SECTION - A

1. If the coefficients of x and x^2 in $(1 + x)^p (1- x)^q$ are 4 and – 5 respectively, then 2p + 3q is equal to

(1) 60 (2) 63 (3) 66 (4) 69

Sol. **(2)**

$(1+x)^p(1-x)^q$

$\left(1+px+\frac{p(p-1)}{2!}x^2+\ldots\right)$

$\left(1-qx+\frac{q(q-1)}{2!}x^2-\ldots\right)$

$p-q=4$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$p^2+q^2-p-q-2pq=-10$

$(q+4)^2+q^2-(q+4)-q-2(4+q)q=-10$

$q^2+8q+16-q^2-q-4-q-8q-2q^2=-10$

$-2q=-22$

$q=11$

$p=15$

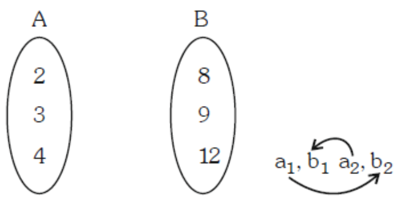
$2(15)+3(11)$

$30+33=63$

2. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R= $\{((a_1, b_1), (a_2,b_2)) \in (A \times B, A \times B) : a_1$ divides b_2 and a_2 divides $b_1\}$ is

(1) 18 (2) 24 (3) 12 (4) 36

Sol. **(4)**



a_1 divides b_2

Each elements has 2 choices

$\Rightarrow 3\times 2=6$

a_2 divides b_1

Each elements has 2 choices

$\Rightarrow 3\times 2=6$

Total = 6 × 6 = 36

3. Let time image of the point P(1,2,6) in the plane passing through the points A(1,2,0), B (1, 4, 1) and C (0, 5, 1) be Q (α, β, γ). Then $(\alpha^2 + \beta^2 + \gamma^2)$ is equal to

(1) 70 (2) 76 (3) 62 (4) 65

Sol. **(4)**

Equation of plane $A(x-1)+B(y-2)+C(z-0)=0$

Put $(1,4,1)\Rightarrow 2B+C=0$

Put $(0,5,1)\Rightarrow -A+3B+C=0$

Sub $:\overline{B-A}=0\Rightarrow A=B, C=-2B$

$$1(x-1)+1(y-2)-2(z-0)=0$$
$$x+y-2z-3=0$$
Image is $(\alpha,\beta,\gamma)pt \equiv (1,2,6)$
$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=\frac{-2(1+2-12-3)}{6}$$
$$\frac{\alpha-1}{1}=\frac{\beta-2}{1}=\frac{\gamma-6}{-2}=4$$
$$\alpha=5,\beta=6,\gamma=-2 \Rightarrow \alpha^2+\beta^2+\gamma^2$$
$$=25+36+4=65$$

4. The statement ~ [pV(~ (p $\wedge$ q))] is equivalent to

(1) (~ (p $\wedge$ q)) $\wedge$ q (2) ~ (p $\vee$ q) (3) ~ (p $\wedge$ q) (4) (p $\wedge$ q) $\wedge$ (~p)

Sol. **(4)**
$$\sim[pv(\sim(p\wedge q))]$$
$$\sim p\wedge(p\wedge q)$$

5. Let $S=\left\{x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right):9^{1-\tan^2 x}+9^{\tan^2 x}=10\right\}$ and

$b=\sum_{x\in S}\tan^2\left(\frac{x}{3}\right)$, then $\frac{1}{6}$ $(\beta-14)^2$ is equal to

(1) 16 (2) 32 (3) 8 (4) 64

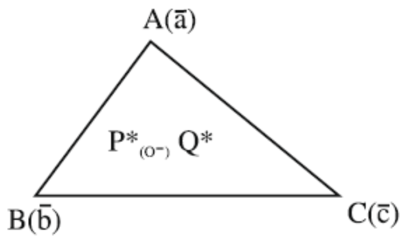
Sol. **(2)**

Let $9^{\tan^2 x}=P$
$$\frac{9}{P}+P=10$$
$$P^2-10P+9=0$$
$$(P-9)(P-1)=0$$
$$P=1,9$$
$$9^{\tan^2 x}=1, 9^{\tan^2 x}=9$$
$$\tan^2 x=0, \tan^2 x=1$$
$$x=0,\pm\frac{\pi}{4} \quad \therefore x\in\left(-\frac{\pi}{2},\frac{p}{2}\right)$$
$$\beta=\tan^2(0)+\tan^2\left(+\frac{\pi}{12}\right)+\tan^2\left(-\frac{\pi}{12}\right)$$
$$=0+2\left(\tan 15^\circ\right)^2$$
$$2(2-\sqrt{3})^2$$
$$2(7-4\sqrt{3})$$
Than $\frac{1}{6}(14-8\sqrt{3}-14)^2=32$

6. If the points P and Q are respectively the circumecenter and the orthocentre of a ΔABC, the $\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}$ is equal to

(1) $2\overrightarrow{QP}$ (2) $\overrightarrow{PQ}$ (3) $2\overrightarrow{PQ}$ (4) $\overrightarrow{PQ}$

Sol. **(4)**



$$\overrightarrow{PA}+\overrightarrow{PB}+\overrightarrow{PC}=\vec{a}+\vec{b}+\vec{c}$$

$$\overrightarrow{PG}=\frac{\vec{a}+\vec{b}+\vec{c}}{3}$$

$$\Rightarrow \vec{a}+\vec{b}+\vec{c}=3\overrightarrow{PG}=\overrightarrow{PQ}$$

Ans. (4)

7. Let A be the point (1,2) and B be any point onthe curve $x^2 + y^2 = 16$. If the centre of the locus of the point P, which divides the line segment AB in the ratio 3 : 2 is the point C (α,β) then the length of the line segment AC is

(1) $\frac{6\sqrt{5}}{5}$ (2) $\frac{2\sqrt{5}}{5}$ (3) $\frac{3\sqrt{5}}{5}$ (4) $\frac{4\sqrt{5}}{5}$

Sol. **(3)**

$$\frac{12\cos\theta+2}{5}=h \quad \Rightarrow \quad 12\cos\theta=5h-2$$

sq & add

$$144=(5h-2)^2+(5k-4)^2$$

$$\left(x-\frac{2}{5}\right)^2+\left(y-\frac{4}{5}\right)^2=\frac{144}{25}$$

$$\text{Centre}\equiv\left(\frac{2}{5},\frac{4}{5}\right)\equiv(\alpha,\beta)$$

$$AC=\sqrt{\left(1-\frac{2}{5}\right)^2+\left(2-\frac{4}{5}\right)^2}$$

$$=\sqrt{\frac{9}{25}+\frac{36}{25}}=\frac{\sqrt{45}}{5}=\frac{3\sqrt{5}}{5}$$

8. Let m be the mean and σ be the standard deviation of the distribution

x_i	0	1	2	3	4	5
f_i	$k+2$	$2k$	k^2-1	k^2-1	k^2+1	$k-3$

where $\sum f_i = 62$. If [x] denotes the greatest integer ≤ x, then [μ2 + σ^2] is equal to

(1) 8 (2) 7 (3) 6 (4) 9

Sol. **(1)**

$$\sum f_i = 62$$

$$3k^2+16k-12k-64=0$$

$$k=4 \text{ or } -\frac{16}{3} \text{(rejected)}$$

$$\mu = \frac{\sum f_i x_i}{\sum f_i}$$

$$\mu = \frac{8+2(15)+3(15)+4(17)+5}{62} = \frac{156}{62}$$

$$\sigma^2 = \sum f_i x_i^2 - \left(\sum f_i x_i\right)^2$$

$$= \frac{8\times1^2+15\times13+17\times16+25}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2 = \frac{500}{62} - \left(\frac{156}{62}\right)^2$$

$$\sigma^2+\mu^2 = \frac{500}{62}$$

$$\left[\sigma^2+\mu^2\right] = 8$$

9. If S_n = 4+ 11 + 21 + 34 + 50 + ...to n terms, then $\frac{1}{60}$ (S_{29} – S_9) is equal to

(1) 220 (2) 227 (3) 226 (4) 223

Sol. **(4)**

$S_n = 4+11+21+34+50+\ldots+n$ terms

Difference are in A.P.

Let $T_n = an^2+bn+c$

$T_1 = a+b+c = 4$

$T_2 = 4a+2b+c = 11$

$T_3 = 9a+3b+c = 21$

By solving these 3 equations

$$a = \frac{3}{2}, b = \frac{5}{2}, c = 0$$

So $T_n = \frac{3}{2}n^2+\frac{5}{2}n$

$S_n = \Sigma T_n$

$$= \frac{3}{2}\sum n^2 + \frac{5}{2}\sum n$$

$$= \frac{3}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2}\frac{(n)(n+1)}{2}$$

$$= \frac{n(n+1)}{4}[2n+1+5]$$

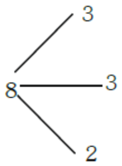
$$S_n = \frac{n(n+1)}{4}(2n+6) = \frac{n(n+1)(n+3)}{2}$$

$$\frac{1}{60}\left(\frac{29\times30\times32}{2} - \frac{9\times10\times12}{2}\right) = 223$$

10. Eight persons are tobe transported from city A to city B in three cars different makes. If each car can accomodate at most three persons, then the number of ways, in which they can be transported, is

(1) 1120 (2) 560 (3) 3360 (4) 1680

Sol. **(4)**



$\text{Ways} = \dfrac{8!}{3!3!2!2!} \times 3!$

$= \dfrac{8 \times 7 \times 6 \times 5 \times 4}{4}$

$= 56 \times 30$

$= 1680$

11. If A = $\dfrac{1}{5!6!7!}\begin{bmatrix} 5! & 6! & 7! \\ 6! & 7! & 8! \\ 7! & 8! & 9! \end{bmatrix}$, then |adj(adj(2A))| is equal to

(1) 2^{16} (2) 2^{8} (3) 2^{12} (4) 2^{20}

Sol. **(1)**

$|A| = \dfrac{1}{5!6!7!} 5!6!7! \begin{vmatrix} 1 & 6 & 42 \\ 1 & 7 & 56 \\ 1 & 8 & 72 \end{vmatrix}$

$R_3 \to R_3 \to R_2$

$R_2 \to R_2 \to R_1$

$|A| = \begin{vmatrix} 1 & 8 & 42 \\ 0 & 1 & 14 \\ 0 & 1 & 16 \end{vmatrix} = 2$

$|\text{adjadj}(2A)| = |2A|^{(n-1)^2}$

$= |2A|^4$

$= \left(2^3 |A|\right)^4$

$= 2^{12} |A|^4 \Rightarrow 2^{16}$

12. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(1) 13 (2) 20 (3) 10 (4) 5

Sol. **(4)**

$(22)^{2022} + (2022)^{22}$

divided byy 3

$(21+1)^{2022} + (2022)^{22}$

$= 3k + 1$

$(\alpha = 1)$

Divided by 7

$(21+1)^{2022} + (2023-1)^{22}$

$7k + 1 + 1 \qquad (\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

13. Let g(x) = f(x) + f(1-x) and $f^{n}(x) > 0$, $x \in (0,1)$. If g is decreasing in the interval $(0, \alpha)$ and increasing in the interval $(\alpha,1)$, then $\tan^{-1}(2\alpha) + \tan^{-1}\left(\frac{\alpha+1}{\alpha}\right)$ is equal to

(1) $\frac{5\pi}{4}$ (2) π (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

