SECTION – A

1. The vernier scale used for measurement has a positive zero error of 0.2 mm. If while taking a measurement it was noted that 'o' on the vernier scale lies between 8.5 cm and 8.6 cm, vernier coincidence is 6, then the correct value of measurement is ……………………. cm.
(least count = 0.01 cm)

(1) 8.36 cm (2) 8.56 cm (3) 8.58 cm (4) 8.54 cm

Sol. (4)

Reading = MSR + VSD × LC – zero error

Reading = 8.5 + $\frac{(0.1)\times 6}{10} - \frac{0.2}{10}$ =8.54 cm

2. For what value of displacement the kinetic energy and potential energy of a simple harmonic oscillation become equal ?

(1) $x = \frac{A}{2}$ (2) x = 0 (3) $x = \pm A$ (4) $x = \pm \frac{A}{\sqrt{2}}$

Sol. (4)

KE = PE

$$\frac{1}{2}k(A^2 - X^2) = \frac{1}{2}KX^2$$

$$A^2 - X^2 = X^2$$

$$2X^2 = A^2$$

$$X^2 = \frac{A^2}{\sqrt{2}}$$

$$X = \pm\frac{A}{\sqrt{2}}$$

3. An electron of mass m and a photon have same energy E. The ratio of wavelength of electron to that of photon is : (c being the velocity of light)

(1) $\left(\frac{E}{2m}\right)^{1/2}$ (2) $\frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$ (3) $c(2mE)^{1/2}$ (4) $\frac{1}{c}\left(\frac{2m}{E}\right)^{1/2}$

Sol. (2)

For photon $E = \frac{hc}{\lambda}$

$$\lambda_P = \frac{hc}{E} \quad \text{...(i)}$$

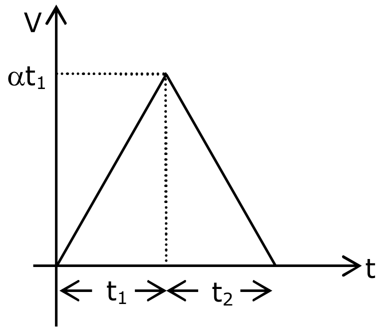
For electron $\lambda_e = \frac{hc}{\sqrt{2mE}}$ ...(ii)

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{hc}{\sqrt{2mE}}}{\frac{hc}{E}} = \sqrt{\frac{E}{2mc^2}} = \frac{1}{c}\left(\frac{E}{2m}\right)^{1/2}$$

4. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t seconds, the total distance travelled is :

(1) $\frac{\alpha\beta}{2(\alpha+\beta)}t^2$ (2) $\frac{\alpha\beta}{4(\alpha+\beta)}t^2$ (3) $\frac{4\alpha\beta}{(\alpha+\beta)}t^2$ (4) $\frac{2\alpha\beta}{(\alpha+\beta)}t^2$

Sol. **(1)**



$t_1 + t_2 = t$, $V' = 0 + \alpha t_1$

$V = u + at$

$0 = \alpha t_1 - \beta t_2$

$t_2 = \frac{\alpha}{\beta} t_1$

$t_1 + \frac{\alpha}{\beta} t_1 = t$

$t_1 = \left(\frac{\beta}{\alpha+\beta}\right) t$

Distance $= \frac{1}{2}(t_1 + t_2) \times \alpha t_1$ area of triangle

$= \frac{1}{2} t \times \alpha \left(\frac{\beta}{a+\beta}\right) t$

$= \frac{\alpha\beta}{2(\alpha+\beta)} t^2$

5. A Carnot's engine working between 400 K and 800 K has a work output of 1200 J per cycle. The amount of heat energy supplied to the engine from the source in each cycle is :

