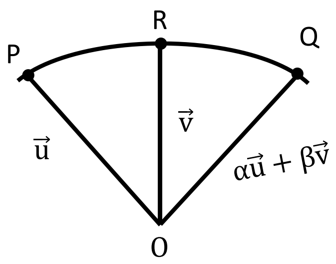
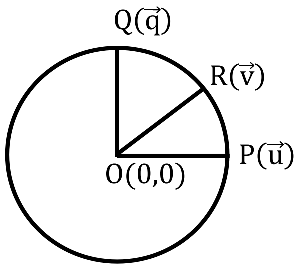
SECTION-A

1. An arc PQ of a circle subtends a right angle at its centre O. The mid point of the arc PQ is R. If $\overrightarrow{OP} = \vec{u}, \overrightarrow{OR} = \vec{v}$ and $\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$, then α, β^2 are the roots of the equation :

(1) $3x^2 - 2x - 1 = 0$ (2) $3x^2 + 2x - 1 = 0$ (3) $x^2 - x - 2 = 0$ (4) $x^2 + x - 2 = 0$

Sol. **(3)**





Let $\overrightarrow{OP} = \vec{u} = \hat{i}$

$\overrightarrow{OQ} = \vec{q} = \hat{j}$

$\because$ R is the mid point of $\overrightarrow{PQ}$

Then $\overrightarrow{OR} = \vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Now

$$\overrightarrow{OQ} = \alpha\vec{u} + \beta\vec{v}$$

$$\hat{j} = \alpha\hat{i} + \beta\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$\beta = \sqrt{2}, \alpha + \frac{\beta}{\sqrt{2}} = 0 \Rightarrow \alpha = -1$$

Now equation

$$x^2 - \left(\alpha + \beta^2\right)x + \alpha\beta^2 = 0$$

$$x^2 - \left(-1 + 2\right)x + (-1)(2) = 0$$

$$\boxed{x^2 - x - 2 = 0}$$

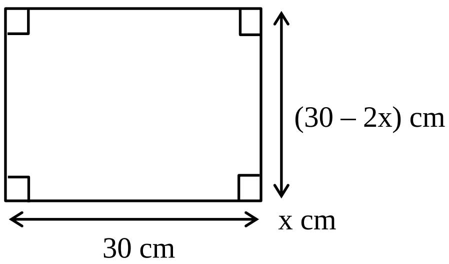
2. A square piece of tin of side 30 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. If the volume of the box is maximum, then its surface area (in cm^2) is equal to :

(1) 800 (2) 1025 (3) 900 (4) 675

Sol. **(1)**

Let the side of the square to be cut off be x cm.

Then, the length and breadth of the box will be (30 – 2x) cm each and the height of the box is x cm therefore,



The volume V(x) of the box is given by

$V(x) = x(30 - 2x)^2$

$\frac{dv}{dx} = (30 - 2x)^2 + 2x \times (30 - 2x)(-2)$

$0 = (30 - 2x)^2 - 4x(30 - 2x)$

$0 = (30 - 2x)[(30 - 2x) - 4x]$

$0 = (30 - 2x)(30 - 6x)$

$x = 15, 5$

$x \neq 15$ (Not possible)

$\{\because V = 0\}$

Surface area without top of the box = $\ell b + 2(bh + h\ell)$

$= (30 - 2x)(30 - 2x) + 2[(30 - 2x)x + (30 - 2x)x]$

$= [(30 - 2x)^2 + 4\{(30 - 2x)x\}$

$= [(30 - 10)^2 + 4(5)(30 - 10)]$

$= 400 + 400$

$= 800 \text{ cm}^2$

3. Let O be the origin and the position vector of the point P be $-\hat{i} - 2\hat{j} + 3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i} + \hat{j} - 3\hat{k}, 2\hat{i} + 4\hat{j} - 2\hat{k}$ and $-4\hat{i} + 2\hat{j} - \hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is :

(1) $\frac{10}{3}$ (2) $\frac{8}{3}$ (3) $\frac{7}{3}$ (4) 3

Sol. **(4)**

Position vector of the point $P(-1, -2, 3)$, $A(-2, 1, -3)$ $B(2, 4, -2)$, and $C(-4, 2, -1)$

Then $\overrightarrow{OP} \cdot \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|(\overrightarrow{AB} \times \overrightarrow{AC})\right|}$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 3 & 1 \\ -2 & 1 & 2 \end{vmatrix}$$

$= \hat{i}(5) - \hat{j}(8 + 2) + \hat{k}(4 + 6)$

$= 5\hat{i} - 10\hat{j} + 10\hat{k}$

Now

$$\overrightarrow{OP}.\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left|(\overrightarrow{AB}\times\overrightarrow{AC})\right|}=(-\hat{i}-2\hat{j}+3\hat{k}).\frac{(5\hat{i}-10\hat{j}+10\hat{k})}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$

$$=\frac{-5+20+30}{\sqrt{25+100+100}}$$

$$=\frac{45}{\sqrt{225}}=\frac{45}{15}=3$$

4. If A is a 3 × 3 matrix and |A| = 2, then $\left|3\text{adj}\left(|3A|\,A^2\right)\right|$ is equal to :

(1) $3^{12}.\ 6^{10}$ (2) $3^{11}.6^{10}$ (3) $3^{12}.6^{11}$ (4) $3^{10}.6^{11}$

Sol. **(2)**

Given |A|= **2**

Now, $|3\text{adj}\ (\ |\ 3A|\ A^2)\ |$

$|3A| = 3^3.|A|$

$= 3^3.(2)$

$\text{Adj.}\ (|3A|\ A^2) = \text{adj}\ \{(3^3.2)\ A^2\}$

$= (2.3^3)^2\ (\text{adj}\ A)^2$

$= 2^2.3^6\ .(\text{adj}\ A)^2$

$|3\ \text{adj}\ (|\ 3A\ |\ A^2)\ | = |2^2.3.3^6\ (\text{adj}\ A)^2|$

$= (2^2.3^7)^3\ |\ \text{adj}\ A|^2$

$= 2^6.3^{21}\ (|A|^2)^2$

$= 2^6.3^{21}(2^2)^2$

$= 2^{10}.3^{21}$

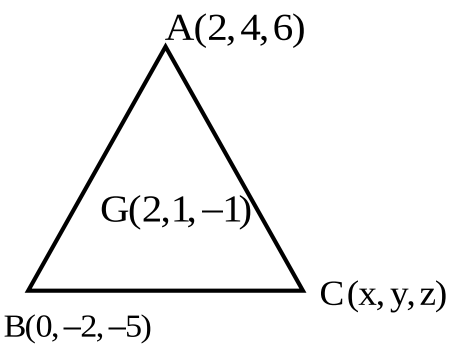
$= 2^{10}.3^{10}.3^{11}$

$|3\ \text{adj}\ (|\ 3A\ |\ A^2\ )| = 6^{10}.3^{11}$

5. Let two vertices of a triangle ABC be (2, 4, 6) and (0, –2, –5), and its centroid be (2, 1, –1). If the image of the third vertex in the plane x + 2y + 4z = 11 is (α, β, γ), then αβ + βγ + γα is equal to :

(1) 76 (2) 74 (3) 70 (4) 72

Sol. **(2)**



Given Two vertices of Triangle $A(2,4,6)$ and $B(0,-2,-5)$ and if centroid $G(2,1,-1)$

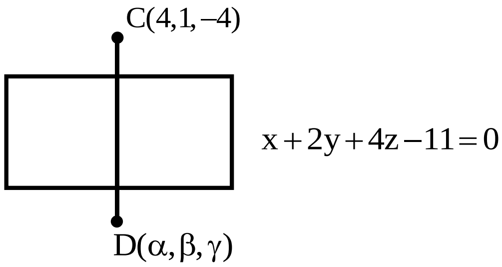
Let Third vertices be (x,y,z)

Now $\frac{2+0+x}{3}=2,\frac{4-2+y}{3}=1,\frac{6-5+z}{3}=-1$

$x=4,\ \ y=1,\ \ \ z=-1$

Third vertices $C(4,1,-4)$

Now, Image of vertices C(4,1,–4) in the given plane is D(α,β,γ)



Now

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=-2\frac{(4+2-16-11)}{1+4+16}$$

$$\frac{\alpha-4}{1}=\frac{\beta-1}{2}=\frac{\gamma+4}{4}=\frac{42}{21}\Rightarrow 2$$

$\alpha=6,\ \beta=5,\ \gamma=4$

Then $\alpha\beta+\beta\gamma+\gamma\alpha$

$=(6\times5)+(5\times4)+(4\times6)$

$=30+20+24$

$=74$

6. The negation of the statement :

$(p \vee q) \wedge (q \vee (\sim r))$ is

(1) $((\sim p) \vee r)) \wedge (\sim q)$

(2) $((\sim p) \vee (\sim q)) \wedge (\sim r)$

(3) $((\sim p) \vee (\sim q)) \vee (\sim r)$

(4) $(p \vee r) \wedge (\sim q)$

Sol. **(1)**

$(p\vee q)\wedge(q\vee(\sim r))$

$\sim\left[(p\vee q)\wedge\left(q\vee(\sim r)\right)\right]$

$=\sim(p\vee q)\wedge(\sim q\wedge r)$

$=\left(\sim p\wedge\sim q\right)\vee(\sim q\wedge r)$

$=(\sim p\vee r)\wedge(\sim q)$

7. The shortest distance between the lines $\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}$ and $\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$ is :

(1) 8

(2) 7

(3) 6

(4) 9

Sol. **(4)**

$$\frac{x+2}{1}=\frac{y}{-2}=\frac{z-5}{2}\text{ and }\frac{x-4}{1}=\frac{y-1}{2}=\frac{z+3}{0}$$

$$\text{Shortest distance } (d) = \frac{\left\|\begin{matrix} a_2 - a_1 & b_2 - b_1 & c_2 - c_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & k \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{matrix}\right\|}$$

$$= \frac{\left\|\begin{matrix} 4+2 & 1-0 & -3-5 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & k \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right\|}$$

$$= \frac{\left|\begin{matrix} 6 & 1 & -8 \\ 1 & -2 & 2 \\ 1 & 2 & 0 \end{matrix}\right|}{\left|\hat{i}(-4) - \hat{j}(-2) + k(2+2)\right|}$$

$$= \frac{|-54|}{\left|-4\hat{i} + 2\hat{j} + 4k\right|}$$

$$= \frac{54}{\sqrt{16+4+16}}$$

$$= \frac{54}{6}$$

$$= 9$$

8. If the coefficient of x^7 in $\left(ax - \frac{1}{bx^2}\right)^{13}$ and the coefficient of x^{-5} in $\left(ax + \frac{1}{bx^2}\right)^{13}$ are equal, then a^4b^4 is equal to :

(1) 22 (2) 44 (3) 11 (4) 33

Sol. **(1)**

$$\left(ax - \frac{1}{bx^2}\right)^{13}$$

We have,

$T_{r+1} = {}^nC_r\,(p)^{n-r}\,(q)^r$

$T_{r+1} = {}^{13}C_r\,(ax)^{13-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r\,(a)^{13-r}\left(-\frac{1}{b}\right)^r (x)^{13-r}.(x)^{-2r}$

$= {}^{13}C_r\,(a)^{13-r}\left(-\frac{1}{b}\right)^r (x)^{13-3r}$ …(1)

Coefficient of x^7

$\Rightarrow 13 - 3r = 7$

$r = 2$

r in equation (1)

