SECTION - A

1. $\int_0^{\infty} \frac{6}{e^{3x}+6e^{2x}+11e^{x}+6}dx$

(1) $\log_e\left(\frac{32}{27}\right)$ (2) $\log_e\left(\frac{256}{81}\right)$ (3) $\log_e\left(\frac{512}{81}\right)$ (4) $\log_e\left(\frac{64}{27}\right)$

Sol. **1**

$$I=\int_0^{\infty}\frac{6}{(e^x+1)(e^x+2)(e^x+3)}dx$$

$$=6\int_0^{\infty}\left(\frac{\frac{1}{2}}{e^x+1}+\frac{-1}{e^x+2}+\frac{\frac{1}{2}}{e^x+3}\right)dx$$

$$=3\int_0^{\infty}\frac{e^{-x}}{1+e^{-x}}dx-6\int_0^{\infty}\frac{e^{-x}dx}{1+2e^{-x}}+3\int_0^{\infty}\frac{e^{-x}}{1+3e^{-x}}dx$$

$$=3\left[-\ln\left(1+e^{-x}\right)\right]_0^{\infty}+6\frac{1}{2}\left[\ln\left(1+2e^{-x}\right)\right]_0^{\infty}$$

$$-\frac{3}{3}\left[\ln\left(1+3e^{-x}\right)\right]_0^{\infty}$$

$$=3\ln 2-3\ln 3+\ln 4$$

$$=3\ln\frac{2}{3}+\ln 4$$

$$=\ln\frac{32}{27}$$

2. Among

(S1) : $\lim_{n\to\infty}\frac{1}{n^2}(2+4+6+.....+2n)=1$

(S2) : $\lim_{n\to\infty}\frac{1}{n^{16}}\left(1^{15}+2^{15}+3^{15}+.....+n^{15}\right)=\frac{1}{16}$

(1) Only (S1) is true (2) Both (S1) and (S2) are true
(3) Both (S1) and (S2) are false (4) Only (S2) is true

Sol. **2**

$$S_1:\lim_{n\to\infty}\frac{n(n+1)}{n^2}=1\Rightarrow \text{True}$$

$$S_2:\lim_{n\to\infty}\frac{1}{n^{16}}\left(\sum r^{15}\right)=\lim_{n\to\infty}\frac{1}{n}\sum\left(\frac{r}{n}\right)^{15}$$

$$=\int_0^1 x^{15}dx=\frac{1}{16}\Rightarrow \text{True}$$

3. The number of symmetric matrices of order 3, with all the entries from the set {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}, is

(1) 10^9 (2) 10^6 (3) 9^{10} (4) 6^{10}

Sol. **2**

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, a, b, c, d, e, f \in \{0, 1, 2, \ldots .9\}$$,Number of matrices = 10^6

4. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. If a vector $\vec{d}$ satisfies $\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{d}.\vec{a} = 24$, then $|\vec{d}|^2$ is equal to -

(1) 323 (2) 423 (3) 413 (4) 313

Sol. **3**

$\vec{d} \times \vec{b} = \vec{c} \times \vec{b}$

$\Rightarrow (\vec{d} - \vec{c}) \times \vec{b} = 0$

$\Rightarrow \vec{d} = \vec{c} + \lambda\vec{b}$

Also $\vec{d}\vec{a} = 24$

$\Rightarrow (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 24$

$$\lambda = \frac{24 - \vec{a} \cdot \vec{c}}{\vec{b} \cdot \vec{a}} = \frac{24 - 6}{9} = 2$$

$\Rightarrow \vec{d} = \vec{c} + 2(\vec{b})$

$= 8\hat{i} - 5\hat{j} + 18\hat{k}$

$\Rightarrow |\vec{d}|^2 = 64 + 25 + 324 = 413$

5. A coin is biased so that the head is 3 times as likely to occur as tail. This coin is tossed until a head or three tails occur. If X denotes the number of tosses of the coin, then the mean of X is-

(1) $\frac{21}{16}$ (2) $\frac{15}{16}$ (3) $\frac{81}{64}$ (4) $\frac{37}{16}$

Sol. **1**

P(H) = $\frac{3}{4}$

P(T) = $\frac{1}{4}$

X	1	2	3
P(X)	$\frac{3}{4}$	$\frac{1}{4} \times \frac{3}{4}$	$\left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^2 \times \frac{3}{4}$

$$\text{Mean}\bar{X} = \frac{3}{4} + \frac{3}{8} + 3\left(\frac{1}{64} + \frac{3}{64}\right)$$

$$= \frac{3}{4} + \frac{3}{8} + \frac{3}{16}$$

$$= 3\left(\frac{7}{16}\right) = \frac{21}{16}$$

6. $\max_{0\le x\le \pi}\left\{x-2\sin x\cos x+\frac{1}{3}\sin 3x\right\}=$

(1) 0 (2) π (3) $\frac{5\pi+2+3\sqrt{3}}{6}$ (4) $\frac{\pi+2-3\sqrt{3}}{6}$

Sol. **3**

$f(x)=x-\sin 2x+\frac{1}{3}\sin 3x$

$f'(x)=1-2\cos 2x+\cos 3x=0$

$x=\frac{5\pi}{6},\frac{\pi}{6}$

$\therefore f''(x)=4\sin 2x-3\sin 3x$

$f''\left(\frac{5\pi}{6}\right)<0$

$\Rightarrow\left(\frac{5\pi}{6}\right)$ is point of maxima

$f\left(\frac{5\pi}{6}\right)=\frac{5\pi}{6}+\frac{\sqrt{3}}{2}+\frac{1}{3}$

7. The set of all a $\in$ R for which the equation x |x – 1| + |x + 2| + a = 0 has exactly one real root, is

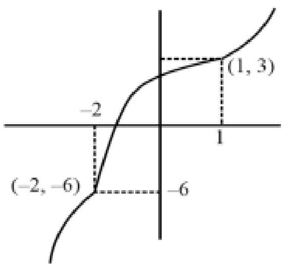
(1) $(-\infty,-3)$ (2) $(-\infty,\infty)$ (3) $(-6,\infty)$ (4) $(-6,-3)$

Sol. **2**

$f(x)=x|x-1|+|x+2|$

$x|x-1|+|x+2|+a=0$

$x|x-1|+|x+2|=-a$



All values are increasing.

8. Let PQ be a focal chord of the parabola $y^2 = 36x$ of length 100, making an acute angle with the positive x-axis. Let the ordinate of P be positive and M be the point on the line segment PQ such that PM:MQ = 3:1. Then which of the following points does <u>NOT</u> lie on the line passing through M and perpendicular to the line PQ?

(1) (3, 33) (2) (6, 29) (3) (– 6, 45) (4) (– 3, 43)

Sol. **4**

$9\left(t+\frac{1}{t}\right)^2=100$

$t=3$

$\Rightarrow P(81,54)\ \&\ Q(1,-6)$

$M(21,9)$

$$\Rightarrow \text{L is } (y-9)=\frac{-4}{3}(x-21)$$

$$3y-27=-4x+84$$

$$4x+3y=111$$

9. For the system of linear equations

2x + 4y + 2az = b

x + 2y + 3z = 4

2x – 5y + 2z = 8

which of the following is NOT correct?

(1) It has infinitely many solutions if a = 3, b = 8

(2) It has unique solution if a = b=8

(3) It has unique solution if a = b = 6

(4) It has infinitely many solutions if a = 3, b=6

Sol. **4**

$$\Delta=\begin{vmatrix}2 & 4 & 2a\\ 1 & 2 & 3\\ 2 & -5 & 2\end{vmatrix}=18(3-a)$$

$$\Delta_x=\begin{vmatrix}b & 4 & 2a\\ 4 & 2 & 3\\ 8 & -5 & 2\end{vmatrix}=(64+19b-72a)$$

For unique solution $\Delta = 0$

$$\Rightarrow a \neq 3 \text{ and } b \in R$$

For infinitely many solution :

$$\Delta=\Delta_x=\Delta_y=\Delta_z=0$$

$$\Rightarrow a=3 \quad \because \Delta=0$$

$$\text{and } b=8 \quad \because \Delta_x=0$$

10. Let s_1, s_2, s_3, ..., s_{10} respectively be the sum to 12 terms of 10 A.P. s whose first terms are 1, 2, 3, ..., 10 and the common differences are 1, 3, 5, ..., 19 respectively. Then $\sum_{i=1}^{10} s_i$ is equal to

(1) 7260 (2) 7380 (3) 7220 (4) 7360

Sol. **1**

$$S_k=6(2k+(11)(2k-1))$$

$$S_k=6(2k+22k-11)$$

$$S_k=144k-66$$

$$\sum_{1}^{10} S_k=144\sum_{k=1}^{10} k-66\times 10$$

$$=144\times\frac{10\times 11}{2}-660$$

$$=7920-660$$

$$=7260$$

11. For the differentiable function f: R - {0} $\rightarrow$R, let 3f(x)+2f$\left(\frac{1}{x}\right)=\frac{1}{x}-10$, then $\left|f(3)+f'\left(\frac{1}{4}\right)\right|$ is equal to

(1) 13 (2) $\frac{29}{5}$ (3) $\frac{33}{5}$ (4) 7

Sol. **1**

$$\left[3f(x)+2f\left(\frac{1}{x}\right)=\frac{1}{x}-10\right]\times 3$$

$$\left[2f(x)+3f\left(\frac{1}{x}\right)=x-10\right]\times 2$$

$$5f(x)=\frac{3}{x}-2x-10$$

$$f(x)=\frac{1}{5}\left(\frac{3}{x}-2x-10\right)$$

$$f'(x)=\frac{1}{5}\left(-\frac{3}{x^2}-2\right)$$

$$\left|f(3)+f'\left(\frac{1}{4}\right)\right|=\left|\frac{1}{5}(1-6-10)+\frac{1}{5}(-48-2)\right|$$

$$=|-3-10|=13$$

12. The negation of the statement $((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$ is

(1) equivalent to $B\vee\sim C$ (2) a fallacy
(3) equivalent to $\sim C$ (4) equivalent to $\sim A$

Sol. **4**

$$p:((A\wedge(B\vee C))\Rightarrow(A\vee B))\Rightarrow A$$

$$[\sim(A\wedge(B\vee C))\vee(A\vee B)]\Rightarrow A$$

$$[(A\wedge(B\vee C))\wedge\sim(A\vee B)]\vee A$$

$$(f\vee A)=A$$

$$\sim p\equiv\sim A$$

13. Let the tangent and normal at the point $(3\sqrt{3},1)$ on the ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$ meet the y-axis at the points A and B respectively. Let the circle C be drawn taking AB as a diameter and the line x = $2\sqrt{5}$ intersect C at the points P and Q. If the tangents at the points P and Q on the circle intersect at the point (α, β), then $\alpha^2-\beta^2$ is equal to