(1) 1800 J (2) 3200 J (3) 2400 J (4) 1600 J

Sol. **(3)**

$\eta = 1 - \frac{1}{2}$

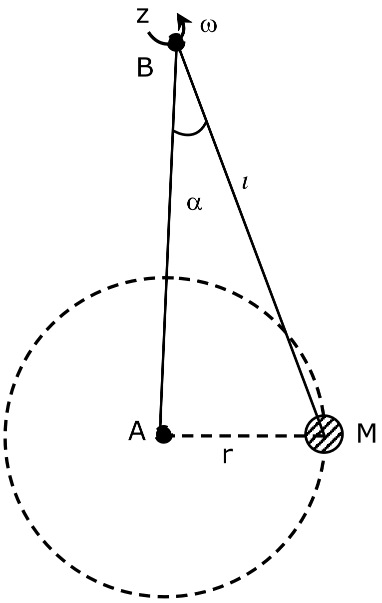
$\eta = \frac{1}{2}$

$\frac{W}{Q} = \eta$

$\frac{1200}{Q} = \frac{1}{2}$

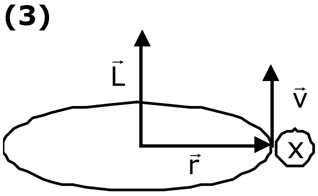
Q = 2400 J

6. A mass M hangs on a massless rod of length l which rotates at a constant angular frequency. The mass M moves with steady speed in a circular path of constant radius. Assume that the system is in steady circular motion with constant angular velocity ω. The angular momentum of M about point A is L_A which lies in the positive z direction and the angular momentum of M about point B is L_B. The correct statement for this system is :



(1) L_A and L_B are both constant in magnitude and direction
(2) L_B is constant, both in magnitude and direction
(3) L_A is contant, both in magnitude and direction
(4) L_B is constant in direction with varying magnitude

Sol. **(3)**



$\vec{L}_A = \vec{r} \times \vec{p}$

as $r \perp p$ so L_A = constant

7. Two ideal polyatomic gases at temperatures T_1 and T_2 are mixed so that there is no loss of energy. If F_1 and F_2 , m_1 and m_2, n_1 and n_2 be the degress of freedom, masses, number of molecules of the first and second gas respectively, the temperature of mixture of these two gases is :

(1) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1F_1 + n_2F_2}$ (2) $\frac{n_1F_1T_1 + n_2F_2T_2}{F_1 + F_2}$ (3) $\frac{n_1F_1T_1 + n_2F_2T_2}{n_1 + n_2}$ (4) $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

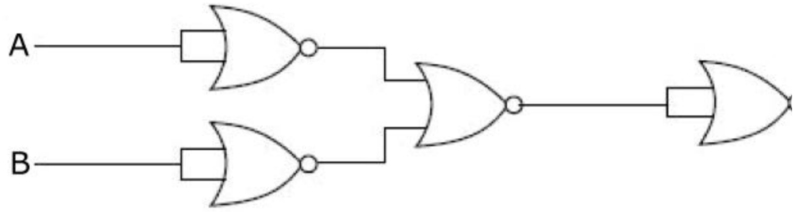
Sol. **(1)**

initial internal energy = final internal energy

$$\frac{F_1}{2}n_1RT_1 + \frac{F_2}{2}n_2RT_2 = \frac{F_1}{2}n_1RT + \frac{F_2}{2}n_2RT$$

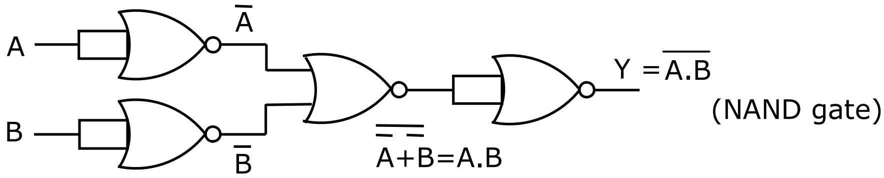
$$T = \frac{F_1n_1T_1 + F_2n_2T_2}{F_1n_1 + F_2n_2}$$