$T_3 = {}^{13}C_2 (a)^{13-2} \left(-\frac{1}{b}\right)^2 (x)^{13-6}$

$= {}^{13}C_2 (a)^{11} \left(\frac{1}{b}\right)^2 (x)^7$

Coefficient of x^7 is ${}^{13}C_2 \frac{(a)^{11}}{b^2}$

Now, $\left(ax + \frac{1}{bx^2}\right)^{13}$

$T_{r+1} = {}^{13}C_r (ax)^{13-r} \left(\frac{1}{bx^2}\right)^r$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-r}(x)^{-2r}$

$= {}^{13}C_r (a)^{13-r} \left(\frac{1}{b}\right)^r (x)^{13-3r}$ …(2)

Coefficient of x^{-5}

$\Rightarrow 13 - 3r = -5$

$r = 6$

r in equation

$T_7 = {}^{13}C_6 (a)^{13-6} \left(\frac{1}{b}\right)^6 (x)^{13-18}$

$T_7 = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6 (x)^{-5}$

Coefficient of x^{-5} is ${}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

ATQ

Coefficient of x^7 = coefficient of x^{-5}

$T_3 = T_7$

${}^{13}C_2 \left(\frac{a^{11}}{b^2}\right) = {}^{13}C_6 (a)^7 \left(\frac{1}{b}\right)^6$

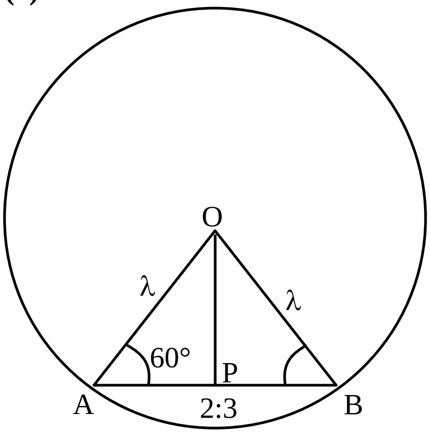
$a^4.b^4 = \frac{{}^{13}C_6}{{}^{13}C_2}$

$= \frac{13\times12\times11\times10\times9\times8\times1}{13\times12\times6\times5\times4\times3} = 22$

9. A line segment AB of length λ moves such that the points A and B remain on the periphery of a circle of radius λ. Then the locus of the point, that divides the line segment AB in the ratio 2 : 3, is a circle of radius :

(1) $\frac{2}{3}\lambda$ (2) $\frac{\sqrt{19}}{7}\lambda$ (3) $\frac{3}{5}\lambda$ (4) $\frac{\sqrt{19}}{5}\lambda$

Sol. **(4)**



Since OAB form equilateral Δ

$\therefore \angle OAP = 60^{o}$

$AP = \frac{2\lambda}{5}$

$\cos 60° = \frac{OA^2 + AP^2 - OP^2}{2OA.AP}$

$\Rightarrow \frac{1}{2} = \frac{\lambda^2 + \frac{4\lambda^2}{25} - OP^2}{2\lambda\left(\frac{2\lambda}{5}\right)}$

$\Rightarrow \frac{2\lambda^2}{5} = \lambda^2 + \frac{4\lambda^2}{25} - OP^2$

$\Rightarrow OP^2 = \frac{19\lambda^2}{25}$

$\Rightarrow OP = \frac{\sqrt{19}}{5}\lambda$

Therefore, locus of point P is $\frac{\sqrt{19}}{5}\lambda$

10. For the system of linear equations

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$,

which of the following is NOT correct ?

(1) The system in inconsistent for $\alpha = -5$ and $\beta = 8$

(2) The system has infinitely many solutions for $\alpha = -6$ and $\beta = 9$

(3) The system has a unique solution for $\alpha \neq -5$ and $\beta = 8$

(4) The system has infinitely many solutions for $\alpha = -5$ and $\beta = 9$

Sol. **(2)**

Given

$2x - y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \alpha z = \beta$

$$\Delta = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \alpha \end{vmatrix} = 7\alpha + 35$$

$\Delta = 7(\alpha + 5)$

For unique solution $\Delta \neq 0$

$\alpha \neq -5$

For inconsistent & Infinite solution

$\Delta = 0$

$\alpha + 5 = 0 \Rightarrow \alpha = -5$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & 3 \\ 7 & 2 & -1 \\ \beta & 5 & -5 \end{vmatrix} = -5(\beta - 9)$$

$$\Delta_2 = \begin{vmatrix} 2 & 5 & 3 \\ 3 & 7 & -1 \\ 4 & \beta & -5 \end{vmatrix} = 11(\beta - 9)$$

$$\Delta_3 = \begin{vmatrix} 2 & -1 & 5 \\ 3 & 2 & 7 \\ 4 & 5 & \beta \end{vmatrix}$$

$\Delta_3 = 7(\beta - 9)$

For Inconsistent system : -

At least one Δ_1, Δ_2 & Δ_3 is not zero $\alpha = -5$, $\beta = 8$ option (A) True

Infinite solution:

$\Delta_1 = \Delta_2 = \Delta_3 = 0$

From here $\beta - 9 = 0 \Rightarrow \beta = 9$

$\alpha = -5$ & option (D) True

$\beta = 9$

Unique solution

$\alpha \neq -5$, $\beta = 8 \rightarrow$ option (C) True

Option (B) False

For Infinitely many solution α must be -5.

11. Let the first term a and the common ratio r of a geometric progression be positive integers. If the sum of squares of its first three is 33033, then the sum of these terms is equal to :

(1) 210 (2) 220 (3) 231 (4) 241

Sol. **(3)**

Let a, ar , ar^2 be three terms of GP

Given : $a^2 + (ar)^2 + (ar^2)^2 = 33033$

$a^2(1 + r^2 + r^4) = 11^2.3.7.13$

$\Rightarrow a = 11$ and $1 + r^2 + r^4 = 3.7.13$

$\Rightarrow r^2(1 + r^2) = 273 - 1$

$\Rightarrow r^2(r^2 + 1) = 272 = 16 \times 17$

$\Rightarrow r^2 = 16$

$\therefore r = 4$ $[\because r > 0]$

Sum of three terms $= a + ar + ar^2 = a(1 + r + r^2)$

$= 11(1 + 4 + 16)$

$= 11 \times 21 = 231$

12. Let P be the point of intersection of the line $\frac{x+3}{3}=\frac{y+2}{1}=\frac{1-z}{2}$ and the plane x + y + z = 2. If the distance of the point P from the plane 3x – 4y + 12z = 32 is q, then q and 2q are the roots of the equation :

(1) $x^2 + 18x - 72 = 0$ (2) $x^2 + 18x + 72 = 0$ (3) $x^2 - 18x - 72 = 0$ (4) $x^2 - 18x + 72 = 0$

Sol. **(4)**

$\frac{x+3}{3} = \frac{y+2}{1} = \frac{1-z}{2} = \lambda$

$x = 3\lambda - 3,\ y = \lambda - 2,\ z = 1 - 2\lambda$

$P(3\lambda - 3, \lambda - 2, 1 - 2\lambda)$ will satisfy the equation of plane x + y + z = 2.

$3\lambda - 3 + \lambda - 2 + 1 - 2\lambda = 2$

$2\lambda - 4 = 2$

$\lambda = 3$

P(6, 1, – 5)

Perpendicular distance of P from plane 3x – 4y + 12 z – 32 = 0 is

$q = \left|\frac{3(6) - 4(1) + 12(-5) - 32}{\sqrt{9+16+144}}\right|$

q = 6.

2q = 12

Sum of roots = 6 + 12 = 18

Product of roots = 6(12) = 72

$\therefore$ Quadratic equation having q and 2q as roots is $x^2 - 18x + 72$.

13. Let f be a differentiable function such that $x^2 f(x) - x = 4\int_0^x tf(t)\,dt$, $f(1) = \frac{2}{3}$. Then 18 f (3) is equal to :

(1) 180 (2) 150 (3) 210 (4) 160

Sol. **(4)**

$x^2 f(x) - x = 4\int_0^x tf(t)dt$

Differentiate w.r.t. x

$x^2 f'(x) + 2x f(x) - 1 = 4xf(x)$

Let y = f(x)

$\Rightarrow x^2 \frac{dy}{dx} - 2xy - 1 = 0$

$\frac{dy}{dx} - \frac{2}{x} y = \frac{1}{x^2}$

I.F. $= e^{\int \frac{-2}{x}dx} = \frac{1}{x^2}$

Its solution is

$\frac{y}{x^2} = \int \frac{1}{x^4}\,dx + C$

$\frac{y}{x^2} = \frac{-1}{3x^3} + C$

$\because f(1) = \frac{2}{3} \Rightarrow y(1) = \frac{2}{3}$

$\Rightarrow \frac{2}{3} = -\frac{1}{3} + C$

$\Rightarrow C = 1$

$\because y = -\frac{1}{3x} + x^2$

$f(x) = -\frac{1}{3x} + x^2$

$f(3) = -\frac{1}{9} + 9 = \frac{80}{9} \Rightarrow 18f(3) = 160$

14. Let N denote the sum of the numbers obtained when two dice are rolled. If the probability that $2^N < N!$ is $\frac{m}{n}$, where m and n are coprime, then 4m –3n equal to :

(1) 12 (2) 8 (3) 10 (4) 6

Sol. **(2)**

$2^N < N!$ is satisfied for $N \geq 4$

Required probability $P(N \geq 4) = 1 - P(N < 4)$

N = 1 (Not possible)

N = 2 (1, 1)

$\Rightarrow P(N = 2) = \frac{1}{36}$

N = 3 (1, 2), (2, 1)

$\Rightarrow P(N = 3) = \frac{2}{36}$

$P(N < 4) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$

$\therefore P(N \geq 4) = 1 - \frac{3}{36} = \frac{33}{36} = \frac{11}{12} = \frac{m}{n}$

$\Rightarrow m = 11, n = 12$

$\therefore 4m - 3n = 4(11) - 3(12) = 8$

15. If $I(x) = \int e^{\sin^2 x} (\cos x \sin 2x - \sin x)\, dx$ and I (0) = 1, then $I\left(\frac{\pi}{3}\right)$ is equal to :

(1) $e^{\frac{3}{4}}$ (2) $-e^{\frac{3}{4}}$ (3) $\frac{1}{2}e^{\frac{3}{4}}$ (4) $-\frac{1}{2}e^{\frac{3}{4}}$

Sol. **(3)**

$I = \int \underbrace{e^{\sin^2 x} \sin 2x}_{II} \underbrace{\cos x dx}_{I} - \int e^{\sin^2 x} \sin x dx$

$= \cos x \int e^{\sin^2 x} \sin 2x dx - \int \left((-\sin x) \int e^{\sin^2 x} \sin 2x dx\right) dx - \int e^{\sin^2 x} \sin x dx$

$\sin^2 x = t$

$\sin 2x\, dx = dt$

$= \cos x \int e^t dt + \int (\sin x \int e^t dt) dx - \int e^{\sin^2 x} \sin x dx$

$= e^{\sin^2 x} \cos x + \int e^{\sin^2 x} \sin x dx - \int e^{\sin^2 x} \sin x dx$

$I = e^{\sin^2 x}\cos x + C$

I (0) = 1

$\Rightarrow$ 1 = 1 + C

$\Rightarrow$ C = 0

$\therefore$ $I = e^{\sin^2 x}\cos x$

$I\left(\frac{\pi}{3}\right) = e^{\sin^2\frac{\pi}{3}}\cos\frac{\pi}{3}$

$= \frac{e^{\frac{3}{4}}}{2}$

16. $96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33}$ is equal to :

