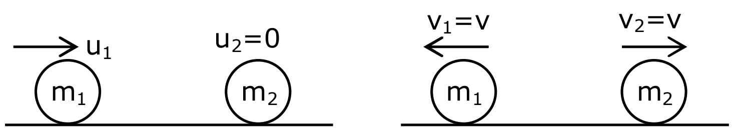
SECTION – A

1. The decay of a proton to neutron is :
(1) Not possible as proton mass is less than the neutron mass
(2) Always possible as it is associated only with β^+ decay
(3) Possible only inside the nucleus
(4) Not possible but neutron to proton conversion is possible

Sol. **(3)**
Positron emission or Beta plus decay is a subtype of radioactive decay called Beta decay, in which a proton inside a radionuclide nucleus is converted into a neutron while releasing a positron and an electron neutrino.
So, decay of a proton to neutron is possible only inside the nucleus.

2. An object of mass m_1 collides with another object of mass m_2, which is at rest. After the collision the objects move with equal speeds in opposite direction. The ratio of the masses $m_2 : m_1$ is :
(1) 2 : 1 (2) 1 : 1 (3) 1 : 2 (4) 3 : 1

Sol. **(4)**

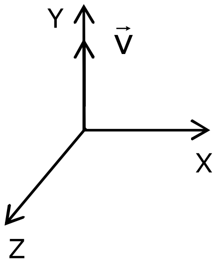


Before Collision After Collision

From conservation of linear momentum;
$P_i = P_f$
$\Rightarrow m_1u_1 + m_2(0) = m_1(-v) + m_2 v$
$\Rightarrow m_1u_1 = v(m_2 - m_1)$ ...(i)
$\because e = 1 = \frac{V_{sep}}{V_{app}} = \frac{v-(-v)}{u_1} = \frac{2v}{u_1}$
$\Rightarrow u_1 = 2V$ ...(ii)
from (i) & (ii)
$m_1(2v) = v(m_2 - m_1)$
$\Rightarrow 2m_1 = m_2 - m_1$
$\Rightarrow 3m_1 = m_2$
$\Rightarrow \frac{m_2}{m_1} = \frac{3}{1}$

3. A plane electromagnetic wave propagating along y-direction can have the following pair of electric field $(\vec{E})$ and magnetic field $(\vec{B})$ components.
(1) E_x, B_z or E_z, B_x (2) E_y, B_x or E_x, B_y
(3) E_x, B_y or E_y, B_x (4) E_y, B_y or E_z, B_z

Sol. **(1)**



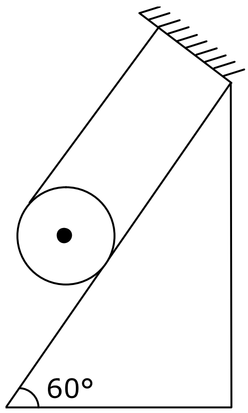
$\because \hat{E} \times \hat{B} = \hat{C}$ for electromagnetic waves.

$\because \hat{E} \times \hat{B}$ should point in the direction of propagation of wave (y-direction here)

$\therefore$ possible combinations are

(E_x, B_z) or (E_z, B_x)

4. A solid cylinder of mass m is wrapped with an inextensible light string and, is placed on a rough inclined plane as shown in the figure. The frictional force acting between the cylinder and the inclined plane is :

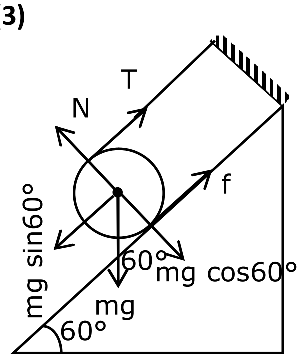


[The coefficient of static friction, μ_s, is 0.4]

(1) $\frac{7}{2}$mg (2) 0

(3) $\frac{mg}{5}$ (4) 5 mg

Sol. **(3)**



Let's assume equilibrium condition of cylinder

$\therefore\ T + f = mg\sin 60^\circ$ ....(i)

