SECTION - A

1. Let A = $\left\{\theta \in (0, 2\pi) : \dfrac{1+2i\sin\theta}{1-i\sin\theta} \text{ is purely imaginary}\right\}$. Then the sum of the elements in A is.

(1) π (2) 3π (3) 4π (4) 2π

Sol. **(3)**

$$z = \frac{1+2i\sin\theta}{1-i\sin\theta} \times \frac{1+i\sin\theta}{1+i\sin\theta}$$

$$z = \frac{1-2\sin^2\theta + i(3\sin\theta)}{1+\sin^2\theta}$$

Re(z) = 0

$$\frac{1-2\sin^2\theta}{1+\sin^2\theta} = 0$$

$$\sin\theta = \frac{\pm 1}{\sqrt{2}}$$

$$A = \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$$

sum = 4π (Option 3)

2. Let P be the plane passing through the line $\dfrac{x-1}{1} = \dfrac{y-2}{-3} = \dfrac{z+5}{7}$ and the point (2, 4, –3). If the image of the point (-1, 3, 4) in the plane P is (α, β, γ) then $\alpha + \beta + \gamma$ is equal to

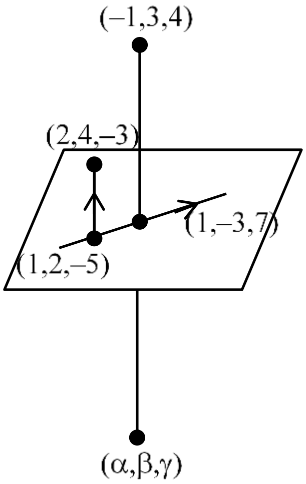
(1) 12 (2) 9 (3) 10 (4) 11

Sol. **(3)**

Equation of plane is given by

$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$

$4x - y - z = 7$



$$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$\alpha = 7, \beta = 1, \gamma = 2$

$\alpha + \beta + \gamma = 10$ (Option 3)

3. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to :

(1) 14 (2) 12 (3) 19 (4) 10

Sol. **(1)**

$$|A - xI| = 0 \Rightarrow \begin{vmatrix} 1-x & 5 \\ \lambda & 10-x \end{vmatrix} = 0 \Rightarrow x^2 - 11x + 10 - 5\lambda = 0$$

$$\Rightarrow (10-5\lambda)A^{-1} = -A + 11I$$

$$\therefore \alpha = \frac{-1}{10-5\lambda} \quad \text{and} \quad \beta = \frac{+11}{10-5\lambda}$$

$$\alpha+\beta=-2 \Rightarrow \frac{10}{10-5\lambda}=-2 \Rightarrow 10\text{-}5\lambda=-5 \Rightarrow \lambda=3$$

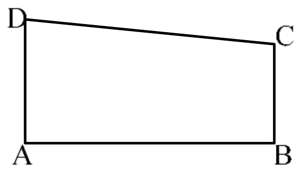
$$\therefore \alpha=\frac{1}{5} \qquad \& \qquad \beta=\frac{-11}{5}$$

$$\therefore 4a^2+\beta^2+\lambda^2=\frac{4}{25}+\frac{121}{25}+3^2=14 \text{ Ans.}$$

4. The area of the quadrilateral ABCD with vertices A(2,1,1), B (1,2, 5), C(–2,–3, 5) and D (1, -6, -7) is equal to

(1) 54 (2) $9\sqrt{38}$ (3) 48 (4) $8\sqrt{38}$

Sol. **(4)**



Vector Area = $\vec{v}$

$$=\frac{1}{2}\overrightarrow{AB}\times\overrightarrow{AC}+\frac{1}{2}\overrightarrow{AC}\times\overrightarrow{AD}$$

$$=\frac{1}{2}\left(\overrightarrow{AB}-\overrightarrow{AD}\right)\times\overrightarrow{AC} \qquad \left(\begin{array}{l}\overrightarrow{AB}=-\hat{i}+\hat{j}+4\hat{k}\\ \overrightarrow{AD}=-\hat{i}-7\hat{j}-8\hat{k}\\ \overrightarrow{AC}=-4\hat{i}-4\hat{j}+4\hat{k}\end{array}\right)$$

$$=\frac{1}{2}\left(8\hat{j}+12\hat{k}\right)\times(-4)\left(\hat{i}+\hat{j}-\hat{k}\right)$$

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 8 & 12\\ 1 & 1 & -1\end{vmatrix}$$

$$=(-2)\left(-20\hat{i}+12\hat{j}-8\hat{k}\right)$$

$$=8\left(5\hat{i}-3\hat{j}+2\hat{k}\right)$$

$$\therefore \text{ Area }=|\vec{v}|=8\sqrt{25+9+4}=8\sqrt{38} \text{ Ans.}$$

5. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by

(1) 34 but not by 14 (2) 14 but not by 34 (3) Both 14 and 34 (4) Neither 14 nor 34

Sol. **(1)**

$25^{190}-8^{190}$ is divisible by 25 – 8 = 17

$19^{190}-2^{190}$ is divisible by 19 – 2 = 17

$25^{190}-19^{190}$ is divisible by 25 – 19 = 6

$8^{190}-2^{190}$ is divisible by 8 – 2 = 6

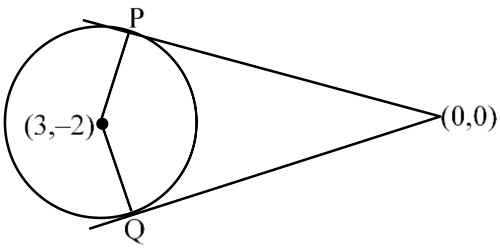
L.C.M. of 1746 = 34

$\therefore$ divisible by 34 but not by 14

6. Let O be the origin and OP and OQ be the tangents to the circle $x^2 + y^2 - 6x + 4y + 8 = 0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha, \frac{1}{2}\right)$, then a value of α is.

(1) $-\frac{1}{2}$ (2) $\frac{5}{2}$ (3) 1 (4) $\frac{3}{2}$

Sol. **(2)**



Circumcircle of ΔOPQ

$(x - 0)(x - 3) + (y - 0)(y + 2) = 0$

$x^2 + y^2 - 3x + 2y = 0$

passes through $\left(\alpha, \frac{1}{2}\right)$

$\therefore \alpha^2 + \frac{1}{4} - 3\alpha + 1 = 0$

$\Rightarrow \alpha^2 - 3\alpha + \frac{5}{4} = 0 \Rightarrow 4\alpha^2 - 12\alpha + 5 = 0$

$\Rightarrow 4\alpha^2 - 10\alpha - 2\alpha + 5 = 0$

$(2\alpha - 1)(2\alpha - 5) = 0 \;\therefore \alpha = \frac{1}{2}, \frac{5}{2}$ Ans.

7. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 +.... and $Sn = \sum_{k=1}^{n} a_K$. Then $S_{30} - a_{40}$ is equal to

(1) 11260 (2) 11280 (3) 11290 (4) 11310

Sol. **(3)**

$S_n = 5 + 8 + 14 + 23 + 35 + 50 + ...+ a_n$

$S_n = \quad 5 + 8 + 14 + 23 + 35 + ...+ a_n$

$O = 5 + 3 + 6 + 9 + 12 + 15 + ... - a_n$

$a_n = 5 + (3 + 6 + 9 + ...\ (n-1) \text{ terms})$

$$a_n = \frac{3n^2 - 3n + 10}{2}$$

$$a_{40} = \frac{3(40)^2 - 3(40) + 10}{2} = 2345$$

$$S_{30} = \frac{3\sum_{n=1}^{30} n^2 - 3\sum_{n=1}^{30} n + 10\sum_{n=1}^{30} 1}{2}$$

$$= \frac{\frac{3 \times 30 \times 31 \times 61}{6} - \frac{3 \times 30 \times 31}{2} + 10 \times 30}{2}$$

$S_{30} = 13635$

$S_{30} - a_{40} = 13635 - 2345$

$= 11290$ (Option (3))

8. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim\limits_{x\to\frac{1}{\alpha}}\left(\dfrac{1-\cos(x^2+bx+a)}{2(1-ax)^2}\right)^{\frac{1}{2}} = \dfrac{1}{k}\left(\dfrac{1}{\beta}-\dfrac{1}{\alpha}\right)$, then k is equal to

(1) β (2) 2α (3) 2β (4) α

Sol. **(2)**

$\therefore ax^2 + bx + 1 = a(x-\alpha)(x-\beta)\ \therefore \alpha\beta = \dfrac{1}{a}$

$\therefore\ x^2 + bx + a = a(1-\alpha x)(1-\beta x)$

$$\therefore \lim_{x\to\frac{1}{\alpha}}\left\{\frac{1-\cos\left(x^2+bx+a\right)}{2\left(1-\alpha x\right)^2}\right\}^{\frac{1}{2}} = \lim_{x\to\frac{1}{2}}\left\{\frac{1-\cos a\left(1-\alpha x\right)\left(1-\beta x\right)}{2\left\{a\left(1-\alpha x\right)\left(1-\beta x\right)\right\}^2}\cdot a^2\left(1-\beta x\right)^2\right\}^{\frac{1}{2}}$$

$$= \left[\frac{1}{2}\cdot\frac{1}{2}a^2\left(1-\frac{\beta}{\alpha}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}\frac{1}{\alpha\beta}\left(1-\frac{\beta}{\alpha}\right) = \frac{1}{2}\left(\frac{1}{\alpha\beta}-\frac{1}{\alpha^2}\right)$$

$$= \frac{1}{2\alpha}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right) = \frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$

$\therefore k = 2\alpha$ Ans.

9. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, is equal to

(1) 1890 (2) 945 (3) 2835 (4) 5670

Sol. **(4)**

$M_2A_2T_2HEICS$

= total words – when C & S are together

$$\frac{\lfloor\underline{11}}{\lfloor\underline{2}\ \lfloor\underline{2}\ \lfloor\underline{2}} - \frac{\lfloor\underline{10}}{\lfloor\underline{2}\ \lfloor\underline{2}\ \lfloor\underline{2}}\times\lfloor\underline{2}$$

$$\frac{\lfloor\underline{10}}{\lfloor\underline{2}\ \lfloor\underline{2}\ \lfloor\underline{2}}\times 9$$

$$= \frac{9\times10\times9\times8\times7}{8}\lfloor\underline{6}$$

$= 5670\ \lfloor\underline{6}$

k = 5670 (Option 4)

10. Let S be the set of all values of $\theta \in [-\pi,\pi]$ for which the system of linear equations

$x + y + \sqrt{3}z = 0$

$-x + \left(\tan\theta\right)y + \sqrt{7}z = 0$

$x + y + \left(\tan\theta\right)z = 0$

has non-trivial solution. Then $\dfrac{120}{\pi}\sum\limits_{\theta\in S}\theta$ is equal to

(1) 20 (2) 40 (3) 30 (4) 10

Sol. **(1)**

For non trivial solutions

D = 0

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$$\tan^2\theta - \left(\sqrt{3}-1\right) - \sqrt{3} = 0$$

$$\tan\theta = \sqrt{3}, -1$$

$$\theta = \left\{\frac{\pi}{3}, \frac{-2\pi}{3}, \frac{-\pi}{4}, \frac{3\pi}{4}\right\}$$

$$\frac{120}{\pi}(\Sigma\theta) = \frac{120}{\pi} \times \frac{\pi}{6} = 20 \text{ (Option 1)}$$

11. For a, b $\in$ Z and $|a - b| \le 10$, let the angle between the plane P : ax + y – z = b and the line l : x –1 = a – y = z +1 be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then $a^4 + b^2$ is equal to

(1) 85 (2) 48 (3) 25 (4) 32

Sol. **(4)**

$$\theta = \cos^{-1}\frac{1}{3} \therefore \sin\theta = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

$$\sin\theta = \frac{a\cdot 1 + 1(-1) + (-1)\cdot 1}{\sqrt{a^2+1+1}\cdot\sqrt{3}} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \{3(a-2)\}^2 = 24(a^2+2)$$

$$\Rightarrow 3(a^2-4a+4) = 8a^2+16$$

$$\Rightarrow 5a^2+12a+4=0$$

$$\Rightarrow 5a^2+10a+2a+4=0$$

$$\therefore a = -2, \frac{-2}{5} \because a \in z$$

$$\therefore a = -2$$

Distance of (6, –6, 4) from

–2x + y – z – b = 0 is $3\sqrt{6}$

$$\therefore \left|\frac{-12-6-4-b}{\sqrt{4+1+1}}\right| = 3\sqrt{6}$$

$$\Rightarrow |b+22| = 18 \therefore b = -40, -4$$

$$\because |a-b| \le 10$$

$$\therefore b = -4$$

$$\therefore a^4 + b^2$$

= 32 Ans.

12. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}, \vec{\mu}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}, \vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then 6 (a + b + c) is equal to

(1) 4 (2) 12 (3) 6 (4) 0

Sol. **(2)**

$$[\vec{u}_1\ \vec{u}_2\ \vec{u}_3] = 0 \quad \therefore \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow$ b – 1 + c – 1 + a (1 – bc) = 0

$\therefore$ abc = a + b + c – 2

$$[\vec{v}_1\ \vec{v}_2\ \vec{v}_3] = 0 \quad \therefore \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$$R_3 \to R_3 - (R_1 + R_2) \Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0$$

$\Rightarrow -2a(ac - bc - c^2) + 2c(a^2 + ab - ac) = 0$

$\Rightarrow -2a^2c + 2abc + 2ac^2 + 2a^2c + 2abc - 2ac^2 = 0$

$\Rightarrow$ 4abc = 0 $\therefore$ abc = 0

$\therefore$ a + b + c = 2 $\therefore$ 6(a+b+c) = 12 Ans.

13. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2 + \frac{1}{2x}\right)^{11}$ is equal to

(1) $10^3 - 10$ (2) $11^3 - 11$ (3) $12^3 - 12$ (4) $13^3 - 13$

Sol. **(3)**

$T_{r+1} = {}^{11}C_r(2x^2)^{11-r}\left(\frac{1}{2x}\right)^r$

$= {}^{11}C_r\, 2^{11-2r}\, x^{22-3r}$

22 – 3r = 10 and 22 – 3r = 7

r = 4 and r = 5

Coefficient of $x^{10} = {}^{11}C_4 \cdot 2^3$

Coefficient of $x^7 = {}^{11}C_5 \cdot 2^1$

difference $= {}^{11}C_4 \cdot 2^3 - {}^{11}C_5 \cdot 2$

$= \frac{11\times10\times9\times8}{24}\times8 - \frac{11\times10\times9\times8\times7}{120}\times2$

$= 11 \times 10 \times 3 \times 8 - 11 \times 3 \times 4 \times 7$

$= 11 \times 3 \times 4 \times (20 - 7)$

$= 11 \times 12 \times 13$

$= 12(12 - 1)(12 + 1)$

$= 12(12^2 - 1)$

$= 12^3 - 12$ (Option 3)

14. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {(x,y) $\in$A × A : x + y =7} is
(1) Symmetric but neither reflexive nor transitive
(2) Transitive but neither symmetric nor reflexive
(3) An equivalence relation
(4) Reflexive but neither symmetric nor transitive

Sol. **(1)**
R = {(1,6), (2,5), (3,4), (4,3),(5,2), (6,1)}

15. If the probability that the random variable X takes values x is given by P(X=x) = k (x+1)3^{-x}, x = 0, 1,2, 3, ..., where k is a constant, then P(X $\geq$ 2) is equal to

(1) $\frac{7}{27}$ (2) $\frac{11}{18}$ (3) $\frac{7}{18}$ (4) $\frac{20}{27}$

Sol. **(1)**

$$\sum_{x=0}^{\infty} P(X=x)=1$$

k(1 + 2·3^{-1} + 3·3^{-2} + 4·3^{-3} + ... ∞) = 1

Let $s=1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\ldots\infty$

$$\frac{s}{3}=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^2}+\ldots\infty$$

$$\frac{2s}{3}=1+\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^2}+\ldots\infty$$

$$\frac{2s}{3}-\frac{1}{1-\frac{1}{3}}=\frac{3}{2}$$

$$s=\frac{9}{4}$$

so $\boxed{k=\frac{4}{9}}$

$$P(X\geq 2)=1-P(x=0)-P(x=1)$$

$$=1-\frac{4}{9}\left(1+\frac{2}{3}\right)$$

$$=\frac{7}{27}$$ (Option 1)

16. The integral $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)$ $\log_2$ x dx is equal to

(1) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{2}{x}\right)+C$ (2) $\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x+C$

(3) $\left(\frac{x}{2}\right)^x \log_2\left(\frac{x}{2}\right)+C$ (4) $\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x+C$

Sol. **(2) Bonus**

$$\int\left(x^x 2^{-x}+2^x x^{-x}\right)\log_2^x dx$$

$$\int\left(e^{x\ln x}\cdot e^{-x\ln 2}+e^{x\ln 2}\cdot e^{-x\ln x}\right)dx$$

$$\int\left(e^{x\ln x-x\ln 2}+e^{x\ln 2-x\ln x}\right)\frac{\ln x}{\ln 2}dx$$

let $\quad x\ln x - x\ln 2 = t$

$\quad (\ln x + 1 - \ln 2)\, dx = dt$

17. The value of 36 $(4\cos^2 9^o - 1)(4\cos^2 27^o - 1)(4\cos^2 81^o - 1)(4\cos^2 243^o - 1)$ is

(1) 27 (2) 54 (3) 18 (4) 36

Sol. **(4)**

$4\cos^2\theta - 1 = 4(1-\sin^2\theta) - 1 = 3 - 4\sin^2\theta = \dfrac{\sin 3\theta}{\sin\theta}$

so given expression can be written as

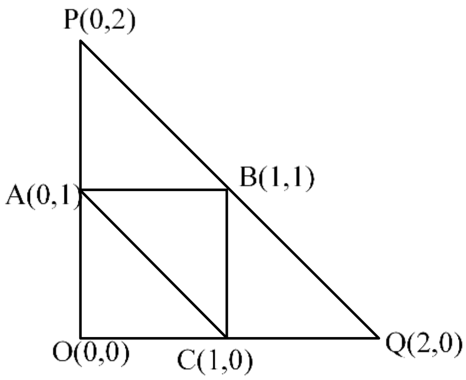
$$36\times\frac{\sin 27^o}{\sin 9^o}\times\frac{\sin 81^o}{\sin 27^o}\times\frac{\sin 243^o}{\sin 81^o}\times\frac{\sin 729^o}{\sin 243^o}$$

$$36\times\frac{\sin 729^o}{\sin 9^o}=36$$

18. Let A (0, 1), B(1,1) and C (1,0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $\left(\alpha+\beta\sqrt{3},0\right)$, where α and β are rational numbers, then $\dfrac{\alpha}{\beta^2}$ is equal to

(1) 6 (2) 8 (3) $\dfrac{9}{2}$ (4) 12

Ans. **(2)**



a = OP = 2 $\quad$ b = OQ = 2 $\quad$ c = PQ = $2\sqrt{2}$

(2,0) $\quad$ (0,2) $\quad$ (0,0)

$$D\left(\frac{4}{2+2+2\sqrt{2}},\frac{4}{2+2+2\sqrt{2}}\right)\equiv D\left(\frac{2}{2+\sqrt{2}},\frac{2}{2+\sqrt{2}}\right)$$

$$y^2=4ax \Rightarrow \left(\frac{2}{2+\sqrt{2}}\right)^2 = 4a\cdot\left(\frac{2}{2+\sqrt{2}}\right)$$

$$\therefore 4a=\frac{2}{2+\sqrt{2}}\ \therefore a=\frac{1}{2}\cdot\frac{2-\sqrt{2}}{4-2}=\frac{1}{4}\left(2-\sqrt{2}\right)$$