(1) 4 (2) 2 (3) 3 (4) 1

Sol. **(3)**

$96\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{2^2\pi}{33}\cos\frac{2^3\pi}{33}\cos\frac{2^4\pi}{33}$

$\because$ cos A cos 2A cos 2^2A …. Cos 2^{n-1} A = $\frac{\sin(2^n A)}{2^n \sin A}$

Here A = $\frac{\pi}{33}$, n = 5

$= \frac{96\sin\left(2^5\frac{\pi}{33}\right)}{2^5\sin\left(\frac{\pi}{33}\right)}$

$= \frac{96\sin\left(\frac{32\pi}{33}\right)}{32\sin\left(\frac{\pi}{33}\right)}$

$= \frac{3\sin\left(\pi-\frac{\pi}{33}\right)}{\sin\left(\frac{\pi}{33}\right)} = 3$

17. Let the complex number z = x + iy be such that $\frac{2z-3i}{2z+i}$ is purely imaginary. If $x + y^2 = 0$, then $y^4 + y^2 - y$ is equal to :

(1) $\frac{3}{2}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{3}{4}$

Sol. **(4)**

z = x + iy

$\frac{(2z-3i)}{2z+i}$ = purely imaginary

Means Re $\left(\frac{2z-3i}{2z+i}\right) = 0$

$$\Rightarrow \frac{(2z-3i)}{(2z+i)} = \frac{2(x+iy)-3i}{2(x+iy)+i}$$

$$= \frac{2x+2yi-3i}{2x+i2y+i}$$

$$= \frac{2x+i(2y-3)}{2x+i(2y+1)} \times \frac{2x-i(2y+1)}{2x-i(2y+1)}$$

$$\text{Re}\left[\frac{2z-3i}{2z+i}\right] = \frac{4x^2+(2y-3)(2y+1)}{4x^2+(2y+1)^2} = 0$$

$\Rightarrow 4x^2 + (2y - 3)(2y + 1) = 0$

$\Rightarrow 4x^2 + [4y^2 + 2y - 6y - 3] = 0$

$\because x + y^2 = 0 \Rightarrow x = -y^2$

$\Rightarrow 4(-y^2)^2 + 4y^2 - 4y - 3 = 0$

$\Rightarrow 4y^4 + 4y^2 - 4y = 3$

$\Rightarrow y^4 + y^2 - y = \frac{3}{4}$

Therefore, correct answer is option (4).

18. If $f(x) = \frac{(\tan 1°)x + \log_e(123)}{x\log_e(1234) - (\tan 1°)}$, $x > 0$, then the least value of $f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$ is :

(1) 2 (2) 4 (3) 8 (4) 0

Sol. **(2)**

$$f(x) = \frac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$$

Let $A = \tan 1, \quad B = \log 123, \quad C = \log 1234$

$$f(x) = \frac{Ax+B}{xC-A}$$

$$f(f(x)) = \frac{A\left(\frac{Ax+B}{xC-A}\right)+B}{C\left(\frac{Ax+B}{CX-A}\right)-A}$$

$$= \frac{A^2x+AB+xBC-AB}{ACx+BC-ACx+A^2}$$

$$= \frac{x(A^2+BC)}{(A^2+BC)} = x$$

$$f(f(x)) = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$$

$AM \geq GM$

$$x + \frac{4}{x} \geq 4$$

19. The slope of tangent at any point (x, y) on a curve y = y(x) is $\frac{x^2+y^2}{2xy}, x > 0$. If y (2) = 0, then a value of y (8) is :

(1) $4\sqrt{3}$ (2) $-4\sqrt{2}$ (3) $-2\sqrt{3}$ (4) $2\sqrt{3}$

Sol. **(1)**

$\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

y = vx

y(2) = 0

y(8) = ?

$\frac{dv}{dx} = v + x\frac{dv}{dx}$

$v + \frac{xdv}{dx} = \frac{x^2+v^2x^2}{2vx^2}$

$x.\frac{dv}{dx} = \left(\frac{v^2+1}{2v} - v\right)$

$\frac{2vdv}{(1-v^2)} = \frac{dx}{x}$

$-\ln(1-v^2) = \ln x + C$

$\ln x + \ln(1-v^2) = C$

$\ln\left[x\left(1-\frac{y^2}{x^2}\right)\right] = C$

$\ln\left[\left(\frac{x^2-y^2}{x}\right)\right] = C$

$x^2 - y^2 = cx$

y(2) = 0 at x = 2, y = 0

$4 = 2C \Rightarrow C = 2$

$x^2 - y^2 = 2x$

Hence, at x = 8

$64 - y^2 = 16$

$y = \sqrt{48} = 4\sqrt{3}$

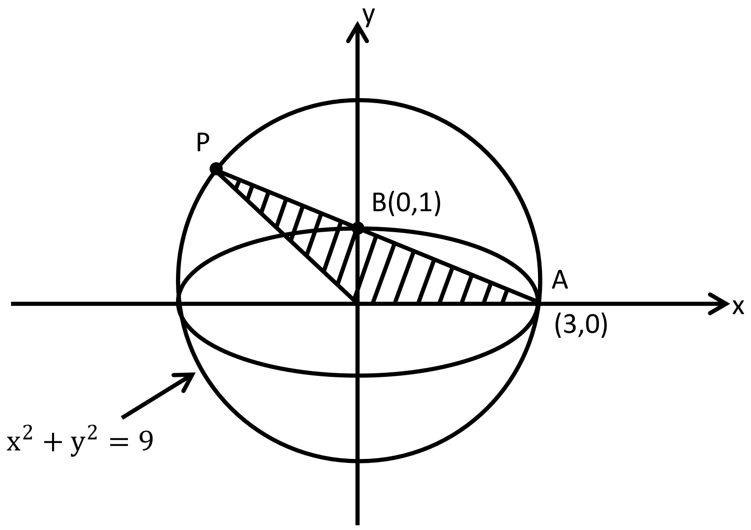
$y(8) = 4\sqrt{3}$

Option (1)

20. Let the ellipse E : $x^2 + 9y^2 = 9$ intersect the positive x-and y-axes at the points A and B respectively. Let the major axis of E be a diameter of the circle C. Let the line passing through A and B meet the circle C at the point P. If the area of the triangle with vertices A, P and the origin O is $\frac{m}{n}$, where m and n are coprime, then m – n is equal to :

(1) 16 (2) 15 (3) 18 (4) 17

Sol. **(4)**



Equation of line AB or AP is

$$\frac{x}{3}+\frac{y}{1}=1$$

$$x+3y=3$$

$$x=(3-3y)$$

Intersection point of line AP & circle is $P(x_0, y_0)$

$$x^2+y^2=9 \Rightarrow (3-3y)^2+y^2=9$$

$$\Rightarrow 3^2(1+y^2-2y)+y^2=9$$

$$\Rightarrow 5y^2-9y=0 \Rightarrow y(5y-9)=0$$

$$\Rightarrow y=9/5$$

Hence $x=3(1-y)=3\left(1-\frac{9}{5}\right)=3\left(\frac{-4}{5}\right)$

$$x=\frac{-12}{5}$$

$$P(x_0, y_0)=\left(\frac{-12}{5}, \frac{9}{5}\right)$$

Area of ΔAOP is $=\frac{1}{2}\times OA\times \text{height}$

Height $=9/5$, $OA=3$

$$=\frac{1}{2}\times 3\times \frac{9}{5}=\frac{27}{10}=\frac{m}{n}$$

Compare both side $m=27, \quad n=10 \quad \Rightarrow m-n=17$

Therefore, correct answer is option-D

SECTION-B

21. Some couples participated in a mixed doubles badminton tournament. If the number of matches played, so that no couple in a match, is 840, then the total numbers of persons, who participated in the tournament, is ____.

Sol. **16**

Let number of couples = n

$\therefore\ {}^{n}C_2 \times {}^{n-2}C_2 \times 2 = 840$

$\Rightarrow n(n-1)(n-2)(n-3) = 840 \times 2$

$= 21 \times 40 \times 2$

$= 7 \times 3 \times 8 \times 5 \times 2$

$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$

$\therefore\ n = 8$

Hence, number of persons = 16.

22. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is _____.

Sol. **6**

$-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$ and $\quad n^2 - 10n + 13 < 0$

$(n-5)^2 > 0 \quad 5 - 3\sqrt{2} < n < 5 + 3\sqrt{2}$

$N \in Z - \{5\} \quad n = \{2, 3, 4, 5, 6, 7, 8\}$

…(i) …(ii)

From (i) $\cap$ (ii)

$N = \{2, 3, 4, 5, 6, 8,\}$

Number of values of n = 6

23. The number of permutations of the digits 1, 2, 3, …., 7 without repetition, which neither contain the string 153 nor the string 2467, is _____.

Sol. **4898**

Numbers are 1, 2, 3, 4, 5, 6, 7

Numbers having string (154) = (154), 2, 3, 6, 7 = 5!

Numbers having string (2467) = (2467), 1,3, 5 = 4!

Number having string (154) and (2467)

= (154), (2467) = 2!

Now n $(154 \cup 2467)$ = 5! + 4! – 2!

= 120 + 24 – 2 = 142

Again total numbers = 7! = 5040

Now required numbers = n (neither 154 nor 2467)

= 5040 – 142

= 4898

24. Let f: $(-2, 2) \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x[x] & , -2 < x < 0 \\ (x-1)[x], & 0 \le x < 2 \end{cases}$$

where [x] denotes the greatest integer function. If m and n respectively are the number of points in (–2, 2) at which y = |f(x)| is not continuous and not differentiable, then m + n is equal to ______.

Sol. **4**

$$f(x) = \begin{cases} -2x, & -2 < x < -1 \\ -x, & -1 \le x < 0 \\ 0, & 0 \le x < 1 \\ x-1, & 1 \le x < 2 \end{cases}$$

Clearly f(x) is discontinuous at x = –1 also non differentiable.

$\therefore$ m = 1

Now for differentiability

$$f'(x) = \begin{cases} -2 & -2 < x < -1 \\ -1 & -1 < x < 0 \\ 0 & 0 < x < 1 \\ -1 & 1 < x < 2 \end{cases}$$

Clearly f(x) is non-differentiable at x = –1, 0, 1

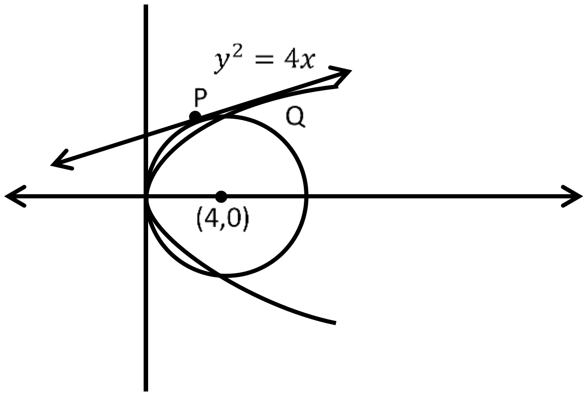
Also, $|f(x)|$ remains same.