& $TR - fR = 0$ ....(ii)

from (i) & (ii)

$$T = f_{req} = \frac{mg\sin 60^\circ}{2}$$

But limiting friction < required friction.

$$\therefore\ \mu mg\cos 60^\circ < \frac{mg\sin 60^\circ}{2}$$

$\therefore$ cylinder won't be in equilibrium

$\therefore$ f will be kinetic

& $f = \mu_K N$

$= \mu_k mg\cos 60^\circ$

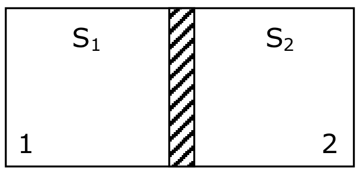
$$= 0.4 \times mg \times \frac{1}{2} = \frac{mg}{5}$$

5. An ideal gas in a cylinder is separated by a piston in such a way that the entropy of one part is S_1 and that of the other part is S_2. Given that $S_1 > S_2$. If the piston is removed then the total entropy of the system will be :

(1) $S_1 + S_2$ (2) $S_1 - S_2$ (3) $S_1 \times S_2$ (4) $\frac{S_1}{S_2}$

Sol. **(1)**

$(S_1>S_2)$



Piston

for gas 1 , $S_1 = \frac{f}{2}n_1R$

for gas 2, $S_2 = \frac{f}{2}n_2R$ } identical gas, so f will be same.

after removal of piston,

$$s = \frac{f}{2}(n_1 + n_2)R = S_1 + S_2$$

6. The time taken for the magnetic energy to reach 25% of its maximum value, when a solenoid of resistance R, inductance L is connected to a battery, is :

(1) $\frac{L}{R}\ln 2$ (2) $\frac{L}{R}\ln 10$ (3) Infinite (4) $\frac{L}{R}\ln 5$

Sol. **(1)**

$\therefore$ Magnetic energy, $U = \frac{1}{2}LI^2$, when current in circuit is I.

$\rightarrow$ When circuit has maximum current,

maximum value of Magnetic energy, $U_0 = \frac{1}{2}LI_0^2$

Given : U = 25% of U_0.

$\Rightarrow \frac{1}{2}LI^2 = \frac{1}{4} \times \frac{1}{2}LI_0^2$

$\Rightarrow I^2 = \frac{I_0^2}{4} \Rightarrow I = \frac{I_0}{2}$

$\therefore\ I = I_0(1 - e^{-t/\tau})$

$\Rightarrow \frac{I_0}{2} = I_0(1 - e^{-t/\tau})$

$\Rightarrow \frac{1}{2} = e^{-t/\tau}$

$\Rightarrow e^{t/\tau} = 2$

$\Rightarrow t = \tau \ell n2$

$\Rightarrow t = \frac{L}{R}\ell n2$

7. For an adiabatic expansion of an ideal gas, the fractional change in its pressure is equal to (where λ is the ratio of specific heats) :

(1) $-\gamma\frac{dV}{V}$ (2) $\frac{dV}{V}$ (3) $-\frac{1}{\gamma}\frac{dV}{V}$ (4) $-\gamma\frac{V}{dV}$

Sol. **(1)**

for adiabatic expansion :

$PV^{\gamma} = \text{const.}$

$\Rightarrow \ell nP + \gamma \ln v = \text{const.}$

$\Rightarrow$ differentiating both sides;

$\frac{dp}{p} + \gamma\frac{dv}{v} = 0$

$\Rightarrow \frac{dp}{p} = -\gamma\frac{dv}{V}$

8. The correct relation between α (ratio of collector current to emitter current) and β (ratio of collector current to base current) of a transistor is :

(1) $\alpha = \frac{\beta}{1+\beta}$ (2) $\alpha = \frac{\beta}{1-\alpha}$ (3) $\beta = \frac{1}{1-\alpha}$ (4) $\beta = \frac{\alpha}{1+\alpha}$

Sol. **(1)**

$\alpha = \frac{I_C}{I_E}$ & $\beta = \frac{I_C}{I_B}$

& $I_E = I_B + I_C$

$\therefore \frac{I_E}{I_C} = \frac{I_B}{I_C} + \frac{I_C}{I_C}$

$\Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1 = \frac{1+\beta}{\beta}$

$\Rightarrow \alpha = \frac{\beta}{1+\beta}$

9. In a series LCR circuit, the inductive reactance (x_L) is 10Ω and the capacitive reactance (X_C) is 4Ω. The resistance (R) in the circuit is 6Ω.