(1) $\frac{304}{5}$ (2) 60 (3) $\frac{314}{5}$ (4) 61

Sol. **1**

Given ellipse $\frac{x^2}{36}+\frac{y^2}{4}=1$

$$\frac{x}{4\sqrt{3}}+\frac{y}{4}=1$$

y = 4

$\frac{x}{4}-\frac{4}{4\sqrt{3}}=\frac{2}{\sqrt{3}}$

$y = -8$

$x^2+y^2+4y-32=0$

$hx+ky+2(y+k)-32=0$

$k=-2$

$hx+2k-32=0$

$hx=36$

$\alpha = h = \frac{36}{2\sqrt{5}}$

$\beta = k = -2$

$\alpha^2-\beta^2 = \frac{304}{5}$

14. The distance of the point (-1, 2, 3) from the plane $\vec{r}.\left(\hat{i}-2\hat{j}+3\hat{k}\right)=10$ parallel to the line of the shortest distance between the lines $\vec{r}=\left(\hat{i}-\hat{j}\right)+\lambda\left(2\hat{i}+\hat{k}\right)$ and $\vec{r}=\left(2\hat{i}-\hat{j}\right)+\mu\left(\hat{i}-\hat{j}+\hat{k}\right)$ is

(1) $2\sqrt{5}$ (2) $3\sqrt{5}$ (3) $3\sqrt{6}$ (4) $2\sqrt{6}$

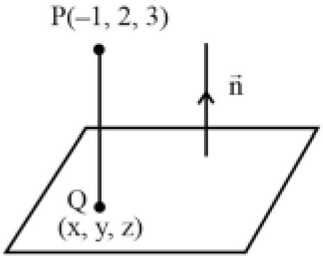
Sol. **4**

Let $L_1:\vec{r}=(\hat{i}-\hat{j})+\lambda(2\hat{i}+\hat{k})$

$L_2:\vec{r}=(2\hat{i}-\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})$

$$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 0 & 1\\ 1 & -1 & 1\end{vmatrix}$$

$\vec{n}=\hat{i}-\hat{j}-2\hat{k}$



Equation of line along shortest distance of L_1 and L_2

$\frac{x+1}{1}=\frac{y-2}{-1}=\frac{z-3}{-2}=r$

$\Rightarrow (x,y,z)\equiv(r-1,2-r,3-2r)$

$\Rightarrow (r-1)-2(2-r)+3(3-2r)=10$

$\Rightarrow r=-2$

$\Rightarrow Q(x,y,z)\equiv(-3,4,7)$

$\Rightarrow PQ=\sqrt{4+4+16}=2\sqrt{6}$

15. Let $B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$, α >2 be the adjoint of a matrix A and |A|=2. then

$\begin{bmatrix}\alpha & -2\alpha & \alpha\end{bmatrix}B\begin{bmatrix}\alpha\\ -2\alpha\\ \alpha\end{bmatrix}$ is equal to

(1) 16 (2) 32 (3) 0 (4) –16

Sol. **4**

$$\text{Given}, B=\begin{bmatrix}1 & 3 & \alpha\\ 1 & 2 & 3\\ \alpha & \alpha & 4\end{bmatrix}$$

$$|B|=4$$

$$1(8-3\alpha)-3(4-3\alpha)+\alpha(\alpha-2\alpha)=4$$

$$-\alpha^2+6\alpha-8=0$$

$$\alpha=2,4$$

Given $\alpha > 2$

So, $\alpha = 2$ is rejected

$$\begin{bmatrix}4 & -8 & 4\end{bmatrix}\begin{bmatrix}1 & 3 & 4\\ 1 & 2 & 3\\ 4 & 4 & 4\end{bmatrix}\begin{bmatrix}4\\ -8\\ 4\end{bmatrix}=[-16]_{1\times 1}$$

16. For x $\in$R, two real valued functions f(x) and g(x) are such that, g(x)= $\sqrt{x}$ +1 and fog(x)=x+3 - $\sqrt{x}$. Then f(0) is equal to

(1) 5 (2) 0 (3) –3 (4) 1

Sol. **1**

$$g(x)=\sqrt{x}+1$$

$$fog(x)=x+3-\sqrt{x}$$

$$=(\sqrt{x}+1)^2-3(\sqrt{x}+1)+5$$

$$=g^2(x)-3g(x)+5$$

$$\Rightarrow f(x)=x^2-3x+5$$

$$\therefore f(0)=5$$

But, if we consider the domain of the composite function fog(x) then in that case f(0) will be not defined as g(x) cannot be equal to zero.

17. Let the equation of plane passing through the line of intersection of the planes x+2y+az=2 and x-y+z=3 be 5x – 11y + bz = 6a – 1. For c $\in$ Z, if the distance of this plane from the point (a, –c, c) is $\frac{2}{\sqrt{a}}$, then $\frac{a+b}{c}$ is equal to

(1) – 4 (2) 2 (3) – 2 (4) 4

Sol. **1**

$$(x+2y+az-2)+\lambda(x-y+z-3)=0$$

$$\frac{1+\lambda}{5}=\frac{2-\lambda}{-11}=\frac{a+\lambda}{b}=\frac{2+3\lambda}{6a-1}$$

$$\lambda=-\frac{7}{2}, a=3, b=1$$

$$\frac{2}{\sqrt{a}}=\left|\frac{5a+11c+bc-6a+1}{\sqrt{25+121+1}}\right|$$

$$c=-1$$

$$\therefore \frac{a+b}{c}=\frac{3+1}{-1}=-4$$

18. Fractional part of the number is $\frac{4^{2022}}{15}$ equal to

(1) $\frac{4}{15}$ (2) $\frac{8}{15}$ (3) $\frac{1}{15}$ (4) $\frac{14}{15}$

Sol. **3**

$$\left\{\frac{4^{2022}}{15}\right\}=\left\{\frac{2^{4044}}{15}\right\}=\left\{\frac{(1+15)^{1011}}{15}\right\}=\frac{1}{15}$$

19. Let y = y_1 (x) and y = y_2(x) be the solution curves of the differential equation $\frac{dy}{dx}$ = y+7 with initial conditions y_1(0) = 0 and y_2(0)=1 respectively. Then the curves y = y_1(x) and y = y_2(x) intersect at

(1) no point (2) infinite number of points

(3) one point (4) two points

Sol. **1**

$$\frac{dy}{dx}=y+7 \Rightarrow \frac{dy}{dx}-y=7$$

$$\text{I.F.}=e^{-x}$$

$$ye^{-x}=\int 7e^{-x}dx$$

$$\Rightarrow ye^{-x}=-7e^{-x}+c$$

$$\Rightarrow y=-7+ce^{x}$$

$$-7+7e^{x}=-7+8e^{x} \Rightarrow e^{x}=0.$$

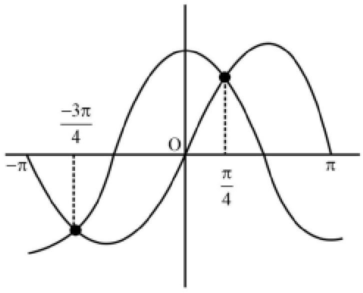
No solution

20. The area of the region enclosed by the curve f(x)= max{sin x, cosx}, $-\pi \le x \le \pi$ and the x-axis is

(1) $2\sqrt{2}\left(\sqrt{2}+1\right)$ (2) $4\left(\sqrt{2}\right)$

(3) 4 (4) $2\left(\sqrt{2}+1\right)$

Sol. **3**



Area =

$$\left|\int_{-\pi}^{\frac{-3\pi}{4}} \sin x dx\right| + \left|\int_{\frac{-3\pi}{4}}^{\frac{-\pi}{2}} \cos x dx\right| + \int_{\frac{-\pi}{2}}^{\frac{\pi}{4}} \cos x dx + \int_{\frac{\pi}{4}}^{\pi} \sin x dx = 4$$

SECTION - B

21. The sum to 20 terms of the series $2.2^2 - 3^2 + 2.4^2 - 5^2 + 2.6^2 -$ .... is equal to _______.

Sol. **1310**

$$\left(2^2 - 3^2 + 4^2 - 5^2 + 20\text{terms}\right) + \left(2^2 + 4^2 + \ldots + 10\text{terms}\right)$$

$$-(2+3+4+5+\ldots..+11) + 4\left[1+2^2+\ldots..10^2\right]$$

$$-\left[\frac{21\times 22}{2} - 1\right] + 4\times\frac{10\times 11\times 21}{6}$$

$$= 1 - 231 + 14\times 11\times 10$$

$$= 1540 + 1 - 231$$

$$= 1310$$

22. Let the mean of the data

x	1	3	5	7	9
Frequency (f)	4	24	28	α	8

be 5. If m and σ^2 are respectively the mean deviation about the mean and the variance of the data, then $\frac{3\alpha}{m+\sigma^2}$ is equal to ________.

Sol. **8**

$$5 = \bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{4+72+140+7\alpha+72}{64+\alpha}$$

$$\Rightarrow 320 + 5\alpha = 288 + 7\alpha \Rightarrow 2\alpha = 32 \Rightarrow \alpha = 16$$

$$\text{M.D.}(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} \text{ where} \sum f_i = 64 + 16 = 80$$

$$\text{M.D. } (\bar{x}) = \frac{4\times 4 + 24\times 2 + 28\times 0 + 16\times 2 + 8\times 4}{80} = \frac{8}{5}$$

$$\text{Variance} = \frac{\sum f_i (x_i - \bar{x})^2}{\sum f_i}$$

$$= \frac{4\times16+24\times4+0+16\times4+8\times16}{80} = \frac{352}{80}$$

$$\therefore \frac{3\alpha}{m+\sigma^2} = \frac{3\times16}{\frac{128}{80}+\frac{352}{80}} = 8$$

23. Let α be the constant term in the binomial expansion of $\left(\sqrt{x} - \frac{6}{x^{\frac{3}{2}}}\right)^n$, $n \le 15$. If the sum of the coefficients of the remaining terms in the expansion is 649 and the coefficient of x^{-n} is λα, then λ is equal to ________.

Sol. **36**

$$T_{k+1} = {}^nC_k (x)^{\frac{n-k}{2}} (-6)^k (x)^{\frac{-3}{2}k}$$

$$\frac{n-k}{2} - \frac{3}{2}k = 0$$

$$n - 4k = 0$$

$$(-5)^n - \left({}_nC_{\frac{n}{4}} (-6)^{\frac{n}{4}}\right) = 649$$

By observation (625 + 24 = 649) , we get n = 4

$\because n = 4 \& k = 1$

Required is coefficient of

$$x^{-4} \text{ is} \left(\sqrt{4} - \frac{6}{x^{\frac{3}{2}}}\right)^4$$

$${}^4C_1(-6)^3$$

By calculating we will get $\lambda = 36$

24. Let $\omega = z\bar{z} + k_1 z + k_2 iz + \lambda(1+i)$, $k_1, k_2 \in R$. Let Re(ω) = 0 be the circle C of radius 1 in the first quadrant touching the line y = 1 and the y-axis. If the curve Im(ω) = 0 intersects C at A and B, then $30(AB)^2$ is equal to ______.

