Name	: _______________
Date of Exam.	: _______________
Duration	: **3 hours**
Max. Marks	: **80**
Study Centre	: _______________

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

- This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
- **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
- **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
- **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
- **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Math Sample Paper - 1

SECTION A

Q1.A pot contain 5 red and 2 green balls. At random a ball is drawn from this pot. If a drawn ball is green then put a red ball in the pot and if a drawn ball is red, then put a green ball in the pot, while drawn ball is not replace in the pot. Now we draw another ball randomly, the probability of second ball to be red is

(a) $\frac{27}{49}$
(b) $\frac{26}{49}$
(c) $\frac{3}{7}$
(d) $\frac{32}{49}$

Q2. The intergral $\int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^{x}\right\}\log_e x\, dx$ is

(a) $\frac{3}{2} - e - \frac{1}{2e^2}$
(b) $-\frac{1}{2} + \frac{1}{e} - \frac{1}{2e^2}$
(c) $\frac{1}{2} - e - \frac{1}{e^2}$
(d) $\frac{3}{2} - \frac{1}{e} - \frac{1}{2e^2}$

Q3. If A is a symmetric matrix and B is a skew-symmetric matrix such that, $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ then AB is equal to

(a) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

(b) $\begin{bmatrix} 4 & 2 \\ 1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -1 \\ -1 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 \\ 1 & 4 \end{bmatrix}$

Q4. The values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles is

(a)$3\sqrt{10}$

(b) $\frac{70}{11}$

(c)$\frac{10}{\sqrt{3}}$

(d) $\frac{20}{3}$

Q5. Range of the function $f(x) = \frac{x^2+x+2}{x^2+x+1}$; $x \in R$ is

(a)$(1, \infty)$

(b)$\left(1, \frac{11}{7}\right)$

(c)$\left(1, \frac{7}{3}\right]$

(d)$\left(1, \frac{7}{5}\right)$

Q6. The maximum value of the function $f(x) = x\sqrt{1-x}, x \leq 1$ is

(a) $\frac{2}{3\sqrt{3}}$

(b) $\frac{3}{2\sqrt{2}}$

(c) $\frac{4}{3\sqrt{2}}$

(d) $\frac{5}{3\sqrt{3}}$

Q7. If $\lim_{x\to 1} \frac{x^2 - ax + b}{x - 1} = 5$, then $a + b$ is equal to

(a)-4

(b)1

(c)-7

(d)5

Q8. If the function $f(x) = \begin{cases} a|\pi - x| + 1, x \leq 5 \\ b|x - \pi| + 3, x > 5 \end{cases}$ is continuous at x = 5, then the value of a - b is

(a)$\frac{-2}{\pi+5}$

(b) $\frac{2}{\pi+5}$

(c) $\frac{2}{\pi - 5}$

(d) $\frac{2}{5-\pi}$

Q9. If f is a differentiable function satisfying $f\left(\frac{1}{n}\right) = 0, \forall\, n \in I$, then

(a)$f(x) = 0, x \in (0,1]$

(b)$f'(0) = 0 = f(0)$

(c)$f(0) = 0$ but $f'(0)$ not necessarily zero

(d)$|f(x)| \leq 1, x \in (0,1]$

Q10. The integrating factor of the differential equation $x\frac{dy}{dx} - 2y = x^4$ is

(a) x^2

(b) $-x^2$

(c) $\frac{1}{x^2}$

(d) $-\frac{1}{x^2}$

Q11. If $f: R \rightarrow R$ defined by $f(x) = 3x - 5$, then $f(x)$ is
(a)is one – one but not onto
(b) onto but not one – one
(c) One – one and onto
(d) Neither one – one nor onto

Q12. If $\int \frac{x+1}{\sqrt{2x-1}}\, dx = f(x)\sqrt{2x-1} + C$,where C is a constant of intergration , then $f(x)$ is equal to
(a) $\frac{2}{3}(x+2)$
(b)$\frac{1}{3}(x+4)$
(c)$\frac{2}{3}(x-4)$
(d)$\frac{1}{3}(x+1)$

Q13. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then, $P(X=1) + P(X=2)$ equals.
(a)$\frac{25}{169}$
(b) $\frac{52}{169}$
(c) $\frac{49}{169}$
(d) $\frac{24}{169}$

Q14.The area of the quadrilateral formed by the tangents at the end points of latusrectum to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$, is
(a)$\frac{27}{4}$ sq. units
(b)9 sq. units
(c)$\frac{27}{2}$ sq. units
(d)27 sq. units

Q15. If $y = y(x)$ and $\frac{2+\sin x}{y+1} . \left(\frac{dy}{dx}\right) = -\cos x$, $y(0) = 1$ then $y\left(\frac{\pi}{2}\right)$ equals
(a)$\frac{1}{3}$
(b)$\frac{2}{3}$
(c)$-\frac{1}{3}$
(d)1

Q16. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a} + 2\,\vec{b}$ and $5\vec{a} - 4\vec{b}$, are perpendicular to each other, then the angle between $\vec{a}$ and $\vec{b}$ is
(a)45°
(b)60°
(c)$\cos^{-1}\left(\frac{1}{3}\right)$
(d) $\cos^{-1}\left(\frac{2}{7}\right)$

Q17.If the line , $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane , $lx + my - z = 9$, then $l^2 + m^2$ is equal to
(a)26

(b)18
(c)5
(d)2

Q18. If $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$, then $(A^2 + A)$ is equal to

(a) $\begin{bmatrix} 10 & 4 \\ 2 & 14 \end{bmatrix}$
(b) $\begin{bmatrix} 10 & -4 \\ -2 & 14 \end{bmatrix}$
(c) $\begin{bmatrix} 10 & 4 \\ -2 & 14 \end{bmatrix}$
(d) $\begin{bmatrix} 10 & -4 \\ 2 & 14 \end{bmatrix}$

Direction : (Q19 – 20) For the following question, choose the correct answer from the codes (a), (b), (c) and (d) defined as follows

Q19.Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2 - 1}dy - y\sqrt{y^2 - 1}\, dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$

Assertion(A): $y(x) = sec\left(sec^{-1}x - \frac{\pi}{6}\right)$ and

Reason(R) : $y(x)$ is given by $\frac{1}{y} = \frac{\sqrt{3}}{2x} - \sqrt{1 - \frac{1}{x^2}}$

(a) Assertion(A) is true, Reason(R) is true; Reason(R) is correct explanation of Assertion(A)
(b) Assertion(A) is true, Reason(R) is true, Reason(R) is not correct explanation of Assertion(A) .
(c) Assertion(A) is true, Reason(R) is false.
(d) Assertion(A) is false, Reason(R) is true.