$\therefore$ n = 3

$\therefore$ m + n = 4

25. Let a common tangent to the curves $y^2 = 4x$ and $(x - 4)^2 + y^2 = 16$ touch the curves at the points P and Q. Then $(PQ)^2$ is equal to _____ :

Sol. **32**



$y^2 = 4x$

$(x-4)^2 + y^2 = 16$

Let equation of tangent of parabola

$y = mx + 1/m$ ..(1)

Now equation 1 also touches the circle

$$\therefore \left|\frac{4m + 1/m}{\sqrt{1+m^2}}\right| = 4$$

$(4m + 1/m)^2 = 16 + 16m^2$

$16m^4 + 8m^2 + 1 = 16m^2 + 16m^4$

$8m^2 = 1$

$\boxed{m^2 = 1/8}$ $\{m^4 = 0\}(m \to \infty)$

For distinet points consider only $m^2 = 1/8$.

Point of contact of parabola

$P(8, 4\sqrt{2})$

$\therefore PQ = \sqrt{S_1} \Rightarrow (PQ)^2 = S_1$

$= 16 + 32 - 16 = 32$

26. If the mean of the frequency distribution

Class :	0-10	10-20	20-30	30-40	40-50
Frequency :	2	3	x	5	4

is 28, then its variance is ______.

Sol. **151**

C.I.	f	x	$f_i x_i$	x_i^2
0-10	2	5	10	25
10-20	3	15	45	225
20-30	x	25	25x	625
30-40	5	35	175	1225
40-50	4	45	180	2025

$\bar{x} = \frac{\Sigma f_i x_i}{N}$

$28 = \frac{10 + 45 + 25x + 175 + 130}{14 + x}$

$28 \times 14 + 28x = 410 + 25x$

$\Rightarrow 3x = 410 - 392$

$\Rightarrow x = \frac{18}{3} = 6$

$\therefore$ Variance $= \frac{1}{N} \Sigma f_i x_i^2 - (\bar{x})^2$

$= \frac{1}{20}\ 18700 - (28)^2$

$= 935 - 784 = 151$

27. The coefficient of x^7 in $(1 - x + 2x^3)^{10}$ is ______.

Sol. **960**

$(1 - x + 2x^3)^{10}$

a	b	c
3	7	0
5	4	1
7	1	2

$T_n = \frac{10!}{a!b!c!} (-2x)^b (x^3)^c$

$= \frac{10!}{a!b!c!} (-2)^b x^{b+3c}$

$\Rightarrow b + 3c = 7, a + b + c = 10$

$\therefore$ Coefficient of $x^7 = \frac{10!}{3!7!0!}(-1)^7 + \frac{10!}{5!4!1!}(-1)^4$ **(2)**

$+ \frac{10!}{7!1!2!}(-1)^1(2)^2$

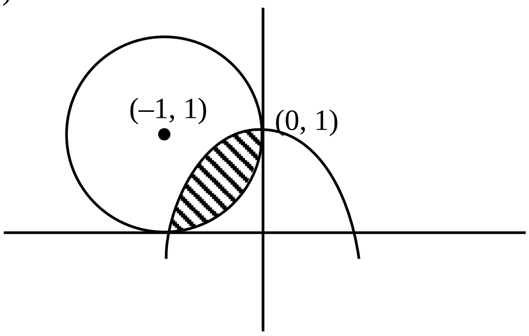
$= -120 + 2520 - 1440 = 960$

28. Let y = p(x) be the parabola passing through the points (–1, 0), (0, 1) and (1, 0). If the area of the region $\left\{(x, y) : (x+1)^2 + (y-1)^2 \le 1, y \le p(x)\right\}$ is A, then 12(π – 4A) is equal to _____:

Sol. **16**

There can be infinitely many parabolas through given points.

Let parabola $x^2 = -4a(y-1)$



Passes through (1, 0)

$\therefore\ b = -4a(-1) \Rightarrow a = \frac{1}{4}$

$\therefore\ x^2 = -(y-1)$

Now area covered by parabola = $\int_{-1}^{0}(1-x^2)dx$

$= \left(x - \frac{x^3}{3}\right)_1^b = (0-0) - \left\{-1 + \frac{1}{3}\right\}$

$= \frac{2}{3}$

Required Area = Area of sector – {Area of square – Area covered by Parabola}

$= \frac{\pi}{4} - \left\{1 - \frac{2}{3}\right\}$

$= \frac{\pi}{4} - \frac{1}{3}$

$\therefore\ 12(\pi - 4A) = 12\left[\pi - 4\left(\frac{\pi}{4} - \frac{1}{3}\right)\right]$

$= 12\left[\pi - \pi + \frac{4}{3}\right]$

$= 16$

29. Let a, b, c be three distinct positive real numbers such that $(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then 6a + 5bc is equal to ______.

Sol. **Bouns**

$(2a)^{\ln a} = (bc)^{\ln b} \quad 2a > 0, bc > 0$

$\ln a(\ln 2 + \ln a) = \ln b(\ln b + \ln c)$

$\ln 2 \cdot \ln b = \ln c \cdot \ln a$

$\ln 2 = \alpha, \ln a = x, \ln b = y, \ln c = z$

$\alpha y = xz$

$x(\alpha + x) = y(y + z)$

$\alpha = \frac{xz}{y}$

$x\left(\frac{xz}{y} + x\right) = y(y+z)$

$x^2(z+y) = y^2(y+z)$

$y + z = 0 \text{ or } x^2 = y^2 \Rightarrow x = -y$

$bc = 1 \text{ or } ab = 1$

bc = 1 or ab = 1

(1) if bc = 1 ⇒ $(2a)^{\ln a} = 1$ → a = 1 ; → a = 1/2

$(a, b, c) = \left(\frac{1}{2}, \lambda, \frac{1}{\lambda}\right), \lambda \neq 1, 2, \frac{1}{2}$

then

6a + 5bc = 3 + 5 = 8

$(II)(a, b, c) = \left(\lambda, \frac{1}{\lambda}, \frac{1}{2}\right), \lambda \neq 1, 2, \frac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

30. The sum of all those terms, of the arithmetic progression 3, 8, 13, …., 373, which are not divisible by 3, is equal to ______.

Sol. **9525**

$A.P: 3, 8, 13.....373$

$T_n = a + (n-1)d$

$373 = 3 + (n-1)5$

$\Rightarrow n = \frac{370}{5}$

$\Rightarrow \boxed{n = 75}$

Now Sum $= \frac{n}{2}[a + l]$

$= \frac{75}{2}[3 + 373] = 14100$

Now numbers divisible by 3 are,

$3, 18, 33........363$

$363 = 3 + (k - 1)15$

$\Rightarrow k - 1 = \frac{360}{15} = 24 \Rightarrow \boxed{k = 25}$

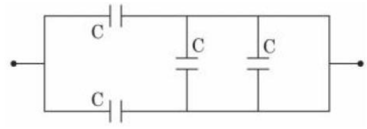
Now, sum $= \frac{25}{2}(3 + 363) = 4575$ s

$\therefore$ req. sum $= 14100 - 4575$

$= 9525$

SECTION - A

31. The equivalent capacitance of the combination shown is



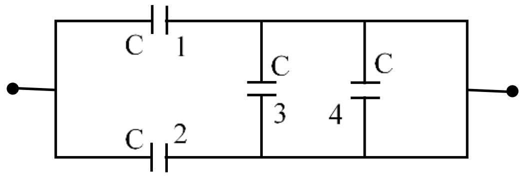
(1) 4C (2) $\frac{5}{3}$C (3) $\frac{C}{2}$ (4) 2C

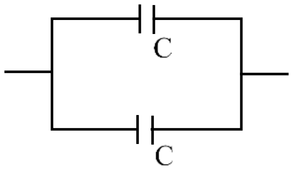
Sol. **(4)**

Capacitor (3) & (4) are short ckt

$\therefore$ C_1 & C_2 are in parallel

$C_{eq.} = C + C = 2C$





32. Match List I with List II :

List I	List II
(A) 3 Translational degrees of freedom	(I) Monoatomic gases
(B) 3 Translational, 2 rotational degrees of freedoms	(II) Polyatomic gases
(C) 3 Translational, 2 rotational and 1 vibrational degrees of freedom	(III) Rigid diatomic gases
(D) 3 Translational, 3 rotational and more than one vibrational degrees of freedom	(IV) Nonrigid diatomic gases

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)
(2) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)
(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)
(4) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

Sol. **(1)**

Fact Based

Type of gas	No of degree of freedom
1 Monoatomic 2. Diatomic + rigid	3 (Translational) 3 (Translational + 2 Rotational = 5)
3. Diatomic + non – rigid	3 (Trans) + 2 (Rotational) + 1 (vibrational)
4. Polyatomic	3 (Trans) + 2(Rotational) + more than 1 (vibrational)

33. Given below are two statements :

Statements I : If the number of turns in the coil of a moving coil galvanometer is doubled then the current sensitivity becomes double.

Statements II : Increasing current sensitivity of a moving coil galvanometer by only increasing the number of turns in the coil will also increase its voltage sensitivity in the same ratio

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true
(2) Both Statement I and Statement II are false
(3) Statement I is true but Statement II is false
(4) Statement I is false but Statement II is true

Sol. **(3)**

$I = \frac{k\theta}{NBA}$

$C \cdot S = \frac{\theta}{I} = \frac{NBA}{K}$

$N \to 2N \quad C \cdot S \to 2CS$

But $V.S. = \frac{\theta}{V} = \frac{NBA}{KR}$

$N \to 2N C \cdot S \to 2CS$

But $V.S. = \frac{\theta}{v} = \frac{\theta}{IR} = \frac{NBA}{RK}$

As $N \to 2N, R \to 2R$ So V.S = constant

34. Given below are two statements :

Statement I : Maximum power is dissipated in a circuit containing an inductor, a capacitor and a resistor connected in series with an AC source, when resonance occurs

Statement II : Maximum power is dissipated in a circuit containing pure resistor due to zero phase difference between current and voltage.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false
(2) Both Statement I and Statement II are false
(3) Statement I is false but Statement II is true
(4) Both Statement I and Statement II are true

Sol. **(4)**

Power is more when total impendence of ckt in minimum

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\because X_L = X_C$ (conductor of resonance)

$\therefore Z_{min} = R \quad \therefore V \,\&\, I$ in same phase

35. The range of the projectile projected at an angle of 15^o with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45^o with horizontal, then its range will be

(1) $100\sqrt{2}$m (2) 50 m (3) 100 m (4) $50\sqrt{2}$m

Sol. **(3)**

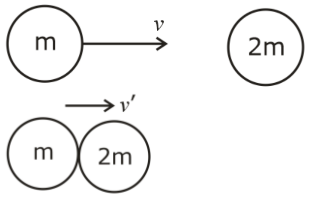
So R = $\frac{u^2 \sin(2\times15)}{g} = \frac{u^2}{2g} =$ So $\Rightarrow \frac{u^2}{g} = 100$

$R' = \frac{u^2 \sin(2\times45)}{g} = \frac{u^2}{g} = 100\,m$

36. A particle of mass m moving with velocity ν collides with a stationary particle of mass 2m. After collision, they stick together and continue to move together with velocity

(1) $\frac{\nu}{2}$ (2) $\frac{\nu}{3}$ (3) $\frac{\nu}{4}$ (4) ν

Sol. **(2)**



pi = pf $\Rightarrow$ mv + 2m (0) = 3m (v')

$v' = \frac{v}{3}$

37. Two satellites of masses m and 3m revolve around the earth in circular orbits of radii r & 3 r respectively. The ratio of orbital speeds of the satellites respectively is

(1) 3 : 1 (2) 1 : 1 (3) $\sqrt{3}:1$ (4) 9 : 1

Sol. **(3)**

$v = \sqrt{\frac{GM}{r}} \Rightarrow v \times \frac{1}{\sqrt{r}}$; M = mass of earth, r = radius of earth

