}\dfrac{f(1-h)-f(1)}{-h} = \lim\limits_{h\to 0}\dfrac{(1-h)^6-1}{-h} = 6$

Number of points of non-differentiability = 1

6. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to ____

(1) 5
(2) 8
(3) 6
(4) 7

Ans. **(4)**

Sol. $\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 - 2|z| + 1$

$|z|^2 - 4|z| - 21 \leq 0$

$(|z| - 7)(|z| + 3) \leq 0$

$\Rightarrow |z| \leq 7$

$\therefore |z|_{max} = 7$

7. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$

(2) $\frac{52}{867}$

(3) $\frac{39}{50}$

(4) $\frac{22}{425}$

Ans. **(3)**

Sol. $P\left(\overline{S}_{missing} \;/\; \text{both found spade}\right) = \frac{P\left(\overline{S_m} \cap BFS\right)}{P(BFS)}$

$= \frac{\left(1-\frac{13}{52}\right)\times\frac{13}{51}\times\frac{12}{50}}{\left(1-\frac{13}{52}\right)\times\frac{13}{51}\times\frac{12}{50}+\frac{13}{52}\times\frac{12}{51}\times\frac{11}{50}}$

$= \frac{39}{50}$

8. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}}+5^{\frac{1}{8}}\right)^{60}$, then (n–1) is divisible by :

(1) 8
(2) 26
(3) 7
(4) 30

Ans. **(2)**

Sol. $T_{r+1} = {}^{60}C_r \left(3^{1/4}\right)^{60-r} \left(5^{1/8}\right)^r$

rational if $\frac{60-r}{4}, \frac{r}{8}$, both are whole numbers, $r \in \{0,1,2,....60\}$

$\frac{60-r}{4} \in W \Rightarrow r \in \{0,4,8,....60\}$

and $\frac{r}{8} \in W \Rightarrow r \in \{0,8,16,...56\}$

$\therefore$ Common terms $r \in \{0,8,16,....56\}$

So 8 terms are rational

Then Irrational terms = 61 – 8 = 53 = n

$\therefore n - 1 = 52 = 13 \times 2^2$

factors 1,2,4,13,26,52

9. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2) respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{5}$

(2) $\sqrt{171}$

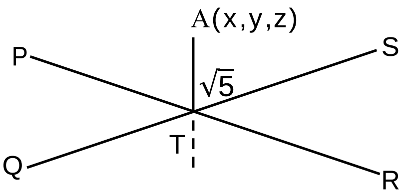
(3) $\sqrt{227}$

(4) $\sqrt{482}$

Ans. **(2)**

Sol. $\vec{p} = 3\hat{i} - \hat{j} + 2\hat{k}$ & $\vec{\theta} = \hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{v}_{PR} = \langle 4, -1, 2 \rangle$ & $\vec{v}_{QS} = \langle -2, 1, -2 \rangle$



$L_{PR} : \vec{r} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) + \lambda \langle 4, -1, 2 \rangle$

$L_{QS} : \vec{r} = \left\langle \hat{i} + 2\hat{j} - 4\hat{k} \right\rangle + \mu \langle -2, 1, -2 \rangle$

Now T on PR = $\langle 3 + 4\lambda, -1 - \lambda, 2 + 2\lambda \rangle$

Similarly T on QS = $\langle 1 - 2\mu, 2 + \mu, -4 - 2\mu \rangle$

For λ & μ: $\left.\begin{array}{l} 3 + 4\lambda = 1 - 2\mu \Rightarrow \mu + 2\lambda = -1 \\ -1 - \lambda = 2 + \mu \Rightarrow \mu + \lambda = -3 \end{array}\right\} \begin{array}{l} \lambda = 2 \\ \mu = -5 \end{array}$

$2 + 2\lambda = -4 - 2\mu$

$\Rightarrow T : \langle 11, -3, 6 \rangle$

D.R. of TA = $\vec{v}_{QS} \times \vec{v}_{PR}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -2 \\ 4 & -1 & 2 \end{vmatrix} = 0\hat{i} - 4\hat{j} - 2\hat{k}$$