Sol. **(2)**

$g(x) = f(x) + f(1-x) \& f''(x) > 0, x \in (0,1)$

$g'(x) = f'(x) - f'(1-x) = 0$

$\Rightarrow f'(x) = f'(1-x)$

$x = 1 - x$

$x = \frac{1}{2}$

$g'(x) = 0$

at $x = \frac{1}{2}$

$g''(x) = f''(x) + f''(1-x) > 0$

g is concave up

hence $\alpha = \frac{1}{2}$

$\tan^{-1} 2\alpha + \tan^{-1}\frac{1}{\alpha} + \tan^{-1}\frac{\alpha+1}{\alpha}$

$\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3 = \pi$

14. For $\alpha, \beta, \gamma, \delta \in N$, if $\int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx = \frac{1}{\alpha}\left(\frac{x}{e}\right)^{\beta x} - \frac{1}{\gamma}\left(\frac{e}{x}\right)^{\delta x} + C$, where $e = \sum_{n=0}^{\infty}\frac{1}{n!}$ and C is constant of integration, then $\alpha + 2\beta + 3\gamma - 4\delta$ is equal to

(1) 4 (2) – 4 (3) – 8 (4) 1

Sol. **(1)**

$x = e^{\ln x}$

$\int\left(\left(\frac{x}{e}\right)^{2x} + \left(\frac{e}{x}\right)^{2x}\right)\log_e x\,dx = \int\left[e^{2(x\ln x - x)} + e^{-2(x\ln x - x)}\right]\ln x\,dx$

$x\ln x - x = t$

$\ln x \cdot dx = dt$

$\int\left(e^{2t} + e^{-2t}\right)dt$

$\frac{e^{2t}}{2} - \frac{e^{-2t}}{2} + C$

$= \frac{1}{2}\left(\frac{x}{e}\right)^{2x} - \frac{1}{2}\left(\frac{e}{x}\right)^{2x} + C$

$\alpha = \beta = \gamma = \delta = 2$

$\alpha + 2\beta + 3\gamma - 4\delta = 4$

15. Let f be a continuous function satisying $\int_0^{t^2}(f(x)+x^2)dx=\frac{4}{3}t^3, \forall t>0$. Then $f\left(\frac{\pi^2}{4}\right)$ is equal to

(1) $-\pi^2\left(1+\frac{\pi^2}{16}\right)$ (2) $\pi\left(1-\frac{\pi^3}{16}\right)$ (3) $-\pi\left(1+\frac{\pi^3}{16}\right)$ (4) $\pi^2\left(1-\frac{\pi^3}{16}\right)$

Sol. **(2)**

$$\int_0^{t^2}\left(f(x)+x^2\right)dx=\frac{4}{3}t^3, \forall t>0$$

$$\left(f\left(t^2\right)+t^4\right)=2t$$

$$f\left(t^2\right)=2t-t^4$$

$$t=\frac{\pi}{2}\Rightarrow f\left(\frac{\pi^2}{4}\right)=\frac{2\pi}{2}-\frac{\pi^4}{16}$$

$$=\pi-\frac{\pi^4}{16}=\pi\left(1-\frac{\pi^3}{16}\right)$$

16. Let a dic be rolled n times. Let the probability of getting odd numbers seven times be equal to the probability of getting odd numbers nine times. If the probability of getting even numbers twice is $\frac{k}{2^{15}}$, then k is equal to

(1) 60 (2) 30 (3) 90 (4) 15

Sol. **(1)**

$P(\text{odd number7times}) = P(\text{odd number9times})$

$${}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7}={}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$${}^nC_7={}^nC_9$$

$$\Rightarrow n=16$$

Required

$$P={}^{16}C_2\times\left(\frac{1}{2}\right)^{16}$$

$$=\frac{16\cdot 15}{2}\times\frac{1}{2^{16}}=\frac{15}{2^{13}}$$

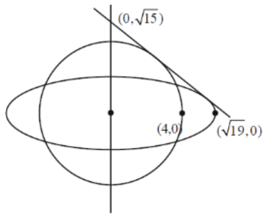
$$\Rightarrow \frac{60}{2^{15}}\Rightarrow k=60$$

17. Let a circle of radius 4 be concentric to the ellipse $15x^2 + 19y^2 = 285$. Then the common tangents are inclined to the minor axis ofthe ellipse at the angle.

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Sol. **(3)**

$$\frac{x^2}{19}+\frac{y^2}{15}=1$$



Let tang be

$y = mx \pm \sqrt{19m^2 + 15}$

$mx - y \pm \sqrt{19m^2 + 15} = 0$

Parallel from (0, 0) = 4

$\left| \dfrac{\pm\sqrt{19m^2 + 15}}{\sqrt{m^2 + 1}} \right| = 4$

$19m^2 + 15 = 16m^2 + 16$

$3m^2 = 1$

$m = \pm \dfrac{1}{\sqrt{3}}$

$\theta = \dfrac{\pi}{6}$ with x-axis

Required angle $\dfrac{\pi}{3}$.

18. Let $\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}, \vec{b} = 3\hat{i} + 5\hat{k}$ and $\vec{C} = \hat{i} + \hat{j} + 2\hat{k}$, Let $\vec{d}$ -be a vector which is perpendicular to both $\vec{a}$, and $\vec{b}$, and $\vec{c}.\vec{d} = 12$. The $\left(-\hat{i} + \hat{j} - \hat{k}\right).\left(\vec{c} \times \vec{d}\right)$ -is equal to

(1) 24 (2) 42 (3) 48 (4) 44

Sol. **(4)**

$\vec{a} = 2\hat{i} + 7\hat{j} - \hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{k}$

$\vec{c} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{d} = \lambda(\vec{a} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 7 & -1 \\ 3 & 0 & 5 \end{vmatrix}$

$\vec{d} = \lambda(35\hat{i} - 13\hat{j} - 21\hat{k})$

$\lambda(35 + 13 - 42) = 12$

$\lambda = 2$

$\vec{d} = 2(35\hat{i} - 13\hat{j} - 21\hat{k})$

$(\hat{i} + \hat{j} - \hat{k})(\vec{c} \times \vec{d})$

$= \begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & 2 \\ 70 & -26 & -42 \end{vmatrix} = 44$

19. Let S = $\left\{z = x + iy : \frac{2z-3i}{4z+2i} \text{ is a real number}\right\}$. Then which of the following is NOT correct ?

(1) y ∈ $\left(-\infty,-\frac{1}{2}\right)\bigcup\left(-\frac{1}{2},\infty\right)$

(2) $(x,y)=\left(0,-\frac{1}{2}\right)$

(3) x = 0

(4) $y + x^2 + y^2 \neq -\frac{1}{4}$

Sol. **(2)**

$$\frac{2z-3i}{4z+2i}\in R$$

$$\frac{2(x+iy)-3i}{4(x+iy)+2i}=\frac{2x+(2y-3)i}{4x+(4y+2)i}\times\frac{4x-(4y+2)i}{4x-(4y+2)i}$$

$$4x(2y-3)-2x(4y+2)=0$$

$$x=0 \qquad y\neq-\frac{1}{2}$$

Ans. = 2

20. Let the line $\frac{x}{1}=\frac{6-y}{2}=\frac{z+8}{5}$ intersect the lines $\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}$ and $\frac{x+3}{6}=\frac{3-y}{3}=\frac{z-6}{1}$ at the points A and B respectively. Then the distance of the mid-point of the line segment AB from the plane 2x – 2y + z = 14 is

(1) 3

(2) $\frac{10}{3}$

(3) 4

(4) $\frac{11}{3}$

Sol. **(3)**

$$\frac{x}{1}=\frac{y-6}{-2}=\frac{z+8}{5}=\lambda \qquad \ldots.(1)$$

$$\frac{x-5}{4}=\frac{y-7}{3}=\frac{z+2}{1}=\mu \qquad \ldots.(2)$$

$$\frac{x+3}{6}=\frac{y-3}{-3}=\frac{z-6}{1}=\gamma \qquad \ldots.(3)$$

Intersection of (1) & (2) "A"

$$(\lambda,-2\lambda+6,5\lambda-8)\&(4\mu+5,3\mu+7,\mu-2)$$

$$\lambda=1,\mu=-1$$

$$A(1,4,-3)$$

Intersection (1) & (3) "B"

$$(\lambda,-2\lambda+6,5\lambda-8)\&(6\gamma-3,-3\gamma+3,\gamma+6)$$

$$\lambda=3$$

$$\gamma=1$$

$$B(3,0,7)$$

Mod point of A & B ⇒ (2, 2, 2)

Perpendicular distance from the plane

$$2x-2y+z=14$$

$$\left|\frac{2(2)-2(2)+2-14}{\sqrt{4+4+1}}\right|=4$$

SECTION - B

21. The sum of all the four-digit numbers that can be formed using all the digits 2, 1, 2, 3 is equal to _______.

Sol. **(26664)**

$2,1,2,3$

$- \quad - \quad \underline{1} \quad \frac{3!}{2!}=3$

$- \quad - \quad \underline{2} \quad 3!=6$

$- \quad - \quad \underline{3} \quad \frac{3!}{2!}=3$

Sum of digits of unit place $=3\times1+6\times2+3\times3=24$

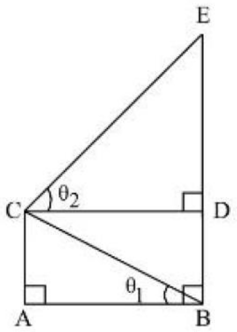
Required sum

$=24\times1000+24\times100+24\times10+24\times1$

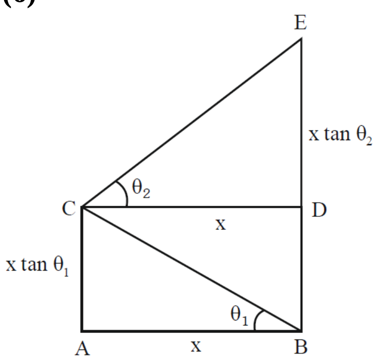
$=24\times1111$

= 26664

22. In the figure, $\theta_1+\theta_2=\frac{\pi}{2}$ and $\sqrt{3}$ (BE) = 4 (AB). If the area of ΔCAB is $2\sqrt{3}-3$ unit^2, when $\frac{\theta_2}{\theta_1}$ is the largest, thenthe perimeter (in unit) of ΔCED is equal to _______.



Sol. **(6)**



$\sqrt{3}BE=4AB$

$Ar(\Delta CAB)=2\sqrt{3}-3$

$\frac{1}{2}x^2\tan\theta_1=2\sqrt{3}-3$

$BE=BD+DE$

$=x(\tan\theta_1+\tan\theta_2)$

$BE=AB(\tan\theta_1+\cot\theta_1)$

$$\frac{4}{\sqrt{3}}\tan\theta_1+\cot\theta_1 \Rightarrow \tan\theta_1=\sqrt{3},\frac{1}{\sqrt{3}}$$

$$\theta_1=\frac{\pi}{6} \quad \theta_2=\frac{\pi}{3}$$

$$\theta_1=\frac{\pi}{3} \quad \theta_2=\frac{\pi}{6}$$

$$\text{as}\frac{\theta_2}{\theta_1}\text{is largest} \therefore \theta_1=\frac{\pi}{6}\theta_2=\frac{\pi}{3}$$

$$\therefore x^2=\frac{(2\sqrt{3}-3)\times 2}{\tan\theta_1}=\frac{\sqrt{3}(2-\sqrt{3})\times 2}{\tan\frac{\pi}{6}}$$

$$x^2=12-6\sqrt{3}=(3-\sqrt{3})^2$$

$$x=3-\sqrt{3}$$

Perimeter of ΔCED

$=CD+DE+CE$

$=3\sqrt{3}+(3-\sqrt{3})\sqrt{3}+(3-\sqrt{3})\times 2=6$

Ans. (6)

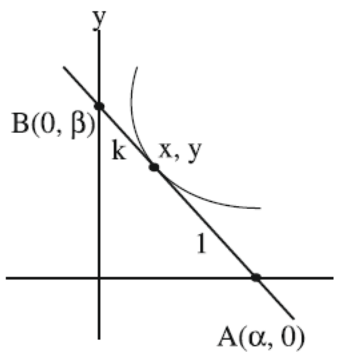
23. Let the tangent at any point P on a curve passing through the points (1,1) and $\left(\frac{1}{10},100\right)$, intersect positive x-axis and y-axis at the points A and B respectively . If PA : PB =1 : k and y = y (x) is the solution of the differential equation $e^{\frac{dy}{dx}} = kx+\frac{k}{2}$, y(0) = k, then 4y then 4y(1) – 5loge^3 is equal to________.