Sol. **24**

$$\omega = z\bar{z} + k_1 z + k_2 iz + \lambda(1+i)$$

$$\text{Re}(\omega) = x^2 + y^2 + k_1 x - k_2 y + \lambda = 0$$

$$\text{Centre} \equiv \left(\frac{-k_1}{2}, \frac{k_2}{2}\right) \equiv (1, 2)$$

$$\Rightarrow k_1 = -2, k_2 = 4$$

$$\text{radius} = 1 \Rightarrow \lambda = 4$$

$$\text{Im} = k_1 y + k_2 x + \lambda = 0$$

$$\therefore 2x - y + 2 = 0$$

$d = \frac{2}{\sqrt{5}}$

$\frac{1^2}{4} = 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore 301^2 = 24$

25. Let $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} = \vec{b} \times \vec{c}$ and $\left|\vec{b}\right|^2 = 50$, then $\left|72 - \left|\vec{b} + \vec{c}\right|^2\right|$ is equal to ______.

Sol. **66**

$|\vec{a}| = \sqrt{11}, |\vec{c}| = \sqrt{22}$

$|\vec{a}| = |\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}|\sin\theta$

$\sqrt{11} = \sqrt{50}\sqrt{22}\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{10}$

$|\vec{b} + \vec{c}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b}\cdot\vec{c}$

$= |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{b}||\vec{c}|\cos\theta$

$= 50 + 22 + 2 \times \sqrt{50} \times \sqrt{22} \times \frac{\sqrt{99}}{10}$

$= 72 + 66$

$\left|72 - \left|\vec{b} + \vec{c}\right|^2\right| = 66$

26. Let m_1, and m_2 be the slopes of the tangents drawn from the point P(4,1) to the hyperbola H: $\frac{y^2}{25} - \frac{x^2}{16} = 1$. If Q is the point from which the tangents drawn to H have slopes $|m_1|$ and $|m_2|$ and they make positive intercepts α and β on the x-axis, then $\frac{(PQ)^2}{\alpha\beta}$ is equal to _______.

Sol. **8**

Equation of tangent to the hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

$y = mx \pm \sqrt{a^2 - b^2m^2}$

passing through (4, 1)

$1 = 4m \pm \sqrt{25 - 16m^2} \Rightarrow 4m^2 - m - 3 = 0$

$\Rightarrow m = 1, \frac{-3}{4}$

Equation of tangent with positive slopes $1 \& \frac{3}{4}$

$\left.\begin{array}{r} 4y = 3x - 16 \\ y = x - 3 \end{array}\right\}$ with positive intercept on x-axis.

$\alpha = \frac{16}{3}, \beta = 3$

Intersection points:

Q : (–4, –7)

P : (4, 1)

$PQ^2 = 128$

$\frac{PQ^2}{\alpha\beta} = \frac{128}{16} = 8$

27. Let the image of the point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$ in the plane x – 2y + z – 2 = 0 be P. If the distance of the point Q(6, –2, α), α > 0, from P is 13, then α is equal to ______.

Sol. **15**

Image of point $\left(\frac{5}{3}, \frac{5}{3}, \frac{8}{3}\right)$

$$\frac{x-\frac{5}{3}}{1} = \frac{y-\frac{5}{3}}{-2} = \frac{z-\frac{8}{3}}{1} = \frac{-2\left(1\times\frac{5}{3}+(-2)\times\frac{8}{3}+1\times\frac{8}{3}-2\right)}{1^2+2^2+1^2}$$

$$= \frac{1}{3}$$

$\therefore x = 2, y = 1, z = 3$

$13^2 = (6-2)^2 + (-2-1)^2 + (\alpha-3)^2$

$\Rightarrow (\alpha-3)^2 = 144 \Rightarrow \alpha = 15 (\because \alpha > 0)$

28. Let for $x \in R$, $S_0(x) = x, S_k(x) = C_k x + k\int_0^x S_{k-1}(t)dt$ where $C_0 = 1$, $C_k = 1 - \int_0^1 S_{k-1}(x)dx$, $k = 1, 2, 3, ...$ Then $S_2(3) + 6C_3$ is equal to ___________.

Sol. **18**

Given, $S_k(x) = C_k x + k\int_0^x S_{k-1}(t)dt$

put $k = 2$ and x = 3

$S_2(3) = C_2(3) + 2\int_0^3 S_1(t)dt$ ……(1)

Also, $S_1(x) = C_1(x) + \int_0^x S_0(t)dt$

$= C_1 x + \frac{x^2}{2}$

$S_2(3) = 3C_2 + 2\int_0^3 \left(C_1 t + \frac{t^2}{2}\right)dt$

$= 3C_2 + 9C_1 + 9$

Also,

$C_1 = 1 - \int_0^1 S_0(x)dx = \frac{1}{2}$

$$C_2 = 1 - \int_0^1 S_1(x)dx = 0$$

$$C_3 = 1 - \int_0^1 S_2(x)dx$$

$$= 1 - \int_0^1 \left(C_2 x + C_1 x^2 + \frac{x^3}{3} \right) dx = \frac{3}{4}$$

$$S_2(x) = C_2 x + 2\int_0^x S_1(t)dt$$

$$= C_2 x + C_1 x^2 + \frac{x^3}{3}$$

$$\Rightarrow S_2(3) + 6C_3 = 6C_3 + 3C_2 + 9C_1 + 9 = 18$$

29. If $S = \left\{ x \in R : \sin^{-1}\left(\frac{x+1}{\sqrt{x^2+2x+2}} \right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}} \right) = \frac{\pi}{4} \right\}$, then is equal to _________.

Sol. **4**

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}} \right) - \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}} \right) = \frac{\pi}{4}$$

$$\because \frac{t}{\sqrt{t^2+1}} \in (-1,1)$$

$$\sin^{-1}\left(\frac{(x+1)}{\sqrt{(x+1)^2+1}} \right) = \sin^{-1}\left(\frac{x}{\sqrt{x^2+1}} \right) + \frac{\pi}{4}$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \left(\frac{1}{\sqrt{2}} \right) \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{x^2+1}} \right) \right) + \frac{1}{\sqrt{2}} \left(\frac{x}{\sqrt{x^2+1}} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\frac{1}{\sqrt{x^2+1}} + \frac{x}{\sqrt{x^2+1}} \right)$$

$$\frac{(x+1)}{\sqrt{(x+1)^2+1}} = \frac{1}{\sqrt{2}} \left(\frac{1+x}{\sqrt{x^2+1}} \right)$$

After solving this equation, we get

x = - 1 or x = 0

$$S = \{-1, 0\}$$

$$\sum_{x\in R}\left(\sin\left(\left(x^2+x+5\right)\frac{\pi}{2}\right)-\cos\left(\left(x^2+x+5\right)\pi\right)\right)$$

$$=\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right]+\left[\sin\left(\frac{5\pi}{2}\right)-\cos(5\pi)\right] = 4$$

30. The number of seven digit positive integers formed using the digits 1, 2, 3 and 4 only and sum of the digits equal to 12 is _______.

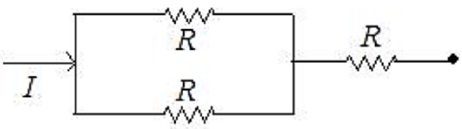
Sol. **413**

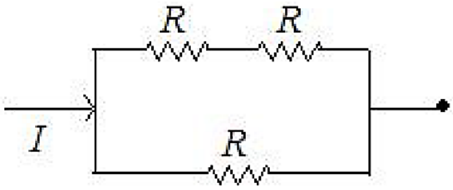
$$x_1+x_2+x_3+x_4+x_5+x_6+x_7=12, x_i\in\{1,2,3,4\}$$

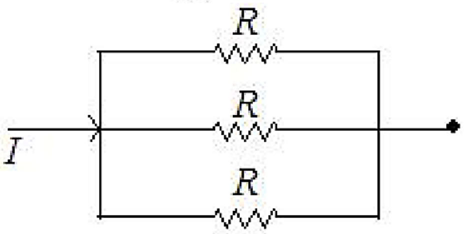
No. of solutions $={}^{5+7-1}C_{7-1}-\frac{7!}{6!}-\frac{7!}{5!}=413$

SECTION - A

31. Different combination of 3 resistors of equal resistance R are shown in the figures. The increasing order for power dissipation is:

(A) 

(B) 

(C) 

(D) 

(1) $P_C < P_B < P_A < P_D$ (2) $P_C < P_D < P_A < P_B$ (3) $P_B < P_C < P_D < P_A$ (4) $P_A < P_B < P_C < P_D$

Sol. **(1)**

Power dissipation, $P = I^2R$

(A) $R_{eq} = \frac{R}{2} + R = \frac{3R}{2}$

(B) $R_{eq} = \frac{(2R)(R)}{2R + R} = \frac{2R}{3}$

(C) $R_{eq} = \frac{R}{3}$

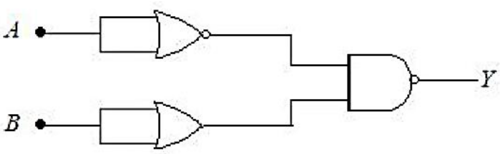
(D) $R_{eq} = 3R$

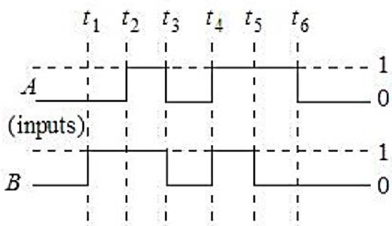
$R_D > R_A > R_B > R_C$

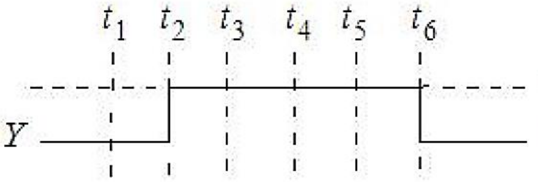
Since, $P \propto R_{eq}$

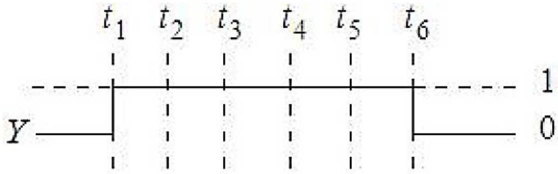
$P_D > P_A > P_B > P_C$

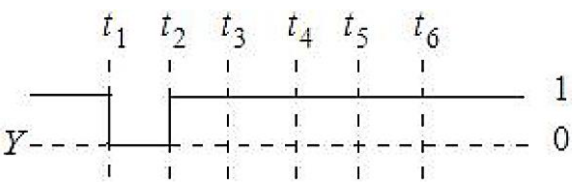
32. For the following circuit and given inputs A and B, chose the correct option for output 'Y'