8. The output of the given combination gates represents :

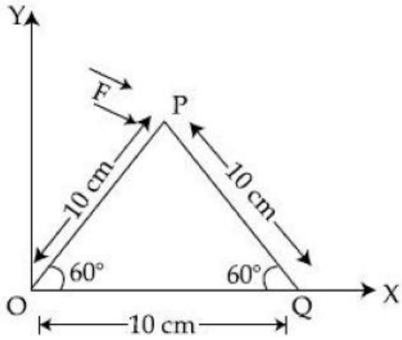


(1) XOR Gate (2)NOR Gate (3) NAND Gate (4) AND Gate

Sol. **(3)**



9. A triangular plate is shown. A force $\vec{F} = 4\hat{i} - 3\hat{j}$ is applied at point P. The torque at point P with respect to point 'O' and 'Q' are :



(1) $15 - 20\sqrt{3}$, $15 + 20\sqrt{3}$ (2) $15 + 20\sqrt{3}$, $15 - 20\sqrt{3}$

(3) $-15 + 20\sqrt{3}$, $15 + 20\sqrt{3}$ (4) $-15-20\sqrt{3}$, $15 - 20\sqrt{3}$

Sol. **(4)**

$\vec{r}_0 = (5\hat{i}+5\sqrt{3}\hat{j})\times(4\hat{i}-3\hat{j})$

$= -15\hat{k} - 20\sqrt{3}\hat{k}$

$\vec{r}_{PQ} = \vec{r}_0 - \vec{r}_Q$

$= 5\hat{i} + 5\sqrt{3}\hat{j} - 10\hat{i}$

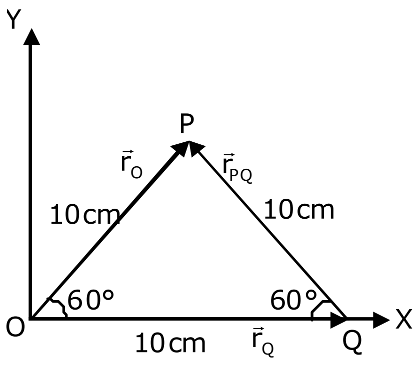
$= -5\hat{i} + 5\sqrt{3}\hat{j}$

$\vec{\tau}_Q = \vec{r}_{PQ} \times \vec{F}$

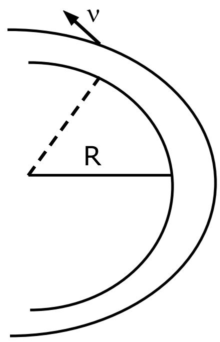
$= \left(-5\hat{i} + 5\sqrt{3}\hat{j}\right)\times\left(4\hat{i} - 3\hat{j}\right)$

$= 15\hat{k} - 20\sqrt{3}\hat{k}$

$= \left(15 - 20\sqrt{3}\right)\hat{k}$

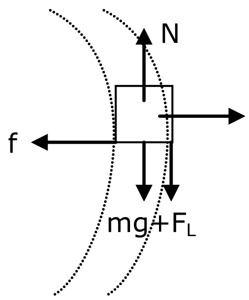


10. A modem grand-prix racing car of mas m is travelling on a flat track in a circular arc of radius R with a speed v, if the coefficient of static friction between the tyres and the track is μ_s, then the magnitude of negative lift F_1 acting downwards on the car is : (Assume forces on the four tyres are identical and g = acceleration due to gravity)



(1) $m\left(\frac{v^2}{\mu_s R} - g\right)$ (2) $m\left(\frac{v^2}{\mu_s R} + g\right)$ (3) $m\left(g - \frac{v^2}{\mu_s R}\right)$ (4) $-m\left(g + \frac{v^2}{\mu_s R}\right)$

Sol. **(1)**



$f = \frac{mv^2}{R}$

$\mu N = \frac{mv^2}{R}$

$\mu(mg + F_L) = \frac{mv^2}{R}$

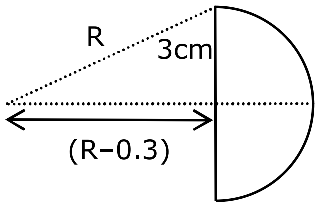
$mg + F_L = \frac{mv^2}{R\mu}$

$F_L = \frac{mv^2}{\mu R} - mg$

$= m\left(\frac{v^2}{\mu R} - g\right)$

11. The thickness at the centre of a plano convex lens is 3 mm and the diameter is 6 cm. If the speed of light in the material of the lens is 2×10^8 ms^{-1}. The focal length of the lens is _____.
(1) 0.30 cm (2) 1.5 cm (3) 15 cm (4) 30 cm