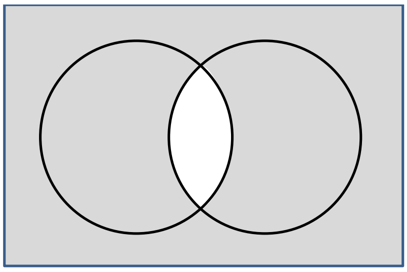
$$\therefore \alpha=\frac{2}{4}=\frac{1}{2}\qquad \beta=\frac{-1}{4}$$

$$\therefore \frac{\alpha}{\beta^2}=8\ \text{Ans.}$$

19. The negation of $(p \wedge (\sim q)) \vee (\sim p)$ is equivalent to

(1) $p \wedge (\sim q)$ (2) $p \wedge (q \wedge (\sim p))$ (3) $p \vee (q \vee (\sim p))$ (4) $p \wedge q$

Sol. **4**



20. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to

(1) 316 (2) 317 (3) 315 (4) 314

Sol. **2**

$\frac{\Sigma x}{12} = \frac{9}{2}$

$\sum x = 54$

$\frac{\Sigma x^2}{12} - \left(\frac{9}{2}\right)^2 = 4$

$\sum x^2 = 291$

$\sum x_{new} = 54 - (9 + 10) + 7 + 14 = 56$

$\sum x^2_{new} = 291 - (81 + 100) + 49 + 196 = 355$

$\sigma^2_{new} = \frac{355}{12} - \left(\frac{56}{12}\right)^2$

$\sigma^2_{new} = \frac{281}{36} = \frac{m}{n}$

$\boxed{m + n = 317}$ Option (2)

SECTION - B

21. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions f : R $\to$ S such that f(a) $\neq$ 1, is equal to ________.

Sol. **180**

Total onto function

$\frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \lfloor 4 = 240$

Now when f(a) = 1

$\lfloor 4 + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \lfloor 3 = 24 + 36 = 60$

so required $f^n = 240 - 60 = 180$

22. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer ≤ x. Then $m^2 + mn + n^2$ is equal to ________.

Sol. **9**

$x^2 - 12x + [x] + 31 = 0$

$\{x\} = x^2 - 11x + 31$

$0 \le x^2 - 11x + 31 < 1$

$x^2 - 11x + 30 < 0$

$x \in (5, 6)$

so $\quad [x] = 5$

$x^2 - 12x + 5 + 31 = 0$

$x^2 - 12x + 36 = 0$

$\boxed{x = 6}$ but $x \in (5, 6)$

so $\quad x \in \phi$

$\boxed{m = 0}$

Now $\quad x^2 - 5|x+2| - 4 = 0$

$x \ge -2$	$x < -2$
$x^2 - 5x - 14 = 0$	$x^2 + 5x + 6 = 0$
$(x-7)(x+2) = 0$	$(x+3)(x+2) = 0$
$x = 7, -2$	$x = -3, -2$

$x = \{7, -2, -3\}$

$n = 3$

$m^2 + mn + n^2 = n^2 = 9$

23. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0)$, $(2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _____.

Sol. **11**

P_2 is given by

$$\begin{vmatrix} x-5 & y-1 & z-1 \\ 3 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix} = 0$$

$\boxed{x - y - z = 3}$

DR of line intersection of P_1 & P_2

$$\begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 3 & -1 & -7 \end{vmatrix}$$

$+6\hat{i} + 4\hat{j} + 2\hat{k}$

Let $\quad z = 0, \quad x - y = 3$

$3x - y = 11$

$2x = 8$

$x = 4$

$y = 1$

So Line is

$$\frac{x-4}{6}=\frac{y-1}{4}=\frac{z-0}{2}=r$$

$(\alpha,\beta,\gamma) = (6r + 4, 4r + 1, 2r)$

$6(\alpha-7)+4(\beta-4)+2(\gamma+1)=0$

$6\alpha-42+4\beta-16+2\gamma+2=0$

$36r+24+16r+4+4r-56=0$

$56r=28$

$r=\frac{1}{2}$

$\alpha + \beta + \gamma = 12r + 5$

$= 6 + 5 = 11$

24. If domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha,\beta)\cup(\gamma,\delta]$, then, $18(\alpha^2+\beta^2+\gamma^2+\delta^2)$ is equal to

Sol. **20**

$$\frac{6x^2+5x+1}{2x-1}>0$$

$$\frac{(3x+1)(2x+1)}{2x-1}>0$$

$-$ | $+$ | $-$ | $+$

$-\frac{1}{2}$ $-\frac{1}{3}$ $\frac{1}{2}$

$x\in\left(\frac{-1}{2},\frac{-1}{3}\right)\cup\left(\frac{1}{2},\infty\right)$ ....(A)

$$-1\le\frac{2x^2-3x+4}{3x-5}\le1$$

and

$\frac{2x^2-1}{3x-5}\ge0$ and $\frac{2x^2-6x+9}{3x-5}\le0$

$-$ | $+$ | $-$ | $+$

$-\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}}$ $\frac{5}{3}$ and $3x-5<0$

$x\in\left[\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]\cup\left(\frac{5}{3},\infty\right)$ ...(B)

$x<\frac{5}{3}$ ...(C)

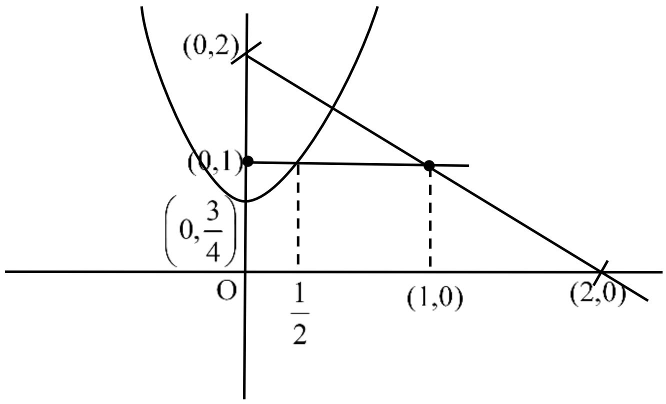
$A\cap B\cap C\equiv\left(\frac{-1}{2},\frac{-1}{3}\right)\cup\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right]$

So $18(\alpha^2+\beta^2+\gamma^2+\delta^2)=18\left(\frac{1}{4}+\frac{1}{9}+\frac{1}{4}+\frac{1}{2}\right)$

$= 18 + 2 = 20$

25. Let the area enclosed by the lines x + y = 2, y = 0 x = 0 and the curve $f(x)=\min\left\{x^2+\frac{3}{4},1+[x]\right\}$ where [x] denotes the greatest integer ≤ x, be A. Then the value of 12A is______.

Sol. **17**



$$\int_0^{\frac{1}{2}}\left(x^2+\frac{3}{4}\right)dx+\frac{1}{2}\times\left(\frac{3}{2}+\frac{1}{2}\right)\times 1$$

$$=\left[\frac{x^3}{3}+\frac{3x}{4}\right]_0^{\frac{1}{2}}+1$$

$$A=\frac{1}{24}+\frac{3}{8}+1$$

$$12A=\frac{1}{2}+\frac{36}{8}+12$$

$$=\frac{1}{2}+\frac{9}{2}+12$$

= 5 + 12

= 17

26. Let 0 < z < y < x be three real numbers such that $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in an arithmetic progression and x, $\sqrt{2}y, z$ are in a geometric progression. If xy + yz + zx = $\frac{3}{\sqrt{2}}$xyz , then $3(x + y + z)^2$ is equal to______.

Sol. **150**

$$\frac{2}{y}=\frac{1}{x}+\frac{1}{z}$$

$$2y^2=xz$$

$$\frac{2}{y}=\frac{x+z}{xz}=\frac{x+z}{2y^2}$$

x + z = 4y

xy + yz + zx = $\frac{3}{\sqrt{2}}$xyz

y(x+z) + zx = $\frac{3}{\sqrt{2}}xz\cdot y$

$$4y^2 + 2y^2 = \frac{3}{\sqrt{2}} y \cdot 2y^2$$
$$6y^2 = 3\sqrt{2}y^3$$
$$y = \sqrt{2}$$
$x + y + z = 5y = 5\sqrt{2}$
$3(x + y + z)^2 = 3 \times 50 = 150$

27. Let the solution curve $x = x(y)$, $0 < y < \frac{\pi}{2}$, of the differential equation $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x \log_e(\cos y)) \sin y\, dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are co-prime, then mn is equal to

Sol. **12**

Cos $y \ln^2 \cos y\, dx = (1 + 3x \ln \cos y) \sin y\, dy$

$$\frac{dx}{dy} = \tan y\left(\frac{3x}{\ln \cos y} + \frac{1}{\ln^2 \cos y}\right)$$
$$\frac{dx}{dy} - \left(\frac{3\tan y}{\ln \cos y}\right)x = \frac{\tan y}{\ln^2 \cos y}$$
$$\text{If} = e^{-\frac{3}{\ln \cos y}\frac{\sin y}{\cos y}dy}$$
$\ln \cos y = t$
$$\frac{1}{\cos y} - \sin y dy = dt$$
$$\text{If} = e^{\frac{3}{t}dt} = e^{3\ln t} = t^3 = \ln^3 \cos y$$
$$\text{solution is } x \cdot \ln^3 \cos y = \left\{\frac{\sin y}{\cos y} \cdot \ln \cos y dy + C\right.$$
$$x \ln^3 \cos y = \frac{-\ln^2 \cos y}{2} + C$$
$$x\left(\frac{\pi}{3}\right) = \frac{1}{2\ln 2} \text{ so } \frac{1}{2\ln 2} \times \ln^3\left(\frac{1}{2}\right) = -\frac{\ln^3\left(\frac{1}{2}\right)}{2} + C$$
$C = 0$
$$y = \frac{\pi}{6} \qquad x \ln^3 \frac{\sqrt{3}}{2} = -\frac{1}{2}\ln^2 \frac{\sqrt{3}}{2} + 0$$
$$x = -\frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$$
$$x = \frac{1}{\ln\frac{4}{3}} = \frac{1}{\ln 4 - \ln 3}$$
$mn = 12$

28. Let [t] denote the greatest integer function. If $\int_0^{2.4}[x^2]dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to ________.