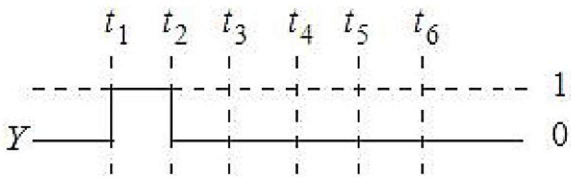




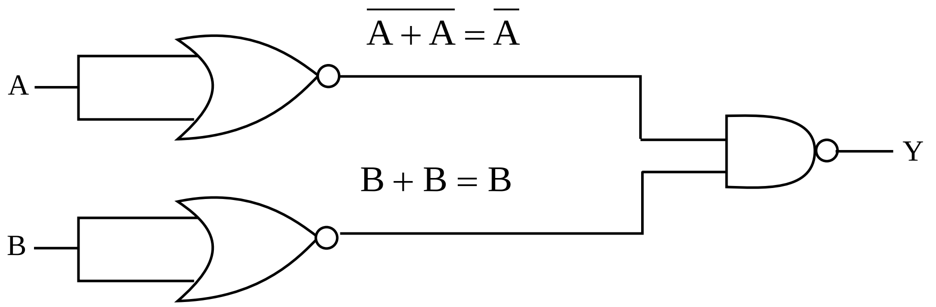
(1) 

(2) 

(3) 

(4) 

Sol. **(3)**



Output, $y = \overline{\overline{A}.B} = \overline{\overline{A}} + B$

$y = A + \overline{B}$

t_1 to t_2 , A = 0, B = 1, Y = 0
t_2 to t_3 A=1, B = 1, Y = 1
t_3 to t_4 A = 0, B = 0, Y = 1
t_4 to t_5, A = 1, B = 1, Y = 1
t_5 to t_6, A = 1, B = 0, Y = 1
After t_6, A = 0, B = 0, Y = 1

33. A bullet of 10 g leaves the barrel of gun with a velocity of 600 m/s. If the barrel of gun is 50 cm long and mass of gun is 3 kg, then value of impulse supplied to the gun will be:

(1) 12 Ns (2) 6 Ns (3) 3 Ns (4) 36 Ns

Sol. **(2)**

Impulse, $|\vec{I}| = |\Delta \vec{p}|$

$= mV - 0$

$= (10 \times 10^{-3}\text{ kg})\ (600\text{ m/s})$

I = 6 N–S

34. Which of the following Maxwell's equation is valid for time varying conditions but not valid for static conditions:

(1) $\oint \vec{D}.\vec{dA} = Q$ (2) $\oint \vec{E}.\vec{dl} = -\frac{\partial \phi_B}{\partial t}$ (3) $\oint \vec{E}.\vec{dl} = 0$ (4) $\oint \vec{B}.\vec{dl} = \mu_0 I$

Sol. **(2)**

For static conditions

$\oint \vec{E}.d\vec{l} = 0$

For time varying condition,

$\oint \vec{E}.d\vec{l} = -\frac{\partial \phi_B}{\partial t}$

35. Match List – I with List – II

List – I **(Layer of atmosphere)**	**List – II** **(Approximate height over earth's surface)**
(A) F1 – Layer	(I) 10 km
(B) D – Layer	(II) 170 – 190 km
(C) Troposphere	(III) 100 km
(D) E – layer	(IV) 65 – 75 km

Choose the correct answer from the options given below:

(1) A – II, B – I, C – IV, D – III
(2) A – II, B – IV, C – III, D – I
(3) A – II, B – IV, C – I, D – III
(4) A – III, B – IV, C – I, D – II

Sol. **(3)**

$F_1 \rightarrow$ Lower part of F layer of ionosphere (170 – 190Km)

D $\rightarrow$ Lowest layer of ionosphere (65 – 75 Km)

Troposphere $\rightarrow$ Lowest layer of atmosphere (10 Km)

E $\rightarrow$ Middle part of ionosphere (100 Km)

36. The rms speed of oxygen molecule in a vessel at particular temperature is $\left(1+\frac{5}{x}\right)^{\frac{1}{2}} \nu$, where ν is the average speed of the molecule. The value of x will be: (Take $\pi = \frac{22}{7}$)

(1) 28
(2) 27
(3) 8
(4) 4

Sol. **(1)**

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$$V_{avg} = \nu = \sqrt{\frac{8RT}{\pi M}}$$

$$V_{rms} = \sqrt{\frac{3\pi}{8}}\nu$$

$$V_{rms} = \sqrt{\frac{3}{8}\times\frac{22}{7}}\nu = \left(\frac{33}{28}\right)^{1/2}\nu$$

$$V_{rms} = \left(1+\frac{5}{28}\right)^{1/2}\nu$$

x = 28

37. The ratio of powers of two motors is $\frac{3\sqrt{x}}{\sqrt{x}+1}$, that are capable of raising 300 kg water in 5 minutes and 50 kg water in 2 minutes respectively from a well of 100 m deep. The value of x will be

(1) 16
(2) 2
(3) 4
(4) 2.4

Sol. **(1)**

$$P = \frac{\text{Work}}{\text{Time}}$$

$$P_1 = \frac{mgh}{t_1} = \frac{(300)g(100)}{5}$$

$$P_2 = \frac{(50)g(100)}{2}$$

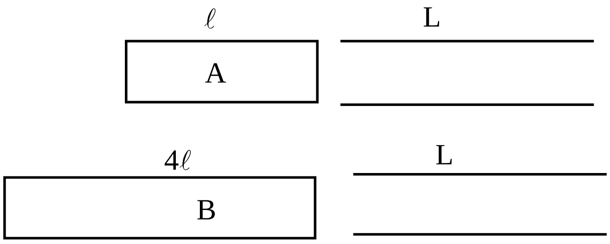
$$\frac{P_1}{P_2} = \frac{600}{250} = \frac{12}{5} = \frac{3\times 4}{4+1}$$

$$\frac{P_1}{P_2} = \frac{3\sqrt{16}}{\sqrt{16}+1}$$

x = 16

38. Two trains 'A' and 'B' of length '*l*' and '4*l*' are travelling into a tunnel of length 'L' in parallel tracks from opposite directions with velocities 108 km/h and 72 km/h, respectively. If train 'A' takes 35s less time than train 'B' to cross the tunnel then, length 'L' of tunnel is:
(Given L = 60 *l*)
(1) 2700 m (2) 1800 m (3) 1200 m (4) 900 m

Sol. **(2)**



$$V_A = 108 \times \frac{5}{18} = 30 m/s$$

$$V_B = 72 \times \frac{5}{18} = 20 m/s$$

$$T_A = \frac{\ell + L}{30}, T_B = \frac{4\ell + L}{20}$$

$$T_A = T_B - 35$$

$$\frac{\ell + L}{30} = \frac{4\ell + L}{20} - 35$$

Given, L = 60 ℓ

$$\frac{61\ell}{30} = \frac{64\ell}{20} - 35$$

$$\frac{192\ell - 122\ell}{60} = 35$$

$$70\ell = 60 \times 35$$

$$\ell = 30m$$

$$L = 60\ell = 1800m$$

39. Two bodies are having kinetic energies in the ratio 16 : 9. If they have same linear momentum, the ratio of their masses respectively is:
(1) 16 : 9 (2) 4 : 3 (3) 9 : 16 (4) 3 : 4

Sol. **(3)**

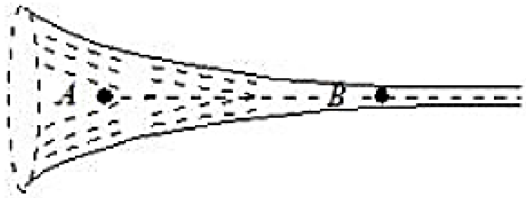
Kinetic energy, $KE = \frac{P^2}{2m}$

$$\frac{k_1}{k_2} = \frac{m_2}{m_1}$$

$$\frac{16}{9} = \frac{m_2}{m_1}$$

$$\frac{m_1}{m_2} = \frac{9}{16}$$

40. The figure shows a liquid of given density flowing steadily in horizontal tube of varying cross – section. Cross sectional areas at A is 1.5 cm^2, and B is 25 mm^2, if the speed of liquid at B is 60 cm/s then $(P_A - P_B)$ is:
(Given P_A and P_B are liquid pressures at A and B points)
Density ρ = 1000 kg m^{-3}
A and B are on the axis of tube



(1) 175 Pa (2) 36 Pa (3) 27 Pa (4) 135 Pa

Sol. **(1)**
By equation of continuity,
$A_1V_1 = A_2V_2$
$(1.5 \times 10^{-4})\ V_A = (25 \times 10^{-6})\ 60$ cm/s
$V_A = 10$ cm/s
By Bernoulli's theorem,

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho V_B^2$$

$$P_A - P_B = \frac{\rho}{2}(V_B^2 - V_A^2)$$

$$P_A - P_B = \frac{1000}{2}(60^2 - 10^2) \times 10^{-4}$$

$$P_A - P_B = 175\text{Pa}$$

41. $^{238}_{92}A \rightarrow {}^{234}_{90}B + {}^{4}_{2}D + Q$
In the given nuclear reaction, the approximate amount of energy released will be:
[Given, mass of $^{238}_{92}A$ = 238.05079 × 931.5 MeV/c^2,
mass of $^{234}_{90}B$ = 234.04363 × 931.5 MeV/c^2,
mass of $^{4}_{2}D$ = 4.00260 × 931.5 MeV/c^2]

(1) 4.25 MeV (2) 5.9 MeV (3) 3.82 MeV (4) 2.12 MeV

Sol. **(1)**

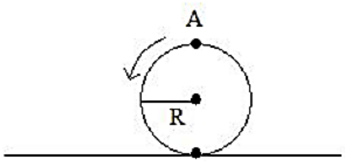
$$Q = \Delta m C^2$$

$$Q = (238.05079 - 234.04363 - 4.00260) \times 931.5\text{MeV}$$

$$Q = 0.00456 \times 931.5\text{MeV}$$

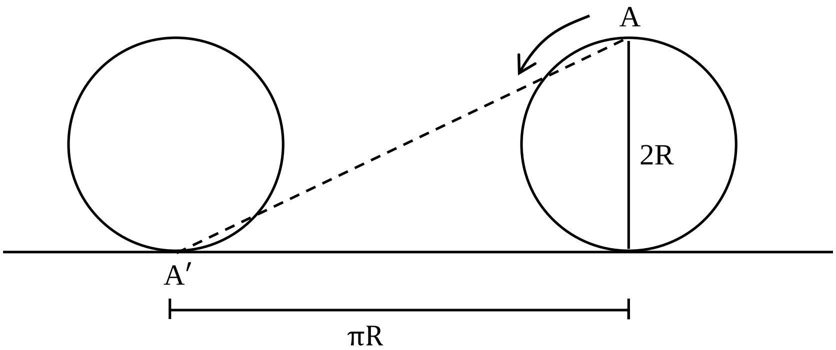
Q = 4.25 MeV

42. A disc is rolling without slipping on a surface. The radius of the disc is R. At t = 0, the top most point on the disc is A as shown in figure. When the disc completes half of its rotation, the displacement of point A from its initial position is



(1) $2R\sqrt{(1+4\pi^2)}$ (2) $R\sqrt{(\pi^2+4)}$ (3) 2R (4) $R\sqrt{(\pi^2+1)}$

Sol. **(2)**

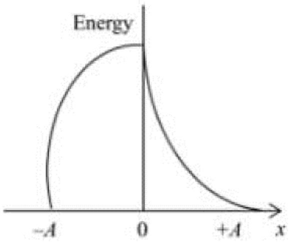


Displacement = A'A

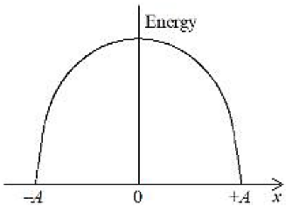
$$A'A = \sqrt{(\pi R)^2 + (2R)^2}$$

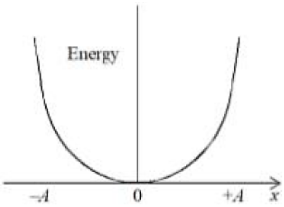
$$A'A = R\sqrt{\pi^2 + 4}$$

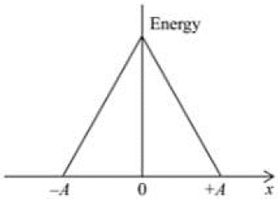
43. Which graph represents the difference between total energy and potential energy of a particle executing SHM vs it's distance from mean position?