Sol. **(4)**



$R^2 = 3^2 + (R - 0.3)^2$

$R^2 = 9 + R^2 + 0.09 - 2 \times 0.3R$

$2 \times 0.3\,R = 9.09$

$R = 15.15$ cm

$\mu = \frac{C}{V} = 1.5$

$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{R}\right)$

$f \simeq 30$cm

12. If an electron is moving in the n^{th} orbit of the hydrogen atom, then its velocity (ν_n) for the n^{th} orbit is given as :

(1) $\nu_n \propto n$ (2) $\nu_n \propto \frac{1}{n}$ (3) $\nu_n \propto n^2$ (4) $\nu_n \propto \frac{1}{n^2}$

Sol. **(2)**

$v = 2.16 \times 10^6 \text{ m/s} \times \frac{z}{n}$

$\therefore v \propto \frac{1}{n}$ (as z = 1)

13. An AC current is given by $I = I_1 \sin\omega t + I_2 \cos\omega t$. A hot wire ammeter will give a reading :

(1) $\frac{I_1 + I_2}{\sqrt{2}}$ (2) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (3) $\sqrt{\frac{I_1^2 - I_2^2}{2}}$ (4) $\frac{I_1 + I_2}{2\sqrt{2}}$

Sol. **(2)**

$I_{RMS} = \sqrt{\frac{\int I^2 dt}{\int dt}}$

$I^2_{RMS} = \int_0^T \frac{(I_1 \sin\omega t + I_2 \cos\omega t)^2 dt}{T}$

$= \frac{1}{T} \int_0^T (I_1^2 \sin^2\omega t + I_2^2 \cos^2\omega t + 2I_1 I_2 \sin\omega t \cos\omega t)dt$

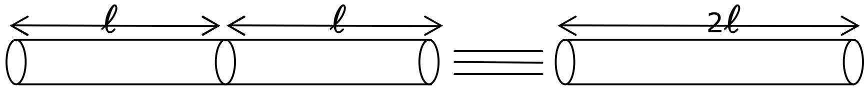
$= \frac{I_1^2}{2} + \frac{I_2^2}{2} + 0$

$I_{RMS} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$

14. Two identical metal wires of thermal conductivities K_1 and K_2 respectively are connected in series. The effective thermal conductivity of the combination is :

(1) $\frac{2K_1 K_2}{K_1 + K_2}$ (2) $\frac{K_1 + K_2}{K_1 K_2}$ (3) $\frac{K_1 + K_2}{2K_1 K_2}$ (4) $\frac{K_1 K_2}{K_1 + K_2}$

Sol. **(1)**



$$R_{eq} = R_1 + R_2$$

$$\frac{1}{K_{eq}}\frac{2\ell}{A} = \frac{\ell}{K_1 A} + \frac{\ell}{K_2 A}$$

$$\frac{2}{K_{eq}} = \frac{\ell}{K_1} + \frac{\ell}{K_2}$$

$$\frac{2}{K_{eq}} = \frac{K_1 + K_2}{K_1 K_2}$$

$$K_{eq} = \frac{2K_1 K_2}{K_1 + K_2}$$

15. A boy is rolling a 0.5 kg ball on the frictionless floor with the speed of 20 ms^{-1}. The ball gets deflected by an obstacle on the way. After deflection it moves with 5% of its initial kinetic energy. What is the speed of the ball now ?