$L_{TA} : \vec{r} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) + \lambda\left\langle -4\hat{j} - 2\hat{k}\right\rangle$

Now $A = \langle 11, -3 - 4\lambda, 6 - 2\lambda\rangle$

$TA = \sqrt{5}$

$(-3 + 4\lambda + 3)^2 + (6 + 2\lambda - 6)^2 = 5$

$16\lambda^2 + 4\lambda^2 = 5$

$20\lambda^2 = 5$

$\lambda = \pm\frac{1}{2}$

A: (11, –3 –2, 6–1) ; A: (11, –3 + 2, 6 + 1)

$|A| = \sqrt{121 + 25 + 25}$; $|A| = \sqrt{121 + 1 + 49}$

$= \sqrt{171}$; $\sqrt{171}$

10. If the three normals drawn to the parabola, $y^2=2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than:

(1) 1

(2) $\frac{1}{2}$

(3) $-\frac{1}{2}$

(4) –1

Ans. **(1)**

Sol. Let the equation of the normal is

$y = mx - 2am - am^3$

here $4a = 2 \Rightarrow a = \frac{1}{2}$

$y = mx - m - \frac{1}{2}m^3$

It passing through A(a, 0) then

$0 = am - m - \frac{1}{2}m^3$

$m = 0,\ a - 1 - \frac{1}{2}m^2 = 0$

$m^2 = 2(a - 1) > 0$

$\therefore a > 1$

11. Let $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1} + 3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$

(2) $\cot^{-1}\left(\frac{3}{2}\right)$

(3) $\frac{\pi}{2}$

(4) $\tan^{-1}(3)$

Ans. **(2)**

Sol. $$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{6^r(3-2)}{\left(1+\left(\frac{3}{2}\right)^{2r+1}\right)2^{2r+1}}\right)$$

$$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{2^r\cdot 3^{r+1}-3^r 2^{r+1}}{\left(1+\left(\frac{3}{2}\right)^{2r+1}\right)2^{2r+1}}\right)$$

$$\sum_{r=1}^{\infty}\tan^{-1}\left(\frac{\left(\frac{3}{2}\right)^{r+1}-\left(\frac{3}{2}\right)^{r}}{1+\left(\frac{3}{2}\right)^{r+1}\left(\frac{3}{2}\right)^{r}}\right)=\sum_{r=1}^{\infty}\left[\tan^{-1}\left(\frac{3}{2}\right)^{r+1}-\tan^{-1}\left(\frac{3}{2}\right)^{r}\right]=\frac{\pi}{2}-\tan^{-1}\frac{3}{2}=\cot^{-1}\frac{3}{2}$$

12. The number of roots of the equation, $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$ in the interval $[0, \pi]$ is equal to :

(1) 3

(2) 2

(3) 4

(4) 8

Ans. **(3)**

Sol. $(81)^{\sin^2 x}+(81)^{1-\sin^2 x}=30$

$$(81)^{\sin^2 x}+\frac{81}{(81)^{\sin^2 x}}=30$$

Let $(81)^{\sin^2 x}=t$

$t+\frac{81}{t}=30 \Rightarrow t^2+81=30t$

$t^2-30t+81=0$

$t^2-27t-3t+81=0$

$(t-3)(t-27)=0$

$t=3, 27$

$(81)^{\sin^2 x}=3, 3^3$

$3^{4\sin^2 x} = 3^1, 3^3$

$4\sin^2 x = 1, 3$

$\sin^2 x = \frac{1}{4}, \frac{3}{4}$

in $[0, \pi]$ $\sin x > 0$

$\sin x = \frac{1}{2}, \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}$

Number of solution = 4

13. If y=y(x) is the solution of the differential equation, $\frac{dy}{dx} + 2y\tan x = \sin x, y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of the function y(x) over R is equal to :

(1) 8

(2) $\frac{1}{2}$

(3) $-\frac{15}{4}$

(4) $\frac{1}{8}$

Ans. (4)