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{2\sqrt{2}}$

Sol. **(1)**

Given : $X_L = 10\Omega$

$X_C = 4\Omega$

$R = 6\Omega$

$\therefore$ Power factor = $\cos\theta = \frac{R}{Z}$

$$= \frac{R}{\sqrt{R^2 + (x_L - X_C)^2}}$$

$$= \frac{6}{\sqrt{6^2 + (10-4)^2}}$$

$$= \frac{6}{6\sqrt{2}} = \frac{1}{\sqrt{2}}$$

10. A proton and an α-particle, having kinetic energies K_p and K_α repectively, enter into a magnetic field at right angles.
The ratio of the radii of trajectory of proton to that α-particle is 2 : 1. The ratio of $K_P : K_\alpha$ is :

(1) 1 : 8 (2) 1 : 4 (3) 8 : 1 (4) 4 : 1

Sol. **(4)**

$\because r = \frac{mv}{qB} = \frac{p}{qB}$

& $\frac{m\alpha}{m_p} = \frac{4}{1}$

$\because \frac{r_p}{r_\alpha} = \frac{P_P}{P_\alpha} \cdot \frac{q_\alpha}{q_P} = \frac{2}{1}$

$\Rightarrow \frac{P_P}{P_\alpha} = \frac{2q_P}{q_\alpha} = 2\left(\frac{1}{2}\right) = 1$

Now, $\frac{K_p}{K_\alpha} = \left(\frac{P_p}{P_\alpha}\right)^2 \cdot \frac{m_\alpha}{m_p} = 1 \times 4 = \frac{4}{1}$

11. The function of time representing a simple harmonic motion with a period of $\frac{\pi}{\omega}$ is :

(1) $\cos(\omega t) + \cos(2\omega t) + \cos(3\omega t)$ (2) $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

(3) $\sin^2(\omega t)$ (4) $\sin(\omega t) + \cos(\omega t)$

Sol. **(2)**

for expression, $3\cos\left(\frac{\pi}{4} - 2\omega t\right)$

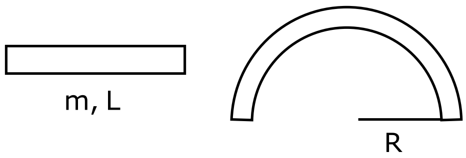
Angular frequency $=2\omega$

$\therefore$ Time period, $T = \dfrac{2\pi}{(2\omega)} = \dfrac{\pi}{\omega}$

12. Consider a uniform wire of mass M and length L. It is bent into a semicircle. Its moment of inertia about a line perpendicular to the plane of the wire passing through the centre is :

(1) $\frac{1}{2}\frac{ML^2}{\pi^2}$ (2) $\frac{2}{5}\frac{ML^2}{\pi^2}$ (3) $\frac{1}{4}\frac{ML^2}{\pi^2}$ (4) $\frac{ML^2}{\pi^2}$

Sol. **(4)**



$\because L = \pi R \Rightarrow R = \dfrac{L}{\pi}$

& moment of inertia = $mR^2 = m\left(\dfrac{L}{\pi}\right)^2 = \dfrac{mL^2}{\pi^2}$

13. The angular momentum of a planet of mass M moving around the sun in an elliptical orbit is $\vec{L}$. The magnitude of the areal velocity of the planet is :

(1) $\frac{L}{M}$ (2) $\frac{2L}{M}$ (3) $\frac{L}{2M}$ (4) $\frac{4L}{M}$

Sol. **(3)**

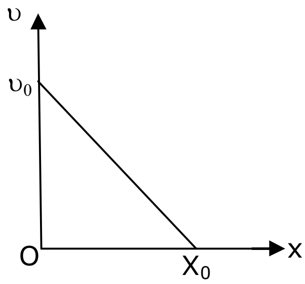
theoretical concept.