(1) 14.41 ms^{-1} (2) 1.00 ms^{-1} (3) 19.0 ms^{-1} (4) 4.47 ms^{-1}

Sol. **(4)**

$K.E._f = 5\%\ KE_i$

$$\frac{1}{2}mv^2 = \frac{5}{100} \times \frac{1}{2} \times m \times 20^2$$

$$v^2 = \frac{1}{20} \times 20^2 = 20$$

$$v = \sqrt{20} = 2\sqrt{5} m/s$$

$= 4.47$ m/s

16. A polyatomic ideal gas has 24 vibrational modes. What is the value of γ ?

(1) 1.03 (2) 1.30 (3) 10.3 (4) 1.37

Sol. **(1)**

f = 3T + 3R + 24V

=30

$$\gamma = 1 + \frac{2}{f}$$

$$\gamma = 1 + \frac{2}{30}$$

= 1.066

Nearest Ans. = 1.03

17. A current of 10 A exists in a wire of crosssectional area of 5 mm^2 with a drift velocity of 2×10^{-3} ms^{-1}. The number of free electrons in each cubic meter of the wire is ________

(1) 1×10^{23} (2) 2×10^{6} (3) 2×10^{25} (4) 625×10^{25}

Sol. **(4)**

$I = neAV_d$

$n = \frac{I}{eAV_d}$

$= \frac{10}{1.6 \times 10^{-9} \times 5 \times 10^{-6} \times 2 \times 10^{-3}}$

$= \frac{10^{25}}{16} = 6.25 \times 10^{27} = 625 \times 10^{25}$

18. A solenoid of 1000 turns per metre has a core with relative permeability 500. Insulated windings of the solenoid carry an electric current of 5 A. The magnetic flux density produced by the solenoid is : (Permeability of free space = $4\pi \times 10^{-7}$ H/m)

(1) $2 \times 10^{-3}\pi T$ (2) $\frac{\pi}{5}$ T (3) $10^{-4}\ \pi T$ (4) πT

Sol. **(4)**

$B = \mu\ n\ i$

$B = \mu_r\ \mu_0\ n\ i$

$B = 500 \times 4\pi \times 10^{-7} \times 10^3 \times 5$

$B = \pi \times 10^{-3} \times 10^3$

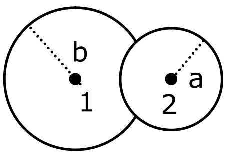
$B = \pi$

19. When two soap bubbles of radii a and b (b > a) coalesce, the radius of curvature of common surface is -

(1) $\frac{b-a}{ab}$ (2) $\frac{ab}{b-a}$ (3) $\frac{ab}{a+b}$ (4) $\frac{a+b}{ab}$

Sol. **(2)**

$\frac{1}{R} = \frac{1}{a} - \frac{1}{b}$; $R = \frac{ab}{b-a}$



$P_1 - P_0 = \frac{4S}{b}$

$P_2 - P_0 = \frac{4S}{a}$

$P_2 - P_1 = \frac{4S}{R}$

$eq(2) - eq(1) = eq(3)$

$\frac{1}{a} - \frac{1}{b} = \frac{1}{R}$

$\therefore R = \frac{ab}{b-a}$

20. Which level of the single ionized carbon has the same energy as the ground state energy of hydrogen atom ?

(1) 8 (2) 1 (3) 6 (4) 4

Sol. **(3)**

$$E_n = -13.6\frac{Z^2}{n^2}$$

$E_{n^{th}}$ of Carbon = $E_{1^{st}}$ of Hydrogen

$$-13.6 \times \frac{6^2}{n^2} = -13.6 \times \frac{1^2}{1^2}$$

n = 6

Section - B

1. A parallel plate capacitor whose capacitance C is 14pF is charged by a battery to a potential difference V = 12 V between its plates. The charging battery is now disconnected and a porcelin plate with k = 7 is inserted between the plates, then the plate would oscillate back and forth between the plates, with a constant mechanical energy of ____ pJ.