Sol. **(5)**

$$Y-y=\frac{dy}{dx}(X-x)$$

$$Y=0$$

$$X=\frac{-ydx}{dy}+x$$



$$\frac{k\alpha+0}{k+1}=x\,,\ \alpha=\frac{k+1}{k}x$$

$$\frac{k+1}{k}x=-y\frac{dx}{dy}+x$$

$$x+\frac{x}{k}=-y\frac{dx}{dy}+x$$

$$x\frac{dy}{dx}+ky=0$$
$$\frac{dy}{dx}+\frac{k}{x}y=0$$
$$y\cdot x^{k}=C$$
$$C=1$$
$$100\cdot\left(\frac{1}{10}\right)^{k}=1$$
$$K=2$$
$$\frac{dy}{dx}=\ln(2x+1)$$
$$y=\frac{(2x+1)}{2}(\ln(2x+1)-1)+c$$
$$2=\frac{1}{2}(0-1)+C$$
$$C=2+\frac{1}{2}=\frac{5}{2}$$
$$y(1)=\frac{3}{2}(\ell\ln 3-1)+\frac{5}{2}$$
$$=\frac{3}{2}\ln 3+1$$
$$4y(1)=6\ln 3+4$$
$$4y(1)-5\ln 3=4+\ln 3$$

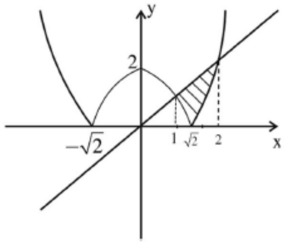
24. Suppose a_1, a_2, 2, a_3, a_4 be in an arithemetico-geometric progression. If the common ratio of the corresponding geometric progression in 2 and the sum of all 5 terms of the arithmetico-geometric progression is $\frac{49}{2}$, then a_4 is equal to________.

Sol. **(16)**

$$\frac{(a-2d)}{4},\frac{(a-d)}{2},a,2(a+d),4(a+2d)$$
$$a=2$$
$$\left(\frac{1}{4}+\frac{1}{2}+1+6\right)\times 2+(-1+2+8)d=\frac{49}{2}$$
$$2\left(\frac{3}{4}+7\right)+9d=\frac{49}{2}$$
$$9d=\frac{49}{2}-\frac{62}{4}=\frac{98-62}{4}=9$$
$$d=1$$
$$\Rightarrow a_4=4(a+2d)$$
$$=16$$

25. If the area of the region $\{(x,y) : |x^2 - 2| \le x\}$ is A, then $6A + 16\sqrt{2}$ is equal to ______.

Sol. **(27)**



$$A = \int_{1}^{\sqrt{2}} \left(x - \left(2 - x^2\right)\right)dx + \int_{\sqrt{2}}^{2} \left(x - \left(x^2 - 2\right)\right)dx$$

$$= \left(1 - 2\sqrt{2} + \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) + \left(2 - \frac{8}{3} + 4\right) - \left(1 - \frac{2\sqrt{2}}{3} + 2\sqrt{2}\right)$$

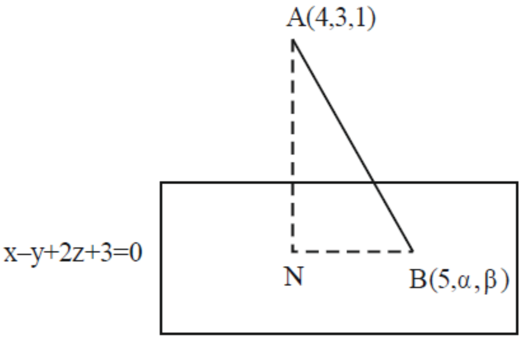
$$= -4\sqrt{2} + \frac{4\sqrt{2}}{3} + \frac{7}{6} + \frac{10}{3} = \frac{-8\sqrt{2}}{3} + \frac{9}{2}$$

$$6A = -16\sqrt{2} + 27 \therefore 6A + 16\sqrt{2} = 27$$

Ans. 27

26. Let the food of perpendicular from the point A (4, 3, 1) on the plane P : $x - y + 2z + 3 = 0$ be N. If B(5, α, β), $\alpha, \beta \in Z$ is a point on plane P such that the area of the triangle ABN is $3\sqrt{2}$, then $\alpha^2 + \beta^2 + \alpha\beta$ is equal to ________.

Sol. **(7)**



$$AN = \sqrt{6}$$

$$5 - \alpha + 2\beta + 3 = 0$$

$$\Rightarrow \alpha = 8 + 2\beta$$

N is given by

$$\frac{x-4}{1} = \frac{y-3}{-1} = \frac{z-1}{2} = \frac{-(4-3+2+3)}{1+1+4}$$

$$x = 3, y = 4, z = -1$$

N

$$(3, 4, -1)$$

$$BN = \sqrt{4 + (\alpha - 4)^2 + (\beta + 1)^2}$$

$$= \sqrt{4 + (2\beta + 4)^2 + (\beta + 1)^2}$$

$$\text{Area of } \Delta ABN = \frac{1}{2} AN \times BN = 3\sqrt{2}$$

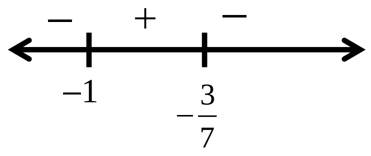
$$\frac{1}{2}\times\sqrt{6}\times BN = 3\sqrt{2}$$
$$BN = 2\sqrt{3}$$
$$4+(2\beta+4)^2+(\beta+1)^2 = 12$$
$$(2\beta+4)^2+(\beta+1)^2-8=0$$
$$5\beta^2+18\beta+9=0$$
$$(5\beta+3)(\beta+3)=0$$
$$\beta=-3$$
$$\alpha=2$$
$$\alpha^2+\beta^2+\alpha\beta=9+4-6=7$$

27. Let S be the set of values of λ, for which the system of equations
$6\lambda x - 3y + 3z = 4\lambda^2$,
$2x + 6\lambda y + 4z = 1$,
$3x + 2y + 3\lambda z = \lambda$ has no solution. Then $12\sum_{l\in S}|\lambda|$ is equal to ________.

Sol. **(24)**
$$\Delta = \begin{vmatrix} 6\lambda & -3 & 3 \\ 2 & 6\lambda & 4 \\ 3 & 2 & 3\lambda \end{vmatrix} = 0$$
$$2\lambda\left(9\lambda^2-4\right)+(3\lambda-6)+(2-9\lambda)=0$$
$$18\lambda^3-14\lambda-4=0$$
$$(\lambda-1)(3\lambda+1)(3\lambda+2)=0$$
$$\Rightarrow \lambda = 1, -1/3, -2/3$$
For each values of $\lambda, \Delta_1 = \begin{vmatrix} 6\lambda & -3 & 4\lambda^2 \\ 2 & 6\lambda & 1 \\ 3 & 2 & \lambda \end{vmatrix} \neq 0$
$$12\left(1+\frac{1}{3}+\frac{2}{3}\right)=24$$

28. If the domain of the function $f(x) = \sec^{-1}\left(\frac{2x}{5x+3}\right)$ is $[\alpha, \beta) \cup (\gamma, \delta]$, then $|3\alpha+10(\beta+\gamma)+21\delta|$ is equal to ______.

Sol. **(24)**
$$f(x)=\sec^{-1}\frac{2x}{5x+3}$$
$$\left|\frac{2x}{5x+3}\right|$$
$$\left|\frac{2x}{5x+3}\right|\geq 1 \Rightarrow |2x| \geq |5x+3|$$
$$(2x)^2-(5x+3)^2 \geq 0$$
$$(7x+3)(-3x-3)\geq 0$$



$$\therefore \text{ domain}\left[-1,\frac{-3}{5}\right)\cup\left(\frac{-3}{5},\frac{-3}{7}\right]$$

$$\alpha=-1,\beta=\frac{-3}{5},\gamma=\frac{-3}{5},\delta=\frac{-3}{7}$$

$$3\alpha+10(\beta+\gamma)+21\delta=-3$$

$$-3+10\left(\frac{-6}{5}\right)+\left(\frac{-3}{7}\right)21=-24$$

29. Let the quadratic curve passing through the point (–1,0) and touching the line y = x at (1,1) be y = f(x). Then the x-intercept of the normal to the curve at the point (α, α + 1) in the first quadrant is _______.

Sol. **(11)**

$$f(x)=(x+1)(ax+b)$$

$$1=2a+2b$$

$$f'(x)=(ax+b)+a(x+1)$$

$$1=(3a+b)$$

$$\Rightarrow b=1/4, a=1/4$$

$$f(x)=\frac{(x+1)^2}{4}$$

$$f'(x)=\frac{x}{2}+\frac{1}{2} \quad \alpha+1=\frac{(\alpha+1)^2}{4},\alpha>-1$$

$$\alpha+1=4$$

$$\alpha=3$$

normal at (3, 4)

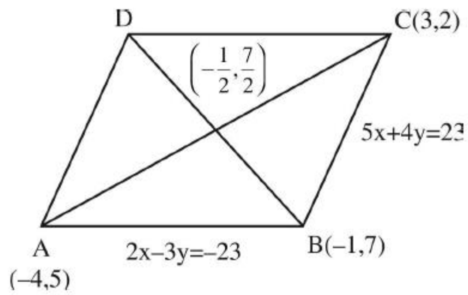
$$y-4=-\frac{1}{2}(x-3)$$

$$y=0 \quad x=8+3$$

Ans. 11

30. Let the equations of two adjacent sides of a parallelogram ABCD be 2x–3y =–23 and 5x + 4y = 23. If the equation of its one diagonal AC is 3x + 7y = 23 and the distance of A from the other diagonal is d, then 50 d^2 is equal to _______.