$\therefore$ Areal velocity, $\dfrac{dA}{dt} = \dfrac{L}{2M}$

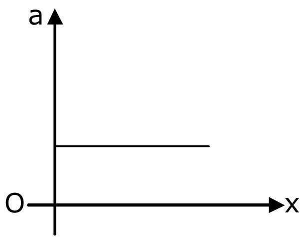
$\rightarrow$ L = Angular momentum of planet

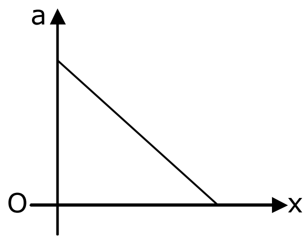
M = Mass of planet.

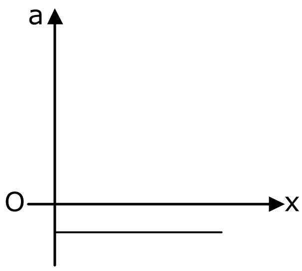
14. The velocity – displacement graph of a particle is shown in the figure.

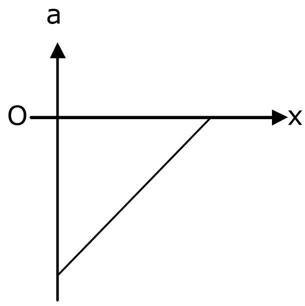


The acceleration – displacement graph of the same particle is represented by :

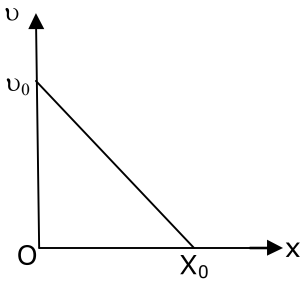
(1) 

(2) 

(3) 

(4) 

Sol. **(4)**



Slope of given graph, $m = -\frac{v_0}{x_0}$

$\therefore v = \left(-\frac{v_0}{x_0}\right)x + v_0$

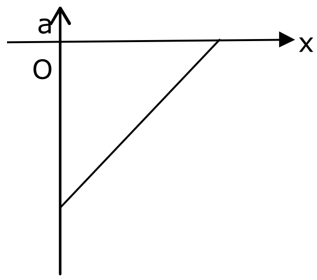
& $\because a = \frac{vdv}{dx}$

$= \left[\left(-\frac{v_0}{x_0}\right)x + v_0\right] . \frac{d}{dx}\left[\left(\frac{-v_0}{x_0}\right)x + v_0\right]$

$= \left[\left(-\frac{v_0}{x_0}\right)x + v_0\right] . \left[-\frac{v_0}{x_0} + 0\right]$

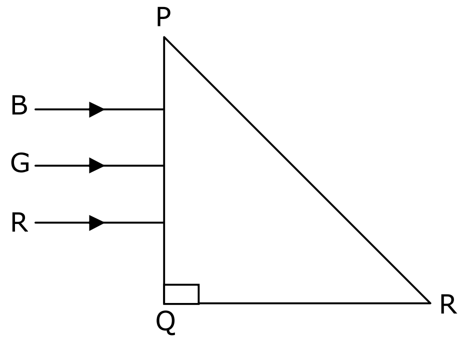
$= \frac{v_0^2}{x_0^2} \cdot x \;\; - \frac{v_0^2}{x_0}$

Again comparing with standard equation of straight line (y = mx + c)
Here, m = +ve & c = -ve



$\therefore$ only graph (4) is possible.

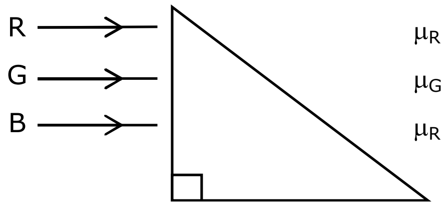
15. Three rays of light, namely red (R), green (G) and blue (B) are incident on the face PQ of a right angled prism PQR as shown in the figure.