Q20.**Assertion(A):** The value of the integral $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{tan\, x}}$ is equal to $\frac{\pi}{6}$

Reason(R) : $\int_a^b f(x)dx = \int_a^b f(a + b - x)dx$

(a) Assertion(A) is true, Reason(R) is true; Reason(R) is correct explanation of Assertion(A)
(b) Assertion(A) is true, Reason(R) is true, Reason(R) is not correct explanation of Assertion(A) .
(c) Assertion(A) is true, Reason(R) is false.
(d) Assertion(A) is false, Reason(R) is true.

SECTION B

Q21. If A and B are two independent events, prove that $P(A \cup B).P(A' \cap B') \le P(C)$ where C is an event defined that exactly one of A and B occurs.

Or

If matrix A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a , b , c are real positive numbers, $abc = 1$ and $A^T A = I$, then find the value of $a^3 + b^3 + c^3$.

Q22. Find the maximum number of equivalence relations on the set A = {1, 2, 3}.

Q23. Let $f(x) = \begin{cases} \frac{x^2}{2} , & 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2} , & 1 \le x \le 2 \end{cases}$. Discuss the continuity of f on [0, 2]

Or

If a and b are the order and degree of differential equation $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = K$ respectively, then find the value of a + 2b.

(a) $\frac{3}{2}$
(b) 2
(c) 3
(d) 6

Q24. Evaluate : $\int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}}$

Q25. Find the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$

SECTION C

Q26. The value of x for which $\sin[cot^{-1}(1+x)] = \cos(tan^{-1}x)$ is

Q27. Find the sum of the distinct real values of μ, for which the vectors, $\mu\hat{\imath} + \hat{\jmath} + \hat{k}$, $\hat{\imath} + \mu\hat{\jmath} + \hat{k}$, $\hat{\imath} + \hat{\jmath} + \mu\hat{k}$ are coplanar.

Or

If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect , then what is value of k.

Q28. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn one-by-one with replacement, then what is the variance of the number of green balls drawn .

Or

A is targeting to B, B and C are targeting to A. Probability of hitting the target by A, B and C are $\frac{2}{3}$, $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If A is hit, then find the probability that B hits the target and C does not hit.

Q29. Find the set of all values of λ for which the system of linear equations $2x_1 - 2x_2 + x_3 = \lambda x_1$, $2x_1 - 3x_2 + 2x_3 = \lambda x_2$ and $-x_1 + 2x_2 = \lambda x_3$ has a non – trivial solutions.

Or

Find the inverse of matrix A = $\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$ by the use of adjoint of matrix A.

Q30.Write the set of all points, where the function $f(x) = \frac{x}{1+|x|}$ is differentiable.

Q31. Solve the following linear programming problem (L.P.P) graphically:
Maximize Z = 3x + y
Subject to constraints; x + 2y ≥ 100 , 2x - y ≤ 0 , 2x + y ≤ 200 ,x ,y ≥ 0

SECTION D

Q32. Evaluate : $\int \frac{dx}{(x^2-2x+10)^2}$

Q33. Find the area (in sq units) of the region A = $\left\{(x,y) : \frac{y^2}{2} \le x \le y+4\right\}$ is

Or

Find the area bounded by the curve $y = -x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$.

Q34. Prove that function $f: N \to N$, defined by f$(x) = x^2 + x + 1$ is one – one but not onto. Find inverse of $f : N \to S$, where S is range of f.

Or

Show that $R = \{(a\,, b): a\,, b \in A\,; |a - b|\ is\ divisible\ by\ 4\}$ is an equivalence relation .Find the set of all elements related to 1.Also write the equivalence class [2].

Q35. Find the curve amongst the family of curves represented by the differential equation , $(x^2 - y^2)dx + 2xydy = 0$, which passes through (1 , 1) .

SECTION E

Q36. In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

Based on the above information, answer the following questions.

(i) If P represents the matrix of number of units of each type produced by factory A for both boys and girls, then P is given by

(ii) If Q represents the matrix of number of units of each type produced by factory B for both boys and girls, then Q is given by

(iii) The total- production of sports clothes of each type for boys is given by the matrix

Q37. It is known that, if the interest is compounded continuously, the principal changes at the rate equal to the product of the rate of bank interest per annum and the principal. Let P denotes the principal at any time t and rate of interest be r % per annum.

Based on the above information, answer the following questions.

(i) Find the value of $\frac{dP}{dt}$

(ii) If P_0 be the initial principal, then find the solution of differential equation formed in given situation

(iii) If the interest is compounded continuously at 5% per annum, in how many years will Rs.100 double itself?

Q38. In a play zone, Aastha is playing crane game. It has 12 blue balls, 8 red balls, 10 yellow balls and 5 green balls. If Aastha draws two balls one after the other without replacement, then answer the following questions.

(i)What is the probability that the first ball is blue and the second ball is green?
(ii) What is the probability that the first ball is yellow and the second ball is red?
(iii) What is the probability that both the balls are red?

SOLUTIONS

S1.Ans. (d)
Sol.
Let A be the event that ball drawn is green and B be the event that ball drawn is red.
$\therefore P(A) = \frac{2}{7}$ and $P(B) = \frac{5}{7}$
and Again, let C be the event that second ball drawn is red.
$\therefore P(C) = P(A).P\left(\frac{C}{A}\right) + P(B)P\left(\frac{C}{B}\right)$
$= \frac{2}{7} \times \frac{6}{7} + \frac{5}{7} \times \frac{4}{7}$
$= \frac{12+40}{49}$
$= \frac{32}{49}$
S2.Ans.(a)
Sol.
Let $I = \int_1^e \left\{\left(\frac{x}{e}\right)^{2x} - \left(\frac{e}{x}\right)^x\right\} \log_e x\, dx$
Now , put $\left(\frac{e}{x}\right)^x = t$
$\Rightarrow x \log_e \left(\frac{x}{e}\right) = \log t$
$= x(\log_e x - \log_e e) = \log t$
On differentiating w.r.t x
$= \left[x\left(\frac{1}{x}\right) + (\log_e x - \log_e e)\right] dx = \frac{1}{t} dt$
$= (1 + \log_e x - 1)dx = \frac{1}{t} dt$
$= (\log_e x)dx = \frac{1}{t} dt$
Also , upper limit $x = e$
$\Rightarrow t = 1$ and lower limit $x = 1 \Rightarrow t = \frac{1}{e}$
$\therefore I = \int_{\frac{1}{e}}^{1} \left(t^2 - \frac{1}{t}\right).\frac{1}{t} dt$
$\Rightarrow I = \int_{\frac{1}{e}}^{1} (t - t^2) dt$
$\Rightarrow I = \left[\frac{t^2}{2} + \frac{1}{t}\right]_{\frac{1}{e}}^{1}$
$\Rightarrow I = \left\{\left(\frac{1}{2} + 1\right) - \left(\frac{1}{2e^2} + e\right)\right\}$
$\Rightarrow I = \frac{3}{2} - e - \frac{1}{2e^2}$