Sol. **(529)**



A & C point will be (–4, 5) & (3, 2)

mid point of AC will be $\left(-\frac{1}{2},\frac{7}{2}\right)$

equation of diagonal BD is

$$y-\frac{7}{2}=\frac{\frac{7}{2}}{-\frac{1}{2}}\left(x+\frac{1}{2}\right)$$

$\Rightarrow \quad 7x+y=0$

Distance of A from diagonal BD

$=d=\frac{23}{\sqrt{50}}$

$\Rightarrow \quad 50d^2=(23)^2$

$50d^2=529$

SECTION - A

31. Given below are two statements:

Statement I : Rotation of the earth shows effect on the value of acceleration due to gravity (g)

Statement II : The effect of rotation of the earth on the value of 'g' at the equator is minimum and that at the pole is maximum.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both Statement I and Statement II are true (2) Both Statement I and Statement II are false

(3) Statement I is false but statement II is true (4) Statement I is true but statement II is false

Sol. (4)

Due to rotation of earth, $g_{eff} = g - \omega^2 R \cos^2 \theta$

Where 'θ' is angle made with equator

Also, At poles, $\theta = 90°$

$\Delta g = \omega^2 R \cos^2\theta$

$= \omega^2 R \cos^2 90 = 0$

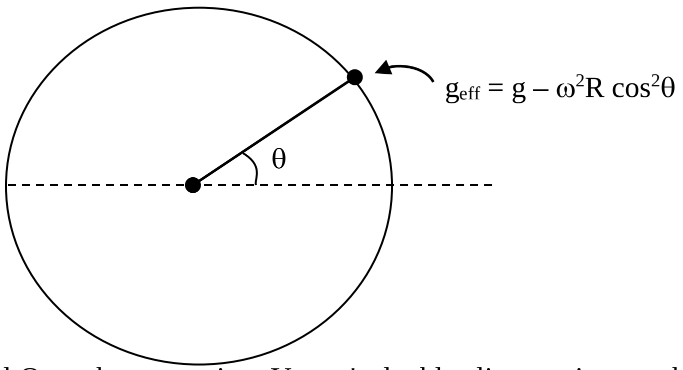
[no effect on poles]

$g_{eff} = g - \omega^2 R \cos^2\theta$

for equator $\theta = 0°$

So, $g_{eff} = g - w^2 R$

& $\Delta g = \omega^2 R$ (Which is maximum change)



32. The ratio of intensities at two points P and Q on the screen in a Young's double slit experiment where phase difference between two waves of same amplitude are π/3 and π/2, respectively are

(1) 3:2 (2) 3:1 (3) 2:3 (4) 1:3

Sol. (1)

$I_{res} = 4I_o \cos^2\left(\frac{\theta}{2}\right)$

If $\theta = \frac{\pi}{3}$, $I_{res} = 4I_o . \cos^2\left[\frac{\pi}{6}\right]$

$= 4I_o . \left(\frac{\sqrt{3}}{2}\right)^2$

$I_1 = (4I_o)\left(\frac{3}{4}\right) = 3I_o$

If $\theta = \frac{\pi}{2}$, $I_{res} = 4I_o . \cos^2\left(\frac{\pi}{2}\right)$

$= 4I_o \left(\frac{1}{\sqrt{2}}\right)^2$

$= (4I_o)\left(\frac{1}{2}\right)$

$I_2 = 2I_o$

$\frac{I_1}{I_2} = \frac{3}{2}$

33. The time period of a satellite, revolving above earth's surface at a height equal to R will be

(Given g = π^2 m/s^2, R = radius of earth)

(1) $\sqrt{32R}$ (2) $\sqrt{4R}$ (3) $\sqrt{2R}$ (4) $\sqrt{8R}$

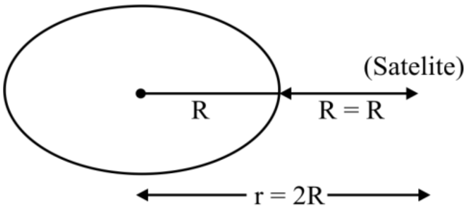
Sol. **(1)**

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T^2 = \frac{4\pi^2 (2R)^3}{GM}$$

$$= 4\times 8\times \frac{\pi 2\times R^3}{GM}$$

$$= 4\times 8\times \frac{(g)(R^3)}{GR^2}$$



At surface of earth $\Rightarrow$ $g = \frac{GM}{R^2}$

So, GM = g. R^2

Also π^2 = g

T^2 = 32R

$\Rightarrow$ $T = \sqrt{32R}$

34. In a metallic conductor, under the effect of applied electric field, the free electrons of the conductor

(1) Move with the uniform velocity throughout from lower potential to higher potential

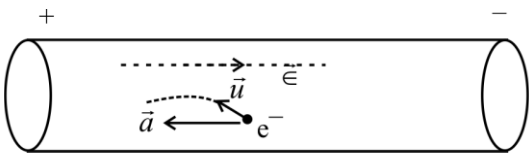
(2) Move in the curved paths from lower potential to higher potential

(3) Move in the straight line paths in the same direction

(4) Drift from higher potential to lower potential.

Sol. **(2)**

Electrons moves in curved path because there velocity $\vec{u}$ may make any angle θ with acceleration $\vec{a}$ between time interval of two successive collisions.



Also electron moves from lower potential to higher potential.

35. A message signal of frequency 3kHz is used to modulate a carrier signal of frequency 1.5 MHz. The bandwidth of the amplitude modulated wave is

(1) 6 kHz (2) 3 kHz (3) 6 MHz (4) 3 MHz

Sol. **(1)**

Bond width of Amplitude modulated signal (AM) = 2 × $f_{(\text{message signal})}$

= 2 × 3 KHz

= 6 KHz

36. In an experiment with vernier calipers of least count 0.1 mm, when two jaws are joined together the zero of vernier scale lies right to the zero of the main scale and 6^{th} division of vernier scale coincides with the main scale division. While measuring the diameter of a spherical bob, the zero of vernier scale lies in between 3.2 cm and 3.3 cm marks, and 4^{th} division of vernier scale coincides with the main scale division. The diameter of bob is measured as

(1) 3.25 cm (2) 3.22 cm (3) 3.18 cm (4) 3.26 cm

Sol. **(3)**

The zero error in verniel scale is = 6 × 0.1mm = 0.6 mm (+ve zero error)

Note: +ve zero error will have to be subtracted

From the reading of the object.

Now, the diameter measured with the help of Vernier scale is

Given by → M.S.D + V.S.D × L.S

$\Rightarrow$ 3.2 cm + 0.1 mm × 4

= 3.24 cm

The actual diameter is ⇒ 3.24 mm – (zero error) = 3.24 – 0.6 = 3.18 cm

37. Two projectiles are projected at 30° and 60° with the horizontal the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(1) $2 : \sqrt{3}$ (2) $1 : \sqrt{3}$ (3) $\sqrt{3} : 1$ (4) 1 : 3

Sol. **(4)**

In projectile motion, $H_{many} = \dfrac{u^2 \sin^2 \theta}{2g}$

at θ = 30°, $H_1 = \dfrac{u^2 \sin^2 30°}{2g}$

at θ = 60°, $H_2 = \dfrac{u^2 \sin^2 60°}{2g}$

$$\frac{H_1}{H_2} = \frac{\sin^2 30°}{\sin^2 60°} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{3}$$

38. Given below are two statements : one is labelled as **Assertion A** and then other is labelled as **Reason R**

Assertion A : An electric fan continues to rotate for some time after the current is switched off.

Reason R : Fan continues to rotate due to inertia of motion.

In the light of above statements, choose the **most appropriate** answer from the options given below.

(1) A is not correct but **R** is correct

(2) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(3) Both **A** and **R** are correct but **R** is **NOT** the correct explanation of **A**

(4) A is correct but **R** is not correct

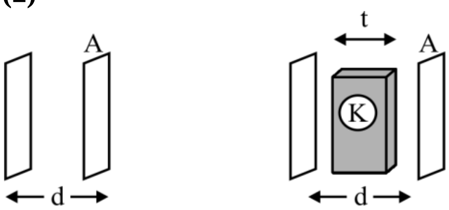
Sol. **(2)**

Inertia is the property of mass due to which the object continues to move until any external force do not stops it. In the case of rotation of fan, if we switch off then also it moves for some time as air resistance takes time to stop it and due to inertia of fan it moves for some time.

39. The distance between two plates of a capacitor is d and its capacitance is C_1, when air is the medium between the plates. If a metal sheet of thickness $\frac{2d}{3}$ and of the same area as plate is introduced between the plates, the capacitance of the capacitor becomes C_2. The ratio $\frac{C_2}{C_1}$ is

(1) 4 : 1 (2) 3 : 1 (3) 2 : 1 (4) 1 : 1

Sol. **(2)**



$t = \frac{2d}{3}$

$K = \infty$ for metals

$C_1 = \frac{\varepsilon_0 A}{d}$

$C_2 = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - \frac{2d}{3} + 0} = \frac{3\varepsilon_0 A}{d}$

$= \frac{C_2}{C_1} = \frac{\frac{3\varepsilon_0 A}{d}}{\frac{\varepsilon_0 A}{d}} = \frac{3}{1}$

40. The amplitude of magnetic field in an electromagnetic wave propagating along y-axis is 6.0×10^{-7} T. The maximum value of electric field in the electromagnetic wave is

(1) 2×10^{15} Vm^{-1} (2) 2×10^{14} Vm^{-1} (3) 6.0×10^{-7} Vm^{-1} (4) 180 Vm^{-1}

Sol. **(4)**

In electromagnetic wave, $E_0 = B_0 C$

$E_0 = 6 \times 10^{-7} \times 3 \times 10^8$

$= 18 \times 10^1$

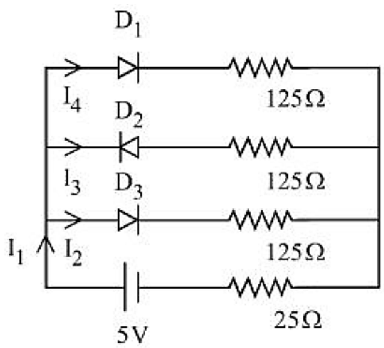
$= 180 \ {}^{V}/_{m}$

$E_0 \rightarrow$ Amplitude of electric field

$B_0 \rightarrow$ Amplitude of magnetic field

$C \rightarrow$ Speed of light

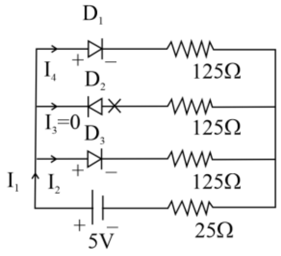
41. If each diode has a forward bias resistance of 25 Ω in the below circuit,



Which of the following options is correct:

(1) $\frac{I_1}{I_2} = 2$ (2) $\frac{I_2}{I_3} = 1$ (3) $\frac{I_3}{I_4} = 1$ (4) $\frac{I_1}{I_2} = 1$

Sol. **(1)**



Here we can see that D_1 and D_3 conducts but D_2 is reversed biased.

Current I_1 will be equally distributed among I_3 and I_4 and $I_3 = 0$

$I_1 = I_2 + I_4 + I_3$

$I_1 = 2I_2$

$$\frac{I_1}{I_2} = 2$$

42. A gas mixture consists of 2 moles of oxygen and 4 moles of neon at temperature T. Neglecting all vibrational modes, the total internal energy of the system will be,

(1) 4RT (2) 11RT (3) 8RT (4) 16RT

Sol. **(2)**

Internal energy of O_2 = $\frac{5}{2}nRT = \frac{5}{2} \times 2RT = 5RT$

Internal energy of Ne $= nRT = \frac{3}{2} \times nRT = \frac{3}{2} \times 4RT = 6RT$

Total energy of mixture (system) = 5RT + 6RT = 11RT

43. For a periodic motion represented by the equation y = sin ωt + cos ωt the amplitude of the motion is

(1) 0.5 (2) 1 (3) 2 (4) $\sqrt{2}$

Sol. **(4)**

If equation of SHM is in the form y = a sin(ωt) + Bcos(ωt)

Then its amplitude is = $\sqrt{A^2 + B^2}$

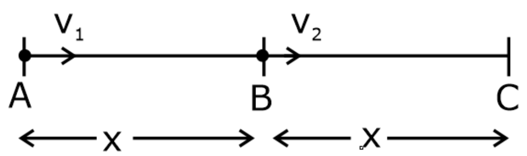
Here A = B = 1 in equation y = sin(ωt) + (cost)

Therefore, Amplitude = $\sqrt{(1)^2 + (1)^2} = \sqrt{2}$

44. A person travels x distance with velocity v_1 and then x distance with velocity v_2 in the same direction. The average velocity of the person is v, then the relation between v, v_1 and v_2 will be.