Sol. **6**

$$\int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx + \int_{2}^{\sqrt{5}} 4dx + \int_{\sqrt{5}}^{2\cdot 4} 5dx$$

$$\sqrt{2}-1+2\left(\sqrt{3}-\sqrt{2}\right)+3\left(2-\sqrt{3}\right)+4\left(\sqrt{5}-2\right)+5\left(\left(2\cdot 4\right)-\sqrt{5}\right)$$

$$=9-\sqrt{2}-\sqrt{3}-\sqrt{5}$$

$$\alpha+\beta+\gamma+\delta=9-1-1-1=6$$

29. The ordinates of the points P and Q on the parabola with focus (3.0) and directrix x=–3 are in the ratio 3 : 1 :. If R (α,β) is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to ________.

Sol. **16**

Parabola is $y^2 = 12x$

Let $Q(3t^2, 6t)$

so $P(27t^2, 18t)$

$$R(\alpha,\beta)=\left(at_1t_2, a\left(t_1+t_2\right)\right)$$

$$= (3t\cdot 3t, 3(t + 3t))$$

$$R(\alpha,\beta) = (9t^2, 12t)$$

$$\frac{\beta^2}{\alpha}=\frac{(12t)^2}{9t^2}=\frac{144}{9}=16$$

30. Let k and m be positive real numbers such that the function $f(x)=\begin{cases} 3x^2+k\sqrt{x+1}, & 0<x<1 \\ mx^2+k^2, & x\ge 1 \end{cases}$ is differentiable for all x > 0. Then $\frac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to_______.

Sol. **309**

function is differentiable $\forall x < 0$

so $f(1^-) = f(1)$

$$3+\sqrt{2}k = m + k^2 \qquad ...(1)$$

and $f_+^1\left(1^-\right)=f_-^1\left(1^+\right)$

$$2mx\Big|_{x=1} = 6x+\frac{k}{2\sqrt{x+1}}\Big|_{x=1}$$

$$2m = 6+\frac{k}{2\sqrt{2}}$$

$$m = 3+\frac{k}{4\sqrt{2}} \qquad ...(2)$$

$$k^2+3+\frac{k}{4\sqrt{2}}=3+\sqrt{2}k$$

$$\boxed{k = \frac{7}{4\sqrt{2}}}, 0$$

$$m = 3 + \frac{7}{32}$$

$$\boxed{m = \frac{103}{32}}$$

So $$\frac{8f'(8)}{f'\left(\frac{1}{8}\right)} = 8 \times \frac{2mx\big|_{x=8}}{6x + \frac{k}{2\sqrt{x+1}}\Big|_{x=\frac{1}{8}}}$$

$$= \frac{8 \times 2 \times 8 \times \frac{103}{32}}{\frac{16}{12}}$$

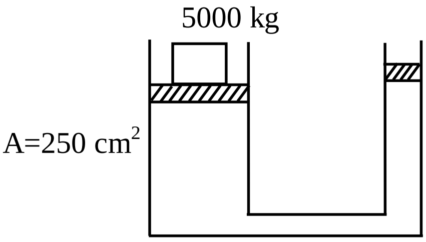
$$= 103 \times 3 = 309$$

SECTION - A

31. A hydraulic automobile lift is designed to lift vehicles of mass 5000 kg. The area of cross section of the cylinder carrying the load is 250 cm^2. The maximum pressure the smaller piston would have to bear is [Assume g = 10 m/s^2] :

(1) $2 \times 10^{+5}$ Pa (2) $20 \times 10^{+6}$ Pa (3) $200 \times 10^{+6}$ Pa (4) $2 \times 10^{+6}$ Pa

Sol. **(4)**



From pascal law same ΔP transmitted through out liquid

$$\Delta P = \frac{F}{A} = \frac{5000 \times 10}{250 \times 10^{-4}}$$

$= 2 \times 10^6$ Pa

32. The orbital angular momentum of a satellite is L, when it is revolving in a circular orbit at height h from earth surface. If the distance of satellite from the earth center is increased by eight times to its initial value, then the new angular momentum will be-

(1) 8L (2) 3L (3) 4L (4) 9L

Sol. **(2)**

$L = mv_0r \left(v_0 = \sqrt{\frac{GM}{h}} \right)$

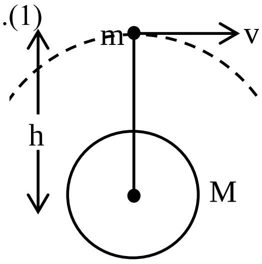
$L = m\sqrt{GMh}$ …….(1)

$h' \rightarrow h + 8h = 9h$

$L' = m\sqrt{GM9h}$ ……..(2)

$\frac{L'}{L} = 3$

$L' = 3L$



33. The waves emitted when a metal target is bombarded with high energy electrons are

(1) Microwaves (2) X-rays (3) Radio Waves (4) Infrared rays

Sol. **(2)**

By theory

34. Match List I with List II

LIST – I		LIST – II	
A.	Torque	I.	$ML^{-2}T^{-2}$
B.	Stress	II.	ML^2T^{-2}
C.	Pressure gradient	III.	$ML^{-1}t^{-1}$
D.	Coefficient of viscosity	IV	$ML^{-1}T^{-2}$

Choose the correct answer from the options given below:

(1) A-III, B-IV, C-I, D-II (2) A-II, B-I, C-IV, D-III

(3) A-IV, B-II, C-III, D-I (4) A-II, B-IV, C-I, D-III

Sol. **(4)**

[Torque] = F.L

$MLT^{-2}.L = ML^{2}T^{-2}$

[Stress] = $\frac{F}{A}$

$\frac{MLT^{-2}}{L^{2}} = ML^{-1}T^{-2}$

[Pressure gradient] = $\frac{\Delta P}{\Delta L} = \frac{F}{A^{2}.L}$

$= \frac{MLT^{-2}}{L^{3}}$

$= ML^{-2}T^{-2}$

$F = nA\frac{dv}{dy}$

$\eta = ML^{-1}T^{-1}$

35. Give below are two statements

Statement I : Area under velocity- time graph gives the distance travelled by the body in a given time.

Statement II : Area under acceleration- time graph is equal to the change in velocity- in the given time.

In the light of given statement, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are true.

(2) Statement I is correct but Statement II is false.

(3) Both Statement I and and Statement II are false.

(4) Statement I is incorrect but Statement II is true.

Sol. **(Official Ans. (1))**

(Motion Ans. (4))

$\vec{v} = \frac{d\vec{s}}{dt} \Rightarrow \int d\vec{s} = \int \vec{v}\,dt$

Area of $\vec{v}$ vs time gives displacement

$\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \int d\vec{v} = \int \vec{a}\,dt$

Area of $\vec{a}$ vs t graph gives change in velocity

36. The power radiated from a linear antenna of length l is proportional to (Given, λ = Wavelength of wave):

(1) $\frac{l}{\lambda}$ (2) $\frac{l^{2}}{\lambda}$ (3) $\frac{l}{\lambda^{2}}$ (4) $\left(\frac{l}{\lambda}\right)^{2}$

Sol. **(4)**

37. Electric potential at a point 'P' due to a point charge of 5×10^{-9}C is 50 V. The distance of 'P' from the point charge is:

(Assume, $\frac{1}{4\pi\varepsilon_0}$ = $9 \times 10^{+9}$ $Nm^{2}C^{-2}$)

(1) 3 cm (2) 9 cm (3) 0.9 cm (4) 90 cm

Sol. **(4)**

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\Rightarrow r = \frac{9\times10^{9}\times5\times10^{-9}}{50}$$

$$\Rightarrow r = \frac{9}{10}\times100\,\text{cm}$$

r = 90 cm

38. The acceleration due to gravity at height h above the earth if h<<R (Radius of earth) is given by

(1) $g' = g\left(1-\frac{h^2}{2R^2}\right)$ (2) $g' = g\left(1-\frac{h}{2R}\right)$ (3) $g' = g\left(1-\frac{2h^2}{R^2}\right)$ (4) $g' = g\left(1-\frac{2h}{R}\right)$

Sol. **(4)**

$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2}$$

using binomial expansion & neglect higher order term

$$\Rightarrow g' = g\left(1-\frac{2h}{R}\right)$$

39. An emf of 0.08 V is induced in a metal rod of length 10 cm held normal to a uniform magnetic field of 0.4 T, when moves with a velocity of:

(1) 2 ms^{-1} (2) 20 ms^{-1} (3) 3.2 ms^{-1} (4) 0.5 ms^{-1}

Sol. **(1)**

$\varepsilon = Blv$

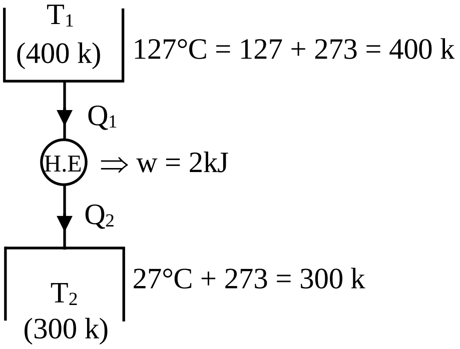
$$\Rightarrow 0.08 = v\times0.4\times\frac{10}{100}$$