The refractive indices of the material of the prism for red, green and blue wavelength are 1.27, 1.42 and 1.49 respectively. The colour of the ray(s) emerging out of the face PR is :

(1) Blue (2) Green (3) Red (4) Blue and green

Sol. **(3)**



$\mu_R = 1.27$

$\mu_G = 1.42$

$\mu_R = 1.49$

for TIR, i = 45°

& i > C $\Rightarrow$ 45°>C

$\therefore$ $\sin c = \frac{1}{\mu}$

$\Rightarrow$ $c = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\mu > \sqrt{2}$

$\mu > 1.414$

$\therefore$ μ_G & μ_B are more than μ.

$\therefore$ only red will come out.

16. Consider a sample of oxygen behaving like an ideal gas. At 300 K, the ratio of root mean square (rms) velocity to the average velocity of gas molecule would be :
(Molecular weight of oxygen is 32 g/mol; R=8.3 JK^{-1} mol^{-1})

(1) $\sqrt{\frac{3}{3}}$ (2) $\sqrt{\frac{8}{3}}$ (3) $\sqrt{\frac{3\pi}{8}}$ (4) $\sqrt{\frac{8\pi}{3}}$

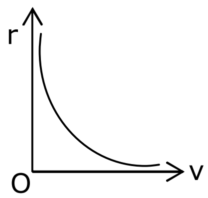
Sol. **(3)**

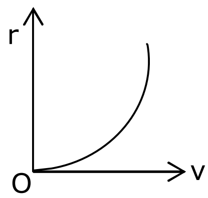
$\because V_{rms} = \sqrt{\frac{3RT}{M}}$

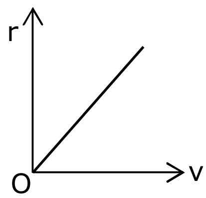
& $V_{avg} = \sqrt{\frac{8RT}{\pi M}}$

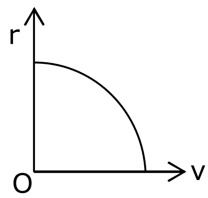
$\therefore \frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3\pi}{8}}$

17. A particle of mass m moves in a circular orbit under the central potential field, $U(r) = \frac{C}{r}$, where C is a positive constant.
The correct radius – velocity graph of the particle's motion is :

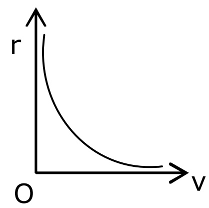
(1) 

(2) 

(3) 

(4) 

Sol. **(1)**



Given potential field. $U(r) = \frac{-C}{r}$

$\because F = -\frac{dU}{dr} = \frac{c}{r^2}$

& $F_C = \frac{mv^2}{r}$

$\therefore \frac{mv^2}{r} = \frac{C}{r^2}$

$\Rightarrow r = \frac{c}{mv^2}$

$\Rightarrow r \alpha \frac{1}{v^2}$

$\therefore$ the graph between r & v will be hyperbolic.

18. Which of the following statement are correct ?

(A) Electric monopoles do not exist whereas magnetic monopoles exist.

(B) Magnetic field lines due to a solenoid at its ends and outside cannot be completely straight and confined.

(C) Magnetic field lines are completely confined withing a toroid.

(D) Magnetic field lines inside a bar magnet are not parallel.

(E) $\chi = -1$ is the condition for a perfect diamagnetic material, where χ is its magnetic susceptibility.

choose the correct answer from the options given below :

(1) (B and (C) only (2) (B) and (D) only

(3) (C) and (E) only (4) (A) and (B) only

Sol. **(3)**

(a) Electric monopoles exist while magnetic monopoles do not exist.

(b) Magnetic field lines at the ends and outside of solenoid cannot be confined.

(c) Magnetic field lines are confined within a toroid.

(d) Magnetic field lines inside a bar magnet are parallel.

(e) For perfectly diamagnetic material $x = -1$.

$\therefore$ (C) & (e) are correct.