(1) 

w

(2) 

(3) 

(4) 

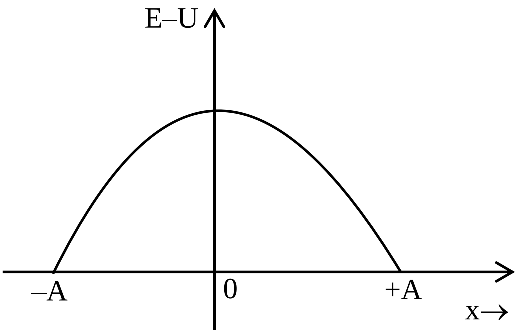
Sol. **(2)**

Total energy in SHM = E

$E = K + U$

$E - U = K$

$$E - U = \frac{1}{2}m\omega^2(A^2 - x^2)$$



44. Two charges each of magnitude 0.01 C and separated by a distance of 0.4 mm constitute an electric dipole. If the dipole is placed in an uniform electric field '$\vec{E}$' of 10 dyne/C making 30° angle with $\vec{E}$, the magnitude of torque acting on dipole is

(1) 1.5×10^{-9} Nm (2) 2.0×10^{-10} Nm (3) 1.0×10^{-8} Nm (4) 4.0×10^{-10} Nm

Sol. **(2)**

Dipole moment, P = qd

$P = 0.01 \times 0.4 \times 10^{-3}$

$P = 4 \times 10^{-6}$ C-m

Torque, $\tau = pE\sin\theta$

$\tau = 4\times10^{-6}\times(10\times10^{-5})\times\sin 30°$

$\tau = 4\times10^{-10}\,N-m$

45. Under isothermal condition, the pressure of a gas is given by $P = aV^{-3}$, where a is a constant and V is the volume of the gas. The bulk modulus at constant temperature is equal to

(1) $\frac{P}{2}$ (2) 2P (3) P (4) 3P

Sol. **(4)**

$P = aV^{-3}$

$\frac{dP}{dV} = -3aV^{-4}$

Bulk modulus, $B = -V\frac{dP}{dV}$

$B = -V\left(\frac{-3a}{V^4}\right)$

$B = 3\frac{a}{V^3} = 3P$

46. A planet having mass 9 Me and radius 4Re, where Me and Re are mass and radius of earth respectively, has escape velocity in km/s given by:

(Given escape velocity on earth $V_e = 11.2 \times 10^3$ m/s)

(1) 11.2 (2) 67.2 (3) 33.6 (4) 16.8

Sol. **(4)**

Escape velocity, $v_e = \sqrt{\frac{2GM}{R}}$

$V_p = \sqrt{\frac{2G(9m_e)}{4R_e}} = \frac{3}{2}(V_e)_{earth}$

$v_p = \frac{3}{2}\times11.2\text{ km/s}$

$v_p = 16.8\text{ km/s}$

47. A body of mass (5 ± 0.5) kg is moving with a velocity of (20 ± 0.4) m/s. Its kinetic energy will be

(1) (1000 ± 140) J (2) (500 ± 140) J (3) (500 ± 0.14) J (4) (1000 ± 0.14) J

Sol. **(1)**

Kinetic energy, $KE = \frac{1}{2}mv^2$

$KE = \frac{1}{2}\times5\times20^2$

$$KE = 1000\,J$$

$$\frac{\Delta K}{K} = \frac{\Delta m}{m} + \frac{2\Delta v}{v}$$

$$\frac{\Delta K}{1000} = \frac{0.5}{5} + 2 \times \frac{0.4}{20}$$

$$\Delta K = 1000(0.1 + 0.04)$$

$$\Delta K = 1000 \times 0.14$$

$$\Delta K = 140\,J$$

$$KE = (1000 \pm 140)\,J$$

48. The difference between threshold wavelengths for two metal surfaces A and B having work function $\phi_A = 9$ eV and $\phi_B = 4.5$ eV in nm is:
{Given, hc = 1242 eV nm}
(1) 276 (2) 264 (3) 540 (4) 138

Sol. (4)

$$\phi = \frac{hc}{\lambda}$$

$$\lambda_A = \frac{1242}{9} = 138\,nm$$

$$\lambda_B = \frac{1242}{4.5} = 276\,nm$$

$$\lambda_B - \lambda_A = 276 - 138 = 138\,nm$$

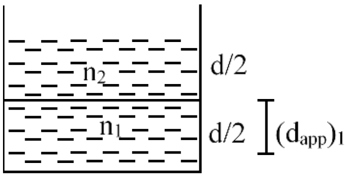
49. The source of time varying magnetic field may be
(A) A permanent magnet
(B) An electric field changing linearly with time
(C) Direct current
(D) A decelerating charge particle
(E) An antenna fed with a digital signal
Choose the correct answer from the options given below:
(1) (B) and (D) only (2) (C) and (E) only (3) (D) only (4) (A) only

Sol. **(3)**
Accelerated charge particle produces EMW which has time varying E and B.
If E is linear function of time then B will be constant.

50. A vessel of depth 'd' is half filled with oil of refractive index n_1 and the other half is filled with water of refractive index n_2. The apparent depth of this vessel when viewed from above will be -

(1) $\frac{d(n_1 + n_2)}{2n_1n_2}$ (2) $\frac{d\,n_1n_2}{(n_1 + n_2)}$ (3) $\frac{d\,n_1n_2}{2(n_1 + n_2)}$ (4) $\frac{2d(n_1 + n_2)}{n_1n_2}$

Sol. (1)



$$\left(d_{app}\right)_1 = \frac{d}{2\left(\frac{n_1}{n_2}\right)} = \frac{n_2 d}{2n_1}$$

$$\left(d_{app}\right)_2 = \frac{\left(d_{app}\right)_1 + \frac{d}{2}}{n_2}$$

$$= \frac{\left(\frac{n_2}{n_1} + 1\right)\frac{d}{2}}{n_2}$$

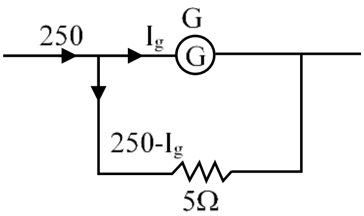
$$\left(d_{app}\right)_2 = \frac{\left(n_1 + n_2\right)d}{2n_1 n_2}$$

SECTION – B

51. When a resistance of 5 Ω is shunted with a moving coil galvanometer, it shows a full scale deflection for a current of 250 mA, however when 1050 Ω resistance is connected with it in series, it gives full scale deflection for 25 volt. The resistance of galvanometer is ___________ Ω.

Sol. **(50)**

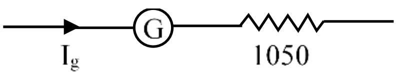
For ammeter,



$$I_g(G) = (250 - I_g)5$$

$$I_g = \frac{1250}{5+G}\text{mA}$$

For voltmeter,



$$V = I_g R$$

$$25 = I_g(G+1050)$$

From equation (1),

$$25 = \frac{1250\times10^{-3}}{G+5}(G+1050)$$

$$20(G+5) = G+1050$$

19 G = 1050 – 100

$$G = \frac{950}{19} = 50\Omega$$

52. The radius of 2^{nd} orbit of He^+ of Bohr's model is r_1 and that of fourth orbit of Be^{3+} is represented as r_2. Now the ratio $\frac{r_2}{r_1}$ is x : 1. The value of x is ___________

Sol. **(2)**

$$r \propto \frac{n^2}{Z}$$

$$\frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2 \times \frac{z_1}{z_2}$$

$$\frac{r_2}{r_1} = \left(\frac{4}{2}\right)^2 \times \frac{2}{4}$$

$$\frac{r_2}{r_1} = 2$$

$$x = 2$$

53. A solid sphere is rolling on a horizontal plane without slipping. If the ratio of angular momentum about axis of rotation of the sphere to the total energy of moving sphere is π : 22 the, the value of its angular speed will be rad/s.

Sol. **(4)**

Angular momentum,

$$L = I\omega$$

$$L = \frac{2}{5}MR^2\omega$$

$$\text{Energy } = \frac{1}{2}MV^2 + \frac{1}{2}I\omega^2$$

$$E = \frac{1}{2}M(\omega R)^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2$$

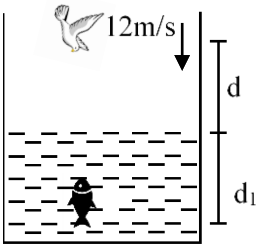
$$= \frac{7}{10}M\omega^2R^2$$

$$\frac{L}{E} = \frac{4}{7\omega} = \frac{\pi}{22}$$

$$\omega = \frac{88}{7\pi} = \frac{88}{7 \times \frac{22}{7}} = 4 \text{ rad / s}$$

54. A fish rising vertically upward with a uniform velocity of 8 ms^{-1}, observes that a bird is diving vertically downward towards the fish with the velocity of 12 ms^{-1}. If the refractive index of water is $\frac{4}{3}$, then the actual velocity of the diving bird to pick the fish, will be ______________ ms^{-1}.