$\frac{v_1}{v_2} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{3r}{r}} = \sqrt{3}$

38. Assuming the earth to be a sphere of uniform mass density, the weight of a body at a depth $d = \frac{R}{2}$ from the surface of earth, if its weight on the surface of earth is 200 N, will be :

(1) 500 N (2) 400 N (3) 100 N (4) 300 N

Sol. **(3)**

mg = 200 N

$g' = g\left(1 - \frac{d}{R}\right) = g\left(1 - \frac{R}{2 \times R}\right) = \frac{g}{2}$

weight = $mg' = \frac{mg}{2} = \frac{200}{2} = 100\,N$

39. The de Broglie wavelength of a molecule in a gas at room temperature (300 K) is λ_1. If the temperature of the gas is increased to 600 K, then the de Broglie wavelength of the same gas molecule becomes

(1) $2\lambda_1$ (2) $\frac{1}{\sqrt{2}}\lambda_1$ (3) $\sqrt{2}\lambda_1$ (4) $\frac{1}{2}\lambda_1$

Sol. **(2)**

$\lambda = \frac{h}{\sqrt{3mK(T)}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{T_2}{T_1}}$

$\lambda_2 = \lambda_1 \sqrt{\frac{T_1}{T_2}}$

$= \lambda_1 \sqrt{\frac{300}{600}} = \frac{\lambda_2}{\sqrt{2}}$

40. A physical quantity P is given as

$P = \frac{a^2 b^3}{c\sqrt{d}}$

The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4% respectively. The percentage error in the measurement of quantity P will be

(1) 14% (2) 13% (3) 16% (4) 12%

Sol. **(2)**

$\frac{dP}{P} \times 100 = \left(2\frac{da}{a} + 3\frac{db}{b} + \frac{dc}{c} + \frac{1}{2}\frac{d(d)}{d}\right) \times 100$

$= 2 \times 1 + 3 \times 2 + 3 + \frac{1}{2} \times 4$

$= 2 + 6 + 3 + 2$

$= 13\%$

41. Consider two containers A and B containing monoatomic gases at the same Pressure (P), Volume (V) and Temperature (T). The gas in A is compressed isothermally to $\frac{1}{8}$ of its original volume while the gas in B is compressed adiabatically to $\frac{1}{8}$ of its original volume. The ratio of final pressure of gas in B to that of gas in A is

(1) 8 (2) 4 (3) $\frac{1}{8}$ (4) $8^{\frac{3}{2}}$

Sol. **(2)**

By Isothermal Process for (A)

$P_1V_1 = P_2V_2$

$PV = P_2\frac{V}{8}$

$P_2 = 8P$

For B adiabatically $\gamma_{mono} = \frac{5}{3}$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$PV^{5/3} = P_2\left(\frac{V}{8}\right)^{5/3}$

$P_2 = (8)^{5/3}P$

$\frac{P_2}{P_1} = \frac{8^{5/3}}{8P} = (8)^{\frac{2}{3}} = 4$

42. Given below are two statements :

Statements I : Pressure in a reservoir of water is same at all points at the same level of water.

Statements II : The pressure applied to enclosed water is transmitted in all directions equally.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statements I and Statements II are false

(2) Both Statements I and Statements II are true

(3) Statements I is true but Statements II is false

(4) Statements I is false but Statements II is true

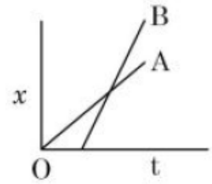
Sol. (2)

Both Statements I and Statements II are true

By Theory

By Pascal law, pressure is equally transmitted to in enclosed water in all direction.

43. The positon-time graphs for two students A and B returning from the school to their homes are shown in figure.



(A) A lives closer to the school

(B) B lives closer to the school

(C) A takes lesser time to reach home

(D) A travels faster than B

(E) B travels faster than A

Choose the correct answer from the options given below :

(1) (A) and (E) only (2) (A), (C) and (E) only

(3) (B) and (E) only (4) (A), (C) and (D) only

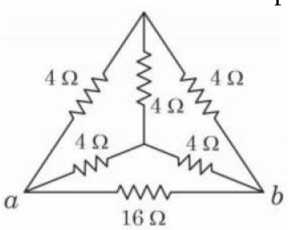
Sol. (1)

(A) and (E) only
Slope of A = V_A
Slope of B = V_B
$(\text{slope})_B > (\text{slope})_A$
$V_B > V_A$
$\therefore t_B < t_A$

44. The energy of an electromagnetic wave contained in a small volume oscillates with
(1) double the frequency of the wave
(2) the frequency of the wave
(3) zero frequency
(4) half the frequency of the wave

Sol. (1)
double the frequency of the wave
$E = E_0 \sin(wt - kx)$
Energy density = $\frac{1}{2}\varepsilon_0 E_{net}$
$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2(wt - kx)$
$= \frac{1}{4}\varepsilon_0 E_0^2 (1 - \cos(2wt - 2kx))$

45. The equivalent resistance of the circuit shown below between points a and b is :

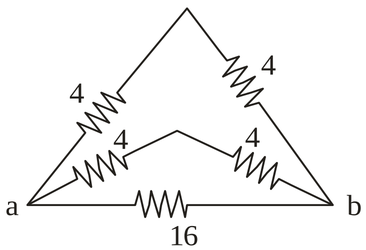


(1) 20 Ω (2) 16 Ω (3) 24Ω (4) 3.2 Ω

Sol. **(4)**

$\frac{1}{R_{ab}} = \frac{1}{16} + \frac{1}{8} + \frac{1}{8}$
$= \frac{1}{R_{ab}} = \frac{1+2+2}{16} = \frac{5}{16}$
$R_{ab} = \frac{16}{5} = 3.2$

Balanced WSB



46. A carrier wave of amplitude 15 V modulated by a sinusoidal base band signal of amplitude 3 V. The ratio of maximum amplitude to minimum amplitude in an amplitude modulated wave is

(1) 2 (2) 1 (3) 5 (4) $\frac{3}{2}$

Sol. **(4)**
$V_C = 15$
$V_m = 3$
$V_{max} = 15 + 3 = 18$
Vmin = 15 – 3 = 12
Vmax = $\frac{18}{12} = \frac{3}{2} = 3 : 2$

47. A particle executes S.H.M. of amplitude A along x-axis. At t = 0, the position of the particle is $x = \frac{A}{2}$ and it moves along positives x-axis. The displacement of particle in time t is $x = A\sin(\omega t + \delta)$, then the value δ will be

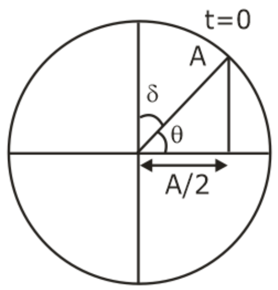
(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

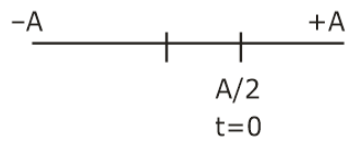
Sol. **(4)**

$\cos\theta = \frac{A}{2 \times A} = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

$\delta = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$





48. The angular momentum for the electron in Bohr's orbit is L. If the electron is assumed to revolve in second orbit of hydrogen atom, them the change in angular momentum will be

(1) $\frac{L}{2}$ (2) zero (3) L (4) 2L

Sol. **(3)**

Angular momentum = $\frac{nh}{2\pi}$

$n = 1, L_1 = \frac{h}{2\pi} = L$

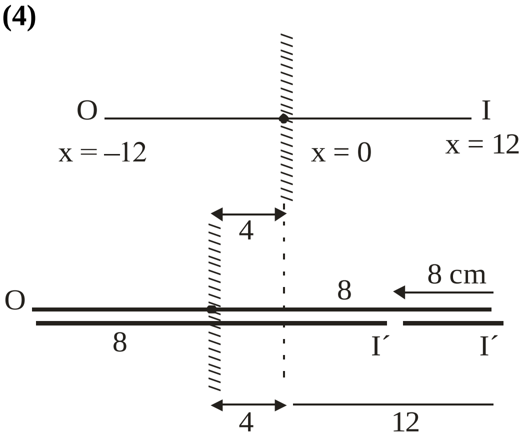
$n = 2, L_2 = \frac{2h}{2\pi} = 2L$

$\Delta L = 2L - L = L$

49. An object is placed at a distance of 12 cm in front of a plane mirror. The virtual and erect image is formed by the mirror. Now the mirror is moved by 4 cm towards the stationary object. The distance by which the position of image would be shifted, will be

(1) 4 cm towards mirror (2) 8 cm away from mirror
(3) 2 cm towards mirror (4) 8 cm towards mirror

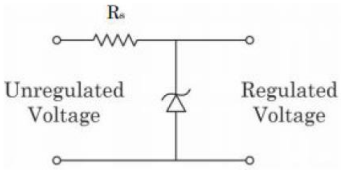
Sol. **(4)**



8 cm towards mirror

Image will be shifted 8 cm towards mirror.

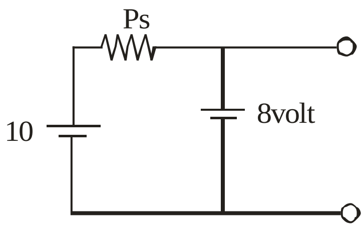
50. A zener diode of power rating 1.6 W is be used as voltage regulator. If the zener diode has a breakdown of 8 V and it has to regulate voltage fluctuating between 3 V and 10 V. The value of resistance R_8 for safe operation of diode will be



(1) 13.3 Ω (2) 13 Ω (3) 10 Ω (4) 12 Ω

Sol. **(3)**

$I_t = \frac{P_t}{V_t} = \frac{1.6}{8} = 0.2A$

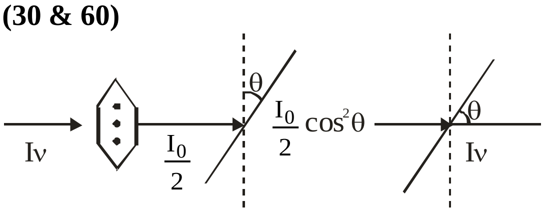


$R_s = \frac{10-8}{I}$

$\boxed{R_s = \frac{2}{0.2} = 10\Omega}$

51. Unpolarised light of intensity 32 Wm^{-2} passes through the combination of three polaroids such that the pass axis of the last polaroid is perpendicular to that of the pass axis of first polaroid. If intensity of emerging light is 3 Wm^{-2}, then the angle between pass axis of first two polaroids is _______°.

Sol. **(30 & 60)**



$I_{net} = 3 = \frac{32}{8}(\sin 2\theta)^2$ $\quad = \frac{I_0}{2}\cos^2\theta \sin^2\theta$

$\text{Sin}(2\theta) = \frac{\sqrt{3}}{2} \Rightarrow 2\theta = 60 \ \& \ 120°$ $\quad = \frac{I_0}{8}(\sin 2\theta)^2$

$\boxed{\theta = 30°}$ & 60°

52. If the earth suddenly shrinks to $\frac{1}{64}$th of its original volume with its mass remaining the same, the period of rotation of earth becomes $\frac{24}{x}h$. The value of x is _________ .

Sol. **(16)**

By AMC

$\frac{2}{5}MR^2\omega_1^2 = \frac{2}{5}M\left(\frac{R}{4}\right)^2 \omega_2$

$\frac{\omega_1}{\omega_2} = \frac{1}{16} = \frac{T_2}{T_1} = \frac{T_2}{24}$

$T_2 = \frac{24}{16}$ $\quad \boxed{\therefore\ x = 16}$ Ans.