S3.Ans.(a)
Sol.
Given matrix A is a symmetric and matrix B is a skew-symmetric.
$\therefore A = A^T$ and $B = -B^T$
Since , $A + B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$ ....(i)
$\Rightarrow (A + B)^T = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}^T$
$A^T + B^T = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$
$A - B = \begin{bmatrix} 2 & 5 \\ 3 & -1 \end{bmatrix}$ ......(ii)
From (i) and (ii)
$\Rightarrow A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

S4.Ans.(b)

Sol. The given equations can be written in the standard form as

$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{2} \text{ and } \frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

The direction ratios of the lines are $-3, \frac{2p}{7}$, 2 and $\frac{-3p}{7}, 1, -5$ respectively.

The direction ratios of the lines are $-3, \frac{2p}{7}$, 2 and $\frac{-3p}{7}, 1, -5$ respectively.

Two lines with direction ratios, a_1, b_1, c_1 and a_2, b_2, c_2, are perpendicular to each other, if $a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\therefore (-3) \cdot \left(\frac{-3p}{7}\right) + \left(\frac{2p}{7}\right) \cdot (1) + 2 \cdot (-5) = 0$$

$$\Rightarrow \frac{9p}{7} + \frac{2p}{7} = 10$$

$$\Rightarrow 11p = 70$$

$$\Rightarrow p = \frac{70}{11}$$

Thus, the value of p is $\frac{70}{11}$.

S5.Ans.(c)

Sol.

Let $y = f(x) = \frac{x^2+x+2}{x^2+x+1}$, $x \in \boldsymbol{R}$

$\therefore y = \frac{x^2+x+2}{x^2+x+1}$

$= y = 1 + \frac{1}{x^2+x+1}$ [i.e., y > 1] ......(i)

$\Rightarrow yx^2 + yx + y = x^2 + x + 2$

$= x^2(y-1) + x(y-1) + (y-2) = 0, \forall x \in \boldsymbol{R}$

Since , x is real , $D \geq 0$

$\Rightarrow (y-1)^2 - 4(y-1)(y-2) \geq 0$

$= (y-1)\{(y-1) - 4(y-2)\} \geq 0$

$= (y-1)(-3y+7) \geq 0$

$= 1 \leq y \leq \frac{7}{3}$ ....(ii)

From equation (i) and (iii) , Range $\in \left(1, \frac{7}{3}\right]$

S6.Ans.(a)

Sol. Given function is

$f(x) = x\sqrt{1-x}, x \leq 1$

$\therefore f'(x) = \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1)$

$= \frac{2(1-x)-x}{2\sqrt{1-x}}$

$= \frac{2-3x}{2\sqrt{1-x}}$

$f''(x) = \frac{2\sqrt{(1-x)}(-3) + \frac{2-3x}{\sqrt{1-x}}}{4(1-x)}$

For maxima and minima, $f'(x) = 0$

$\frac{2-3x}{2\sqrt{1-x}} = 0$

$x = 2/3$

Now $f''(2/3) < 0$

$x = 2/3$ is point of maxima

Hence local max value $= f(2/3) = \frac{2}{3\sqrt{3}}$

S7.Ans.(c)

Sol.

If $\lim_{x\to 1} \frac{x^2 - ax + b}{x - 1} = 5$ ………………. (i)

Since , limit exist and equal to 5 and denominator is zero at x = 1 , so numerator $x^2 - ax + b$ should be zero at x = 1

So , $1 - a + b = 0 \Rightarrow a = 1 + b$ ……..(ii)

On putting the value of 'a' from equation (ii) in equation (i)) ,we get

$\Rightarrow \lim_{x\to 1} \frac{x^2-(1+b)x+b}{x-1} = 5$

$\Rightarrow \lim_{x\to 1} \frac{(x^2-x)-b(x-1)}{x-1} = 5$

$\Rightarrow \lim_{x\to 1} \frac{(x-1)(x-b)}{x-1} = 5$

$\Rightarrow \lim_{x\to 1}(x - b) = 5$

$\Rightarrow 1 - b = 5$

$\Rightarrow b = -4$ …(iii)

On putting value 'b' from equation (iii) to equation (ii) , we get

$a = -3$

So, $a + b = -3 - 4 = -7$

S8. Ans. (d)

Sol.

Given function

$f(x) = \begin{cases} a|\pi - x| + 1, x \le 5 \\ b|x - \pi| + 3, x > 5 \end{cases}$

And it is also given that f(x) is continuous at x = 5

Clearly , $f(5) = a(5 - \pi) + 1$ ………(i)

$\lim_{x\to 5^-} f(x) = \lim_{h\to 0}[a|\pi - (5 - h)| + 1]$

$= a(\pi - 5) + 1$ …..(ii)

$\lim_{x\to 5^+} f(x) = \lim_{h\to 0}[b(|(5 + h) - \pi| + 3]$

$= \lim_{h\to 0}[b(|(5 + h) - \pi| + 3]$

$= b(5 - \pi) + 3$ ….(iii)

$\because$ Function f(x) is continuous at x = 5

$\therefore f(5) = L.H.L = R.H.L$

$\Rightarrow\ a(5-\pi)+1 = b(5-\pi)+3$
$= (a-b)(5-\pi) = 2$
$= a-b = \frac{2}{5-\pi}$
S9. Ans. (b)
Sol.
Given , $f(1) = f\left(\frac{1}{2}\right) = f\left(\frac{1}{3}\right) = \cdots = \lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0$
As $f\left(\frac{1}{n}\right) = 0$; $n \in$ Integers and $n \geq 1$.
$\Rightarrow \lim_{n\to\infty} f\left(\frac{1}{n}\right) = 0$
$\Rightarrow f(0) = 0$
Since , there are infinitely many points in neighbourhood of x = 0
$\therefore f(x) = 0$
$\Rightarrow f'(x) = 0$
$= f'(0) = 0$
Hence , $f(0) = f'(0) = 0$
S10.Ans.(c)
Sol. Given differential equation is
$x\frac{dy}{dx} - 2y = x^4$
$\frac{dy}{dx} - \frac{2}{x}y = x^3$
Integrating factor $= e^{\int -\frac{2}{x}dx} = e^{-2\log x} = \frac{1}{x^2}$
S11.Ans.(c)
Sol. Given function is
$f(x) = 3x - 5$
One – one:
Let $x, y \in R$ such that
$f(x) = f(y)$
$3x - 5 = 3y - 5$
$3x = 3y$
$x = y$
$\Rightarrow f(x)$ is one – one.
Onto: Let $y = 3x - 5$
$3x = y + 5$
$x = \frac{y+5}{3} \in R$
So, $f(x)$ is onto.