Sol. **(3)**



$d_{app} = d_1 + \mu d$

$v_{app} = v_1 + \mu v$

$12 = 8 + \frac{4}{3}v$

$4 = \frac{4}{3}v$

$v = 3 m/s$

55. The elastic potential energy stored in a steel wire of length 20 m stretched through 2 cm is 80 J. The cross sectional area of the wire is ___________ mm^2.
(Given, $y = 2.0 \times 10^{11}\ Nm^{-2}$)

Sol. **(40)**

Energy, $U = \frac{1}{2}kx^2$

$80 = \frac{1}{2}k(2\times10^{-2})^2$

$k = \frac{160}{4\times10^{-4}}$

$k = 4\times10^5\ N/m$

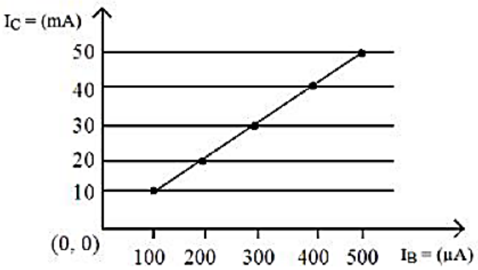
$\frac{yA}{\ell} = 4\times10^5$

$A = \frac{4\times10^5\times20}{2\times10^{11}}$

$A = 40\times10^{-6} m^2$

$A = 40 mm^2$

56. From the given transfer characteristic of a transistor in CE configuration, the value of power gain of this configuration is 10^x, for $R_B = 10\ k\ \Omega$, $R_C = 1\ k\ \Omega$. The value of x is ______________



Sol. **(3)**

Current gain, $\beta = \frac{\Delta I_C}{\Delta I_B}$

$\beta = \frac{10 mA}{100 \mu A}$

$\beta = 100$

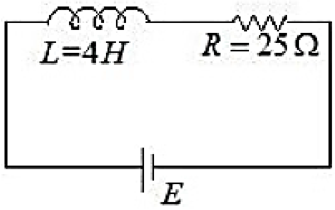
Power gain $\beta^2 \frac{R_C}{R_B}$

$= 10^4 \times \frac{1}{10}$

$= 10^3$

So, $x = 3$

57. In the given figure, an inductor and a resistor are connected in series with a batter of emf E volt. $\frac{E^a}{2b}$ J/s represents the maximum rate at which the energy is stored in the magnetic field (inductor). The numerical value of $\frac{b}{a}$ will be ______________



Sol. **(25)**

$$U = \frac{1}{2}LI^2$$

$$I = I_0\left(1 - e^{-t/\tau}\right)$$

Rate of energy, $P = \frac{dU}{dt}$

$$P = LI\frac{dI}{dt}$$

$$\frac{dP}{dt} = L\left(I\frac{d^2I}{dt^2} + \left(\frac{dI}{dt}\right)^2\right)$$

For maximum rate, $\frac{dP}{dt} = 0$

$$I\frac{d^2I}{dt^2} = -\left(\frac{dI}{dt}\right)^2 \quad \text{..... (1)}$$

$$I = I_0\left(1 - e^{t/\tau}\right)$$

$$\frac{dI}{dt} = \frac{I_0}{\tau}e^{-t/\tau}$$

$$\frac{d^2I}{dt^2} = -\frac{I_0}{\tau^2}e^{-t/\tau}$$

By equation (1),

$$I_0\left(1 - e^{-t/\tau}\right) \times \frac{I_0}{\tau^2}e^{-t/\tau} = \frac{-I_0^2}{\tau^2}e^{-2t/\tau}$$

Let $e^{-t/\tau} = x$

$$x - x^2 = x^2$$

$$x = \frac{1}{2}$$

Maximum power,

$$P = LI\frac{dI}{dt}$$

$$P = L\,I_0\left(1 - \frac{1}{2}\right)\left(\frac{I_0}{\tau} \times \frac{1}{2}\right)$$

$$P=\frac{L I_0^2}{4\times\frac{L}{R}}=\frac{I_0^2 R}{4}$$

$$P=\frac{E^2}{4R}$$

$$a=2,\ 2b=4R$$

$$b=2R=50$$

$$\frac{b}{a}=25$$

58. A potential V_o is applied across a uniform wire of resistance R. The power dissipation is P_1. The wire is then cut into two equal halves and a potential of V_o is applied across the length of each half. The total power dissipation across two wires is P_2. The ratio $P_2 : P_1$ is $\sqrt{x}:1$. The value of x is ________________

Sol. **(16)**

$$P_1=\frac{v_0^2}{R}$$

$$P_2=\frac{v_0^2}{\left(\frac{R}{2}\right)}+\frac{v_0^2}{\left(\frac{R}{2}\right)}$$

$$P_2=4P_1$$

$$\frac{P_2}{P_1}=\frac{4}{1}=\frac{\sqrt{x}}{1}$$

$$x=16$$

59. At a given point of time the value of displacement of a simple harmonic oscillator is given as y = A cos (30°). If amplitude is 40 cm and kinetic energy at that time is 200 J, the value of force constant is $1.0\times10^{x}\ Nm^{-1}$. The value of x is ____________

Sol. **(4)**

$$v=\omega\sqrt{A^2-x^2}$$

$$y=A\times\frac{\sqrt{3}}{2}$$

$$v=\omega\sqrt{A^2-\frac{3A^2}{4}}=\frac{\omega A}{2}$$

Given, $KE=200\,J$

$$\frac{1}{2}m\frac{\omega^2A^2}{4}=200$$

$$KA^2=1600\quad\left(K=m\omega^2\right)$$

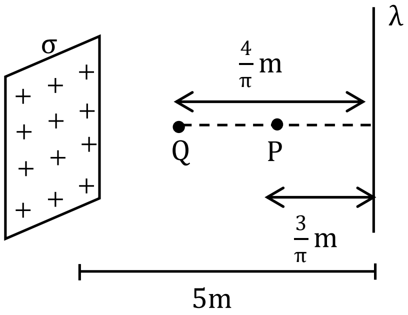
$$K=\frac{1600}{\left(40\times10^{-2}\right)^2}$$

$$K=10^4\ N/m$$

$$x=4$$

60. A thin infinite sheet charge and an infinite line charge of respective charge densities + σ and + λ are placed parallel at 5 m distance from each other. Points 'P' and 'Q' are at $\frac{3}{\pi}$m and $\frac{4}{\pi}$m perpendicular distances from line charge towards sheet charge, respectively. 'E_P' and 'E_Q' are the magnitudes of resultant electric field intensities at point 'P' and 'Q' respectively. If $\frac{E_P}{E_Q} = \frac{4}{a}$ for $2|\sigma| = |\lambda|$, then the value of a is ____________

Sol. **(6)**



$$E_P = \frac{2K\lambda}{r} - \frac{\sigma}{2\varepsilon_0}$$

$$E_P = \frac{\sigma}{2\varepsilon_0} - \frac{\lambda}{2\pi\varepsilon_0\left(\frac{3}{\pi}\right)}$$

$$E_P = \frac{2\sigma}{2\varepsilon_0} - \frac{2\sigma}{6\varepsilon_0} = \frac{\sigma}{6\varepsilon_0}$$

Similarly, $E_Q = \frac{\sigma}{2\varepsilon_0} - \frac{2\sigma}{2\pi\varepsilon_0\left(\frac{4}{\pi}\right)} = \frac{\sigma}{4\varepsilon_0}$

$$\frac{E_P}{E_Q} = \frac{4}{6}$$

a = 6

SECTION - A

61. Given below are two statements :

Statement I: Permutit process is more efficient compared to the synthetic resin method for the softening of water.

Statement II: Synthetic resin method results in the formation of soluble sodium salts.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both the Statements I and II are correct

(2) Statement I is incorrect but Statement II is correct

(3) Statement I is correct but Statement II is incorrect

(4) Both the Statements I and II are incorrect

Sol. **4**

Nowadays hard water is softened by using synthetic ion exchangers. This method is more efficient than zeolite process/Permutit process

62. Which one of the following is most likely a mismatch?

(1) Zinc - Liquation

(2) Copper – Electrolysis

(3) Titanium - van Arkel Method

(4) Nickel - Mond process

Sol. **1**

Zinc is refined by distillation method, which is used for metals having low boiling point.

63. The energy of an electron in the first Bohr orbit of hydrogen atom is -2.18×10^{-18} J . Its energy in the third Bohr orbit is__________.

(1) $\frac{1}{27}$ of this value

(2) $\frac{1}{9}$ th of this value

(3) One third of this value

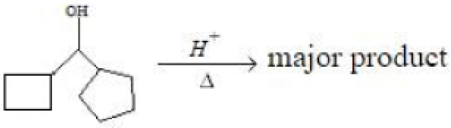
(4) Three times of this value

Sol. **2**

$E_{1,1} = -2.18 \times 10^{-18}$ J

$$E_{3,1} = E_{1,1} \times \frac{1^2}{3^2}$$

$$E_{3,1} = \frac{1}{9} \times E_{1,1}$$

64. 

In the above reaction, left hand side and right hand side rings are named as ' A ' and ' B ' respectively. They undergo ring expansion. The correct statement for this process is:

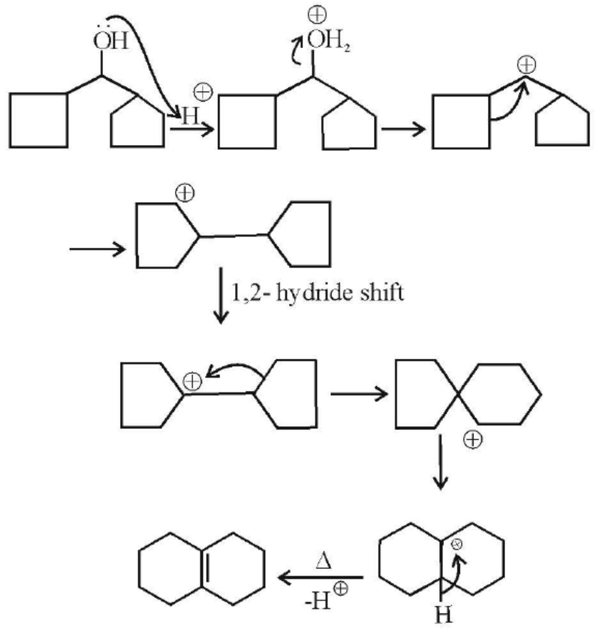
(1) Finally both rings will become six membered each.

(2) Ring expansion can go upto seven membered rings

(3) Finally both rings will become five membered each.

(4) Only A will become 6 membered.

Sol. **1**



65. Match The following

Column–A	Column–B
a) Nylon 6	I. Natural Rubber
b) Vulcanized Rubber	II. Cross Linked
c) cis–1, 4–polyisoprene	III. Caprolactam
d) Polychloroprene	IV. Neoprene

Choose the correct answer from options given below:

(1) a→II, a→III, c→IV, d→I

(2) a→IV, b→III, c→II, d→I

(3) a→III, b→II, c→I, d→IV

(4) a→III, b→IV, c→I, d→II

Sol. **3**

Nylon-6 - Caprolactum (Monomer)

Natural rubber- Isoprene (Monomer)

Vulcanized rubber - Sulphur containing rubber

Neoprene- Chloroprene (Monomer)

66. What happens when a lyophilic sol is added to a lyophobic sol?

(1) Film of lyophobic sol is formed over lyophilic sol.

(2) Lyophilic sol is dispersed in lyophobic sol.

(3) Lyophobic sol is coagulated

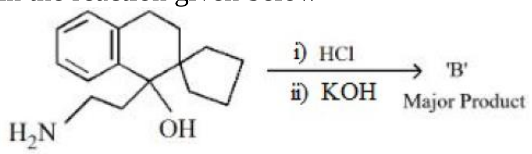
(4) Film of lyophilic sol is formed over lyophobic sol.

Sol. **4**

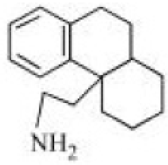
Protective film of lyophilic sol is formed over lyophobic sol.

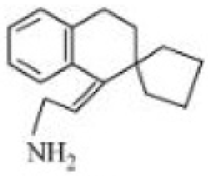
Which protects it from coagulation.