Sol. $\frac{dy}{dx} + 2\tan x \cdot y = \sin x$

I.F. = $e^{2\ell n(\sec x)} = \sec^2 x$

$y \cdot \sec^2 x = \int \sin x \sec^2 x dx = \int \tan x \sec x dx + c$

$y \sec^2 x = \sec x + c$

$y = \cos x + c\cos^2 x$

$x = \frac{\pi}{3}, y = 0$

$\Rightarrow \frac{1}{2} + \frac{c}{4} \Rightarrow c = -2$

$\therefore y = \cos x - 2\cos^2 x$

$y = -2\left(\cos^2 x - \frac{1}{2}\cos x\right) = -2\left(\left(\cos x - \frac{1}{4}\right)^2 - \frac{1}{16}\right)$

$y = \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore \quad y_{max} = \frac{1}{8}$

14. Which of the following Boolean expression is a tautology?

(1) $(p \wedge q) \wedge (p \rightarrow q)$
(2) $(p \wedge q) \vee (p \vee q)$
(3) $(p \wedge q) \vee (p \rightarrow q)$
(4) $(p \wedge q) \rightarrow (p \rightarrow q)$

Ans. **(4)**

Sol.

p	q	$p \wedge q$	$p \vee q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T	T
F	T	F	T	T	T
T	F	F	T	F	T
F	F	F	F	T	T

15. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, i = \sqrt{-1}$. Then, the system of linear equations $A^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) No solution
(2) Exactly two solutions
(3) A unique solution
(4) Infinitely many solutions

Ans. **(1)**

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}\begin{bmatrix} i & -i \\ -i & i \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 4\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix} \Rightarrow 128(x-y) = 8$

$\Rightarrow x - y = \frac{1}{16}$ ....(1)

and $128(-x + y) = 64 \Rightarrow x - y = \frac{-1}{2}$ ...(2)

$\Rightarrow$ no solution (from eq. (1) & (2))

16. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x+\cos x) = \frac{1}{2}(\log_{10} n-1)$, $n>0$, then the value of n is equal to :

(1) 16
(2) 20
(3) 12

(4) 9

Ans. **(3)**

Sol. $\log_{10}(\sin x) + \log_{10}(\cos x) = -1$

$\sin x\cos x = \frac{1}{10}$ ....(1)

and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \sin x + \cos x = \left(\frac{n}{10}\right)^{\frac{1}{2}}$

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = \frac{n}{10}$ (squaring)

$\Rightarrow 1 + 2\left(\frac{1}{10}\right) = \frac{n}{10}$ (using equation(1))

$\Rightarrow \frac{n}{10} = \frac{12}{10} \Rightarrow n = 12$

17. The locus of the midpoints of the chord of the circle, $x^2+y^2=25$ which is tangent to the hyperbola, $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is :

(1) $\left(x^2+y^2\right)^2 - 16x^2 + 9y^2 = 0$

(2) $\left(x^2+y^2\right)^2 - 9x^2 + 144y^2 = 0$

(3) $\left(x^2+y^2\right)^2 - 9x^2 - 16y^2 = 0$

(4) $\left(x^2+y^2\right)^2 - 9x^2 + 16y^2 = 0$

Ans. **(4)**

Sol. tangent of hyperbola

$y = mx \pm \sqrt{9m^2 - 16}$ ...(i)

which is a chord of circle with mid-point (h, k)

so equation of chord $T = S_1$

$hx + ky = h^2 + k^2$

$y = -\frac{hx}{k} + \frac{h^2+k^2}{k}$ ...(ii)

by (i) and (ii)

$m = -\frac{h}{k}$ and $\sqrt{9m^2 - 16} = \frac{h^2+k^2}{k}$

$9\frac{h^2}{k^2} - 16 = \frac{\left(h^2+k^2\right)^2}{k^2}$

locus $9x^2 - 16y^2 = (x^2 + y^2)^2$

18. Let [x] denote greatest integer less than or equal to x. If for $n \in N, \left(1 - x + x^3\right)^n = \sum_{j=0}^{3n} a_j x^j$, then