(Assume no friction)

Sol. 864

$U_i = \frac{1}{2}\ cv^2$

$= \frac{1}{2} \times 14 \times (12)^2$ pJ

= 1008 pJ

$U_f = \frac{Q^2}{2kC}$

$= \frac{(14 \times 12)^2}{2 \times 7 \times 14}$

= 144 pJ

oscillating energy = $U_i - U_f$

= 1008 − 144

= 864 pJ

2. If 2.5 × 10^{-6} N average force is exerted by a light wave on a non-reflecting surface of 30 cm^2 area during 40 minutes of time span, the energy flux of light just before it falls on the surface is _______ W/cm^2. (Round off to the nearest integer)

(Assume complete absorption and normal incidence conditions are there)

Sol. **25**

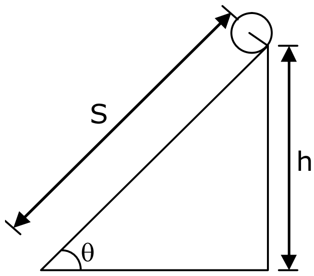
Pressure = $\frac{\text{Intensity}}{C}$ (for absorbing surface)

I = P × C

$I = \frac{2.5 \times 10^{-6}}{30cm^2}$ N × 3 × 10^8 m/s

I = 25 W/cm^2

3. The following bodies
(1) a ring
(2) a disc
(3) a solid cylinder
(4) a solid sphere
of same mass 'm' and radius 'R' are allowed to roll down without slipping simultaneiously from the top of the inclined plane. The body which will reach first at the bottom of the inclined plane is________.
[Mark the body as per their respective numbering given in the question]



Sol. **4**

$$a = \frac{g\sin\theta}{\left(1 + \frac{I}{mR^2}\right)}$$

$I_R = mR^2$, $a_R = g\sin\theta/2$

$I_D = \frac{mR^2}{2}$, $a_D = \frac{2}{3} g\sin\theta$

$I_{SC} = \frac{mR^2}{2}$, $a_{SC} = \frac{2}{3} g\sin\theta$

$I_{SC} = \frac{2}{5} mR^2$, $a_{ss} = \frac{5}{7} g\sin\theta$

$S = ut + \frac{1}{2}at^2$,

$t = \sqrt{\frac{2S}{a}}$

$\therefore t \propto \frac{1}{\sqrt{a}}$

solid sphere will take minimum time.

4. For VHF signal broadcasting,________ km^2 of maximum service area will be covered by an antenna tower of height 30 m, if the receiving antenna is placed at ground. Let radius of the earth be 6400 km. (Round off to the nearest integer) (Take π as 3.14)

Sol. **1206.00**

$d = \sqrt{2hR}$ area = πd^2

Area = $\pi(2hR) = 3.14\times2\times30\times6400\times10^3 \, . \, m^2$

$= 1205.76 \, km^2$

$\approx 1206 \, km^2$

5. Consider two identical springs each of spring constant k and negligible mass compared to the mass M as shown. Fig.1 shows one of them and Fig. 2 shows their series combination. The ratios of time period of oscillation of the two SHM is $T_b/T_a = \sqrt{x}$, where value of x is ______. (Round off to the nearest integer)

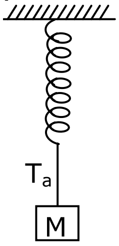


Fig.1

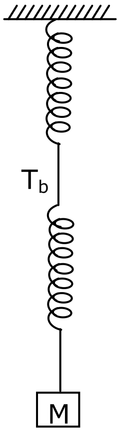


Fig.2

Sol. **2.00**

$$K_{eq_{series}} = \frac{k_1 k_2}{k_1 + k_2} = \frac{k}{2}$$

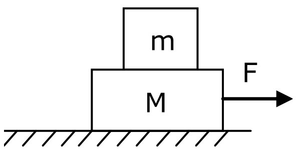
$$T_B = 2\pi\sqrt{\frac{M}{k/2}} = 2\pi\sqrt{\frac{2M}{k}}$$

$$T_B = \sqrt{2}T_A$$

$$\frac{T_B}{T_A} = \sqrt{2}$$

$\therefore$ x = 2

6. Two blocks (m = 0.5 kg and M = 4.5 kg) are arranged on a horizontal frictionless table as shown in figure. The coefficient of static friction between the two blocks is 3/7. Then the maximum horizontal force that can be applied on the larger block so that the blocks move together is _______ N. (Round off to the nearest integer) [Take g as 9.8 ms^{-2}]



Sol. **21**

a_{max} of m_1 (i.e. 0.5 kg)

= Mg

$= \frac{3}{7} \times 9.8$

= 4.2 m/s^2

$\therefore F_{max}$ 4.2×5 = 21 N

7. The radius in kilometer to which the present radius of earth (R = 6400 km) to be compressed so that the escape velocity is increased 10 times is ______.