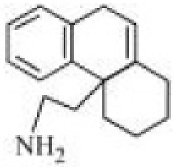
67. In the reaction given below

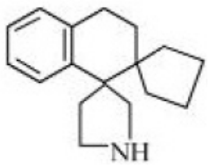


'B' is:

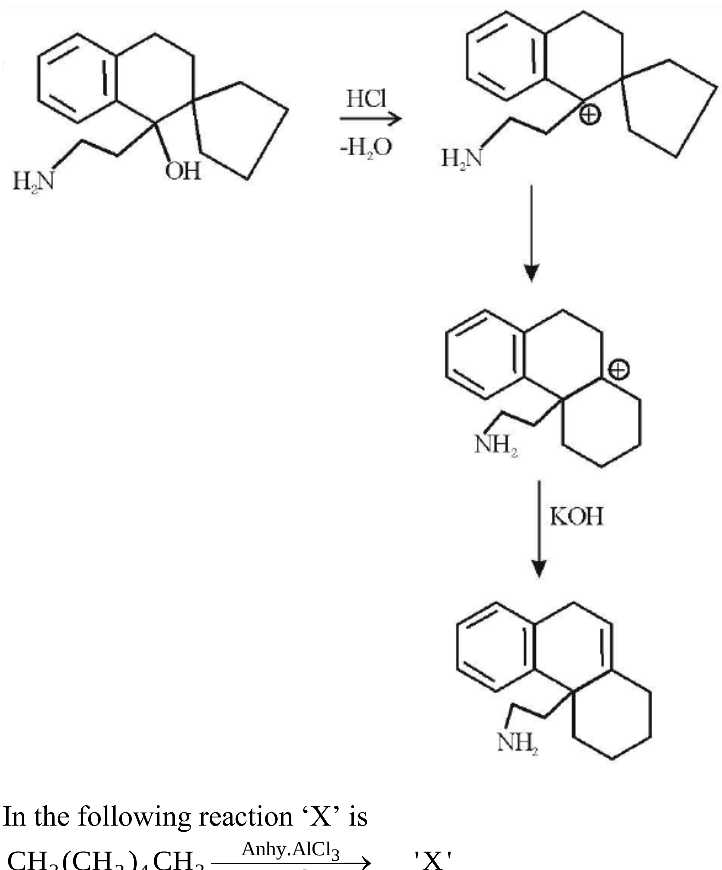
(1) 

(2) 

(3) 

(4) 

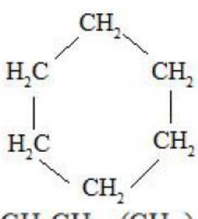
Sol. **3**



68. In the following reaction 'X' is

$$CH_3(CH_2)_4CH_3 \xrightarrow[\text{HCl}\ \Delta]{\text{Anhy.AlCl}_3} \underset{\text{major product}}{\text{'X'}}$$

(1) $CH_3(CH_2)_4CH_2Cl$

(2) 

(3) $Cl-CH_2-(CH_2)_4-CH_2-Cl$

(4) $CH_3\underset{\displaystyle CH_3}{\underset{|}{C}H}-(CH_2)_2CH_3$

Sol. **4**

n-alkanes on heating in this presence of anhydrous A1Cl$_3$ and hydrogen chloride gas isomerise to branched chain alkanes. The major product has one methyl side chain.

$$CH_3-(CH_2)_4-CH_3 \xrightarrow[HCl,\Delta]{Anhy.AlCl_3} CH_3-\underset{\underset{CH_3}{|}}{CH}-(CH_2)_2-CH_3$$

2- methylpentane
(major)

69. 2-Methyl propyl bromide reacts with $C_2H_5O^-$ and gives 'A' whereas on reaction with C_2H_5OH it gives 'B'. The mechanism followed in these reactions and the products 'A' and 'B' respectively are :

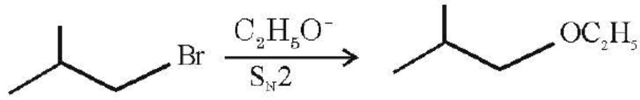
(1) S_N1, A= tert-butyl ethyl ether; S_N1, B= 2-butyl ethyl ether

(2) S_N2, A= 2-butyl ethyl ether; S_N2, B= iso-butyl ethyl ether

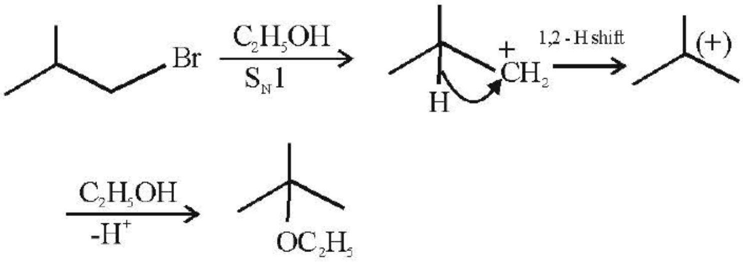
(3) S_N2, A= iso-butyl ethyl ether; S_N1, B= tert-butyl ethyl ether

(4) S_N1, A= tert-butyl ethyl ether; S_N2, B= iso-butyl ethyl ether

Sol. **3**

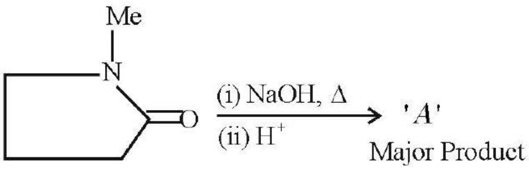
(i) 

$C_2H_5O^-$ is strong nucleophile.

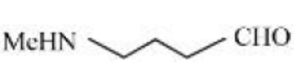
(ii) 

C_2H_5OH is weak nucleophile.

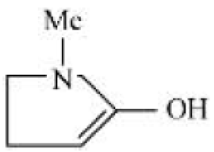
70. In the reaction given below

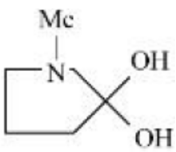


'A' is

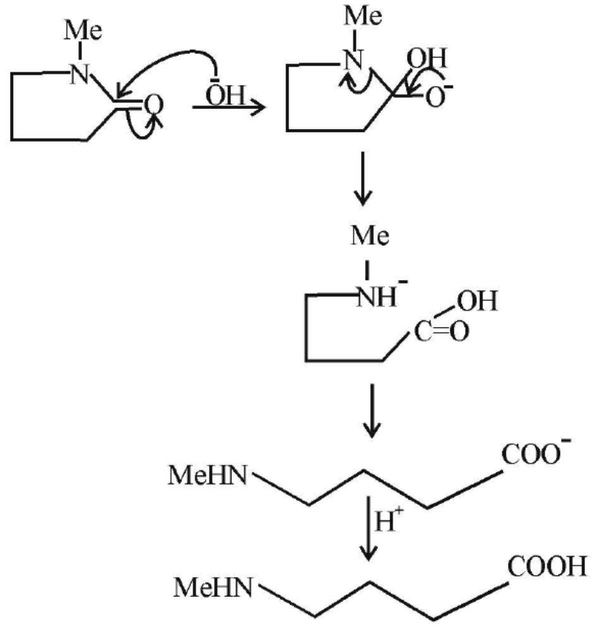
(1) 

(2) 

(3) 

(4) 

Sol. 2



71. D–(+) Glyceraldehyde $\xrightarrow[\text{ii) } H_2O/H^+ \text{, iii) } HNO_3]{\text{i) HCN}}$

The products formed in the above reaction are

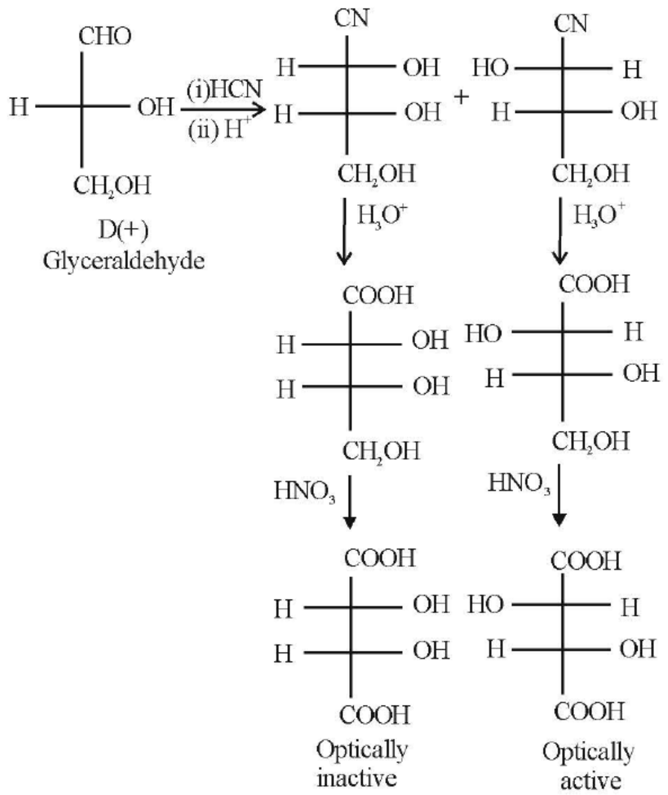
(1) Two optically active products

(2) One optically inactive and one meso product.

(3) One optically active and one meso product

(4) Two optically inactive products

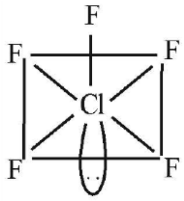
Sol. 3



72. ClF_5 at room temperature is a:

(1) Colourless liquid with square pyramidal geometry
(2) Colourless gas with trigonal bipyramidal geometry
(3) Colourless gas with square pyramidal geometry
(4) Colourless liquid with trigonal bipyramidal geometry

Sol. **1**



ClF_5 is colourless liquid.

73. The pair of lanthanides in which both elements have high third - ionization energy is:

(1) Dy, Gd (2) Eu,Gd (3) Lu,Yb (4) Eu,Yb

Sol. **4**

$$\left.\begin{array}{l} Eu^{+2}:[Xe]4f^{7} \\ Yb^{+2}:[Xe]4f^{14} \end{array}\right\} \text{High IE due to half filled \& fully filled configurations}$$

74. The mismatched combinations are

A. Chlorophyll - Co
B. Water hardness - EDTA
C. Photography -$[Ag(CN)_2]^-$
D. Wilkinson catalyst - $[(Ph_3P)_3 RhCl]$
E. Chelating ligand - D-Penicillamine

Choose the correct answer from the options given below :

(1) A and C Only (2) D and E Only (3) A and E Only (4) A, C, and E Only

Sol. **1**

Mg is present in chlorophyll and in black and white photography the developed film is fixed by washing with hypo solution which dissolves the undecomposed AgBr to form a complex ion $[Ag(S_2O_3)_2]^{3-}$

75. Which of the following statements are not correct ?

A. The electron gain enthalpy of F is more negative than that of Cl.
B. Ionization enthalpy decreases in a group of periodic table.
C. The electronegativity of an atom depends upon the atoms bonded to it.
D. Al_2O_3 and NO are examples of amphoteric oxides.