53. Three concentric spherical metallic shells X, Y and Z of radius a, b and c respectively [a < b < c] have surface charge densities $\sigma, -\sigma$ and σ, respectively. The shells X and Z are at same potential. If the radii of X & Y are 2 cm and 3 cm, respectively. The radius of shell Z is __________ cm.

Sol. **(5)**

$q_x = \sigma 4\pi a^2$

$q_y = -\sigma 4\pi b^2$

$q_z = \sigma 4\pi c^2$

Potential of y

$$\frac{q_x}{4\pi\varepsilon_0 a} + \frac{q_y}{4\pi\varepsilon_0 b} + \frac{q_z}{4\pi\varepsilon_0 c} = \frac{q_x}{4\pi\varepsilon_0 c} + \frac{q_y}{4\pi\varepsilon_0 c} + \frac{q_z}{4\pi\varepsilon_0 c}$$

$$\frac{\sigma 4\pi a^2}{a} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c} = 4\pi\sigma\frac{(a^2 - b^2 + c^2)}{C}$$

$c(a - b + c) = a^2 - b^2 + c^2$

$c(a - b) + c^2 = (a + b)(a - b)$

$c(a - b) = (a + b)(a - b)$

$\boxed{c = a + b} = 2 + 3$

$\boxed{c = 5 \text{ cm}}$ Ans.

54. A transverse harmonic wave on a string is given by

$y(x, t) = 5 \sin (6t + 0.003 x)$

where x and y are in cm and t in sec. The wave velocity is _________ ms^{-1}.

Sol. **(20)**

$$v = \frac{w}{k} = \frac{6}{.003 \times 10^2} = \frac{6}{.3} = \frac{60}{3} = 20 \text{ m/s.}$$

55. 10 resistors each of resistance 10 Ω can be connected in such as to get maximum and minimum equivalent resistance. The ratio of maximum and minimum equivalent resistance will be ______.

Sol. **(100)**

$R_{max} \Rightarrow$ in series $\Rightarrow$ $10R = 10 \times 10 = 100\Omega$

$R_{max} \Rightarrow$ in parallel = $\frac{R}{10} = \frac{10}{10} = 1\Omega$

$$\frac{R_{max}}{R_{min}} = \frac{100}{1} = 100 \text{ Ans.}$$

$R_{min} \Rightarrow \frac{100}{1} = 100$ Ans.

56. The decay constant for a radioactive nuclide is 1.5×10^{-5} s^{-1}. Atomic weight of the substance is 60 g $mole^{-1}$, $(N_A = 6 \times 10^{23})$. The activity of 1.0 μg of the substance is _________ $\times 10^{10}$ Bq.

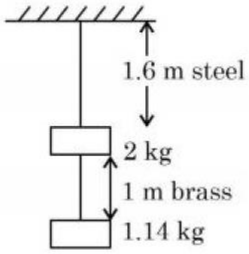
Sol. **(15)**

No. of moles = $\frac{1 \times 10^{-6}}{60} = \frac{10^{-7}}{6}$

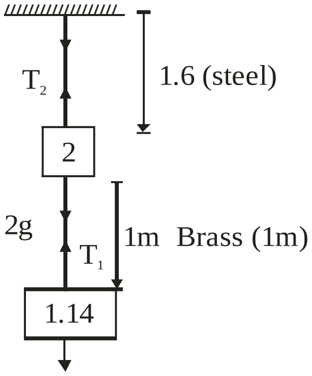
No. of atom = n(NA) = $\frac{10^{-7}}{6} \times 6 \times 10^{23} = 10^{16}$

at (t = 0) $A_0 = No\lambda = 10^{16} \times 1.5 \times 10^{-5} = 15 \times 10^{10}$ Bq

57. Two wires each of radius 0.2 cm and negligible mass, one made of steel and the other made of brass are loaded as shown in the figure. The elongation of the steel wire is _______ $\times 10^{-6}$ m. [Young's modulus for steel = 2×10^{11} Nm^{-2} and g = 10 ms^{-2}



Sol. **(20)**



1.14 = 11.4

$T_2 = T_1 + 20 = 20 + 11.4$

$T_2 = 31.4$

$\therefore$ Elongation in steel wire

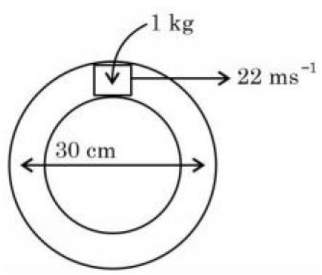
$$\Delta L = \frac{T_2 L}{AY}$$

$$= \frac{31.4 \times 1.6}{\pi(0.2 \times 10^{-2})^2 \times 2 \times 10^{11}}$$

$= 2 \times 10^{-5}$

$\boxed{\Delta L = 20 \times 10^{-6}\,\text{m}}$

58. A closed circular tube of average radius 15 cm, whose inner walls are rough, is kept in vertical plane. A block of mass 1 kg just fit inside the tube. The speed of block is 22 m/s, when it is introduced at the top of tube. After completing five oscillations, the block stops at the bottom region of tube. The work done by the tube on the block is ____ J. (Given : g = 10 m/s^2)



Sol. **(245)**

R_{arg} = 15cm = .15 m

By WET

$W_f + W_{gravity} = \Delta K = Kf - Ki$

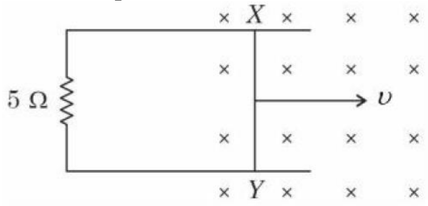
$W_f + 10 \times .3 = 0 - \frac{1}{2} \times 1 \times (22)^2$

$W_f = -3 - \frac{484}{2} = 3 - 242 = -245$

Work by friction = – 245

By NTA (+245)

59. A 1 m long metal rod XY completes the circuit as shown in figure. The plane of the circuit is perpendicular to the magnetic field of flux density 0.15 T. If the resistance of the circuit is $5\,\Omega$, the force needed to move the rod in direction, as indicated, with a constant speed of 4 m/s will be _______ 10^{-3}N.



Sol. **(18)**

$F = I\ell B = \left(\frac{e}{R}\right)\ell B = \frac{(Bv\ell)B\ell}{R} = \frac{B^2\ell^2 v}{R}$

$= \frac{(\bullet 15)^2 \times (1)^2 \times 4}{5} = 180 \times 10^{-4}$

$= 18 \times 10^{-3}$ = **18 Ans.**

60. The current required to be passed through a solenoid of 15 cm length and 60 turns in order to demagnetize a bar magnet of magnetic intensity 2.4×10^3 Am^{-1} is ________A.

Sol. **(6)**

$H = 2.4 \times 10^3$ A/m

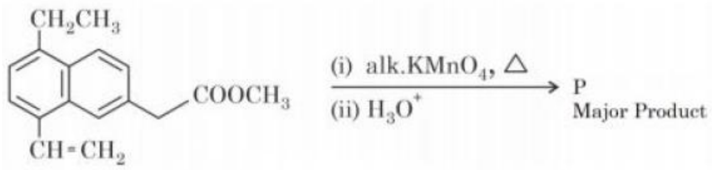
$H = nI = \frac{N}{\ell} I$

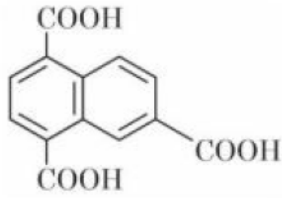
$I = \frac{H\ell}{N} = \frac{2.4 \times 10^3 \times 15 \times 10^{-2}}{60}$

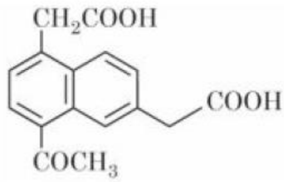
$\boxed{I = 6A}$

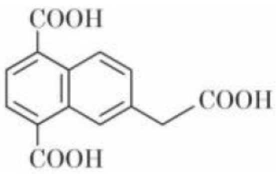
SECTION - A

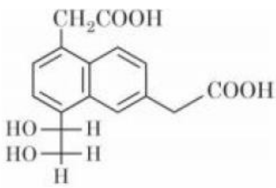
Q.61 The major product 'P' formed in the given reaction is



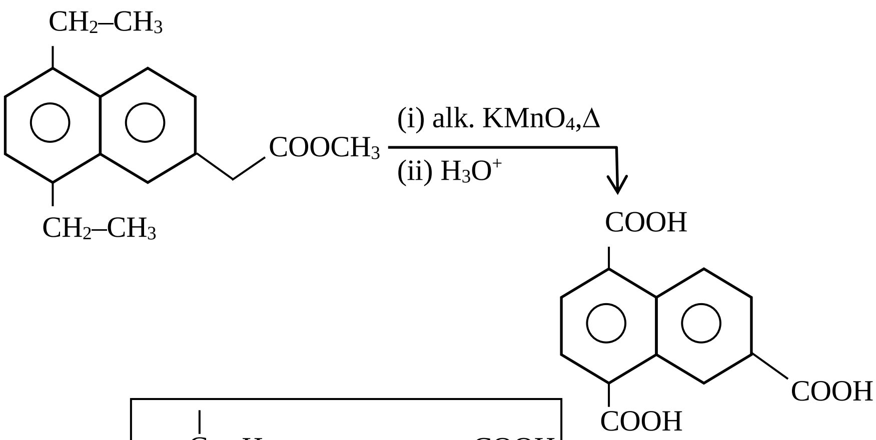
(1) 

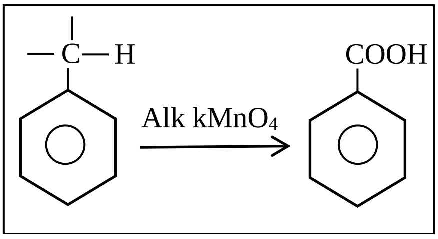
(2) 

(3) 

(4) 

Sol. (1)





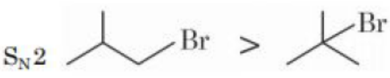
Q.62 Prolonged heating is avoided during the preparation of ferrous ammonium sulphate to

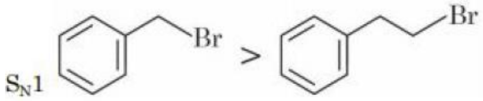
(1) prevent hydrolysis (2) prevent reduction (3) prevent breaking (4) prevent oxidation

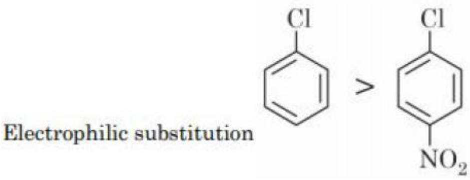
Sol. (4)

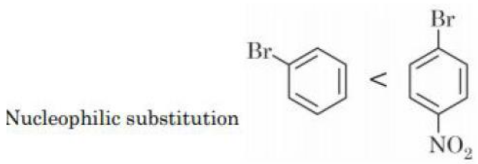
It may oxidise ferrous ion to ferric ions.