S12.Ans. (b)
Sol.
We have ,
$\int \frac{x+1}{\sqrt{2x-1}}\, dx = f(x)\sqrt{2x-1} + C$
Let $I = \int \frac{x+1}{\sqrt{2x-1}} dx$
Put $2x - 1 = t^2$
$\Rightarrow 2dx = 2tdt$
$= dx = dt$
$\Rightarrow I = \int \frac{\frac{t^2+1}{2}+1}{t} . tdt$
$\Rightarrow I = \frac{1}{2}\int (t^2+3)dt$
$\Rightarrow I = \frac{1}{2}\left[\frac{t^3}{3} + 3t\right] + C$
$\Rightarrow I = \frac{t}{6}(t^2 + 9\,) + C$

$\Rightarrow I = \frac{\sqrt{2x-1}}{6}(2x-1+9) + C \qquad [\because t = \sqrt{2x-1}]$

$= \frac{\sqrt{2x-1}}{6}(2x+8) + C$

$= \frac{x+4}{3}.\sqrt{2x-1} + C$

On comparing it with eq. (i), we get

$f(x) = \frac{x+4}{3}$

S13.Ans. (a)

Sol.

Let p = probability of getting an ace in a draw = probability of success

And q = probability of not getting an ace in a draw = probability of failure

Then , $p = \frac{4}{52} = \frac{1}{13}$ and $q = 1 - p = 1 - \frac{1}{13} = \frac{12}{13}$

Here , number of trials , $n = 2$

Clearly , follows binomial distribution with parameter $n = 2$ and $p = \frac{1}{13}$

Now , $P(X = x) = 2_{C_x}\left(\frac{1}{13}\right)^x\left(\frac{12}{13}\right)^{2-x} \quad , x = 0 , 1 , 2$

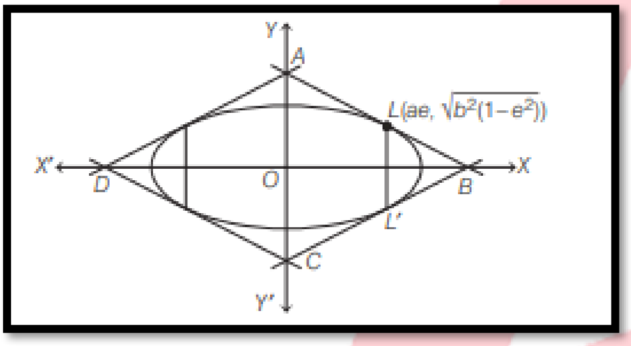
$\therefore P(X=1) + P(X=2) = 2_{C_1}\left(\frac{1}{13}\right)^1\left(\frac{12}{13}\right)^1 + 2_{C_2}\left(\frac{1}{13}\right)^2\left(\frac{12}{13}\right)^0$

$= 2\left(\frac{12}{169}\right) + \frac{1}{169}$

$= \frac{24}{169} + \frac{1}{169} = \frac{25}{169}$

S14.Ans. (d)

Sol.



Given , $\frac{x^2}{9} + \frac{y^2}{5} = 1$

To find tangents at the end points of latusrectum , we find ae .

i.e., $ae = \sqrt{a^2 - b^2} = \sqrt{4} = 2$

and $\sqrt{b^2(1-e^2)} = \sqrt{5\left(1 - \frac{4}{9}\right)} = \frac{5}{3}$

By symmetry , the quadrilateral is a Rhombus .

So , area is four times the area of the right angled triangle formed by the tangent and axes in then 1^{st} quadrant .

$\therefore$ Equation of tangent at $\left(2 , \frac{5}{3}\right)$ is

$\Rightarrow \frac{2}{9}x + \frac{5}{3}.\frac{y}{5} = 1$

$= \frac{x}{\frac{9}{2}} + \frac{y}{3} = 1$

$\therefore$ Area of quadrilateral ABCD = 4(area of Δ AOB)

= $4\left(\frac{1}{2}.\frac{9}{2}.3\right) = 27$ sq. units

S15.Ans.(a)

Sol.

Given , $\frac{dy}{dx} = \frac{-\cos x\,(y+1)}{2+\sin x}$

$\Rightarrow \frac{dy}{y+1} = \frac{-\cos x}{2+\sin x}\,dx$

$\int \frac{dy}{y+1} = \int \frac{-\cos x}{2+\sin x}\,dx$

$\log(y+1) = -\log(2+\sin x) + \log c$

$\log(y+1) + \log(2+\sin x) = \log c$

$\log\big((y+1)(2+\sin x)\big) = \log c$

$(y+1)(2+\sin x) = c$

When $x = 0\,, y = 1 \Rightarrow c = 4$

$\Rightarrow y + 1 = \frac{4}{2+\sin x}$

$\therefore y\left(\frac{\pi}{2}\right) = \frac{4}{3} - 1$

$y\left(\frac{\pi}{2}\right) = \frac{1}{3}$

S16.Ans.(b)

Sol.

Since , $(\vec{a} + 2\vec{b})(5\vec{a} - 4\vec{b}) = 0$

$= 5|\vec{a}|^2 + 6\vec{a}.\vec{b} - 8|\vec{b}|^2 = 0$

$= 6\vec{a}.\vec{b} - 3 = 0 \qquad [\because\ |\vec{a}| = |\vec{b}| = 1\ ]$

$= \cos\theta = \frac{1}{2}$

$= \theta = 60°$

S17.Ans.(d)

Sol.

Since , the line $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane $lx + my - z = 9$, therefore we have

$2l - m - 3 = 0$

$\because$ normal will be perpendicular to the line

$= 2l - m = 3$ ...(i)

And $3l - 2m + 4 = 9$

$\because$ point $(3\,, -2\,, -4)$ lies on the plane

$= 3l - 2m = 5$ ....(ii)

On solving equ. (i) and (ii) , we get

$\therefore l = 1$ and $m = -1$

$\therefore l^2 + m^2 = 2$

S18.Ans.(b)

Sol.