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$$ is equal to :

(1) 1
(2) n
(3) 2^{n-1}
(4) 2

Ans. **(1)**

Sol. $\left(1 - x + x^3\right)^n = \sum_{j=0}^{3n} a_j x^j$

$(1-x + x^3)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_{3n}\, x^{3n}$

Put x = 1

$1 = a_0 + a_1 + a_2 + a_3 + a_4 + \ldots\ldots + a_{3n}$ ...(1)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 + a_4 \ldots\ldots (-1)^{3n} a_{3n}$ ...(2)

Add (1) + (2)

$\Rightarrow a_0 + a_2 + a_4 + a_6 + \ldots\ldots\ldots = 1$

Sub (1) – (2)

$\Rightarrow a_1 + a_3 + a_5 + a_7 + \ldots\ldots\ldots = 0$

Now $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$

$= (a_0 + a_2 + a_4 + \ldots\ldots) + 4(a_1 + a_3 + \ldots.)$

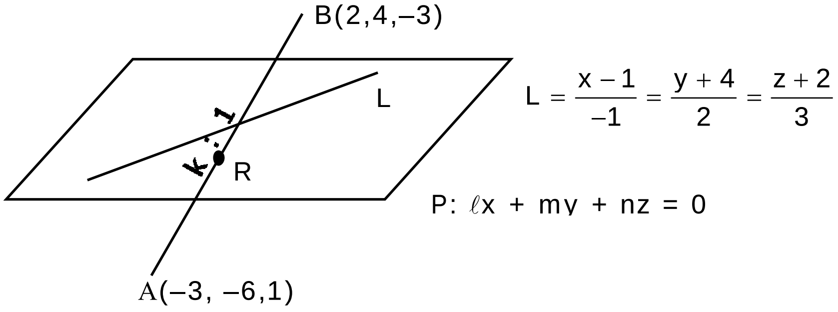
$= 1 + 4 \times 0$

$= 1$

19. Let P be a plane lx+my+nz=0 containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5
(2) 2
(3) 4
(4) 3

Ans. **(2)**



Sol.

Line lies on plane

$-\ell + 2m + 3n = 0$ ...(1)

Point on line (1,–4,–2) lies on plane

$\ell - 4m - 2n = 0$ ...(2)

from (1) & (2)

$-2m + n = 0 \Rightarrow 2m = n$

$\ell = 3n + 2m \Rightarrow \ell = 4n$

$\ell : m : n :: 4n : \frac{n}{2} : n$

$\ell : m : n :: 8n : n : 2n$

$\ell : m : n :: 8 : 1 : 2$

Now equation of plane is $8x + y + 2z = 0$

R divide AB is ratio k : 1

$R : \left(\frac{-3+2k}{k+1}, \frac{-6+4k}{k+1}, \frac{1-3k}{k+1}\right)$ lies on plane

$$8\left(\frac{-3+2k}{k+1}\right) + \left(\frac{-6+4k}{k+1}\right) + 2\left(\frac{1-3k}{k+1}\right) = 0$$

$-24 + 16k - 6 + 4k + 2 - 6k = 0$

$-28 + 14k = 0$

$k = 2$

20. The number of elements in the set $\{x \in R : (|x| - 3)|x + 4| = 6\}$ is equal to :

(1) 2

(2) 1

(3) 3

(4) 4

Ans. **(1)**

Sol. **Case-1** $x \le -4$

$(-x - 3)(-x - 4) = 6$

$\Rightarrow (x + 3)(x + 4) = 6$
$\Rightarrow x^2 + 7x + 6 = 0$
$\Rightarrow x = -1$ or -6
but $x \leq -4$
$x = -6$
Case-2 $x \in (-4, 0)$
$(-x - 3)(x + 4) = 6$
$\Rightarrow -x^2 - 7x - 12 - 6 = 0$
$\Rightarrow x^2 + 7x + 18 = 0$
$D < 0$ No solution
Case-3 $x \geq 0$
$(x - 3)(x + 4) = 6$
$\Rightarrow x^2 + x - 12 - 6 = 0$
$\Rightarrow x^2 + x - 18 = 0$