(1) $v = v_1 + v_2$ (2) $\frac{1}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (3) $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$ (4) $v = \frac{v_1 + v_2}{2}$

Sol. **(3)**



Time taken b/w A & B $\Rightarrow t_1 = \frac{x}{v_1}$

Time between b/w B & C $\Rightarrow t_2 = \frac{x}{v_2}$

Average velocity (v) = $\frac{\text{Total displacement}}{\text{Total time}} = \frac{x+x}{t_1+t_2} = \frac{2x}{\frac{x}{v_1}+\frac{x}{v_2}}$

(v) = $\frac{2v_1v_2}{v_1+v_2}$ or $\frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$

45. The half life of a radioactive substance is T. The time taken, for disintegrating $\frac{7}{8}$th part of its original mass will be:

(1) T (2) 2T (3) 3T (4) 8T

Sol. **(3)**

If $\frac{7}{8}$th is disintegrated it means only $\frac{1}{8}$th part is radioactive active no. of nuclears after 'n' half lives

$\Rightarrow \frac{N_o}{2^n} = \frac{N_o}{8}$

$2^n = 8 = n = 3$

So, the elapsed is 3 half lives = 3T

46. A gas is compressed adiabatically, which one of the following statement is <u>NOT</u> true.

(1) There is no change in the internal energy

(2) The temperature of the gas increases.

(3) The change in the internal energy is equal to the work done on the gas

(4) There is no heat supplied to the system

Sol. (1)

In Adiabatic process, $\Delta Q = 0$

If gas is compressed, then w (by gas) $\neq 0$

Therefore by 1st law $\Delta Q = \Delta u + w$

$0 = \Delta u + w$

$\Delta u = -w \neq 0$

It implies in adiabatic compression, internal energy of gas changes.

47. Given below are two statements:

Statement I : For diamagnetic substance, $-1 \le X < 0$, where X is the magnetic susceptibility.

Statement II : Diamagnetic substances when placed in an external magnetic field, tend to move from stronger to weaker part of the field.

In the light of the above statements, choose the correct answer from the options given below

(1) Both Statement I and Statement II are false

(2) Statement I is incorrect but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is correct but Statement II is false

Sol. **(3)**

Diamagnetic substances have the property due to which they tends to move away from stronger magnetic field to weaker magnetic field, as their magnetic susceptibility is negative.

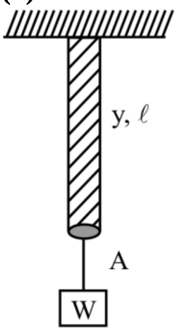
Therefore both statements are correct.

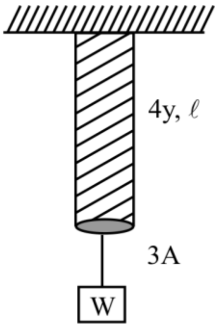
48. Young's moduli of the material of wires A and B are in the ratio of 1:4, while its area of cross sections are in the ratio of 1:3. If the same amount of load is applied to both the wires, the amount of elongation produced in the wires A and B will be in the ratio of

[Assume length of wires A and B are same]

(1) 12 : 1 (2) 1 : 36 (3) 1 : 12 (4) 36 : 1

Sol. **(1)**



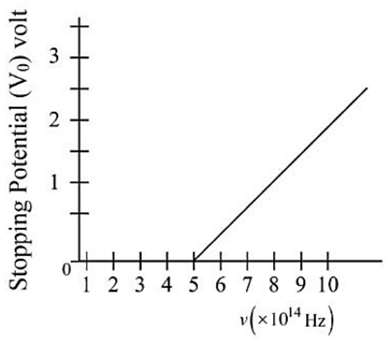


$\frac{W}{A} = Y.\frac{\Delta\ell_1}{\ell}$ $\frac{W}{3A} = 4Y.\frac{\Delta\ell_2}{\ell}$

$\Delta l_1 = \frac{W\ell}{AY}$ $\Delta l_2 = \frac{W\ell}{12AY}$

$\frac{\Delta\ell_1}{\Delta\ell_2} = \frac{12}{1}$

49. The variation of stopping potential (V_0) as a function of the frequency (ν) of the incident light for a metal is shown in figure. The work function of the surface is



(1) 2.07 eV (2) 18.6 eV (3) 2.98 eV (4) 1.36 eV

Sol. **(1)**

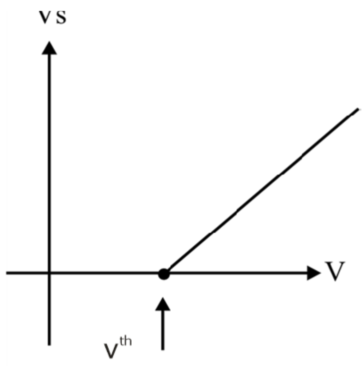
Work function $(\phi) = h\nu^{th}$

$= 6.6 \times 10^{-34} \times 5 \times 10^{14}$

$= 33 \times 10^{-20}$

$\phi = 3.3 \times 10^{-19}$ J

$= \dfrac{3.3\times10^{-19}}{1.6\times10^{-19}} \text{ev} \Rightarrow 2.07$



50. A bar magnet is released from rest along the axis of a very long vertical copper tube. After some time the magnet will

(1) Oscillate inside the tube

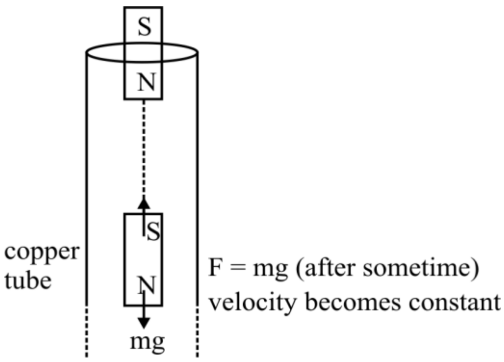
(2) Move down with an acceleration greater than g

(3) Move down with almost constant speed

(4) Move down with an acceleration equal to g

Sol. **(3)**

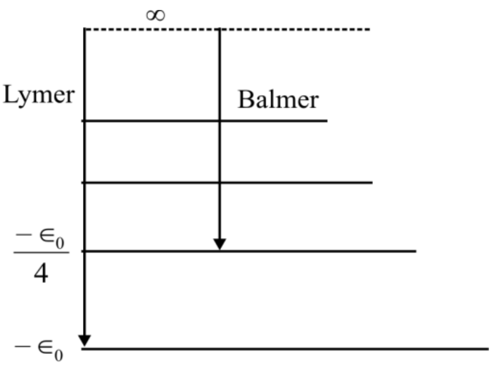
According to lenz's law, the rate of charge of flum produced by bar magnet will be approused by the conducting loops.



SECTION – B

51. If 917 Å be the lowest wavelength of Lyman series then the lowest wavelength of Balmer series will be _________ Å.

Sol. (3668)



Lowest wavelength of by may sense will be obtained for trasition $n = \infty \longrightarrow n = 1$

and for balmer series, Lyman Series n = $\infty \longrightarrow$ n = 2

for Lyman, $E_0 = \dfrac{hC}{917Å}$

for balmer, $\dfrac{E_0}{4} = \dfrac{hC}{\lambda(Å)}$

using this

$\lambda = 917 \times 4 = 3668$

52. A square loop of side 2.0 cm is placed inside a long solenoid that has 50 turns per centimeter and carries a sinusoidally varying current of amplitude 2.5 A and angular frequency 700 rad s^{-1}. The central axes of the loop and solenoid coincide. The amplitude of the emf induced in the loop is x × 10^{-4} V. The value of x is _______ .

(Take, $\pi = \dfrac{22}{7}$)

Sol. **(44)**

emf induced in solenoid $\Rightarrow$ BAWN sin(ωt), w = 700 rad/s

Amplitude $\Rightarrow$ BAWN

Area (A) = 2cm × 2 cm

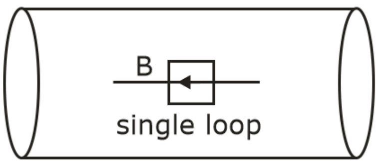
= 4 cm^2

= 4 × 10^{-4} m^2

$(B)_{solenoid} = \mu_0 ni$

= $4\pi \times 10^{-7} \times 5000 \times 2.5$

= $5\pi \times 10^{-3}$



(N = 1)

n = $\dfrac{50 \text{ turns}}{\text{cm}} = \dfrac{5000}{\text{m}}$

i = 2.5

Amplitude of emf = $(5p \times 10^{-3})(4 \times 10^{-4})(700)(1)$

$= 5 \times \dfrac{22}{7} \times 4 \times 700 \times 10^{-7} = 44 \times 10^{-4}$

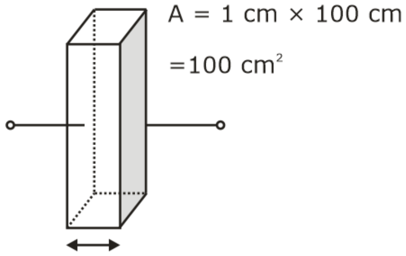
53. A rectangular parallelepiped is measured as 1 cm × 1 cm × 100 cm. If its specific resistance is 3 × 10^{-7} Ω m, then the resistance between its tow opposite rectangular faces will be __________ × 10^{-7} Ω.

Sol. **(3)**

$\rho = 3 \times 10^{-7}$ Ω-cm

$R = \rho . \dfrac{l}{A}$

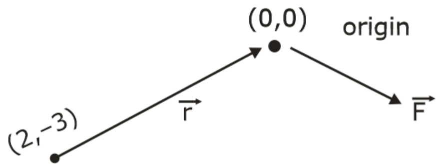
$= \dfrac{3\times10^{-7}\times(10^{-2}m)}{(100\times10^{-4}m^2)} = 3 \times 10^{-7}$



54. A force of $-P\hat{k}$ acts on the origin of the coordinate system. The torque about the point (2, –3) is $P(a\hat{i} + b\hat{j})$, The ratio of $\dfrac{a}{b}$ is $\dfrac{x}{2}$. The value of x is -

Sol. **(3)**

$\vec{\tau} = \vec{r} \times \vec{F}$



$\vec{\tau} = \text{head} - \text{tail}$

$= (0-2)\hat{i} + (0(-3))\hat{j}$

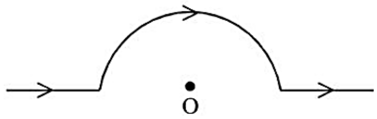
$= -2\hat{i} + 3\hat{j}$

$\tau = (-2\hat{i} + 3\hat{j}) \times (-p\hat{k})$

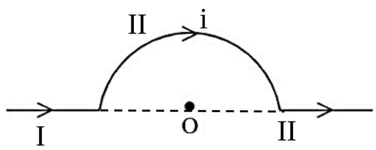
$= -2p\hat{j} - 3p\hat{i}$

$= -p(3p\hat{i} + 2p\hat{j})$ $\frac{a}{b} = \frac{3}{2} = \frac{x}{2}$ x = 3

55. A straight wire carrying a current of 14 A is bent into a semicircular arc of radius 2.2 cm as shown in the figure. The magnetic field produced by the current at the centre (O) of the arc. is __________ $\times 10^{-4}$T



Sol. **(2)**



$B_{total} = B_I + B_{II} + B_{III}$

$B_I = 0$

$B_{III} = 0$

Because $\vec{dl} \times \vec{r} = 0$

Now, magnetic field due to semicirclaur ring at its center is given by

$B_{II} = \frac{\mu_0 i}{4R}$

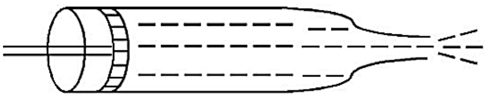
$= \frac{4\pi \times 10^{-7} \times 14}{4 \times 2.2 \times 10^{-2}}$

$= \frac{22}{7} \times \frac{10^{-7} \times 14}{22 \times 10^{-3}}$

$= 2 \times 10^{-7}$

$= 2$

56. Figure below shows a liquid being pushed out of the tube by a piston having area of cross section 2.0 cm^2. The area of cross section at the outlet is 10 mm^2. If the piston is pushed at a speed of 4 cm s^{-1}, the speed of outgoing fluid is _________ cm s^{-1}



Sol. **(80)**

By equation of continuity

$A_1V_1 = A_2V_2$

$(2\ cm^2)\ (4\ cm/s) = (10 \times 10^{-2}\ cm^2)\ (v)$

$$\frac{8cm^3}{s} = 10^{-1} cm^2 (v)$$

V = 80 cm/s

57. A rectangular block of mass 5 kg attached to a horizontal spiral spring executes simple harmonic motion of amplitude 1 m and time period 3.14 s. The maximum force exerted by spring on block is __________N.