$\Rightarrow$ v = 2 m/s

40. Work done by a Carnot engine operating between temperatures 127°C and 27°C is 2 kJ. The amount of heat transferred to the engine by the reservoir is:

(1) 2 kJ (2) 4 kJ (3) 2.67 kJ (4) 8 kJ

Sol. **(4)**



$n = 1 - \frac{300}{400} = \frac{1}{4}$

$n = \frac{w}{Q_1} = \frac{1}{4} \Rightarrow Q_1 = 8kJ$

41. The width of fringe is 2 mm on the screen in a double slits experiment for the light of wavelength of 400 nm. The width of the fringe for the light of wavelength 600 nm will be:

(1) 1.33 mm (2) 3 mm (3) 2 mm (4) 4 mm

Sol. **(2)**

$\beta = \frac{D\lambda}{d}$

$\Rightarrow \beta \propto \lambda$

$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \Rightarrow \frac{2}{\beta} = \frac{400}{600}$

$\beta = 3$ mm

42. The temperature at which the kinetic energy of oxygen molecules becomes double than its value at 27°C is

(1) 1227°C (2) 627°C (3) 327°C (4) 927°C

Sol. **(3)**

KE of O_2 molecules = $5 \times \left(\frac{1}{2}KT\right)$

$(KE)_{27°C} = 5 \times \frac{1}{2}k(27+273) = \frac{5}{2}k \times 300$

$(KE)_T = 2\left(\frac{5}{2}k\right) \times 300 = \frac{5}{2}k(600)$

i.e. T = 600 K

= 600 – 273

T = 327°C

43. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Electromagnets are made of soft iron.

Reason R : Soft iron has high permeability and low retentivity.

In the light of above, statements, chose the **most appropriate** answer from the options given below.

(1) **A** is correct but **R** is not correct

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**
(4) **A** is not correct but **R** is correct

Sol. **(2)**

44. The trajectory of projectile, projected from the ground is given by $y = x - \frac{x^2}{20}$. Where x and y are measured in meter. The maximum height attained by the projectile will be.

(1) 10 m (2) 200 m (3) $10\sqrt{2}$ m (4) 5 m

Sol. **(4)**

$y = x - \frac{x^2}{20}$

$\left(\frac{dy}{dx}\right) = 1 - \frac{x}{10}$ for y_{max} ; $\frac{dy}{dx} = 0$

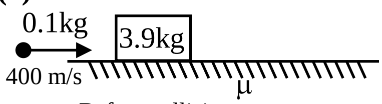
$x = 10$

$y_{max} = 10 - \frac{100}{20} = 5m$

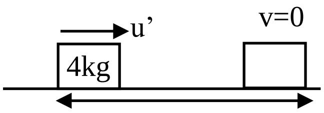
45. A bullet of mass 0.1 kg moving horizontally with speed 400 ms^{-1} hits a wooden block of mass 3.9 kg kept on a horizontal rough surface. The bullet gets embedded into the block and moves 20 m before coming to rest. The coefficient of friction between the block and the surface is _______.
(Given g = 10 m/s^2)

(1) 0.90 (2) 0.65 (3) 0.25 (4) 0.50

Sol. **(3)**



Before collision



After collision

Apply momentum conservation just before and just after the collision

$0.1 \times 400 = (3.9 + .1)\ u'$

$\Rightarrow u = 10$ m/s

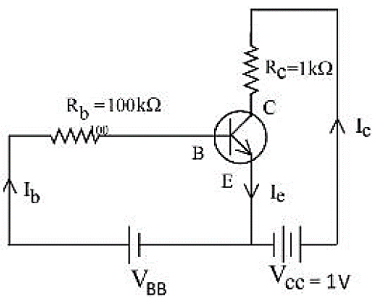
$\Delta KE = W_{\text{all FORCE}}$

$\because f = \mu mg$ (kinetic friction)

$\Rightarrow 0 - \frac{1}{2}(4)(10)^2 = -\mu(4)g \times 20$

$\Rightarrow \mu = 0.25$

46. For a given transistor amplifier circuit in CE configuration $V_{CC} = 1$ V, $R_C = 1$ kΩ, $R_b = 100$ kΩ and $\beta = 100$. Value of base current I_b is



(1) $I_b = 100\ \mu A$ (2) $I_b = 10\ \mu A$ (3) $I_b = 0.1\ \mu A$ (4) $I_b = 1.0\ \mu A$

Sol. **(2)**

$V_{cc} = 1$ V

$R_c I_c = 1$

$I_c = \dfrac{1}{10^3}$ A $= 1$ mA

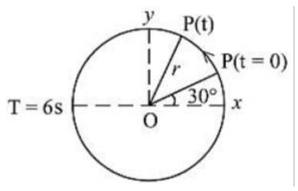
$\beta = \dfrac{I_c}{I_\beta}$

$I_\beta = \dfrac{I_C}{\beta}$

$= 1 \times 10^{-5}$ A

$= 10\ \mu A$

47. For particle P revolving round the centre O with radius of circular path r and angular velocity ω, as shown in below figure, the projection of OP on the x-axis at time t is

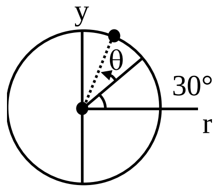


(1) $x(t) = r\cos\left(\omega t + \dfrac{\pi}{6}\right)$ (2) $x(t) = r\cos\left(\omega t - \dfrac{\pi}{6}\omega\right)$

(3) $x(t) = r\cos(\omega t)$ (4) $x(t) = r\sin\left(\omega t + \dfrac{\pi}{6}\right)$

Sol. **(1)**

$\theta = \omega t$



Angle from x axis $= \omega t + \dfrac{\pi}{6}$

Projection of OP on x axis $= r\cos\left(\omega t + \dfrac{\pi}{6}\right)$

48. A radio active material is reduced to 1/8 of its original amount in 3 days. If 8×10^{-3} kg of the material is left after 5 days the initial amount of the material is

(1) 64 g (2) 40 g (3) 32 g (4) 256 g

Sol. **(4)**

$m = m_0\, e^{-\lambda t}$

$\frac{m_0}{8} = m_0\, e^{-\lambda t}$

$-\ln 8 = -\lambda t$

$= \lambda = \frac{\ln 8}{3}$ per day

$m = m_0\, e^{-\lambda t}$

$8 = m_0\, e^{-\frac{\ln 8}{3}\times 5}$

$\Rightarrow 8 = m_0\, e^{-\frac{3\ln 2}{3}\times 5}$

$8 = m_0\, e^{\ln 2^{-5}}$

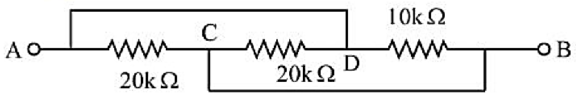
$= 8\ m_0 \left(\frac{1}{2^5}\right)$

$m_0 = 8 \times 2^5$

$= 8 \times 32$

$m_0 = 256$ gm

49. The equivalent resistance between A and B as shown in figure is:



(1) 20 kΩ (2) 30 kΩ (3) 5 kΩ (4) 10 kΩ

Sol. **(3)**

Potential different across all resistor is same

So they are in parallel

$\frac{1}{R} = \frac{1}{20} + + \frac{1}{20} + \frac{1}{10}$

$R_{eq} = 5k\Omega$

50. In photo electric effect

A. The photocurrent is proportional to the intensity of the incident radiation.

B. Maximum Kinetic energy with which photoelectrons are emitted depends on the intensity of incident light.

C. Max K.E with which photoelectrons are emitted depends on the frequency of incident light.

D. The emission of photoelectrons require a minimum threshold intensity of incident radiation.

E. Max. K.E of the photoelectrons is independent of the frequency of the incident light.

Choose the correct answer from the options given below:

(1) B and C only (2) A and C only (3) A and E only (4) A and B only

Sol. **(2)**

$h\nu = \phi + (KE)_{max}$

$(KE)_{max} = h\nu - \phi$

SECTION - B

51. A 600 pF capacitor is charged by 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. Electrostatic energy lost in the process is _________ μJ

Sol. **(6)**

loss of strength = $\frac{1}{2}\frac{c \times c}{c+c}(v_1 - v_2)^2$

$= \frac{1}{2} \times \left[\frac{600 \times 10^{-12}}{2}\right] \times (200)^2$

$= 600 \times 10^{-12} \times 10^4 = 6 \times 10^{-6} = 6\mu J$

52. A series combination of resistor of resistance 100 Ω, inductor of inductance 1 H and capacitor of capacitance 6.25 μF is connected to an ac source. The quality factor of the circuit will be __________

Sol. **(4)**

$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$= \frac{1}{100}\sqrt{\frac{1}{6.25 \times 10^{-6}}}$

$= 4$

53. The number density of free electrons in copper is nearly 8×10^{28} m^{-3}. A copper wire has its area of cross section $= 2 \times 10^{-6}$ m^2 and is carrying a current of 3.2 A. The drift speed of the electrons is ________ $\times 10^{-6}$ ms^{-1}

Sol. **(125)**

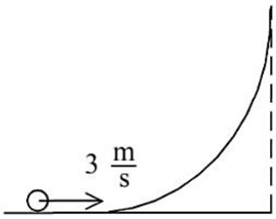
I = neAvd

$\Rightarrow 3.2 = 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2 \times 10^{-6} (v_d)$