$$x = \frac{-1 \pm \sqrt{1 + 72}}{2}$$

$\therefore x = \dfrac{\sqrt{73} - 1}{2}$ only

SECTION –B

1. Let f: (0, 2) → R be defined as $f(x) = \log_2\left(1 + \tan\left(\frac{\pi x}{4}\right)\right)$. Then, $\lim\limits_{n\to\infty} \frac{2}{n}\left(f\left(\frac{1}{n}\right) + f\left(\frac{2}{n}\right) + \ldots + f(1)\right)$ is equal to ______

Ans. **(1)**

Sol. $E = 2 \lim\limits_{x\to\infty} \sum\limits_{r=1}^{n} \frac{1}{n}\, f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1 + \tan\frac{\pi x}{4}\right)dx \qquad \text{...(i)}$$

replacing $x \to 1 - x$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1 + \tan\frac{\pi}{4}(1 - x)\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1 + \tan\left(\frac{\pi}{4} - \frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(1 + \frac{1 - \tan\frac{\pi}{4}x}{1 + \tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n 2}\int_0^1 \ell n\left(\frac{2}{1 + \tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \left(\ell n\,2 - \ell n\left(1 + \tan\frac{\pi x}{4}\right)\right)dx \quad ....(ii)$$

equation (i) + (ii)

E = 1

2. The total number of 3×3 matrices A having entries from the set {0, 1, 2, 3} such that the sum of all the diagonal entries of AA^T is 9, is equal to ____

Ans. (766)

Sol. $$AA^T = \begin{bmatrix} x & y & z \\ a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} x & a & d \\ y & b & e \\ z & c & f \end{bmatrix}$$

$$= \begin{bmatrix} x^2 + y^2 + z^2 & ax + by + cz & dx + ey + fz \\ ax + by + cz & a^2 + b^2 + c^2 & ad + be + cf \\ dx + ey + fz & ad + be + cf & d^2 + e^2 + f^2 \end{bmatrix}$$

$Tr\,(AA^T) = x^2 + y^2 + z^2 + a^2 + b^2 + c^2 + d^2 + e^2 + f^2 = 9$

all→1   1

one 3, rest = 0 $\frac{9!}{8!} = 9$

two 2 , one 1 & rest 0 $\frac{9!}{2!6!} = 63 \times 4 = 252$

one 2 , five 1, rest 0 $\frac{9!}{5!3!} = 63 \times 8 = 504$

= 766

3. Let f:R→R be a continuous function such that f(x)+f(x+1)=2, for all $x \in R$. If $I_1 = \int_0^8 f(x)\,dx$ and $I_2 = \int_{-1}^3 f(x)\,dx$, then the value of $I_1 + 2I_2$ is equal to ____

Ans. (16)

Sol. f(x) + f(x + 1) = 2 .....(i)

x→(x + 1)

f(x + 1) + f(x + 2) = 2 .....(ii)

by (i) & (ii)

f(x) – f(x + 2) = 0

f(x + 2) = f(x)

f(x) is periodic with T = 2

$$I_1 = \int_0^{2\times 4} f(x)dx = 4\int_0^2 f(x)dx$$

$$I_2 = \int_{-1}^3 f(x)dx = \int_0^4 f(x+1)dx = \int_0^4 (2 - f(x))dx$$

$$I_2 = 8 - 2\int_0^2 f(x)dx$$

$I_1 + 2I_2 = 16$

4. Consider an arithmetic series and a geometric series having four initial terms from the set {11,8,21,16,26,32,4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ____

Ans. **(3)**

Sol. AP – 11, 16, 21, 26 ....

GP – 4, 8, 16, 32 ....