If $A = \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}$,

then $(A^2 + A)$ = $\begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix}^2 + \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 9 & -8 \\ -4 & 17 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} 10 & -4 \\ -2 & 14 \end{bmatrix}$

S19.Ans.(b)

Sol.

Given , $\frac{dy}{dx} = \frac{y\sqrt{y^2-1}}{x\sqrt{x^2-1}}$

$= \int \frac{dy}{y\sqrt{y^2-1}} = \int \frac{dx}{x\sqrt{x^2-1}}$

$= \sec^{-1} y = \sec^{-1} x + C$

At $x = 2\,, y = \frac{2}{\sqrt{3}}$

$= \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \sec^{-1}(2) + C$

$= \frac{\pi}{6} = \frac{\pi}{3} + C$

$= C = -\frac{\pi}{6}$

Now , $y = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$

$= \cos\left(\cos^{-1}\frac{1}{x} - \cos^{-1}\frac{\sqrt{3}}{2}\right)$

$= \cos\left(\cos^{-1}\left(\frac{\sqrt{3}}{2x} + \sqrt{1-\frac{1}{x^2}}\sqrt{1-\frac{3}{4}}\right)\right)$

$y = \frac{\sqrt{3}}{2x} + \frac{1}{2}\sqrt{1-\frac{1}{x^2}}$

Assertion(A) is true, Reason(R) is true, Reason(R) is not correct explanation of Assertion(A) .

20.Ans.(d)

Sol. Let $I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$ ....(i)

$\therefore \quad I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan\left(\frac{\pi}{2}-x\right)}}$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\cot x}}$

$\Rightarrow I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}\, dx$ ....(ii)

On adding Equation (i) and (ii) , we get

$\Rightarrow 2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} dx$

$= 2I = [x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= I = \frac{1}{2}\left[\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{\pi}{12}$

Assertion is false

But $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$ is a true statement by property of definite integrals.

SECTION B

(This section comprises of very short answer type questions (VSA) of 2 marks each)

S21.

Sol. Here , $P(A \cup B).P(A' \cap B')$

$\Rightarrow \{P(A) + P(B) - P(A \cap B)\}\{P(A').P(B')\}$

[Since A, B are independent , so A' , B' are independent]

$\therefore P(A \cup B).P(A' \cap B') \leq \{P(A) + P(B)\}.\{P(A').P(B')\}$

$= P(A).P(A').P(B') + P(B)P(A')A(B') \leq P(A).P(B') + P(B).P(A')$ .....(i)

$[\because \;\; P(A') \leq 1 \; and \; P(B') \leq 1]$

$= P(A \cup B).P(A' \cap B') \leq P(C)$

$[\because P(C) = P(A).P(B') + P(B)P(A')]$

Or

Given , A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$, where a , b , c are real positive numbers

$abc = 1 \; and \; A^T A = I$ ......(i)

Now , $A^T A = I$

$\Rightarrow \begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= a^2 + b^2 + c^2 = 1 \; and \; ab + bc + ca = 0$ ...(ii)

We know that ,

$\Rightarrow a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= a^3 + b^3 + c^3 = (a + b + c)(1 - 0) + 3$

Now , $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

$= 1$ ……(iv)

So from equation (iii)

$\Rightarrow a^3 + b^3 + c^3 = 1 + 3 = 4$

S22.

Sol. Given set

$\{1, 2, 3\}$

Equivalence relations on the set are

$\{(1, 1), (2, 2), (3, 3)\}$

$\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$

$\{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$

$\{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$

$\{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (1, 3), (3, 1), (2, 3), (3, 2)\}$

Maximum 5 equivalence relations.

S23.

Sol.

Given , $f(x) = \begin{cases} \frac{x^2}{2} \ , \ 0 \le x < 1 \\ 2x^2 - 3x + \frac{3}{2} \ , \ 1 \le x \ \le 2 \end{cases}$ ……(i)

Clearly , RHL (at x = 1)

$\lim_{x \to 1^+} (2x^2 - 3x + \frac{3}{2}) = \frac{1}{2}$

LHL (at x = 1)

$\lim_{x \to 1^-} \frac{x^2}{2} = \frac{1}{2}$

$f(1) = \frac{1}{2}$

Hence, LHL = RHL = f(1)

Hence, f is continuous at x = 1

Or

Given

$$\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = K$$

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = K\frac{d^2y}{dx^2}$$

Taking square of both sides,

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = K^2\left(\frac{d^2y}{dx^2}\right)^2$$

Here order$(a) = 2$ & degree $(b) = 2$

So, $a + 2b = 2 + 2 \times 2 = 6$

S24.

Sol. Let $I = \int \frac{dx}{x^2(x^4+1)^{\frac{3}{4}}} = \int \frac{dx}{x^2.x^3\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}}$

Put $1 + x^{-4} = t$

$\Rightarrow -\frac{4}{x^5}\ dx = dt$

$\therefore I = -\frac{1}{4}\int \frac{dt}{t^{\frac{3}{4}}} = -\frac{1}{4}.\frac{t^{\frac{1}{4}}}{\frac{1}{4}} + c$

$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}} + c$

$= -\frac{(x^4+1)^{\frac{1}{4}}}{x} + c$

S25.

Sol. Given differential equation can be rewritten as , $x\frac{dy}{dx} + 2y = x^2$ or $\frac{dy}{dx} + \frac{2}{x}.y = x$, which is a linear differential equation of the form $\frac{dy}{dx} + P(x)x = Q(x)$, where $P(x) = \frac{2}{x}$, $Q(x) = x$

Now , integrating factor

I.F. = $e^{\int \frac{2}{x} dx} = e^{2\log x} = e^{\log x^2} = x^2$

Solution is given by

$y \times x^2 = \int (x \times x^2) dx$

$yx^2 = \int x^3\, dx$

$yx^2 = \frac{x^4}{4} + C$

$y = \frac{x^2}{4} + \frac{C}{x^2}$

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

S26.

Sol. Given

$\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$ ....(i)

And we know that ,

$\cot^{-1}\theta = \sin^{-1}\left(\frac{1}{\sqrt{1+(\theta)^2}}\right)$ and $\tan^{-1}\theta = \cos^{-1}\left(\frac{1}{\sqrt{1+(\theta)^2}}\right)$

From Eq . (i) ,

$\Rightarrow \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$= \frac{1}{\sqrt{1+(1+x)^2}} = \frac{1}{\sqrt{1+x^2}}$

$= 1 + x^2 + 2x + 1 = x^2 + 1$

$= x = -\frac{1}{2}$

S27.