$\Rightarrow v_d = \frac{1}{8 \times 10^{-6} \times 10^{9}}$

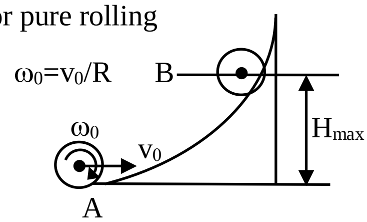
$\Rightarrow v_d = 125 \times 10^{-6}$ m/s

54. A hollow spherical ball of uniform density rolls up a curved surface with an initial velocity 3 m/s (as shown in figure). Maximum height with respect to the initial position covered by it will be _________ cm (take, $g = 10$ m/s^2)



Sol. **(75)**

For pure rolling



$(M.E)_A = (M.E)_B$

$$\Rightarrow \frac{1}{2}mv_0^2 + \frac{1}{2}\times\left(\frac{2}{3}mR^2\right)\left(\frac{v_0}{R}\right)^2 = mgH_{max}$$

$$\Rightarrow H_{max} = \frac{5}{6}\frac{v_0^2}{g} = \frac{5}{6}\times\frac{3^2}{10} = 0.75m$$

$$\Rightarrow H_{max} = 75cm$$

55. A steel rod of length 1 m and cross sectional area 10^{-4} m^2 is heated from 0°C to 200°C without being allowed to extend or bend. The compressive tension produced in the rod is __________ $\times 10^4$N. (Given Young's modulus of steel = 2×10^{11} Nm^{-2}, coefficient of linear expansion = 10^{-5}K^{-1})

Sol. **(4)**

Thermal stress = Y α ΔT

F = Y A α ΔT

$= 2 \times 10^{11} \times 10^{-4} \times 10^{-5} \times 200$

$= 4 \times 10^4$

x = 4

56. The ratio of magnetic field at the centre of a current carrying coil of radius r to the magnetic field at distance r from the centre of coil on its axis is $\sqrt{x}:1$. The value of x is ________

Sol. **(8)**

$$B_{axis} = \frac{\mu_0 i R^2}{2(R^2+x^2)^{3/2}}$$

$$\frac{(B_{axis})x=R}{(B_{axis})x=0} = \frac{\frac{\mu_0 i R^2}{2(R^2+R^2)^{3/2}}}{\frac{\mu_0 i R^2}{2(R^2)^{3/2}}} = \frac{R^3}{2^{3/2}R^3} = \frac{1}{\sqrt{8}}$$

$$\frac{(B)_{At_{centrs}}}{(B)_{At}\,x=R} = \frac{\sqrt{8}}{1}$$

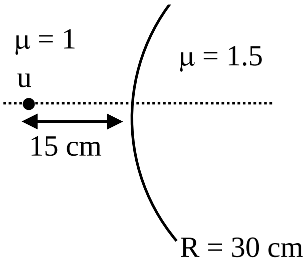
x = 8

57. The ratio of wavelength of spectral lines H_α and H_β in the Balmer series is $\frac{x}{20}$. The value of x is _______

Sol. **(27)**

58. Two transparent media having refractive indices 1.0 and 1.5 are separated by a spherical refracting surface of radius of curvature 30 cm. The centre of curvature of surface is towards denser medium and a point object is placed on the principle axis in rarer medium at a distance of 15 cm from the pole of the surface. The distance of image from the pole of the surface is __________ cm.

Sol. **(30)**



$$\frac{1.5}{v} - \frac{1}{(-15)} = \frac{1.5-1}{30}$$

$$\Rightarrow \frac{1.5}{v} - \frac{1}{60} - \frac{1}{15} = \frac{1-4}{60}$$

V = – 30 cm

= 30 cm

59. A guitar string of length 90 cm vibrates with a fundamental frequency of 120 Hz. The length of the string producing a fundamental frequency of 180 Hz will be ___________ cm.

Sol. **(60)**

$\mathbf{f} = \frac{v}{2L}$ (Fundamental Frequency)

$120 = \frac{v}{2L}$ …….(1)

$180 = \frac{v}{2L'}$ …….(2)

$$\frac{L'}{L} = \frac{120}{180}$$

$$L' = \frac{2}{3} \times 90$$

L' = 60 cm

60. A body of mass 5 kg is moving with a momentum of 10 kg ms^{-1}. Now a force of 2 N acts on the body in the direction of its motion for 5 s. The increase in the Kinetic energy of the body is __________ J.

Sol. **(30)**

$$\mathbf{(KE)} = \frac{P^2}{2M}$$

$$\Rightarrow \frac{1}{2}mu^2 = \frac{(10)^2}{2\times 5}$$

$$= \frac{1}{2} \times 5 \times u^2 = \frac{100}{10}$$

Initial speed u = 2 m/s

$\Delta KE = w_{\text{all forces}}$

$= \vec{F}.\vec{S} \quad (\theta = 0°)$

$$= F\left(ut + \frac{1}{2}at^2\right)$$

$$= \ 2.\left[2\times5+\frac{1}{2}\times\frac{2}{5}\times5^2\right]$$

$= 30$ J

SECTION - A

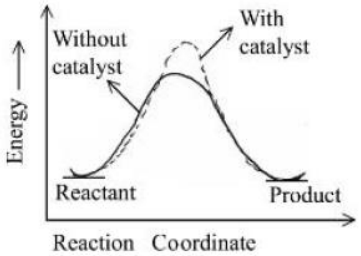
Topic: Chemistry in everyday life
Level: Med

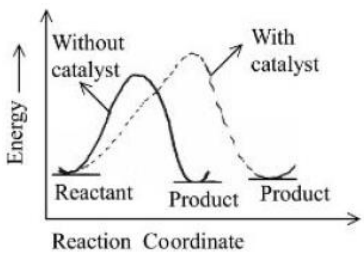
61. The statement/s which are true about antagonists from the following is/are :
A. They bind to the receptor site
B. Get transferred inside the cell for their action
C. Inhibit the natural communication of the body
D. Mimic the natural messenger.
Choose the correct answer from the options given below:
(1) A and B (2) A and C (3) A, C and D (4) B only

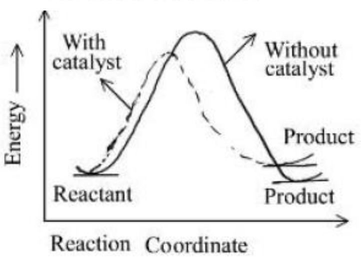
Sol. **2**
Antagonists bind to receptor site and inhibit the natural communication of both

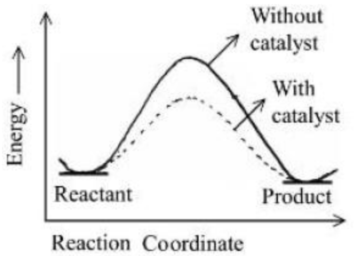
Topic: Chemical kinetics
Sub: collision theory
Level: Easy

62. The correct reaction profile diagram for a positive catalyst reaction.

(1)


(2)


(3)


(4)


Sol. **4**
Catalysts decrease activation energy only.

Topic :
Sub Topic :
Level :

63. Given below are two statements : One is labelled as **Assertion A** and other is labelled as **Reason R**
Assertion A : Sodium is about 30 times as abundant as potassium in the oceans.
Reason R : Potassium is bigger in size than sodium.
In the light of the above statements, choose the correct answer from the options given below
(1) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**
(2) **A** is true but **R** is false
(3) **A** is false but **R** is true
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **1**
Due to bigger size of potassium, it forms more efficient lattices as compared to sodium with silicates.
The abundance of sodium in ocean is more due to the more soluble nature of salt of sodium as compared to potassium salts.

Topic :
Sub Topic :
Level :

64. Which of these reactions is not a part of breakdown of ozone in stratosphere?

(1) $CF_2Cl_2(g) \xrightarrow{uv} \dot{C}l(g) + \dot{C}F_2Cl(g)$ (2) $\dot{C}l(g) + O_3(g) \rightarrow Cl\dot{O}(g) + O_2(g)$

(3) $2Cl\dot{O} \rightarrow ClO_2(g) + \dot{C}l(g)$ (4) $Cl\dot{O}(g) + O(g) \rightarrow \dot{C}l(g) + O_2(g)$

Sol. **3**

Ozone destruction

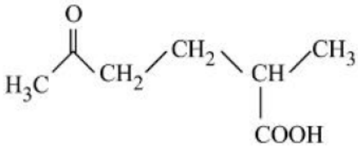
$CF_2Cl_2 \xrightarrow{hv} \dot{C}l + \dot{C}F_2Cl(g)$

$\dot{C}l + O_3 \rightarrow Cl\dot{O} + O_2$

$Cl\dot{O} + \dot{O} \rightarrow \dot{C}l + O_2$

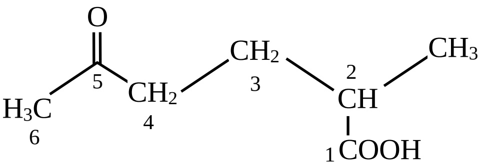
Topic: Nomenclature
Level: Easy

65. The correct IUPAC nomenclature for the following compound is :



(1) 2-Methyl-5-oxohexanoic acid (2) 2-Formyl-5-methylhexan-6-oic acid
(3) 5-Formyl-2-methylhexanoic acid (4) 5-Methyl-2-oxohexan-6-oic acid