Sol. **(20)**

When an object executes S.H.M, its morning acceleration is given by $a_{max} = \omega^2 A$

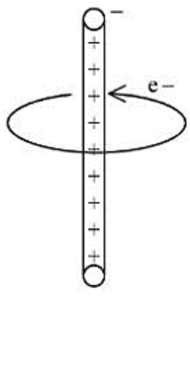
Where $\omega = \frac{2\pi}{T}$

Therefore, $a_{max} = \frac{4\pi^2 A}{T^2}$

(Max force) $F_{max} = ma_{max} = 5 \times \frac{4 \times 3.14 \times 3.14}{3.14 \times 3.14} \times (1)$

= 20 N

58. An electron revolves around an infinite cylindrical wire having uniform linear charge density 2×10^{-8} C m^{-1} in circular path under the influence of attractive electrostatic field as shown in the figure. The velocity of electron with which it is revolving is ________ $\times 10^6$ m s^{-1}. Given mass of electron = 9×10^{-31} kg



Sol. **(8)**

In uniform circular motion

$F_c = ma_c$

$$(q)(E) = \frac{mv^2}{r}$$

$$(e)\left(\frac{2k\lambda}{r}\right) = \frac{mv^2}{r}$$

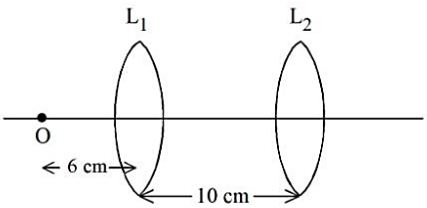
$$v^2 = \frac{(e)(2k\lambda)}{m} = \frac{(1.6 \times 10^{-19}) \times 2 \times (9 \times 10^9) \times (2 \times 10^{-8})}{9 \times 10^{-31}}$$

$= 1.6 \times 4 \times 10^{13}$

$V^2 = 16 \times 4 \times 10^{12} \Rightarrow v = 8 \times 10^6$ m/s

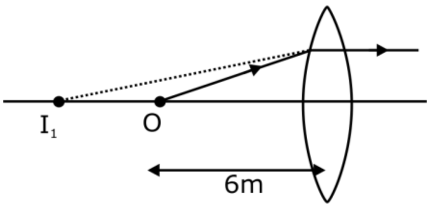
Ans. 8

59. A point object, 'O' is placed in front of two thin symmetrical coaxial convex lenses L_1 and L_2 with focal length 24 cm and 9 cm respectively. The distance between two lenses is 10 cm and the object is placed 6 cm away from lens L_1 as shown in the figure. The distance between the object and the image formed by the system of two lenses is __________ cm.



Sol. **(34)**

Due to lens L_1



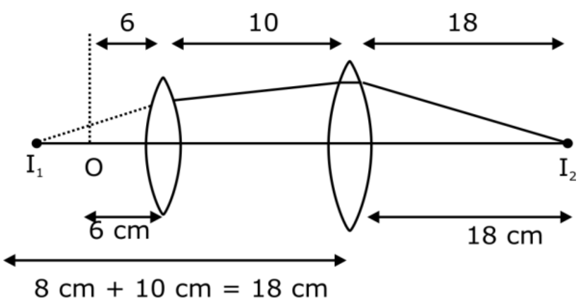
u = –6m

f = +24m

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{24}-\frac{1}{6}=\frac{1-4}{24}\ \Rightarrow v=-8\text{m}$$

Due to lens L_2



U = –18m

F = +9m

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\ x$$

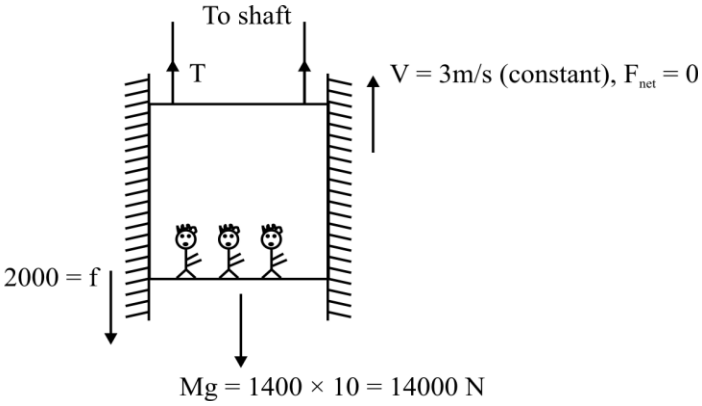
$$\frac{1}{v}-\frac{1}{9}=\frac{1}{18}$$

$$V = \frac{1}{v}=\frac{2-1}{18}$$

V = 18m

60. If the maximum load carried by an elevator is 1400 kg (600 kg – Passengers + 800 kg - elevator), which is moving up with a uniform speed of 3 m s^{-1} and the frictional force acting on it is 2000 N, then the maximum power used by the motor is __________ kW (g = 10 m/s^2)

Sol. **(48)**



Tension in the string $\Rightarrow$ 16000 N

Maximum power = (F)(V)

= 16000 × 3

= 48000

= 48 kw

Ans. 48

SECTION – A

Solid State Easy

61. The correct relationships between unit cell edge length ' a ' and radius of sphere ' r ' for face-centred and body-centred cubic structures respectively are:

(1) $2\sqrt{2}r = a$ and $\sqrt{3}r = 4a$
(2) $r = 2\sqrt{2}a$ and $4r = \sqrt{3}a$
(3) $r = 2\sqrt{2}a$ and $\sqrt{3}r = 4a$
(4) $2\sqrt{2}\,r = a$ and $4r = \sqrt{3}a$

Sol. 4

FCC BCC

$\sqrt{2}a = 4r$ $\sqrt{3}a = 4r$

$a = \frac{4r}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$a = 2\sqrt{2}r$

Chemistry in Everyday life Medium

62. The reaction used for preparation of soap from fat is :

(1) an addition reaction
(2) an oxidation reaction
(3) alkaline hydrolysis reaction
(4) reduction reaction

Sol. 3

The process of making is soap is saponification.

Ester + Base $\longrightarrow$ Alcohol + Soap

In saponification, triglycerides are combine with strong base and form fatty acid so this is alkaline Hydrolysis reaction.

Mole Easy

63. Match List I with List II

LIST I		LIST II	
A	16 g of CH_4 (g)	I.	Weight 28 g
B	1 g of H_2 (g)	II	60.2×10^{23} electrons
C	1 mole of N_2 (g)	III	Weight 32 g
D	0.5 mol of SO_2 (g)	IV	Occupies 11.4 L volume at STP

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III
(2) A-II, B-IV, C-III, D-I
(3) A-II, B-III, C-IV, D-I
(4) A-I, B-III, C-II, D-IV

Sol. 1

16g CH_4 = mole = 1

e– = 60.0×10^{23}

19Hz = 0.5mole = 11.4(L) STP

1 mole N_2 = 2rg

0.5 mol SO_2 = weights 32g.

Periodic Table Medium

64. The correct order of metallic character is =

(1) K>Be>Ca
(2) Be>Ca>K
(3) K>Ca>Be
(4) Ca>K>Be

Sol. **3**

Be
$\wedge$
K > Ca $\rightarrow$ In period Metallic Character decreases

$\downarrow$ In group metallic character Increases

So $K > Ca > Be$

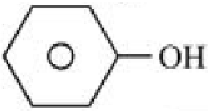
Metallic character decreases

GOC **Medium**

65. The correct order for acidity of the following hydroxyl compound is :

A. CH_3OH

B. $(CH_3)_3COH$

C. 

D. 

E. 

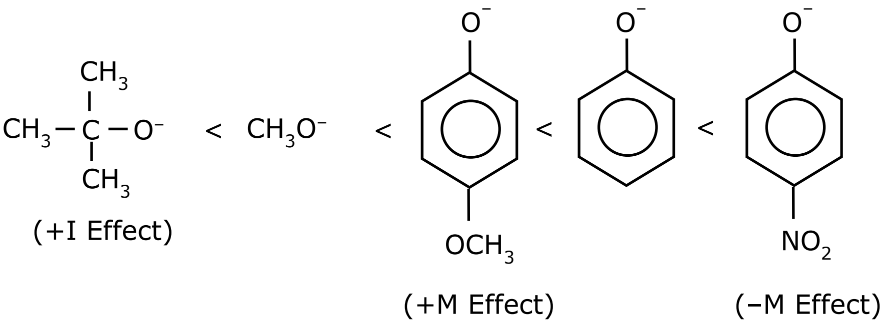
Choose the correct answer from the options given below:

(1) E > C > D > A > B
(2) D > E > C > A > B
(3) E > D > C > B > A
(4) C > E > D > B > A

Sol. **1**

Acidity $\propto$ stability of conjugate base

Stability order



Activity $\rightarrow$ E > C > D > A > B

Coordination Compound **Medium**

66. Match List I with List II

LIST I Complex		**LIST II Crystal Field splitting energy (Δ_0)**	
A	$[Ti(H_2O)_6]^{2+}$	I.	– 1.2
B	$[V(H_2O)_6]^{2+}$	II	– 0.6
C	$[Mn(H_2O)_6]^{3+}$	III	0
D	$[Fe(H_2O)_6]^{3+}$	IV	– 0.8

Choose the correct answer from the options given below:

(1) A-IV, B-I, C-II, D-III
(2) A-IV, B-I, C-III, D-II
(3) A-II, B-IV, C-III, D-I
(4) A-II, B-IV, C-I, D-III

Sol. **1**

$\left[Ti\left(H_2O\right)_6\right]^{+2}$ $\left[V\left(H_2O\right)_6\right]^{+2}$ $\left[Mn\left(H_2O\right)_6\right]^{+3}$ $\left[Fe\left(H_2O\right)_6\right]^{3+}$

$\downarrow$

Ti^{+2}	V^{+2}	Mn^{+3}	Fe^{+3}
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$3d^2$	$3d^3$	$3d^4$	$3d^5$