19. The speed of electrons in a scanning electron microscope is 1×10^{7} ms^{-1} . If the protons having the same speed are used instead of electrons, then the resolving power of scanning proton microscope will be changed by a factor of :

(1) $\frac{1}{\sqrt{1837}}$ (2) $\sqrt{1837}$ (3) 1837 (4) $\frac{1}{1837}$

Sol. **3**

$\because RP \alpha \frac{1}{\lambda}$

& $\lambda \alpha \frac{1}{m}$

$\Rightarrow RP \alpha m$

$\therefore \frac{RP_{pr}}{RP_{e\ell}} = \frac{m_{pr}}{m_{e\ell}} = 1837$

20. If the angular velocity of earth's spin is increased such that the bodies at the equator start floating, the duration of the day would be approximately :

[Take g = 10 ms^{-2}, the radius of earth, R = 6400×10^{3} m, Take $\pi = 3.14$]

(1) 60 minutes (2) does not change (3) 84 minutes (4) 1200 minutes

Sol. **(3)**

At equator, body starts floating i.e. condition of weightlessness.

$\because g_{equ} = g - R\omega^2 = 0$

$\Rightarrow \omega = \sqrt{\frac{g}{R}}$

$\therefore$ Time period, T = $\frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}$

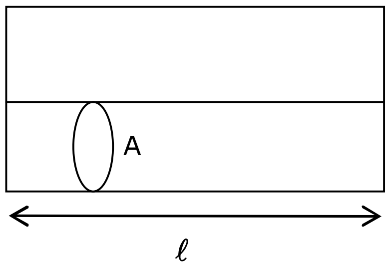
$= 2 \times 3.14\sqrt{\frac{6.4 \times 10^6}{10}}$

$\simeq 84$ minutes

Section – B

1. Two wires of same length and thickness having specific resistances 6Ω cm and 3Ω cm respectively are connected in parallel. The effective resistivity is $\rho\,\Omega$ cm. The value of ρ, to the nearest integer, is _________.

Sol. **4**



In parallel,

$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

$$\because \frac{\rho\ell}{2A} = \frac{\rho_1\frac{\ell}{A} \times \rho_2\frac{\ell}{A}}{\rho_1\frac{\ell}{A} \times \rho_2\frac{\ell}{A}}$$

$$\Rightarrow \frac{\rho}{2} = \frac{6 \times 3}{6 + 3} = 2$$

$$\Rightarrow \rho = 2 \times 2 = 4$$

2. A ball of mass 4 kg, moving with a velocity of $10ms^{-1}$, collides with a spring of length 8m and force constant 100 Nm^{-1}. The length of the compressed spring is x m. The value of x, to the nearest integer, is _______

Sol. **6**

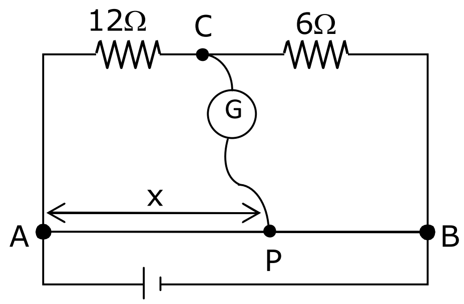
If spring is compressed by y

$$\therefore \frac{1}{2}mv^2 = \frac{1}{2}ky^2$$

$$\Rightarrow y = \sqrt{\frac{m}{k}}.v = \sqrt{\frac{4}{100}} \times 10 = 2m$$

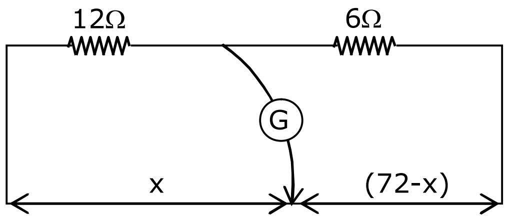
$\therefore$ final length of spring = 8 – 2 = 6m

3. Consider a 72 cm long wire AB as shown in the figure. The galvanometer jockey is placed at P on AB at a distance x cm from A. The galvanometer shows zero deflection.