Sol. **64**

$$V_{es} = \sqrt{\frac{2GM}{R}}$$

$$V_{es}\sqrt{R} = \text{count}$$

$$V_{es.}\sqrt{R} = 10V_{es}\sqrt{R'}$$

$$R' = \frac{R}{100} = 64KM$$

8. The equivalent ressitance of series combination of two resistors is 's'. When they are connected in parallel, the equivalent resistance is 'p'. If s = np, then the maximum value for n is ______. (Round off to the nearest integer)

Sol. **4**

s = np

$$R_1 + R_2 = n\left[\frac{R_1R_2}{R_1 + R_2}\right]$$

$$R_1^2 + R_2^2 + 2R_1R_2 = nR_1R_2$$

$$R_1^2 + (2-n)R_1R_2 = P_2^2 = 0$$

$$\left[(2-n)R_2\right]^2 = 4\times1\times R_2^2$$

$$(2-4)^2R_2^2 = 4R_2^2$$

2-n = ±2

2-n = -2

n = 4

So n = 4

9. The angular speed of truck wheel is increased from 900 rpm to 2460 rpm in 26 seconds. The number of revolutions by the truck engine during this time is ______.
(Assuming the acceleration to be uniform).

Sol. **728**

$$\omega_f = 2460 \times \frac{2\pi}{60}$$

$$= 82\pi$$

$$\omega_i = \frac{900\times2\pi}{60} = 30\ \pi$$

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$= \frac{82\pi - 30\pi}{26}$$

$$= 2\pi \text{ rad/sec}^2$$

$$\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha}$$

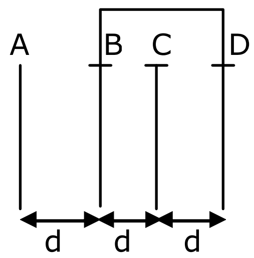
$= \dfrac{(82\pi + 30\pi)(82\pi - 30\pi)}{2 \times 2\pi}$

$= \dfrac{(112 \times 52)\pi^2}{4\pi}$

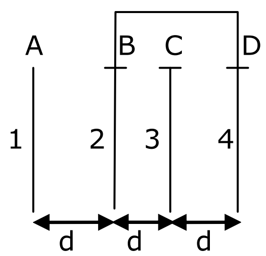
No. of revolution $= \dfrac{(112 \times 13)\pi}{2\pi}$

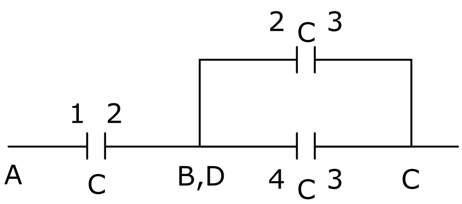
= 728

10. Four identical rectangular plates with length, l =2 cm and breadth, b = 3/2 cm are arranged as shown in figure. The equivalent capacitance between A and C is $\dfrac{x\varepsilon_0}{d}$. The value of x is _______. (Round off to the nearest integer)



Sol. **2.00**





$C = \dfrac{\epsilon_0 A}{d}$

$C_{eq} = \dfrac{2C \times C}{2C + C} = \dfrac{2C}{3} = \dfrac{2}{3}\dfrac{\epsilon_0 A}{d} = \dfrac{2}{3} \times \dfrac{\epsilon_0}{d} \times \left(2 \times \dfrac{3}{2}\right) = \dfrac{2\varepsilon_0}{d}$