CFSE = $-0.4 \times t_{2g} + 0.6 \times eg + xp$

$= -0.4 \times 2 + 0.6 \times 0 + xp$

(A) $= -0.8 \rightarrow Ti^{+2}$

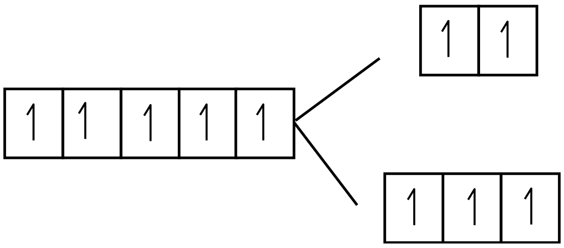
(B) $V^{+2} \rightarrow 3d^3$

CFSE $= -0.4 \times t_{2g} + 0.6 \times eg + xp$

$= -0.4 \times 3 + 0.6 \times 0 + xp$

$= -1.2$

(C)

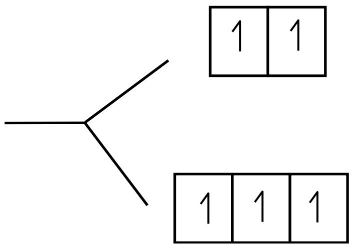
$Mn^{+3} \rightarrow 3d^4$ 

CFSE = $-0.4 \times t_{2g} + 0.6 \times eg + xp$

$-0.4 \times 3 + 0.6 \times 1 + xp$

$= -1.2 + 0.6 = 0.6$

(D)

$Fe(H_2O)_6]^{+3} \rightarrow Fe^{+3} \rightarrow 3d^5$ 

CFSE $= -0.4 \times t_{2g} + 0.6 \times eg + xp$

$= -0.4 \times 3 + 0.6 \times 2$

$= -1.2 + 1.2$

$= 0$

Qualitative analysis **Medium**

67. In Carius tube, an organic compound ' X ' is treated with sodium peroxide to form a mineral acid ' Y '.The solution of $BaCl_2$ is added to ' Y ' to form a precipitate ' Z '.' Z ' is used for the quantitative estimation of an extra element. ' X ' could be

(1) Chloroxylenol

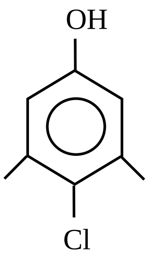
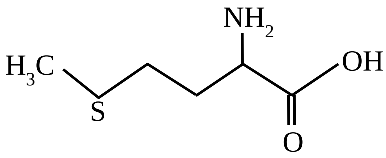
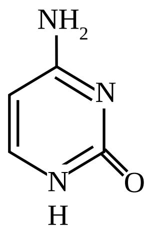
(2) Methionine

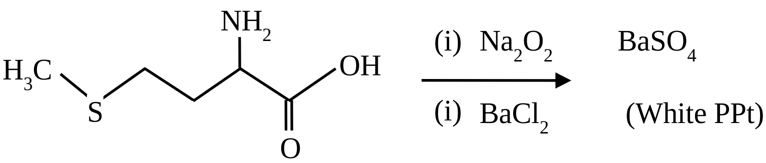
(3) A nucleotide

(4) Cytosine

Sol. 2

Carious method is used for quantitative analysis of sulfur

Chloroxylenol	Methionine	Nucleotide	Cylosine
		Phosphate+ Sugar+ Nitrogeneous base	



So Methionine is correct answer

S-block **Medium**

68. Number of water molecules in washing soda and soda ash respectively are:

(1) 1 and 0 (2) 1 and 10 (3) 10 and 0 (4) 10 and 1

Sol. **3**

Washing Soda $\rightarrow Na_2CO_3.\underline{10}H_2O$

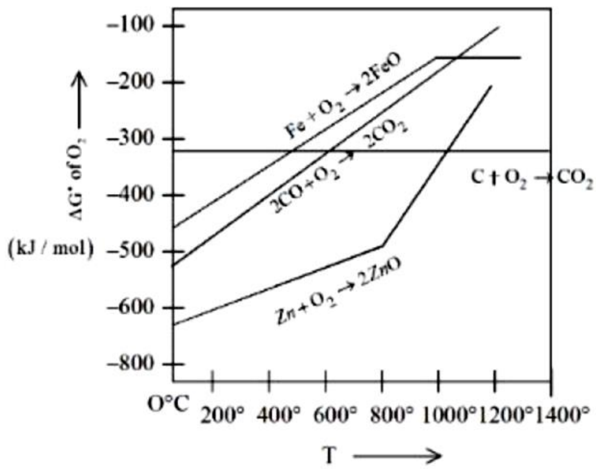
0.2

Soda Ash $\rightarrow Na_2CO_3$

No. of water = 10 + 0 = (10)

Metallurgy **Medium**

69. Gibbs energy vs T plot for the formation of oxides is given below.



For the given diagram, the correct statement is –

(1) At 600 ^oC, C can reduce ZnO (2) At 600 ^oC, C can reduce FeO

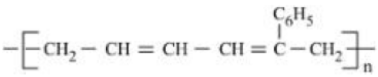
(3) At 600 ^oC, CO cannot reduce FeO (4) At 600 ^oC, CO can reduce ZnO

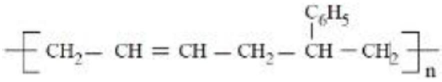
Sol. **2**

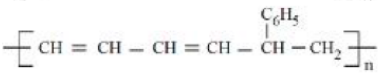
$FeO + C \longrightarrow Fe + CO_2$

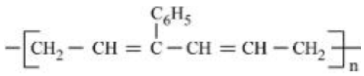
At 600°C ΔG of Reaction is –Ve

70. Buna–S can be represented as:

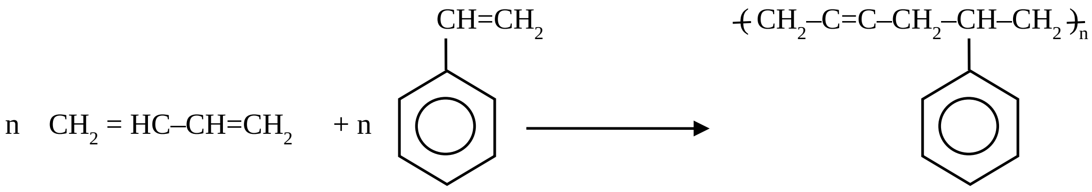
(1) 

(2) 

(3) 

(4) 

Sol. 2



(Butadiene) Styrene (Buna–S)

Hydrogen **Medium**

71. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : Physical properties of isotopes of hydrogen are different.

Reason : Mass difference between isotopes of hydrogen is very large.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) A is false but **R** is true

(3) A is true but **R** is false

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **Correct – (4)**

The Physical properties of isotope of Hydrogen are different due to Large mass difference

Coordination Compound **Medium**

72. The correct order of the number of unpaired electrons in the given complexes is

A. $[Fe(CN)_6]^{3-}$

B. $[FeF_6]^{3-}$

C. $[Co\ F_6]^{3-}$

D. $[Cr(oxalate)_3]^{3-}$

E. $[Ni(CO)_4]$

Choose the correct answer from the options given below:

(1) E < A < D < C < B (2) A < E < C < B < D

(3) A < E < D < C < B (4) E < A < B < D < C

Sol. 1

$(Fe(CN)_6)^{-3} \rightarrow Fe^{+3}$, $3d^5$, CN^- SFL

1	1	1	1	1

pairing

u.e. = 1

$(FeF_6]^{-3} \rightarrow Fe^{+3}$, F^- WF_2, $3d^5$, u.e. = 5

$(COF_6)^{-3} \rightarrow Co^{+3}$, F^- neF_2, $3d^6$, u.e. = 4

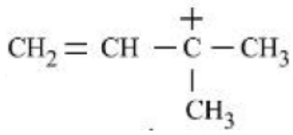
$(Cr(oxalate)_3)^{-3} \rightarrow Cr^{+3}$, $C_2O_1^{-2}$ WFL, $3d^3$, u.e. = 3

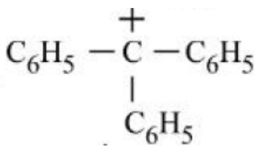
$(Ni(CO)_4) \rightarrow Ni$, $CO \rightarrow SFL$, u.e. = 0, $4S^2\ 3d^8$

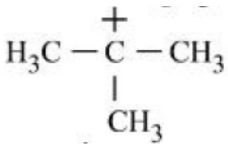
Topic : GOC

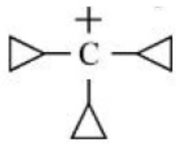
Medium

73. The decreasing order of hydride affinity for following **carbonations** is:

A. 

B. 

C. 

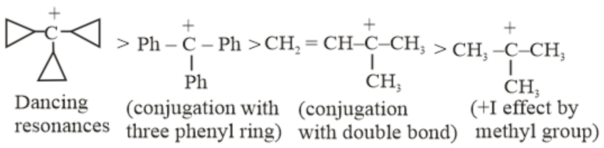
D. 

Choose the correct answer from the options given below:

(1) C, A, D, B (2) A, C, B, D (3) A, C, D, B (4) C, A, B, D

Sol. 4

Stability of carbocation $\propto \dfrac{1}{\text{Hydride affinity}}$



Chapter: carbonyl

Level : Med.

74. Incorrect method of preparation for alcohols from the following is:

(1) Ozonolysis of alkene.

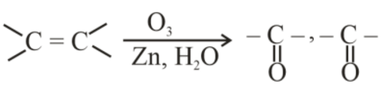
(2) Hydroboration-oxidation of alkene.

(3) Reaction of alkyl halide with aqueous NaOH.

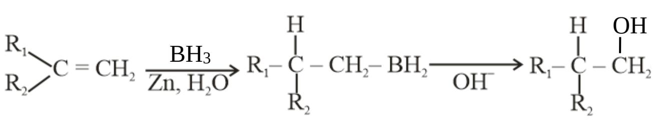
(4) Reaction of Ketone with RMgBr followed by hydrolysis.

Sol. 1

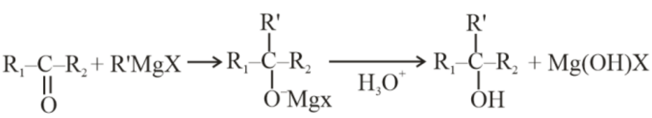
1) Ozonolysis of alkene–



2) Hydroboration – oxidation of alkene

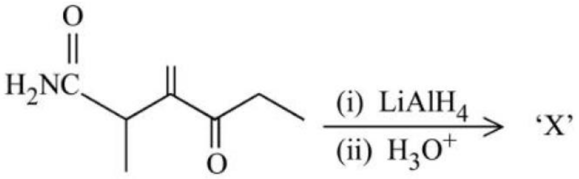


3) $R-X+NaOH \longrightarrow R-OH+NaX$

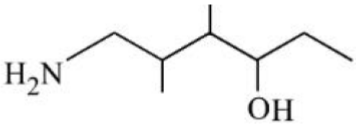


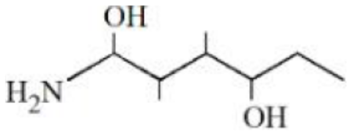
{Chap – Aldehyele, ketone, SO - Med}

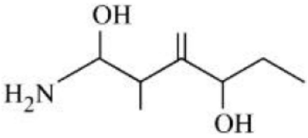
75. In the reaction given below:

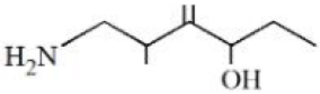


The product 'X' is:

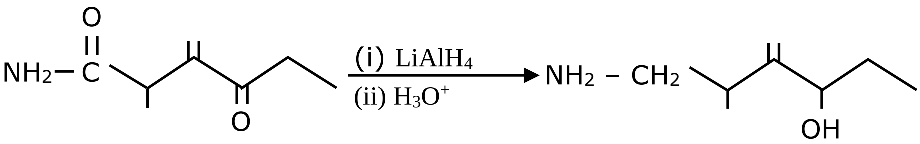
(1) 

(2) 

(3) 

(4) 

Sol. **4**



s-block **Medium**

76. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : The energy required to form Mg^{2+} from Mg is much higher than that required to produce Mg^{+}

Reason R: Mg^{2+} is small ion and carry more charge than Mg^{+}

In the light of the above statements, choose the correct answer from the options given below.