So common terms are 16, 256, 4096

5. If the normal to the curve $y(x)=\int_{0}^{x}\left(2t^2-15t+10\right)dt$ at a point (a, b) is parallel to the line x+3y = –5, a>1, then the value of |a+6b| is equal to ____

Ans. **(406)**

Sol. $y'(x) = (2x^2 - 15x + 10)$

at point P

$3 = (2a^2 - 15a + 10)$

$\Rightarrow 2a^2 - 15a + 7 = 0$

$\Rightarrow 2a^2 - 14a - a + 7 = 0$

$\Rightarrow 2a(a - 7) - 1(a - 7) = 0$

$a = \frac{1}{2}$ or 7,

given $a > 1 \therefore a = 7$

also P lies on curve

$\therefore b=\int_0^a\left(2t^2-15t+10\right)dt$

$b=\int_0^7\left(2t^2-15t+10\right)dt$

$6b = -413$

$\therefore |a + 6b| = 406$

6. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then a+b+c is equal to ______

Ans. **(4)**

Sol. $$\lim_{x\to 0}\frac{\left\{a\left(1+x+\frac{x^2}{2!}+\ldots\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\ldots\right)+c\left(1-x+\frac{x^2}{2!}\ldots\right)\right\}}{x\left(x-\frac{x^3}{3!}+\ldots\right)}=2$$

$$\therefore \lim_{x\to 0}\frac{(a-b+c)+x(a-c)+x^2\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)+\ldots.}{x^2\left(1-\frac{x^2}{6}\ldots\right)}=2$$

$\therefore a - b + c = 0$

$\&a - c = 0$

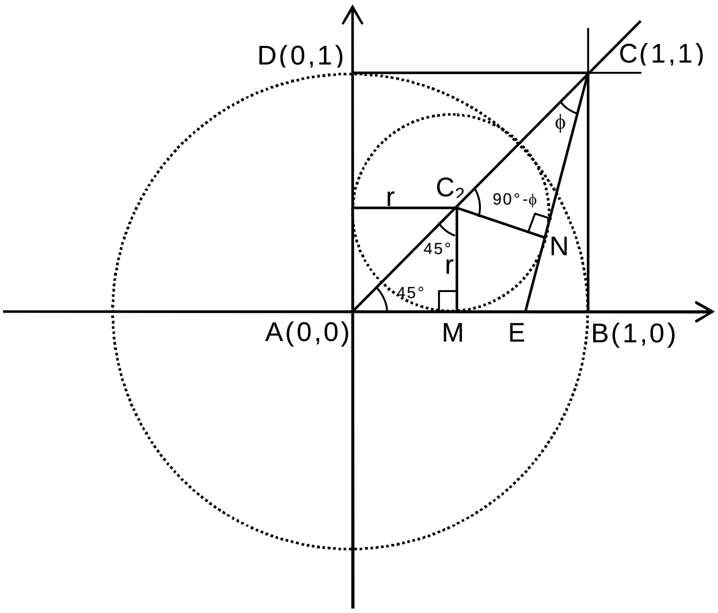
$\& \frac{a}{2}+\frac{b}{2}+\frac{c}{2}=2$

$\Rightarrow$ a + b + c = 4

7. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then $\alpha+\beta$ is equal to ___

Ans. **(1)**

Sol.



(i) $\sqrt{2}r+r=1$

$r=\frac{1}{\sqrt{2}+1}$

$r=\sqrt{2}-1$

(ii) $CC_2=2\sqrt{2}-2=2\left(\sqrt{2}-1\right)$

From ΔCC_2N = $\sin\phi=\frac{\sqrt{2}-1}{2\left(\sqrt{2}-1\right)}$

$\phi=30^\circ$

(iii) In ΔACE are sine law

$\frac{AE}{\sin\phi}=\frac{AC}{\sin 105^\circ}$

$AE=\frac{1}{2}\times\frac{\sqrt{2}}{\sqrt{3}+1}.2\sqrt{2}$

$AE=\frac{2}{\sqrt{3}+1}=\sqrt{3}-1$

$\therefore EB=1-\left(\sqrt{3}-1\right)$

$2-\sqrt{3}$

$\alpha = 2, \beta = -1 \Rightarrow \alpha + \beta = 1$

8. Let z and ω be two complex numbers such that $\omega = z\bar{z} - 2z + 2, \left|\frac{z+i}{z-3i}\right| = 1$ and Re (w) has minimum value. Then, the minimum value of $n \in N$ for Which ω^n is real, is equal to ____