Q.63 Identify the correct order of reactivity for the following pairs towards the respective mechanism

(A) 

(B) 

(C) 

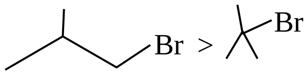
(D) 

Choose the correct answer from the options given below:

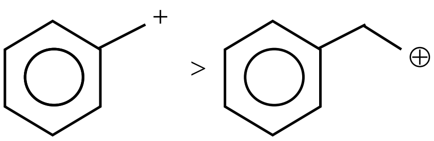
(1) (A), (C) and (D) only
(2) (A), (B) and (D) only
(3) (B), (C) and (D) only
(4) (A), (B), (C) and (D)

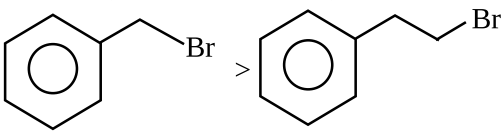
Sol. (4)

(A) $S_N2 \rightarrow$ for S_N2 Reaction $1^o > 2^o > 3^o$



(B) $S_N1 \rightarrow$ reactivity × Stability of Carbocation formed

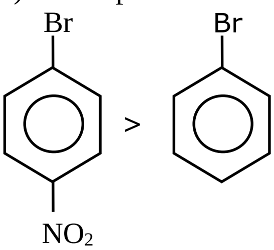


So, 

(C) Electrophilic Substitution reaction

$$\text{rate} \times \frac{1}{\text{EWG}}$$

(D) Nucleophilic substitution :- rate × no. of EWG atloched at benzons



Q.64 Given

(A) $2CO(g) + O_2(g) \rightarrow 2CO_2(g)$ $\Delta H_1^o = -x$ KJ mol^{-1}

(B) $C(\text{graphite}) + O_2(g) \rightarrow CO_2(g)$ $\Delta H_2^o = -y$ KJ mol^{-1}

The ΔH^o for the reaction

$C(\text{graphite}) + \frac{1}{2}O_2(g) \rightarrow CO(g)$ is

(1) $\frac{x-2y}{2}$
(2) $\frac{x+2y}{2}$
(3) $\frac{2x-y}{2}$
(4) $2y - x$

Sol. **(1)**

$$2CO(g) + O_2(g) \rightarrow 2CO_2(g) \qquad \Delta H_1^o = y\ kJ/mol \qquad \ldots\ldots\ldots(1)$$

$$C(graphite) + O_2(g) \rightarrow CO_2(g) \qquad \Delta H_2^o = -y \quad kJ/mol \quad \ldots\ldots\ldots(2)$$

$$C(graphite) + ½O_2(g) \rightarrow CO(g) \qquad \Delta H_3^o = ?$$

$$\Delta H_3^o = H_2^o - \frac{H_1^o}{2} = -y - \frac{-x}{2}$$

$$\Delta H_3^o = \frac{x}{2} - y = \frac{x-2y}{2}$$

Q.65 Using column chromatography mixture of two compounds 'A' and 'B' was separated. 'A' eluted first, this indicates 'B' has

(1) high R_f, weaker adsorption (2) high R_f, stronger adsorption
(3) low R_f, stronger adsorption (4) low R_f, weaker adsorption

Sol. **(3)**

More Polar the compound, the more it will adhere to the adsorbent and the smaller the distance it will travel from baseline, and Lower its R_f value.

B has Low R_f value and strong Adsoption

$$R_f = \frac{\text{distance covered by substance from base line}}{\text{total distance covered by solvent form base line}}$$

Q.66 Lime reacts exothermally with water to give 'A' which has low solubility in water. Aqueous solution of 'A' is often used for the test of CO_2. a test in which insoluble B is formed. If B is further reacted with CO_2 then soluble compound is formed. 'A' is

(1) Quick lime (2) Slaked lime (3) White lime (4) Lime water

Sol. **(2)**

$$CaO + H_2O \rightarrow \underset{A(less\ soluble)}{Ca(OH)_2}$$

$$Ca(OH)_2 + CO_2 \rightarrow \underset{B(insoluble)}{CaCO_3} + H_2O$$

$$\underset{B}{CaCO_3} + H_2O + CO_2 \rightarrow \underset{Soluble}{Ca(HCO_3)_2}$$

Q.67 Match list I with list II

List I Industry		List II Waste Generated	
(A)	Steel plants	(I)	Gypsum
(B)	Thermal power plants	(II)	Fly ash
(C)	Fertilizer industries	(III)	Slag
(D)	Paper mills	(IV)	Bio-degradable wastes

Choose the correct answer from the options given below

(1) (A)-(III), (B)-(IV), (C)-(I), (D)-(II) (2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)
(3) (A)-(II), (B)-(III), (C)-(IV), (D)-(I) (4) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

Sol. **(4)**

Steel plant produces slag from blast furnace. Thermal power plant produces fly ash, Fertilizer industries produces gypsum. Paper mills produces bio degradable waste

Q.68 Suitable reaction condition for preparation of Methyl phenyl ether is

(1) Benzene, MeBr (2) $PhO^{\ominus}Na^{\oplus}$, MeOH (3) Ph–Br,$MeO^{\ominus}Na^{\oplus}$ (4) $PhO^{\ominus}Na^{\oplus}$, MeBr

Sol. **(4)**

Williamson;s synthasis :-

$Ph - O^{\ominus}Na^{\oplus} + Me - Br \rightarrow Ph - O - Me + NaBr$

Q.69 The one that does not stabilize 2° and 3° structures of proteins is

(1) H-bonding (2) –S–S–linkage

(3) van der waals forces (4) –O–O–linkage

Sol. **(4)**

Fact

The main forces which stabilize the secondary and tertiary structure of proteins are

→ Hydrogen bonds

→ S – S Linkages

→ vanderwaals force

→ electrostatic force of attraction

Q.70 The compound which does not exist is

(1) $PbEt_4$ (2) BeH_2 (3) NaO_2 (4) $(NH_4)_2BeF_4$

Sol. **(3)**

Sodium superoxide is not stable

Q.71 Given below are two reactions, involved in the commercial production of dihydrogen (H_2). The two reactions are carried out at temperature "T_1" and "T_2", respectively

$C(s) + H_2O(g) \xrightarrow{T_1} CO(g) + H_2(g)$

$CO(g) + H_2O(g) \xrightarrow[\text{catalyst}]{T_2} CO_2(g) + H_2(g)$

The temperatures T_1 and T_2 are correctly related as

(1) $T_1 = T_2$ (2) $T_1 < T_2$ (3) $T_1 > T_2$ (4) $T_1 = 100$ K, $T_2 = 1270$ K

Sol. **(3)**

$T_1 = 1270$ K $T_2 = 673$ K

$T_1 > T_2$ on the basis of data

Q.72 The enthalpy change for the adsorption process and micelle formation respectively are

(1) $\Delta H_{ads} < 0$ and $\Delta H_{mic} < 0$ (2) $\Delta H_{ads} > 0$ and $\Delta H_{mic} < 0$

(3) $\Delta H_{ads} < 0$ and $\Delta H_{mic} > 0$ (4) $\Delta H_{ads} > 0$ and $\Delta H_{mic} > 0$

Sol. **(3)**

Adsorption → Exothermic $(\Delta H_{ads} = -ve)$

Micelle formation → Endothermic $(\Delta H_{mic} = +ve)$

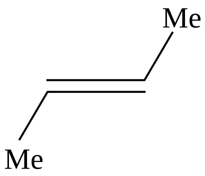
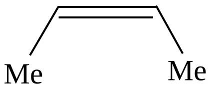
$\Delta H_{ads} < O$ and $\Delta H_{mic} > O$

Q.73 The pair from the following pairs having both compounds with net non-zero dipole moment is

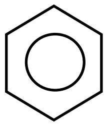
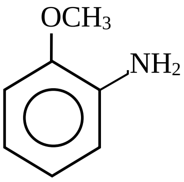
(1) cis-butene, trans-butene (2) Benzene, anisidine

(3) CH_2Cl_2, $CHCl_3$ (4) 1,4-Dichlorohenzene, 1,3-Dichlorobenzene

Sol. **(3)**

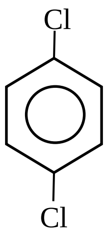
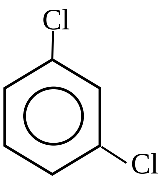
(1)  

$\mu = 0$ $\mu \neq 0$

(2)  

$\mu = 0$ $\mu \neq 0$

(3) CH_2Cl_2 ($\mu \neq 0$) $CHCl_3$ ($\mu \neq 0$)

(4)  

$\mu = 0$ $\mu \neq 0$

Q.74 Which of the following is used as a stabilizer during the concentration of sulphide ores?
(1) Xanthates (2) Fatty acids (3) Pine oils (4) Cresols

Sol. **4**

Cresol is used as stabilizer

Q.75 Which of the following statements are correct ?
(A) The M^{3+}/M^{2+} reduction potential for iron is greater than manganese
(B) The higher oxidation states of first row d-block elements get stabilized by oxide ion.
(C) Aqueous solution of Cr^{2+} can liberate hydrogen from dilute acid.
(D) Magnetic moment of V^{2+} is observed between 4.4-5.2 BM.
Choose the correct answer from the options given below:
(1) (C), (D) only (B) (B), (C) only (C) (A), (B), (D) only (D) (A), (B) only

Sol. **2**

(A) The M^{3+}/M^{2+} reduction potential for manganese is greater than iron

(B) $E^0_{Fe^{+3}/Fe^{+2}} = +0.77$

$E^0_{Mn^{+3}/Mn^{+2}} = +1.57$

(C) $E^0_{Cr^{+3}/Cr^{+2}} = -0.26$

$$\therefore \quad Cr^{2\oplus} + H^{\oplus} \longrightarrow Cr^{3\oplus} + \frac{1}{2}H_2$$

(D) $V^{2\oplus}$ = 3 unpaired electron
Magnetic Moment = 3.87 B.M

Q.76 Given below are two statements :

Statement I : Aqueous solution of $K_2Cr_2O_7$ is preferred as a primary standard in volumetric analysis over $Na_2Cr_2O_7$ aqueous solution.

Statement II : $K_2Cr_2O_7$ has a higher solubility in water than $Na_2Cr_2O_7$

In the light of the above statements, choose the correct answer from the options given below:

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Sol. **(2)**

(1) $K_2Cr_2O_7$ is used as primary standard. The concentration $Na_2Cr_2O_7$ changes in aq. solution.

(2) It is less soluble than $Na_2Cr_2O_7$

Q.77 The octahedral diamagnetic low spin complex among the following is

(1) $[CoF_6]^{3-}$ (2) $[CoCl_6]^{3-}$ (3) $[Co(NH_3)_6]^{3+}$ (4) $[NiCl_4]^{2-}$

Sol. **(3)**

(1) Paramagnetic, High Spin & Tetrahedral

(2) Paramagnetic, High Spin & Octahedral

(3) Paramagnetic, High Spin & Octahedral

(4) Diamagnetic, Low Spin & Octahedral

$[Co(NH_3)_6]^{3+}, CN = 6$ CN = 6 (Octahedral)

$NH_3 = SFL$

$Co^{+3} = [Ar]3d^6$

Diamagnetic & Low spin complex

Q.78 Isomeric amines with molecular formula $C_8H_{11}N$ given the following tests

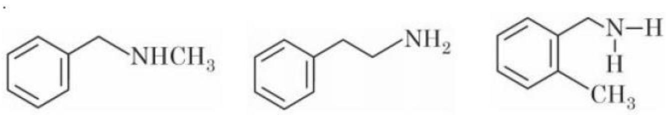
Isomer (P) ⇒ Can be prepared by Gabriel phthalimide synthesis

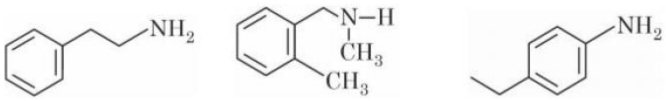
Isomer (Q) ⇒ Reacts with Hinsberg's reagent to give solid insoluble in NaOH

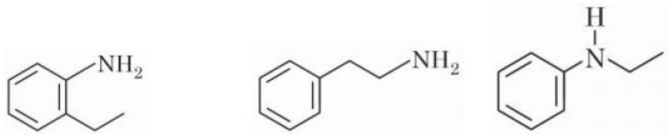
Isomer (R) ⇒ Reacts with HONO followed by β-naphthol in NaOH to given red dye.