(1) Both **A** and **R** are true and **R** is the correct explanation of **A**

(2) **A** is true but **R** is false

(3) **A** is false but **R** is true

(4) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

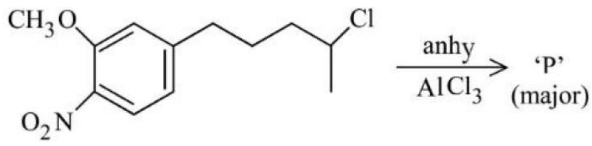
Sol. Correct – (1)

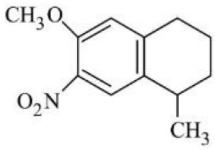
(A) - $$\begin{tikzcd} Mg \arrow[r, "IE_1"] \arrow[rr, bend right, "IE_1 + IE_2"'] & Mg^{+} \arrow[r, "IE_2"] & Mg^{2+} \end{tikzcd}$$

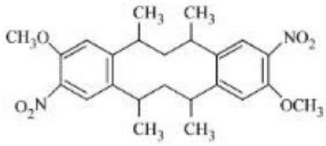
In formation of Mg^{2+} $IE_1 + IE_2$ is required while in formation of Mg^{+} IE_1 is required

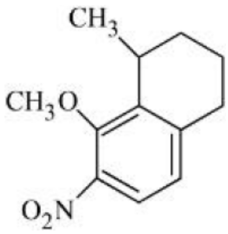
(R) Mg^{2+} is small ion and carry more change than $Mg^{\oplus}$

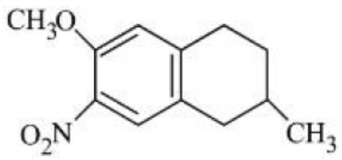
77. The major product 'P' formed in the given reaction is:



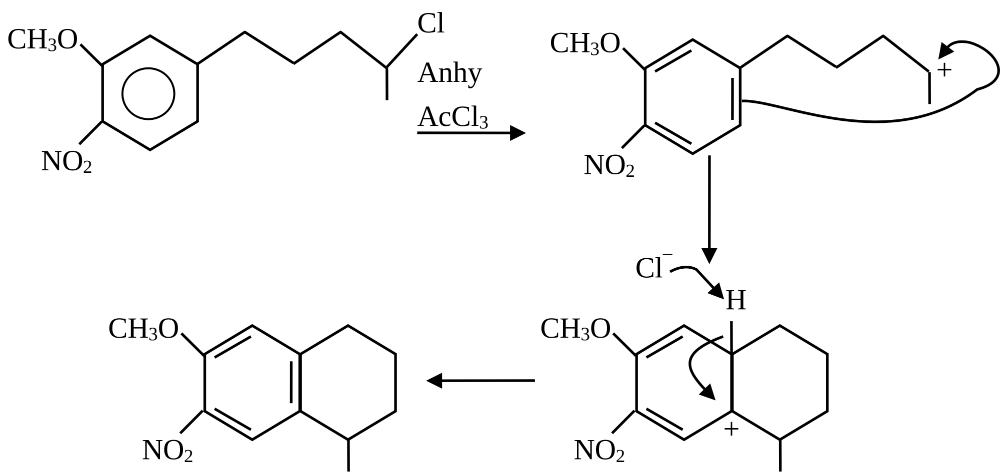
(1) 

(2) 

(3) 

(4) 

Sol. 1



78. Ferric chloride is applied to stop bleeding because -

(1) Blood absorbs $FeCl_3$ and forms a complex.

(2) $FeCl_3$ reacts with the constituents of blood which is a positively charged sol.

(3) Fe^{3+} ions coagulate blood which is a negatively charged sol.

(4) Cl^{-} ions cause coagulation of blood.

Sol. 3

Fe^{3+} coagulation negatively charged sol blood.

Environmental Chemistry **Easy**

79. The delicate balance of CO_2 and O_2 is NOT disturbed by

(1) Burning of Coal (2) Deforestation (3) Burning of petroleum (4) Respiration

Sol. Correct – (4)

The balance of carbon dioxide and oxygen in atmosphere is mainly maintained by the oxygen released and carbon dioxide consumed during photosynthesis by plants.

80. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A : 3.1500 g of hydrated oxalic acid dissolved in water to make 250.0 mL solution will result in 0.1M oxalic acid solution.

Reason R : Molar mass of hydrated oxalic acid is 126 g mol^{-1}

In the light of the above statements, choose the correct answer from the options given below:

(1) **A** is false but **R** is true

(2) **A** is true but **R** is false

(3) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Sol. **4**

Assertion is correct.

$H_2C_2O_4.2H_2O$

$$M = \frac{3.15 \times 1000}{126 \times 250}$$

$$= \frac{12.6}{126} = 0.1$$

Reason is correct. It is used as a fact in explanation of assertion.

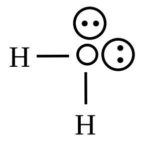
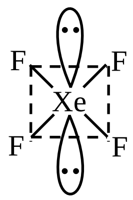
SECTION - B

Chemical bonding **Medium**

81. The number of molecules from the following which contain only two lone pair of electrons is________

$H_2O, N_2, CO, XeF_4, NH_3, NO, CO_2, F_2$

Sol. **4**

	lp
	2
$:N \equiv N:$	2
$:C \equiv O:$	2
	2
$\ddot{N}H_3$	1
$\cdot N \equiv \ddot{O}$	3

82. The specific conductance of 0.0025M acetic acid is $5\times10^{-5}\,S\ cm^{-1}$ at a certain temperature. The dissociation constant of acetic acid is________ $\times10^{-7}$. (Nearest integer)

Consider limiting molar conductivity of CH_3COOH as 400 S $cm^2\ mol^{-1}$.

Sol. **66**

$$\Lambda_m = \frac{k}{C}\times1000$$

Given k = 5 × 10^{-5} S cm^{-1}

C = 0.0025 M

$$\Lambda_m = \frac{5\times10^{-5}\times10^{3}}{0.0025} = \frac{5\times10^{-2}}{2.5\times10^{-3}}$$

$$= 20\,S\,cm^2 mol^{-1}$$

$$\alpha = \frac{20}{400} = \frac{1}{20}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.0025\times\frac{1}{20}\times\frac{1}{20}}{\frac{19}{20}}$$

$$= \frac{0.0025}{19\times20} = 6.6\times10^{-6}$$

$$= 66\times10^{-7}$$

83. An aqueous solution of volume $300\,cm^3$ contains 0.63 g of protein. The osmotic pressure of the solution at 300 K is 1.29 mbar. The molar mass of the protein is _______ g mol^{-1}

Given : R = 0.083 L bar $K^{-1}\,mol^{-1}$

Sol. **40535**

$$\because \pi = CRT$$

$$\pi = \frac{n}{V}RT$$

$$\pi = \frac{\omega}{V}\frac{RT}{M}$$

$$M = \frac{\omega RT}{\pi\times V}$$

$$M = \frac{0.63\times0.083\times300}{1.29\times10^{-3}\times300\times10^{-3}}$$

M = 40535 gm/moL

p-block **Medium**

84. The difference in the oxidation state of Xe between the oxidised product of Xe formed on complete hydrolysis of XeF_4 and XeF_4 is ____________

Sol. **2**

$$\overset{+4}{XeF_4} + H_2O \longrightarrow Xe + \overset{+6}{XeO_3} + O_2 + HF$$

Difference = 6 – 4 = (2)

85. The number of endothermic process/es from the following is

A. $I_2(g) \to 2I(g)$

B. $HCl(g) \to H(g) + Cl(g)$

C. $H_2O(l) \to H_2O(g)$

D. C(s) + $O_2(g) \to CO_2(g)$

E. Dissolution of ammonium chloride in water

Sol. **4**

A → Endothermic (Atomisation) B → Endothermic (Atomisation)

C → Endothermic (Vapourisation) D → Exothermic (Combustion)

E → Endothermic (Dissolution)

86. The number of incorrect statement/s from the following is

A. The successive half lives of zero order reactions decreases with time.

B. A substance appearing as reactant in the chemical equation may not affect the rate of reaction

C. Order and molecularity of a chemical reaction can be a fractional number

D. The rate constant units of zero and second order reaction are mol $L^{-1}s^{-1}$ and $mol^{-1}Ls^{-1}$ respectively.

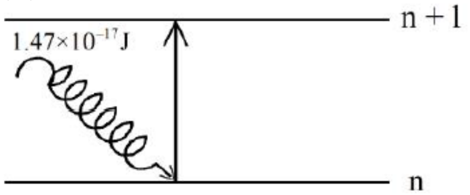
Sol. **1**

(A) For zero order $t_{1/2} = \frac{[A]_0}{2K}$ as concentration decreases half life decreases (Correct statement)

(B) If order w.r.t. that reactant is zero then it will not affect rate of reaction. (Correct statement)

(C) Order can be fractional but molecularity can not be (Incorrect statement)

(D) For zero order reaction unit is mol $L^{-}s^{-1}$ and for second order reaction unit is $mol^{-1}Ls^{-1}$ (Correct statement)

87. 

The electron in the nth orbit of Li^{2+} is excited to (n + 1) orbit using the radiation of energy 1.47 × 10^{-17} J (as shown in the diagram). The value of n is___________

Given: $R_H = 2.18 \times 10^{-18}$ J

Sol. **1**

$$\Delta E = R_H Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$1.47 \times 10^{-17} = 2.18 \times 10^{-18} \times 9 \left(\frac{1}{n^2} - \frac{1}{(n+1)^2} \right)$$

$$\frac{1.47}{1.96} = \frac{3}{4} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$$

So, n = 1

d-block **Medium**

88. For a metal ion, the calculated magnetic moment is 4.90BM. This metal ion has___________ number of unpaired electrons.

Sol. **4**

$\mu = 4.90$BM.

$\mu = \sqrt{n(n+2)}$

So, $\boxed{n = 4}$

89. In alkaline medium, the reduction of permanganate anion involves a gain of —— electrons.

Sol. **3**

$$\underbrace{\overset{+7}{MnO_4^-} \longrightarrow Mn^{+4}}_{(3)}$$

90. $A(g) \rightleftharpoons 2B(g) + C(g)$

For the given reaction, if the initial pressure is 450 mmHg and the pressure at time t is 720 mmHg at a constant temperature T and constant volume V. The fraction of A(g) decomposed under these conditions is $x \times 10^{-1}$. The value of x is _____________ (nearest integer)

Sol. **3**

$A(g) \rightleftharpoons 2B(g) + C(g)$

t = 0 450

time t 450 – x 2x x

$P_T = P_A + P_B + P_C$

720 = 450 – x + 2x + x

2x = 270

x = 135

Fraction of A decomposed = $\frac{135}{450} = 0.3 = 3 \times 10^{-1}$

So, x =3