Part - III

MATHEMATICS, Paper - I (A)

(English Version)

Time : **3 Hours**

Max. Marks : **75**

Note: This question paper consists of THREE Sections A, B and C.

SECTION - A

I. Very Short Answer Type Questions. **10×2=20**

(i) Answer **ALL** questions.

(ii) Each question carries **TWO** marks.

1. If f : R {0} → R is defined by $f(x) = x^3 - \frac{1}{x^3}$, then show that $f(x) + f\left(\frac{1}{x}\right) = 0$

2. Find the domain of the real valued function $f(x) = \frac{1}{(x^2-1)(x+3)}$

3. If $\begin{pmatrix} x-3 & 2y-8 \\ z+2 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ -2 & a-4 \end{pmatrix}$, then find the value of x, y, z and a.

4. If ω is complex (non-real) cube root of 1, then show that -

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = 0$$

5. Let $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\bar{b} = 3\hat{i} + \hat{j}$. Find the unit vector in the direction of $\bar{a} + \bar{b}$.

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6. Find the vector equation of the line passing through the point $2\hat{i} + 3\hat{j} + \hat{k}$ and parallel to the vector $4\hat{i} - 2\hat{j} + 3\hat{k}$.

7. If the vector $2\hat{i} + \lambda\hat{j} - \hat{k}$ and $4\hat{i} - 2\hat{j} + 2\hat{k}$ are perpendicular to each other, find λ.

8. Prove that $\sin 78° + \cos 132° = \frac{\sqrt{5}-1}{4}$

9. Find $\sin^2 82\frac{1}{2}^{\circ} - \sin^2 22\frac{1}{2}^{\circ}$

10. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e^3$

SECTION - B

5×4=20

II. Short Answer Type Questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **FOUR** marks.

11. If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $E = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, then show that $(aI + bE)^3 = a^3I + 3a^2bE$, where I is unit matrix of order 2.

12. $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar vectors. Prove that the following four points are coplanar $-\bar{a} + 4\bar{b} - 3\bar{c}$, $3\bar{a} + 2\bar{b} - 5\bar{c}$, $-3\bar{a} + 8\bar{b} - 5\bar{c}$, $-3\bar{a} + 2\bar{b} + \bar{c}$.

13. Prove that for any three vectors $\bar{a}, \bar{b}, \bar{c}$

$$[\bar{b} + \bar{c} \quad \bar{c} + \bar{a} \quad \bar{a} + \bar{b}] = 2[\bar{a}\,\bar{b}\,\bar{c}].$$

14. Prove that $\tan 70° - \tan 20° = 2\tan 50°$

15. Solve that equation $\sin x + \sqrt{3}\cos x = \sqrt{2}$

16. Prove that $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

17. In ΔABC, if $a : b : c = 7 : 8 : 9$, then find $\cos A : \cos B : \cos C$

SECTION - C

$5 \times 7 = 35$

III. Long Answer Type Questions.

(i) Answer **ANY FIVE** questions.

(ii) Each question carries **SEVEN** marks.

18. If $A = \{1, 2, 3\}$, $B = \{\alpha, \beta, \gamma,\}$ $C = \{p, q, r\}$ and $f : A \rightarrow B$, $g : B \rightarrow C$ are defined by $f = \{(1, \alpha), (2, \gamma), (3, \beta)\}$, $g = \{(\alpha, q), (\beta, r), (\gamma, p)\}$, then show that f and g are bijective functions and $(gof)^{-1} = f^{-1}og^{-1}$

19. Using mathematical induction, show that

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \cdots \text{ upto n terms} = \frac{n}{3n+1} \ \forall\ n \in N.$$

20. Show that

$$\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

21. Solve $x + y + z = 1$, $2x + 2y + 3z = 6$, $x + 4y + 9z = 3$ by using matrix inversion method.

22. If $\bar{a} = 2\hat{i} + \hat{j} - 3\hat{k}$, $\bar{b} = \hat{i} - 2\hat{j} + \hat{k}$, $\bar{c} = -\hat{i} + \hat{j} - 4\hat{k}$ and $\bar{d} = \hat{i} + \hat{j} + \hat{k}$, then compute $|(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d})|$

23. If A, B, C are the angles of a triangle, prove that $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$

24. If $r_1 = 2$, $r_2 = 3$, $r_3 = 6$ and $r = 1$, prove that $a = 3$, $b = 4$ and $c = 5$.