Ans. **(4)**

Sol. Let z = x + iy

$|z + i| = |z-3i|$

$\Rightarrow y = 1$

Now $\omega = x^2 + y^2 - 2x - 2iy + 2$

$\omega = x^2 + 1 - 2x - 2i + 2$

$\text{Re}(\omega) = x^2 - 2x + 3$

$\text{Re}(\omega) = (x-1)^2 + 2$

$\text{Re}(\omega)_{min}$ at $x = 1 \Rightarrow z = 1 + i$

Now $\omega = 1 + 1 - 2 - 2i + 2$

$\omega = 2(1-i) = 2\sqrt{2}e^{i\left(\frac{-\pi}{4}\right)}$

$\omega^n = 2\sqrt{2}e^{i\left(\frac{-n\pi}{4}\right)}$

If ω^n is real $\Rightarrow$ n =4

9. Let $P = \begin{bmatrix} -30 & 20 & 56 \\ 90 & 140 & 112 \\ 120 & 60 & 14 \end{bmatrix}$ and $A = \begin{bmatrix} 2 & 7 & \omega^2 \\ -1 & -\omega & 1 \\ 0 & -\omega & -\omega+1 \end{bmatrix}$ where $\omega = \frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $\left(P^{-1}AP - I_3\right)^2$ is $\alpha\omega^2$, then the value of α is equal to _____

Ans. **(36)**

Sol. $\left|P^{-1}AP - I\right|^2$

$= \left|\left(P^{-1}AP - I\right)\left(P^{-1}AP - I\right)\right|^2$

$= \left|P^{-1}APP^{-1}AP - 2P^{-1}AP + I\right|$

$= \left|P^{-1}A^2P - 2P^{-1}AP + P^{-1}IP\right|$

$= \left|P^{-1}\left(A^2 - 2A + I\right)P\right|$

$= \left|P^{-1}(A - I)^2 P\right|$

$= \left|P^{-1}\right|\left|A - I\right|^2\left|P\right|$

$= \left|A - I\right|^2$

$$= \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega-1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix}^2$$

$$= \left(1\left(\omega(\omega+1)+\omega\right)-7\omega+\omega^2.\omega\right)^2$$

$$= \left(\omega^2+2\omega-7\omega+1\right)^2$$

$$= \left(\omega^2-5\omega+1\right)^2$$

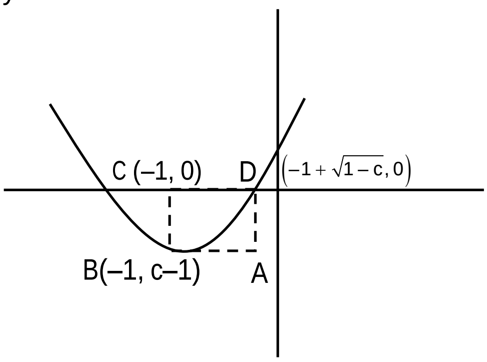
$$= \left(-6\omega\right)^2$$

$$= 36\omega^2 \Rightarrow \alpha = 36$$

10. Let the curve y=y(x) be the solution of the differential equation, $\frac{dy}{dx} = 2(x+1)$. If the numerical value of area bounded by the curve y=y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to ___

Ans. **(2)**

Sol. $y = x^2 + 2x + c$



Area of rectangle (ABCD) $= |(c-1)(\sqrt{1-c})|$

Area of parabola and x-axis $= 2\left(\frac{2}{3}\left((1-c)^{3/2}\right)\right) = \frac{4\sqrt{8}}{3}$

$1 - c = 2 \Rightarrow c = -1$

Equation of $f(x) = x^2 + 2x - 1$

$f(1) = 1 + 2 - 1 = 2$