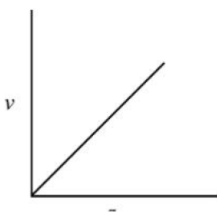
Sol. **1**

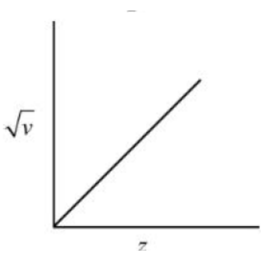


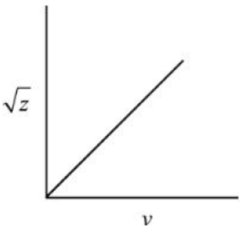
2-Methyl-5-oxohexanoic acid

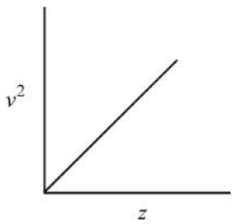
Topic :
Sub Topic :
Level :

66. Henry Moseley studied characteristic X-ray spectra of elements. The graph which represents his observation correctly is
Given ν = frequency of X-ray emitted
Z = atomic number

(1) 

(2) 

(3) 

(4) 

Sol. **2**

$\sqrt{v} \propto Z$

Topic :
Sub Topic :
Level :

67. Match list I with list II

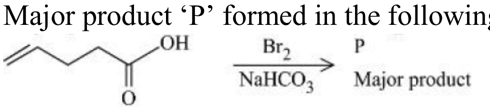
List I Coordination complex		**List II Number of unpaired electrons**	
A.	$[Cr(CN)_6]^{3-}$	I.	0
B.	$[Fe(H_2O)_6]^{2+}$	II.	3
C.	$[Co(NH_3)_6]^{3+}$	III.	2
D.	$[Ni(NH_3)_6]^{2+}$	IV.	4

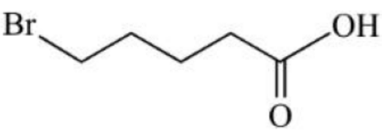
Choose the correct answer from the options given below:
(1) A-II, B-IV, C-I, D-III
(2) A-IV, B-III, C-II, D-I
(3) A-II, B-I, C-IV, D-III
(4) A-III, B-IV, C-I, D-II

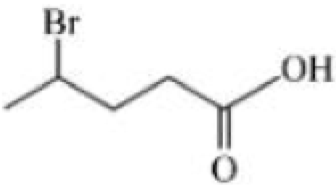
Sol. **1**
For option (A)
$Cr^{+3} : 3d^3$
$CN^- \rightarrow$ SFL
$\Rightarrow$ No. of unpaired electrons = 3
For option (B)
$Fe^{+2} : 3d^6$
H_2O : WFL
No. of unpaired electrons = 4
For option (C)
$Co^{+3} : 3d^6$
NH_3 : SFL
No. of unpaired electrons = 0
For option (D)
$Ni^{+2} : 3d^8$
NH_3 : SFL
No. of unpaired electrons = 2

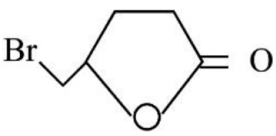
Topic: Hydrocarbon
Level: Med

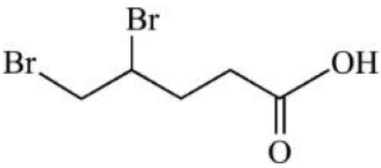
68. Major product 'P' formed in the following reaction is :



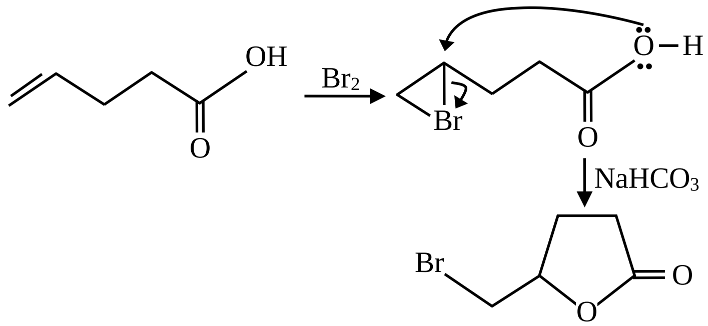
(1) 

(2) 

(3) 

(4) 

Sol. **3**



Topic :
Sub Topic :
Level :

69. For a good quality cement, the ratio of lime to the total of the oxides of Si, Al and Fe should be as close as to

(1) 2 (2) 1 (3) 3 (4) 4

Sol. **1**

$$\frac{\%CaO}{\%SiO_2 + \%Al_2O_3 + \%Fe_2O_3} = 1.9 - 2.1$$

Option (1) is correct.

Topic: Biomolecule
Level: Easy

70. Match list I with list II

List I Natural amino acid		List II One letter code	
A.	Glutamic acid	I.	Q
B.	Glutamine	II.	W
C.	Tyrosine	III.	E
D.	Tryptophan	IV.	Y

Choose the correct answer from the options given below:

(1) A-III, B-I, C-IV, D-II (2) A-IV, B-III, C-I, D-II
(3) A-II, B-I, C-IV, D-III (4) A-III, B-IV, C-I, D-II

Sol. **1**

A-III, B-I, C-IV, D-II
Fact

Topic: Mole concept
Sub: Significant figares
Level: F

71. Which of the following have same number of significant figures ?

A. 0.00253
B. 1. 0003
C. 15.0
D. 163

Choose the correct answer from the options given below

(1) B and C only (2) A, B and C only
(3) A, C and D only (4) C and D only

Sol. **3**

0.00253, 15.0, 163
All have three significant figures.

Topic: Qualitative
Sub:
Level:M

72. Given below are two statements :

Statement I : Methyl orange is a weak acid.

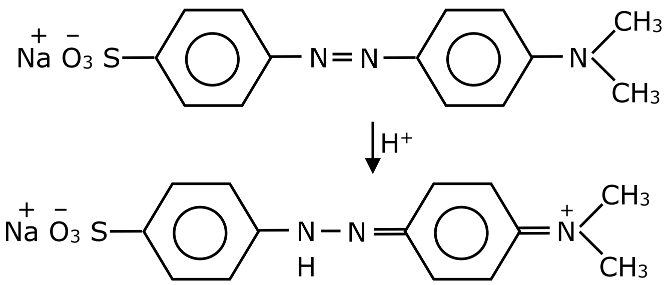
Statement II : The benzenoid form of methyl orange is more intense/deeply coloured than the quinonoid form.

In the light of the above statement, choose the most appropriate answer from the options given below:

(1) Both statement I and statement II are incorrect
(2) Both statement I and Statement II are correct
(3) Statement I is correct but statement II is incorrect
(4) Statement I is incorrect but statement II is correct

Sol. **1**

(i) Methy orange is weak base



(ii) Red color (quinonoid form)

So both statement are false

Topic: GOC
Level: Easy

73. The descending order of acidity for the following carboxylic acid is –

A. CH_3COOH B. $F_3C-COOH$ C. $ClCH_2-COOH$ D. $BrCH_2-COOH$

Choose the correct answer from the options given below:

(1) D > B > A > E > C (2) B > D > C > E > A
(3) E > D > B > A > C (4) B > C > D < E > A

Sol. **2**

Acidity α stability of conjugate base

Stability order

$F_3C-COO^- > F-CH_2-COO^- > Cl-CH_2-COO^- > Br-CH_2-COO^- > CH_3COO^-$

Topic :
Sub Topic :
Level :

74. In Hall-Heroult process, the following is used for reducing Al_2O_3 :-

(1) Magnesium (2) Graphite (3) Na_3AlF_6 (4) CaF_2

Sol. **2**

In case of Hall's process, reduction of Al_2O_3 to Al can be done using graphite.

Topicp: Real gas
Sub: Vanderwals costant
Level: M

75. Arrange the following gases in increasing order of van der waals constant ‘a’

A. Ar B. CH_4 C. H_2O D. C_6H_6

Choose the correct options from the following

(1) A, B, C and D (2) B, C, D and A (3) C, D, B and A (4) D, C, B and A

Sol. **1**

A α force of attraction vanderwaal force depends on molecular size and molecular mass and there is H-bonding in water, so correct option will be A < B < C < D.

Topic: Stoichiometry-II
Sub: titration
Level: M

76. Given below are two statement :

Statement I : In redox titration, the indicators used are sensitive to change in pH of the solution.

Statement II : In acid-base titration, the indicators used are sensitive to change in oxidation potential.

In the light of the above statement, choose the most appropriate answer from the options given below

(1) Both statement I and Statement II are incorrect
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are correct

Sol. **1**

Fact

Topic :
Sub Topic :
Level :

77. Which of the following can reduce decomposition of H_2O_2 on exposure to light

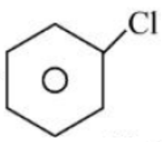
(1) Dust (2) Urea
(3) Glass containers (4) Alkali

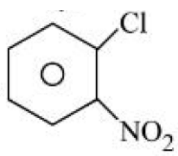
Sol. **2**

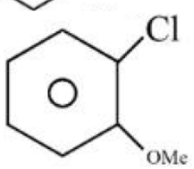
Urea acts as a stabilizer in the decomposition of H_2O_2

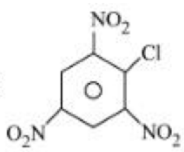
Topic: Alkyl Halide
Level: M

78. The correct order of reactivity of following haloarenes towards nucleophilic substitution with aqueous NaOH is

A. 

B. 

C. 

D. 