Sol. Given vectors , $\mu\hat{\imath} + \hat{\jmath} + \hat{k}$, $\hat{\imath} + \mu\hat{\jmath} + \hat{k}$, $\hat{\imath} + \hat{\jmath} + \mu\hat{k}$ will be coplanar , if

$\begin{vmatrix} \mu & 1 & 1 \\ 1 & \mu & 1 \\ 1 & 1 & \mu \end{vmatrix} = 0$

$\Rightarrow \mu(\mu^2-1) - 1\,(\mu-1) + 1(1-\mu) = 0$

$= (\mu-1)[\mu(\mu+1) - 1 - 1] = 0$

$= (\mu-1)[\mu^2+\mu-2] = 0$

$= (\mu-1)(\mu+2)(\mu-1) = 0$

$\mu = 1 \text{ or} -2$

So , sum of the distinct real values of $\mu = 1 - 2 = -1$

Or

Since , the lines intersect , therefore , they must have a point in common , i.e.

$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

And $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

$\Rightarrow x = 2\lambda + 1$, $y = 3\lambda - 1$, $z = 4\lambda + 1$

And $x = \mu + 3$, $y = 2\mu + k$, $z = \mu$ are same .

$\Rightarrow 2\lambda + 1 = \mu + 3$

$3\lambda - 1 = 2\mu + k$

$4\lambda + 1 = \mu$

On solving 1st and 3rd terms , we get ,

$\lambda = -\frac{3}{2}$ and $\mu = -5$

$\therefore k = 3\lambda - 2\mu - 1$

$\Rightarrow k = 3\left(-\frac{3}{2}\right) - 2(-5) - 1 = \frac{9}{2}$

S28.

Sol. Given box contains 15 green and 10 yellow balls .

∴ Total number of balls = 15 + 10 = 25

$P(green\ balls) = \frac{15}{25} = \frac{3}{5} = p =$ Probability of success

$P(yellow\ balls) = \frac{10}{25} = \frac{2}{5} = q =$ Probability of unsuccess

And $n = 10 =$ Number of trials .

∴ Variance = $npq = 10 \times \frac{3}{5} \times \frac{2}{5} = \frac{12}{5}$

Or

Given , $P(A) =$ probability that A will hit B = $\frac{2}{3}$

$P(C) =$ Probability that C will hit $A = \frac{1}{3}$

$P(E) =$ Probability that A will be hit

$\Rightarrow P(E) = 1 - P(\bar{B}).P(\bar{C}) = 1 - \frac{1}{2}.\frac{2}{3} = \frac{2}{3}$

Probability if A is hit by B and not by C = $P(B \cap \bar{C}/E) = \frac{P(B)P(\bar{C})}{P(E)} = \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{2}{3}} = \frac{1}{2}$

S29.

Sol. Given system of linear equations

$\Rightarrow 2x_1 - 2x_2 + x_3 = \lambda x_1$

$= (2-\lambda)x_1 - 2x_2 + x_3 = 0$ ....(i)

$\Rightarrow 2x_1 - 3x_2 + 2x_3 = \lambda x_2$

$= 2x_1 - (3+\lambda)x_2 + 2x_3 = 0$ ....(ii)

$\Rightarrow -x_1 + 2x_2 = \lambda x_3$

$= -x_1 + 2x_2 - \lambda x_3 = 0$ ....(iii)

Since , the system has non – trivial solutions

$\therefore \begin{bmatrix} 2-\lambda & -2 & 1 \\ 2 & -(3+\lambda) & 2 \\ -1 & 2 & -\lambda \end{bmatrix} = 0$

$\Rightarrow (2-\lambda)[\lambda(3+\lambda) - 4] + 2[-2\lambda + 2] + 1[4 - (3+\lambda)] = 0$

$= (2-\lambda)[3\lambda + (\lambda)^2 - 4] - 4\lambda + 4 + 4 - 3 - \lambda = 0$

$= (2-\lambda)[\lambda^2 + 3\lambda - 4] - 5\lambda + 5 = 0$

$= (2-\lambda)(\lambda^2 + 4\lambda - \lambda - 4] - 5(\lambda - 1) = 0$

$= (2-\lambda)\big(\lambda(\lambda+4) - (\lambda+4)\big) - 5(\lambda-1) = 0$

$= (2-\lambda)(\lambda-1)(\lambda+4) - 5(\lambda-1) = 0$

$= (\lambda-1)[(2-\lambda)(\lambda+4) - 5] = 0$

$= (\lambda-1)[2\lambda + 8 - \lambda^2 - 4\lambda - 5] = 0$

$= \lambda = 1, 1, -3$

Hence , λ contains two elements.

Or

Given : A = $\begin{bmatrix} 2 & 4 \\ -1 & 3 \end{bmatrix}$

As we know that

$\Rightarrow A^{-1} = \frac{adj(A)}{|A|}$

$= |A| = 6 + 4 = 10$

$= adj(A) = \begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow A^{-1} = \frac{adj(A)}{|A|}$

$= A^{-1} = \dfrac{\begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix}}{10}$

$= A^{-1} = \begin{bmatrix} \frac{3}{10} & -\frac{2}{5} \\ \frac{1}{10} & \frac{1}{5} \end{bmatrix}$

S30.

Sol. Given, $f(x) = \frac{x}{1 + |x|} = \begin{cases} \frac{x}{1+x} & , x \geq 0 \\ \frac{x}{1-x} & , x < 0 \end{cases}$

$\therefore f'(x) = \begin{cases} \frac{(1+x).1 - x.1}{(1+x)^2} & , \; x \geq 0 \\ \frac{(1-x).1 - x(-1)}{(1-x)^2} & , \; x > 0 \end{cases}$

$\Rightarrow f'(x) = \begin{cases} \frac{1}{(1+x)^2} & , x \geq 0 \\ \frac{1}{(1-x)^2} & , x < 0 \end{cases}$

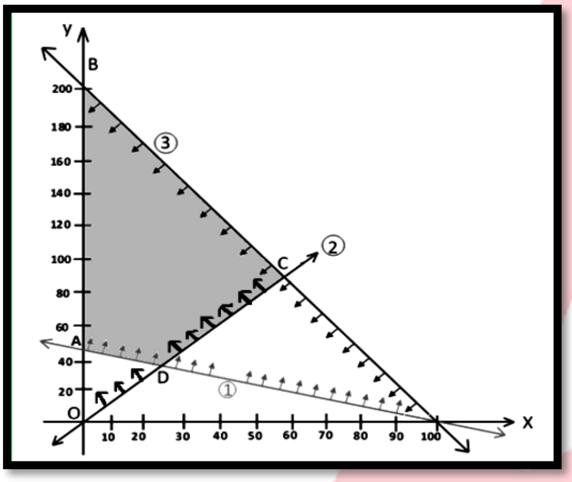
$\therefore$ RHD at x = 0 $\Rightarrow \lim\limits_{x \to 0} \frac{1}{(1+x)^2} = 1$

And LHD at x = 0 $\Rightarrow \lim\limits_{x \to 0} \frac{1}{(1-x)^2} = 1$

Hence , $f(x)$ is differentiable for all x.