The value of x, to the nearest integer, is ____________

Sol. **48**



At balanced condition

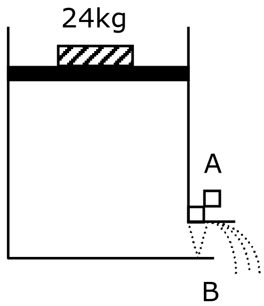
$\Rightarrow \frac{x}{12} = \frac{72-x}{6}$

$\Rightarrow x = 2(72-x)$

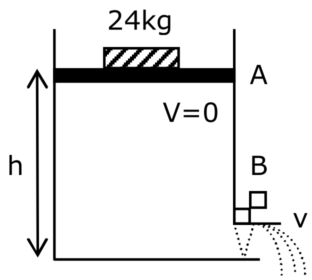
$\Rightarrow 3x = 144$

$\Rightarrow x = \frac{144}{3} = 48\text{cm}$

4. Consider a water tank as shown in the figure. It's cross-sectional area is 0.4 m^2. The tank has an opening B near the bottom whose cross-section area is 1 cm^2. A load of 24 kg is applied on the water at the top when the height of the water level is 40 cm above the bottom, the velocity of water coming out the opening B is v ms^{-1}.
The value of v, to the nearest integer, is ………………….
[Take value of g to be 10 ms^{-2}]



Sol. **3**



On applying Bernoulli's theorem at points A & B

$P_0 + \frac{mg}{A} + \rho gh + \frac{1}{2}\rho V^2 = P_{atm} + \frac{1}{2}\rho v^2$

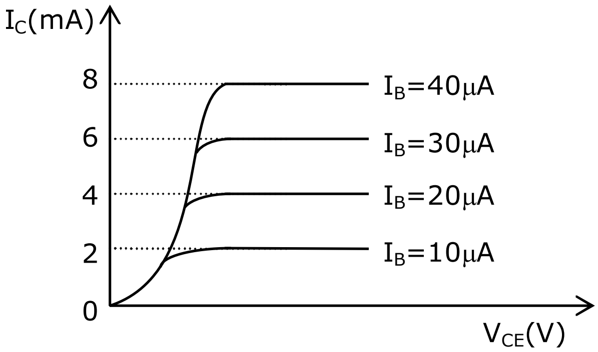
$\because V = 0$ (at A)

$\because \frac{mg}{A} + \rho gh = \frac{1}{2}\rho v^2$

$\Rightarrow \frac{24 \times 10}{0.4} + 10^3 \times 10 \times 0.4 = \frac{1}{2} \times 10^3 v^2$

$\Rightarrow v \simeq 3m/s$

5. The typical output characteristics curve for a transistor working in the common-emitter configuration is shown in the figure.



The estimated current gain from the figure is …………..

Sol. **200**

for common emitter configuration

$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{(4-2)mA}{(20-10)\mu A}$

$= \frac{2}{10} \times \frac{10^{-3}}{10^{-6}} = 200$

6. The radius of a sphere is measured to be (7.50 $\pm$ 0.85) cm. Suppose the percentage error in its volume is x.
The value of x, to the nearest x, is ………………

Sol. **34**

$\therefore$ volume, $V = \frac{4}{3}\pi r^3$

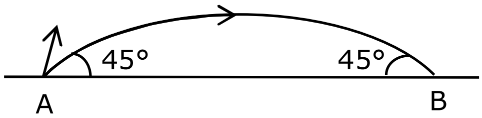
$\frac{\Delta V}{V} \times 100 = 3\frac{\Delta r}{V} \times 100$

$= 3\left(\frac{0.85}{7.5}\right) \times 100$

$= \frac{2.55}{7.5} \times 100$

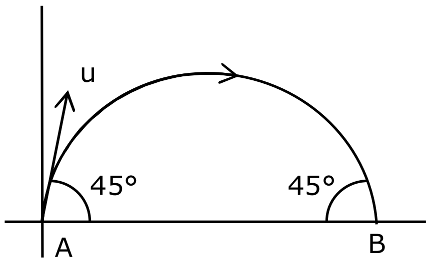
$=34\%$

7. The projectile motion of a particle of mass 5 g is shown in the figure.



The initial velocity of the particle is $5\sqrt{2}$ ms^{-1} and the air resistance is assumed to be negligible. The magnitude of the change in momentum between the points A and B is $x \times 10^{-2}$ $kgms^{-1}$. The value of x, to the nearest integer, is …………………..