Choose the correct answer from the options given below:

(1) D > B > A > C (2) A > B > D > C
(3) C > A > D > B (4) D > C > B > A

Sol. 1

Rate $\alpha\, \text{EWG}\, \alpha \dfrac{1}{\text{EDG}}$

$NO_2 \rightarrow -\text{M effect}$

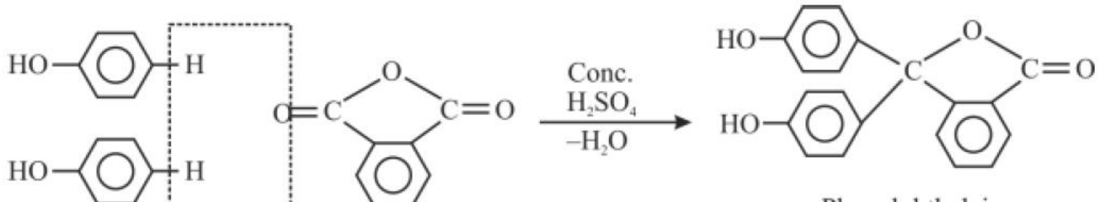
$\text{OMe} \rightarrow +\text{M effect}$

Topic :
Sub Topic :
Level :

79. A compound 'X' when treated with phthalic anhydride in presence of concentrated H_2SO_4 yields 'Y'. 'Y' is used as an acid/base indicator. 'X' and 'Y' are respectively:

(1) Anisole, methyl orange
(2) Toludine, Phenolphthalein
(3) Carbolic acid, Phenolphthalein
(4) Salicylaldehyde, Phenolphthalein

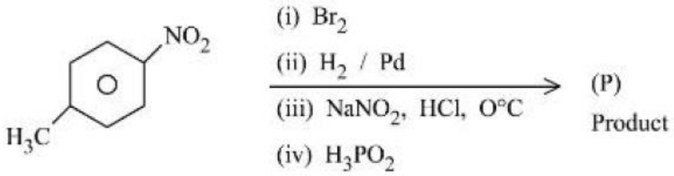
Sol. 3

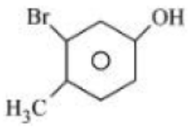


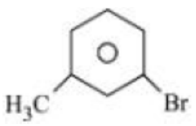
Phenolphthalein

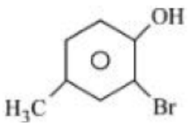
Topic: Nitrogen containg compound
Level: M

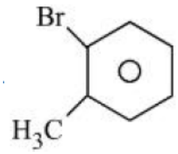
80. The product (P) formed from the following multistep reaction is :



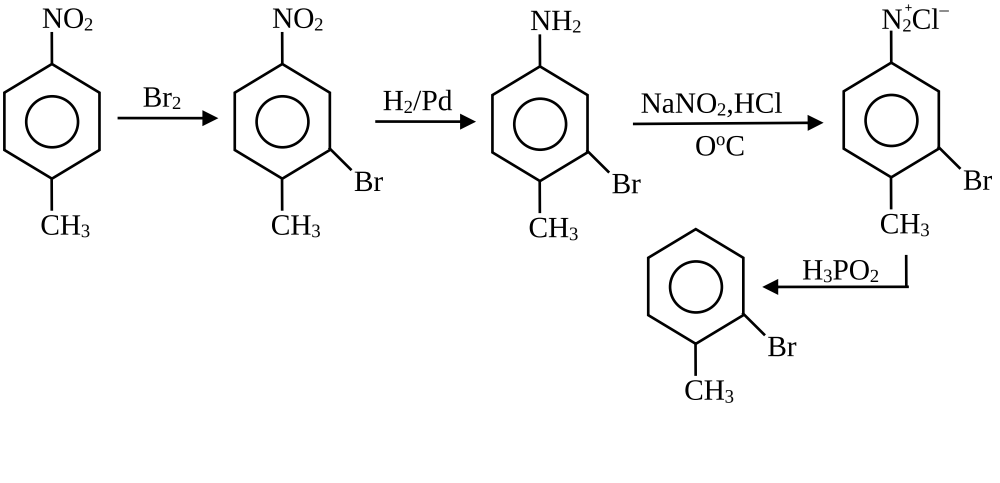
(1) 

(2) 

(3) 

(4) 

Sol. 4



SECTION - B

Topic :
Sub Topic :
Level :

81. The observed magnetic moment of the complex $[Mn(\underline{N}CS)_6]^{x-}$ is 6.06 BM. The numerical value of x is________

Sol. **4**

$[Mn(NCS)_6]^{x-}$

Number of unpaired electron = 5

So, Mn must be in +2 oxidation state (Mn^{+2})

$\Rightarrow 2 + (-6) = -x \qquad \Rightarrow -4 = -x \qquad \Rightarrow x = 4$

Topic: Thermochemistry
Sub: bomb calorimetery
Level: M

82. For complete combustion of ethane,

$C_2H_4(g)+3O_2(g) \rightarrow 2CO_2(g)+2H_2O(l)$

The amount of heat produced as measured in bomb calorimeter is 1406 KJ mol^{-1} at 300K. The minimum value of $T\Delta S$ needed to reach equilibrium is (–)__________KJ (Nearest integer)

Given : R = 8.3 $JK^{-1}mol^{-1}$

Sol. **1411**

$\Delta G = \Delta H - T\Delta S$ at equilibrium:-

$\Delta G = 0$

$T\Delta S = \Delta H = \Delta U + \Delta ngRT = -1406 + (-2)\times 8.3\times 300\times 10^{-3} = -1410.98 \approx 1411$

Topic: Ionic equilibrium
Sub: solubility produce
Level: Easy

83. The solubility product of $BaSO_4$ is 1×10^{-10} at 298 K. The solubility of $BaSO_4$ in 0.1 M K_2SO_4(aq) solut is________$\times10^{-9}$ g L^{-1} (Nearest integer)

Given: Molar mass of $BaSO_4$ is 233 g mol^{-1}

Sol. **233**

$K_{sp} = x(x+0.1) = 10^{-10}$

$0.1x = 10^{-10}$

$x = 10^{-9}M$

$x(\text{in g/l}) = 233\times 10^{-9}$

Topic: Atomic Structure
Sub: wave mechanical model
Level: Easy

84. The number of atomic orbitals from the following having 5 radial nodes is_________

7s, 7p, 6s, 8p, 8d

Sol. **3**

No. of radial node

$= n - \ell - 1$

For 6S $\rightarrow 6 - 0 - 1 = 5$,

7P $\rightarrow 7 - 1 - 1 = 5$

8d $\rightarrow 8 - 2 - 1 = 5$

Topic: Electrochemistry
Sub: Thermodynamics of cell
Level: T

85. The number of incorrect statement from the following is________

(1) The electrical work that a reaction can perform at constant pressure and temperature is equal to the reaction Gibbs energy

(2) E^o_{cell} is dependent on the pressure

(3) $\frac{dE^o_{cell}}{dT} = \frac{\Delta_r S^o}{nF}$

(4) A cell is operating reversibly if the cell potential is exactly balanced by an opposing source of potential difference

Sol. **1**

dG = vdp – sdT

dG = – sdT

$$\frac{dG}{dT} = -S \Rightarrow \frac{d\Delta G}{dT} = -\Delta S$$

$$\frac{dE^o}{dT} = \frac{-\Delta S}{-nF}$$

Topic: Surface chemi.
Sub: coagulation
Level:E

86. Coagulating value of the electrolytes $AlCl_3$ and NaCl for As_2S_3 are 0.09 and 50.04 respectively. The coagulating power of $AlCl_3$ is x times the coagulating power of NaCl. The value of x is______

Sol. **556**

Coagulating power $\propto \frac{1}{\text{coagulation value}}$

$$\frac{(CP)_{AlCl_3}}{(CP)_{NaCl}} = \frac{50.04}{0.09} = 556$$

Topic:Liquid solution
Sub: elvation ir boiling point
Level:E

87. If the boiling points of two solvents X and Y (having same molecular weights) are in the ratio 2 : 1 and their enthalpy of vaporizations are in the ratio 1 : 2, then the boiling point elevation constant of X is m times the boiling point elevation constant of Y. The value of m is________(nearest integer)

Sol. **8**

$$K_b = \frac{RT_b^2 m}{1000\,\Delta H_{vap}}$$

$$\frac{(K_b)_x}{(K_b)_y} = \frac{(T_b^2 M)_x}{(T_b^2 M)_y} \times \frac{(\Delta H)_y}{(\Delta H)_x} = \left(\frac{2}{1}\right)^2 \times \left(\frac{2}{1}\right) = \frac{8}{1}$$

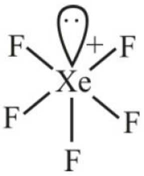
Topic :
Sub Topic :
Level :

88. The number of species from the following carrying a single lone pair on central atom Xenon is_______

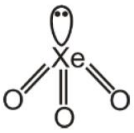
XeF_5^+, XeO_3, XeO_2F_2, XeF_5^-, XeO_3F_2,$XeOF_4$, XeF_4

Sol. **4**

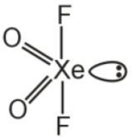
XeF_5^+



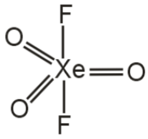
XeO_3



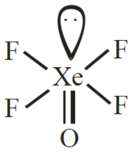
XeO_2F_2



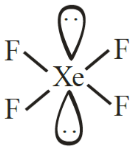
XeO_3F_2



$XeOF_4$



XeF_4

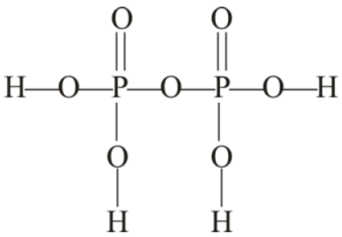


So, Answer is 4

Topic :
Sub Topic :
Level :

89. The ratio of sigma and π bonds present in pyrophosphoric acid is________

Sol. **6**



$$\frac{\sigma}{\pi} = \frac{12}{2} = 6$$

So, Answer is 6

Topic :
Sub Topic :
Level :

90. The sum of oxidation state of the metals in $Fe(CO)_5$, VO^{2+} and WO_3 is_______

Sol. **10**

$\overset{(0)}{Fe}(CO)_5$ $\quad \overset{(+4)}{V}O^{2+}$ $\quad \overset{(+6)}{W}O_3$

So, Sum of oxidation state = 0 + 4 + 6 = 10