S31.

Sol.



Max $Z = 3x + y$

Subject to $x + 2y \geq 100$ .....(i)

$2x - y \leq 0$ ....(ii)

$2x + y \leq 200$ ....(iii)

$x \geq 0 , y \geq 0$

Corner points	Z = 3x + y
A(0 , 50)	50
B(0 , 200)	200
C(50 , 100)	250
D(20 , 40)	100

Max Z = 250 at x = 50 , y = 100

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

S32.

Sol. Let $I = \int \frac{dx}{(x^2-2x+10)^2} = \int \frac{dx}{((x-1)^2+3^2)^2}$

Now , put $x - 1 = 3\tan\theta \Rightarrow dx = 3\sec^2\theta\, d\theta$

So , $I = \int \frac{3\sec^2\theta d\theta}{(3^2\tan^2\theta+3^2)^2} = \int \frac{3\sec^2\theta d\theta}{3^4\sec^4\theta d\theta}$

$I = \frac{1}{27}\int \cos^2\theta\, d\theta = \frac{1}{27}\int \frac{1+\cos 2\theta}{2} d\theta$

$\left[\because \cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$

$I = \frac{1}{54}\int(1+\cos 2\theta)d\theta = \frac{1}{54}\left(\theta + \frac{\sin 2\theta}{2}\right) + C$

$= \frac{1}{54}\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{108}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + C$

$\left[\because \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}\right]$

$= \frac{1}{54}\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{54}\frac{\left(\frac{x-1}{3}\right)}{1+\left(\frac{x-1}{3}\right)^2} + C$

$= \frac{1}{54}\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{18}\left(\frac{x-1}{(x-1)^2+3^2}\right) + C$

$= \frac{1}{54}\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{1}{18}\left(\frac{x-1}{x^2-2x+10}\right) + C$

It is given , that

$I = \frac{1}{54}\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{3(x-1)}{x^2-2x+10}\right] + C$

It is given that

$I = A\left[\tan^{-1}\left(\frac{x-1}{3}\right) + \frac{f(x)}{x^2-2x+10}\right] + C$

On comparing ,we get A = $\frac{1}{54}$ and $f(x) = 3(x-1)$

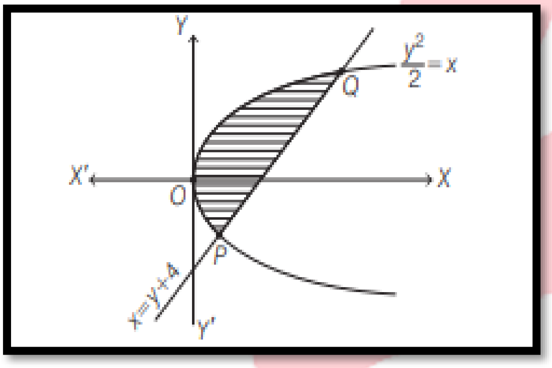
S33.

Sol. Given region A = $\left\{(x,y): \frac{y^2}{2} \le x \le y+4\right\}$

$\therefore \frac{y^2}{2} = x$ ....(i)

$\Rightarrow y^2 = 2x$ and $x = y + 4 \Rightarrow y = x - 4$ ......(ii)

Graphical representation of A



On substitution y = x – 4 from equation (ii) to equation (i) , we get

$\Rightarrow (x-4)^2 = 2x$

$= x^2 - 8x + 16 = 2x$

$= x^2 - 10x + 16 = 0$

$= (x-2)(x-8) = 0$

$= x = 2, 8$

$\therefore y = -2, 4$ [from eq. (ii)]

So, the area enclosed by the region A

$= \int_{-2}^{4}\left[(y+4) - \frac{y^2}{2}\right]dy = \left[\frac{y^2}{2} + 4y - \frac{y^3}{6}\right]_{-2}^{4}$

$= \left(\frac{16}{2} + 16 - \frac{64}{6}\right) - \left(\frac{4}{2} - 8 + \frac{8}{6}\right)$

$= 8 + 16 - \frac{32}{6} - 2 + 8 - \frac{4}{3}$
$= 30 - 12 = 18\ sq.unit$.
Or
Given $y = -x|x|, x = -1\ to\ x = 1$

$$y = \begin{cases} x^2, -1 \le x \le 0 \\ -x^2, 0 \le x \le 1 \end{cases}$$

Area of bounded region $= \int_{-1}^{0} x^2 dx + \int_{0}^{1} -x^2 dx$

$$\left| \left\{ \frac{x^3}{3} \right\}_{-1}^{0} \right| + \left| \left\{ -\frac{x^3}{3} \right\}_{0}^{1} \right| = \left| 0 + \frac{1}{3} \right| + \left| -\frac{1}{3} - 0 \right| = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