Choose the most appropriate answer from the options given below :

(1) A, C and D Only (2) B and D Only (3) A, B and D Only (4) A, B, C and D

Sol. **1**

Electronegativity of an element depends on the atom with which it is attached.
NO = neutral oxide
Al_2O_3 = amphoteric oxide

76. The radical which mainly causes ozone depletion in the presence of UV radiations is :

(1) $NO^{\bullet}$ (2) $\dot{O}H$ (3) $CH_3^{\bullet}$ (4) $Cl^{\bullet}$

Sol. **4**

$$O_2(g) \xrightarrow{UV} O(g) + O(g)$$
$$O_2(g) + O(g) \longrightarrow O_3(g)$$
$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$
$$\dot{C}l(g) + O_3(g) \longrightarrow Cl\dot{O}(g) + O_2(g)$$
$$Cl\dot{O}(g) + O(g) \longrightarrow \dot{C}l(g) + O_2(g)$$

77. In which of the following processes, the bond order increases and paramagnetic character changes to diamagnetic one?

(1) $O_2 \rightarrow O_2^+$ (2) $O_2 \rightarrow O_2^{2-}$ (3) $NO \rightarrow NO^+$ (4) $N_2 \rightarrow N_2^+$

Sol. **3**

NO is paramagnetic with BO = 2.5, NO^+ is diamagnetic with BO = 3

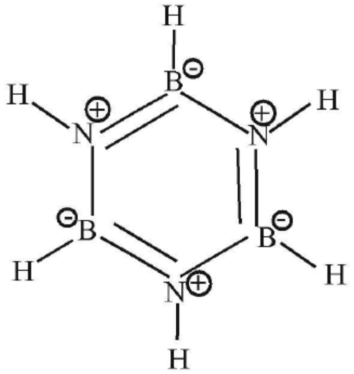
78. The incorrect statement from the following for borazine is:

(1) It is a cyclic compound. (2) It has electronic delocalization.

(3) It can react with water. (4) It contains banana bonds.

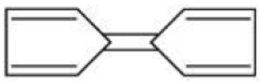
Sol. **4**

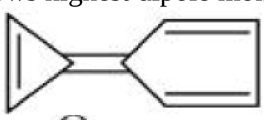
Borazine is $B_3N_3H_6$

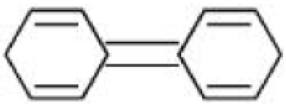


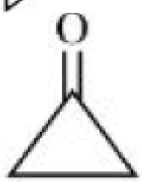
$B_3N_3H_6 + 9H_2O \rightarrow 3NH_3 + 3H_3BO_3 + 3H_2$

79. Among the following compounds, the one which shows highest dipole moment is

(1) 

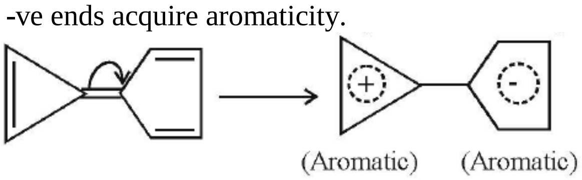
(2) 

(3) 

(4) 

Sol. **2**

Among the given compounds, the following compound has the highest dipole moment because both the +ve and -ve ends acquire aromaticity.



80. $Be(OH)_2$ reacts with $Sr(OH)_2$ to yield an ionic salt. Choose the incorrect option related to this reaction from the following :

(1) Be is tetrahedrally coordinated in the ionic salt.

(2) The reaction is an example of acid - base neutralization reaction.

(3) The element Be is present in the cationic part of the ionic salt.

(4) Both Sr and Be elements are present in the ionic salt.

Sol. **3**

$Be(OH)_2$ is amphoteric in nature.

$Sr(OH)_2$ is basic in nature.

These two undergo acid – base reaction to form a salt.

$$Be(OH)_2 + Sr(OH)_2 \rightarrow \underset{\text{(salt)}}{Sr[Be(OH)_4]}$$

SECTION - B

81. Solution of 12 g of non-electrolyte (A) prepared by dissolving it in 1000 mL of water exerts the same osmotic pressure as that of 0.05M glucose solution at the same temperature. The empirical formula of A is CH_2O. The molecular mass of A is____________ g. (Nearest integer)

Sol. **240**

$\pi_A = \pi_{glucose}$

$C_ART = CRT$

$$\frac{12/M_A}{1} = 0.05$$

M_A(Molar mass of A) = $\frac{12}{0.05} = \frac{1200}{5} = 240gm$

82. $KMnO_4$ is titrated with ferrous ammonium sulphate hexahydrate in presence of dilute H_2SO_4. Number of water molecules produced for 2 molecules of $KMnO_4$ is_________.

Sol. **68**

$2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \cdot (NH_4)_2 SO_4 \cdot 6H_2O \rightarrow K_2SO_4 + 2MnSO_4 + 5\ Fe_2(SO_4)_3 + 10\ (NH_4)_2SO_4 + 68H_2O$

On the basis of above equation,

68 molecules of water will be produced from 2 molecules of $KMnO_4$.

83. 20 mL of calcium hydroxide was consumed when it was reacted with 10 mL of unknown solution of H_2SO_4. Also 20 mL standard solution of 0.5MHCl containing 2 drops of phenolphthalein was titrated with calcium hydroxide, the mixture showed pink colour when burette displayed the value of 35.5 mL whereas the burette showed 25.5 mL initially. The concentration of H_2SO_4 is____________ M.(Nearest integer)

Sol. **1**

miliequivalent of $Ca(OH)_2$ = miliequivalent of H_2SO_4

$M_1 \times 2 \times 20 = M_2 \times 2 \times 10$

$\boxed{2M_1 = M_2}$

miliequivalent of HCl = miliequivalent of $Ca(OH)_2$

$20 \times 0.5 = 10 \times M_1 \times 2$

$M_1 = 0.5$ M

Concentration of $H_2SO_4 = M_2 = 2M_1$

$= 2 \times 0.5$

$= 1$ M

84. $t_{87.5}$ is the time required for the reaction to undergo 87.5% completion and t_{50} is the time required for the reaction to undergo 50% completion. The relation between $t_{87.5}$ and t_{50} for a first order reaction is ____________ $t_{87.5}$ = x × t_{50}. The value of x is____________ . (Nearest integer)

Sol. **3**

$$A_0 \xrightarrow{t_{1/2}} A_0/2 \xrightarrow{t_{1/2}} A_0/4 \xrightarrow{t_{1/2}} A_0/8$$

$$t_{7/8} = t_{87.5\%} = 3t_{1/2}$$

85. A certain quantity of real gas occupies a volume of 0.15 dm^3 at 100 atm and 500 K when its compressibility factor is 1.07 . Its volume at 300 atm and 300 K (When its compressibility factor is 1.4) is $\times 10^{-4}\ dm^3$. (Nearest integer)

Sol. **392**

$$Z = \frac{PV}{nRT}$$

$$\frac{Z_1}{Z_2} = \left(\frac{P_1V_1}{nRT_1}\right) \times \left(\frac{nRT_2}{P_2V_2}\right)$$

$$\frac{1.07}{1.4} = \left(\frac{100 \times 0.15}{500}\right)\left(\frac{300}{300 \times V_2}\right)$$

$$V_2 = \frac{0.03 \times 1.4}{1.07} = 0.03925$$

$$= 392 \times 10^{-4}\ dm^3$$

86. A metal surface of $100 cm^2$ area has to be coated with nickel layer of thickness 0.001 mm. A current of 2A was passed through a solution of $Ni(NO_3)_2$ for 'x' seconds to coat the desired layer. The value of x is____________. (Nearest integer)

(ρ_{Ni} (density of Nickel) is 10 g mL^{-1}, Molar mass of Nickel is 60 g mol^{-1} F = 96500 C mol^{-1})

Sol. **161**

Volume of nickel required = $100 \times 0.001 \times 10^{-3} \times 100$

$= 0.01\ cm^3$

Mass of Nickel required = 0.01×10

$= 0.1$ gm

Moles = $\frac{0.1}{60} = \frac{1}{600}$ mol

$Ni^{2+} + 2e^- \rightarrow Ni(s)$

for coating of 1 mol Ni, charge required = 2×96500 C

for coating of $\frac{1}{600}$ mol, charge required = $2 \times 96500 \times \frac{1}{600}$ C

$= \frac{965}{3}$ C

$$I = \frac{q}{t}$$

$$t = \frac{965/3}{2} = 160.83 \text{ sec} \approx 161$$

87. 25.0 mL of 0.050 $MBa(NO_3)_2$ is mixed with 25.0 mL of 0.020 M NaF. K_{sp} of BaF_2 is 0.5×10^{-6} at 298 K. The ratio of $[Ba^{2+}][F^-]^2$ and K_{sp} is_______________. (Nearest integer)

Sol. **5**

$$\left[Ba^{+2}\right] = \frac{25 \times 0.05}{50} = 0.025M$$

$$\left[F^-\right] = \frac{25 \times 0.02}{50} = 0.01M$$

$$\left[Ba^{+2}\right]\left[F^-\right]^2 = 25 \times 10^{-7}$$

$$K_{sp} = 5 \times 10^{-7} \text{ (given)}$$

$$\text{Ratio} = \frac{\left[Ba^{+2}\right]\left[F^-\right]^2}{K_{sp}} = 5$$

88. $A_2 + B_2 \rightarrow 2AB$. $\Delta H_f^0 = -200\,kJ\,mol^{-1}$ new line AB, A_2 and B_2 are diatomic molecules. If the bond enthalpies of A_2, B_2 and AB are in the ratio 1 : 0.5:1, then the bond enthalpy of A_2 is _________kJ mol^{-1}. (Nearest integer)

Sol. **800**

$A_2 + B_2 \rightarrow 2AB$ $\Delta H_f^\circ = -200 kJ / mol$

Bond enthalpy of A_2 = x

Bond enthalpy of B_2 = 0.5 x

Bond enthalpy of AB = x

ΔH_f° = x + 0.5 x – 2x = –2(200)

–0.5x = – 400

$x = \frac{400}{0.5} = 800 kJ / mol$

Bond enthalpy of A_2 = x = 800 kJ/mol

89. An organic compound gives 0.220 g of CO_2 and 0.126 g of H_2O on complete combustion. If the % of carbon is 24 then the % of hydrogen is ____________$\times 10^{-1}$. (Nearest integer)

Sol. **56**

% of carbon = $\frac{\frac{0.220}{44} \times 12}{x} \times 100$

(x = mass of organic compound)

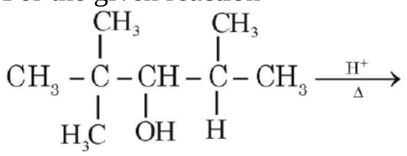
$24 = \frac{6}{x}$

x = 0.25 gm

% of H = $\frac{\frac{0.126}{18} \times 2 \times 1}{0.25} \times 100$

= 5.6 = 56 × 10^{-1}

90. For the given reaction



'A'

The total number of possible products formed by tertiary carbocation of A is___________.

Sol. **5**