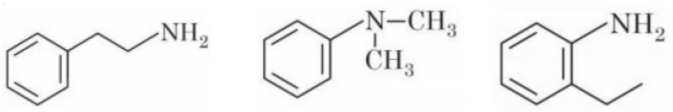
Isomer (P), (Q) and (R) respectively are

(P) (Q) (R)

(1)


(2)


(3)


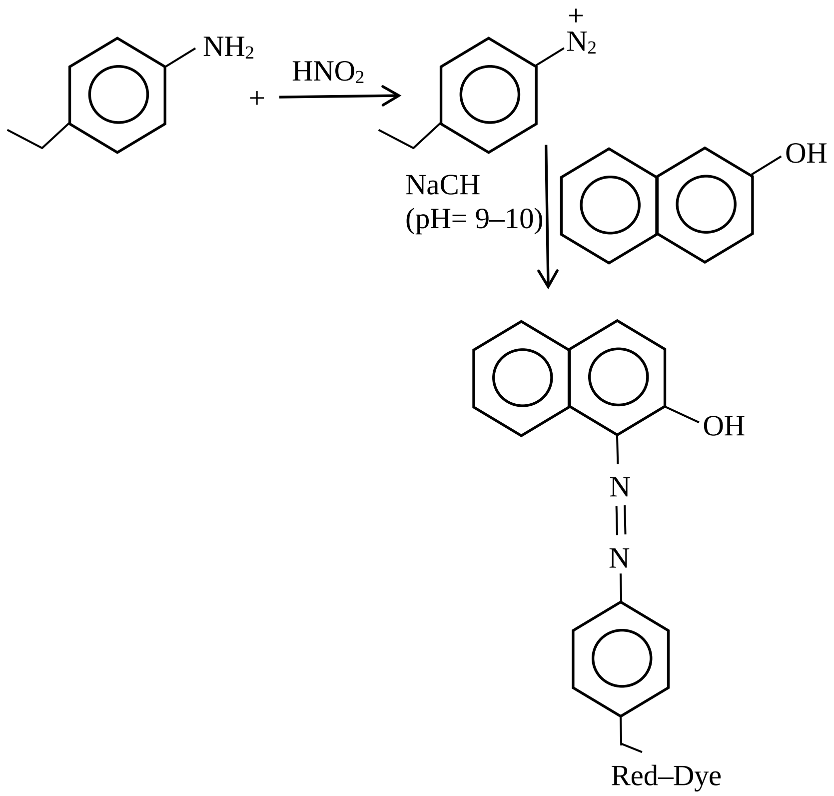
(4)


Sol. **(2)**

P = Can be prepased by Gabriel phthalimide synthesis it should be i-amine

Q = React with Hinsberg's reagent and insoluble in NaOH it should be 2°–amine

R = React with HNO_2 followed by B–Napthol in NaOH it give red dye it must be Aromatic Amine



Q.79 The number of molecules and moles in 2.8375 litres of O_2 at STP are respectively

(1) 7.527×10^{22} and 0.125 mol
(2) 1.505×10^{23} and 0.250 mol
(3) 7.527×10^{23} and 0.125 mol
(4) 7.527×10^{22} and 0.250 mol

Sol. (1)

Moles of $O_2\left(n_{O_2}\right)=\dfrac{\text{Volume of }O_2}{22.7}=0.125\,\text{moles}$

Molecules of $O_2=\text{moles}\times N_A$

$= 0.125 \times 6.022 \times 10^{23}$

$= 7.527 \times 10^{22}$ molecules

Ans (1) 7.527×10^{22} and 0.125 mole

Q.80 Match list I with List II

	List I **polymer**	**List II** **Type/Class**	
(A)	Nylon-2-Nylon-6	(I)	Thermosetting polymer
(B)	Buna-N	(II)	Biodegradable polymer
(C)	Urea-Formaldehyde resin	(III)	Synthetic rubber
(D)	Dacron	(IV)	Polyester

Choose the correct answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)
(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)
(3) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Sol. **(4)**

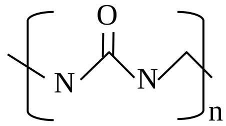
Fact Base

(A) Nylon–2–Nylon–6 $\rightarrow$It is α Biodegradable polymer

(B) Buna – N $\rightarrow$ $\left[CH_2 - CH = CH - CH_2 - CH_2 - \overset{CN}{\overset{|}{CH}} \right]_n$

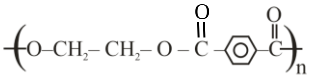
$\rightarrow$ It is a synthetic rubber

(C) Urea – formaldehyde resin



It is a thermos setting polymer

(D) Dacron



It is a polyester.

SECTION - B

Q.81 If the degree of dissociation of aqueous solution of weak monobasic acid is determined to be 0.3, then the observed freezing point will be______________ % higher than the expected/theoretical freezing point. (Nearest integer)

Sol. **30**

For mono basic acid $\rightarrow$ n = 2

$i = 1 + (n-1)\alpha = 1 + (2-1)0.3$

i = 1.3

$$\%\text{increase} = \frac{(\Delta T_f)_{obs} - (\Delta T_f)_{cal}}{(\Delta T_f)_{cal}} \times 100$$

$$= \frac{K_f \times i \times m - K_f \times m}{K_f \times m} \times 100$$

$$= \frac{i-1}{1} \times 100 = 30\%$$

Q.82 In the following reactions, the total number of oxygen atoms in X and Y is___________

$Na_2O + H_2O \rightarrow 2X$

$Cl_2O_7 + H_2O \rightarrow 2Y$

Sol. **5**

$Na_2O + H_2O \rightarrow 2NaOH$

$Cl_2O_7 + H_2O \rightarrow 2HClO_4$

1 + 4 = 5

Q.83 The sum of lone pairs present on the central atom of the interhalogen IF_5 and IF_7 is______

Sol. **1**

IF_5 = 1 lone pair

IF_7 = 0 lone pair

1 + 0 = 1

Q.84 The number of bent-shaped molecule/s from the following is__________

$N_3^-, NO_2^-, I_3^-, O_3, SO_2$

Sol. 3

N_3^- linear

NO_2^- bent

I_3^- linear

O_3 bent

SO_2 bent

Q.85 The number of correct statement/s involving equilibria in physical from the following is__________

(1) Equilibrium is possible only in a closed system at a given temperature.

(2) Both the opposing processes occur at the same rate.

(3) When equilibrium is attained at a given temperature, the value of all its parameters

(4) For dissolution of solids in liquids, the solubility is constant at a given temperature.

Sol. 3

(A) is correct

(B) for equilibrium $r_f = r_b$

$\Rightarrow$(B) is correct

(C) at equilibrium the value of parameters become constant of a given temperature and not equal

$\Rightarrow$ (C) is incorrect

(D) for a given solid solute and a liquid solvent solubility depends upon temperature only

$\Rightarrow$ (D) is correct

Q.86 At constant temperature, a gas is at pressure of 940.3 mm Hg. The pressure at which its volume decreases by 40% is__________ mm Hg. (Nearest integer)

Sol. 1567

$P_{initial}$ = 940.3 mm Hg   Vinitial = 100 (Assume)

P_{final} =?

$P_iV_i = P_f V_f$

$940.3 \times 100 = P_f \times 60$

P_f = 1567.16 mm of Hg

P_f = 1567

Q.87 $FeO_4^{2-} \xrightarrow{+2.2V} Fe^{3+} \xrightarrow{+0.70V} Fe^{2+} \xrightarrow{-0.45V} Fe^o$

$E^o_{FeO_4^{2-}/Fe^{2+}}$ is $x \times 10^{-3}$ V. The value of x is___________

Sol. 1825

$$FeO_4^{2-} + 3e^{\Theta} \to Fe^{+3} \quad \Delta G_1$$

$$Fe^{+3} \quad + e^{\Theta} \to Fe^{+2} \quad \Delta G_2$$

$$FeO_4^{-2} \quad + 4e^{\Theta} \to Fo^{+2} \Delta G_3$$

$$\Delta G_3 = \Delta G_1 + \Delta G_2$$

$$(-)4E_3^o F = (-)3 \times 2.2 \times F + (-)1 \times 0.7 \times f$$

$$4E_3^o = 6.6 + 0.7 = 7.3$$

$$E_3^o = \frac{7.3}{4} = 1.825 = 1825 \times 10^{-3}$$

Q.88 A molecule undergoes two independent first order reactions whose respective half lives are 12 min and 3 min. If both the reactions are occurring then the time taken for the 50% consumption of the reactant is__________ min. (Nearest integer)

Sol. **2**

$$k_{eff} = k_1 + k_2$$

$$\frac{\ell n^2}{t_{eff}} = \frac{\ell n^2}{t_1} + \frac{\ell n^2}{t_2}$$

$$\frac{1}{t_{eff}} = \frac{1}{12} + \frac{1}{3} = \frac{1+4}{12} = 5/12$$

$$t_{eff} = \frac{12}{5} = 2.4 = 2$$

Q.89 The number of incorrect statement/s about the black body from the following is__________

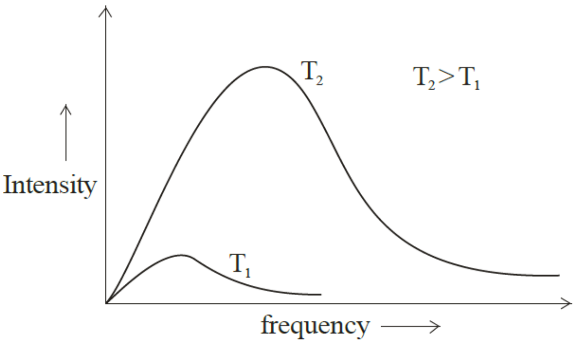
(1) Emit or absorb energy in the form of electromagnetic radiation.

(2) Frequency distribution of the emitted radiation depends on temperature.

(3) At a given temperature, intensity vs frequency curve passes through a maximum value.

(4) The maximum of the intensity vs frequency curve is at a higher frequency at higher temperature compared to that at lower temperature.

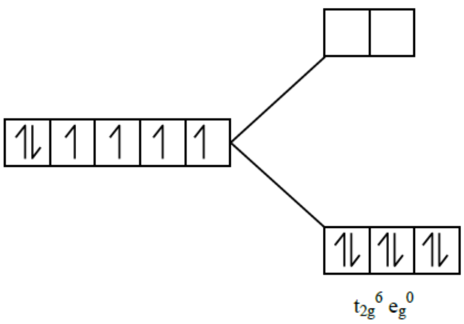
Sol. **0**



Q.90 In potassium ferrocyanide, there are____________ pairs of electrons in the t_{2g} set of orbitals.

Sol. **3**

$K_4[Fe(CN)_6]$



$Fe^{+2} = [Ar]3d^6$

$CN^- = SFL$

t_{2g} contain 6 electron so it become 3 pairs