S34.
Sol. Let $x_1, x_2 \in N$.
Now , $f(x_1) = f(x_2)$
$\Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$
$= x_1^2 + x_1 = x_2^2 + x_2$
$= (x_1^2 - x_2^2) + (x_1 - x_2) = 0$
$= (x_1 - x_2)(x_1 + x_2 + 1) = 0$
$= x_1 - x_2 = 0 \quad [\because x_1 + x_2 + 1 \neq 0]$
$= x_1 = x_2$
Thus , f is one – one
Let $y \in N$, then for any x ,
$f(x) = y$ if $y = x^2 + x + 1$
$\Rightarrow y = \left(x^2 + x + \frac{1}{4}\right) + \frac{3}{4}$
$\Rightarrow y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$
$\Rightarrow x + \frac{1}{2} = \pm\sqrt{y - \frac{3}{4}}$
$\Rightarrow x = \pm\frac{\sqrt{4y-3}}{2} - \frac{1}{2}$
$\Rightarrow x = \frac{\pm\sqrt{4y-3}-1}{2}$
$\left[\frac{-\sqrt{4y-3}-1}{2} \notin N\ for\ any\ value\ of\ y\right]$
Now , for $y = \frac{3}{4}$, $x = -\frac{1}{2} \notin N$.
Thus , f is not onto.
Or
We have :
$R = \{(a, b) : a, b \in A ; |a - b| is\ divisible\ by\ 4\}$
(i)Reflexive : For any $a \in A$,
$\therefore (a, a) \in R$
$|a - a| = 0$, which is divisible by 4 .
Thu , R is reflexive .
(ii)Symmetric : Let $(a, b) \in R$
$\Rightarrow |a - b|$ is divisible by 4
$= |b - a|$ is divisible by 4
Thus , R is symmetric.
(iii)Transitive : Let (a , b) $\in R$ and $(b, c) \in R$
$\Rightarrow |a - b|$ is divisible by 4 and | b - c| is divisible by 4
$\Rightarrow |a - b| = 4\lambda$
$\Rightarrow a - b = \pm 4\lambda$ ....(i)
And $|b - c| = 4\mu$
$\Rightarrow b - c = \pm 4\mu$ ...(ii)
Adding (i) and (ii)

$\Rightarrow (a-b)+(b-c) = \pm 4(\lambda+\mu)$
$\Rightarrow (a-c) = \pm 4(\lambda+\mu)$
$\Rightarrow (a,c) \in R$
Thus , R is transitive
Now , R is reflexive , symmetric and transitive
Hence , R is an equivalence relation
S35 .
Sol. Given differential equation is
$(x^2-y^2)dx+2xydy=0$, which can be written as $\frac{dy}{dx}=\frac{y^2-x^2}{2xy}$
Put $y=vx$ [$\because$ it is in homogenous form]
$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$
Now , differential equation becomes
$\Rightarrow v+x\frac{dv}{dx}=\frac{v^2x^2-x^2}{2x(vx)}$
$\Rightarrow v+x\frac{dv}{dx}=\frac{(v^2-1)x^2}{2vx^2}$
$\Rightarrow x\frac{dv}{dx}=\frac{v^2-1}{2v}-v=\frac{v^2-1-2v^2}{2v}$
$\Rightarrow x\frac{dv}{dx}=-\frac{1+v^2}{2v}$
$\Rightarrow \int\frac{2v}{1+v^2}dx=-\int\frac{dx}{x}$
$\Rightarrow \ln(1+v^2)=-lnx-\ln C$
$\left[\because \int\frac{f\prime(x)}{f(x)}dx \Rightarrow \ln|f(x)|+C\right]$
$\Rightarrow \ln|(1+v^2)Cx|=0$ [$\because \ln A+\ln B=\ln AB$]
$\Rightarrow (1+v^2)Cx=1$ [$\because \log x=0 \Rightarrow e^0=1$]
Now , putting $v=\frac{y}{x}$, we get
$=\left(1+\frac{y^2}{x^2}\right)Cx=1$
$=C(x^2+y^2)=x$
$\because$ The curve passes through (1 , 1) , so
$C(1+1)=1$
$\Rightarrow C=\frac{1}{2}$
Thus , required curve is $x^2+y^2-2x=0$, which represent a circle having centre (1 , 0)
$\therefore$ The solution of given differential equation represents a circle with centre on the x – axis .

SECTION E

S36.
Sol. (i) In factory A, number of units of types I, II and III for boys are 80, 70, 65 respectively and for girls number of units of types I, II and III are 80, 75, 90 respectively.

$$\therefore P = \begin{matrix} & \text{Boys} & \text{Girls} \\ I \\ II \\ III \end{matrix}\begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix}$$

(ii) In factory B, number of units of types I, II and III for boys are 85, 65, 72 respectively and for girls number of units of types I, II and III are 50, 55, 80 respectively.

$$\therefore P = \begin{matrix} & \text{Boys} & \text{Girls} \\ I \\ II \\ III \end{matrix}\begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix}$$

(iii) Let X be the matrix that represent the number of units of each type produced by factory A for boys, and Y be the matrix that represent the number of units of each type produced by factory B for boys.
Then, $I \quad II \quad III$

$X = [170 \quad 130 \quad 130]$ and

$\quad\quad I \quad\quad II \quad\quad III$

$Y = [85 \quad 65 \quad 72]$

Now, required matrix = $X + Y = [80 \quad 70 \quad 65] + [85 \quad 65 \quad 72] = [165 \quad 135 \quad 137]$

S37.

Sol. (i) Here, P denotes the principal at any time t and the rate of interest be r% per annum compounded continuously, then according to the law given in the problem, we get $\frac{dP}{dt} = \frac{Pr}{100}$

(ii)We have

$\frac{dP}{dt} = \frac{Pr}{100}$

$\Rightarrow \frac{dP}{P} = \frac{r}{100} dt$

$= \int \frac{1}{P} dP = \frac{r}{100} \int dt$

$= \log P = \frac{rt}{100} + C$

At t = 0 , $P = P_0$

$\therefore C = \log P_0$

So , log P = $\frac{rt}{100} + C$

$= \log\left(\frac{P}{P_0}\right) = \frac{rt}{100}$

(iii) We have

r = 5, P_0 = Rs.100 and P = Rs. 200 = $2P_0$

Substituting these values in (2), we get

$\Rightarrow \log 2 = \frac{5}{100} t$

$= t = 20 \log_e 2 = 20 \times 0.6931$ years = 13 .862 years

S38.

Sol. Let B, R, Y and G denote the events that ball drawn is blue, red, yellow and green respectively.

$\therefore P(B) = \frac{12}{35}, P(R) = \frac{8}{35}, P(Y) = \frac{10}{35}$ and $P(G) = \frac{5}{35}$

(i)$P(G \cap B) = P(B).P(G|B) = \frac{12}{35}.\frac{5}{34} = \frac{6}{119}$

(ii)$P(R \cap Y) = P(Y).P(R|Y) = \frac{10}{35}.\frac{8}{34} = \frac{8}{119}$

(iii)Let E = event of drawing a first red ball and

F = event of drawing a second red ball

Here , P(E) = $\frac{8}{35}$ and $P(F|E) = \frac{7}{34}$

$\therefore P(F \cap E) = P(E).P(F|E) = \frac{8}{35}.\frac{7}{34} = \frac{4}{85}$