Sol. **5**



$\Delta\vec{P} = \vec{P}_B - \vec{P}_A$

$= m(5\sqrt{2}\cos 45^\circ\,\hat{i} - 5\sqrt{2}\sin 45^\circ\,\hat{j})$

$-m(5\sqrt{2}\cos 45^\circ\,\hat{i} + 5\sqrt{2}\sin 45^\circ\,\hat{j})$

$= -2m \times 5\sqrt{2} \times \frac{1}{\sqrt{2}}\hat{j}$

$= -10 \times 5 \times 10^{-3}$ kg m/s

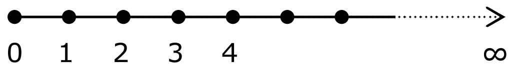
$\therefore |\vec{P}| = 5 \times 10^{-2}$kg m/s

$\therefore x = 5$

8. An infinite number of point charges, each carrying $1\mu C$ charge, are placed along the y-axis at y = 1m, 2m, 4m, 8m …………………..
The total force on a 1C point charge, placed at the origin, is $x \times 10^3$ N.
The value of x, to the nearest integer, is ………

[Take $\frac{1}{4\pi\varepsilon_0}$ = 9×10^9 Nm^2/C^2]

Sol. **12**



$$F_{total} = \frac{kq_1q_2}{r_1^2} + \frac{kq_1q_3}{r_2^2} + \frac{kq_1q_4}{r_3^2} + ...$$

$$= 9 \times 10^9 \times 10^{-6}\left[1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2^2}\right)^2 + \left(\frac{1}{2^3}\right)^2 + \left(\frac{1}{2^\infty}\right)^2\right]$$

$$= 9 \times 10^9 \times 10^{-6}\left[\frac{1}{1-\frac{1}{4}}\right]$$

$\left[\because S_\infty = \frac{a}{1-r}\right]$ for G.P.

$$= 9 \times 10^3 \times \frac{4}{3} = 12 \times 10^3 N$$

9. A TV transmission tower antenna is at a height of 20 m. Suppose that the receiving antenna is at.
(i) Ground level
(ii) a height of 5m
The increase in antenna range in case (ii) relative to case (i) is n%.
The value of n, to the nearest integer, is

Sol. **50**

for calculation of Range from tower.

$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$\therefore$ h_T = height of Tower
& h_R = height of receiver

$\rightarrow$ for 1st case; h_T = 20m, h_R = 0

$\because d_1 = \sqrt{2 \times 6400 \times 10^3 \times 20} = 16km$

$\rightarrow$ for 2nd case; h_T = 20m, h_R = 5m

$\because d_2 = \sqrt{2 \times 6400 \times 10^3 \times 20} + \sqrt{2 \times 6400 \times 10^3 \times 5}$

= 16 + 8 = 24 km

$\therefore$ % change in range

$$= \frac{d_2 - d_1}{d_1} \times 100$$

$$= \frac{24 - 16}{16} \times 100$$

$$= \frac{8}{16} \times 100 = 50\%$$

10. A galaxy is moving away from the earth at a speed of 286 kms^{-1}. The shift in the wavelength of a redline at 630 nm is $x \times 10^{-10}$ m.
The value f x, to the nearest integer, is
[Take the value of speed of light c, as 3×10^8 ms^{-1}]

Sol. **6**

Red shift, $\Delta\lambda = \left(\frac{V_r}{c}\right)\lambda$

$= \frac{286 \times 10^3}{3 \times 10^8} \times 630 \times 10^{-9}$

$= 6.006 \times 10^{-10} m$

$= 6 \times 10^{-10} m$

$\therefore x = 6$