FIRST YEAR EXAMINATION - JUNE 2022
MATHEMATICS

Answer any 6 questions from 1 to 8.
Each carries 3 scores.

1. (i) $A \cap A' = \phi$ (1)

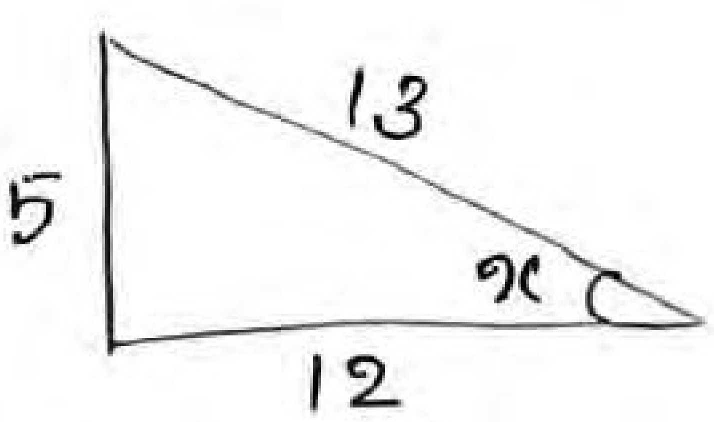
(ii) (a) $A = \{1, 2\}$ 1

(b) Subsets of A are
$\{1\}, \{2\}, \{1,2\}, \phi$. 1 3

2. (i) $25^\circ = 25 \times \frac{\pi}{180} = \frac{5\pi}{36}$ radian. 1

(ii) $\tan x = \frac{5}{12}$, x in III quadrant

$\sin x = \frac{-5}{13}$ 1



$\cos x = \frac{-12}{13}$ 1 3

3. (i) $\frac{4}{3}, x, \frac{3}{4}$ are in G.P

$\therefore \frac{x}{4/3} = \frac{3/4}{x}$ ½

$x^2 = \frac{3}{4} \times \frac{4}{3} = 1$

$x = \underline{\underline{\pm 1}}$ ½

(ii) $\sqrt{3},\ 3,\ 3\sqrt{3},\ \ldots$

$a = \sqrt{3}$

$r = \dfrac{3}{\sqrt{3}} = \sqrt{3}$ | 1

n^{th} term, $a_n = ar^{n-1}$

$= \sqrt{3}\,(\sqrt{3})^{n-1}$

$= \sqrt{3}^{\,n}$ | 1 | 3

4. Lines are $-\sqrt{3}x + y - 5 = 0$ — (1)

$-x + \sqrt{3}y + 6 = 0$ — (2)

Slope of (2), $m_1 = -\dfrac{a}{b} = -\dfrac{-1}{\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

Slope of (1), $m_2 = -\dfrac{a}{b} = -\dfrac{-\sqrt{3}}{1} = \sqrt{3}.$ | 1

If θ is the angle between the lines

$\tan\theta = \left|\dfrac{m_2 - m_1}{1 + m_1 m_2}\right|$

$= \left|\dfrac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3}\cdot\frac{1}{\sqrt{3}}}\right| = \left|\dfrac{3-1}{2\sqrt{3}}\right| = \left|\dfrac{1}{\sqrt{3}}\right|$ | 1

$\tan\theta = \pm\dfrac{1}{\sqrt{3}}$

If $\tan\theta = \dfrac{1}{\sqrt{3}}$, $\theta = \pi/6$

If $\tan\theta = -\dfrac{1}{\sqrt{3}}$, $\theta = \pi - \pi/6 = 5\pi/6$ | 1 | 3

5 (i) $y^2 = 8x$

Parabola is of the form $y^2 = 4ax$

$\therefore \quad 4a = 8$

$a = 2$

Focus of $y^2 = 4ax$ is $(a, 0)$

$\therefore$ Focus $= (2, 0)$ 1

(ii) $x^2 + y^2 + 6x - 4y - 3 = 0$

$(x^2 + 6x + 9) + (y^2 - 4y + 4) = 3 + 9 + 4$

$(x+3)^2 + (y-2)^2 = 16$ 1

$(x+3)^2 + (y-2)^2 = 4^2$

centre $= (-3, 2)$ 1

radius $= 4$ 3

6. Let $A(-2, 4, 7)$ and $B(3, -5, 8)$

Let the ratio be $k : 1$

Since YZ plane divide the line segment, then x-coordinate $= 0$. 1

ie; $\dfrac{mx_2 + nx_1}{m+n} = 0$

$\dfrac{k \cdot 3 + 1 \cdot \bar{2}}{k+1} = 0$ 1

$3k - 2 = 0 \Rightarrow k = \dfrac{2}{3}$

$\therefore$ Ratio is $\frac{2}{3} : 1$ ie; $\underline{\underline{2 : 3}}$ 1 3

7.	(i) $\lim_{x \to 2} x^2 - 4 = 2^2 - 4 = \underline{\underline{0}}$	1	
	(ii) $\lim_{x \to 2} \frac{x^2-4}{x-2} = \lim_{x \to 2} \frac{x^2-2^2}{x-2}$ $= 2 \cdot 2^{2-1}$ $= 2 \cdot 2 = \underline{\underline{4}}$	1	
	(iii) $\lim_{x \to 0} \frac{\sin 4x}{x} = \lim_{x \to 0} \frac{\sin 4x}{4x} \cdot 4$ $= 1 \cdot 4 = \underline{\underline{4}}$	1	3
8.	p : $\sqrt{3}$ is irrational. Assume that $\sqrt{3}$ is rational ie, $\sqrt{3} = \frac{a}{b}$, where a and b have no common factors.	1	
	$a = \sqrt{3}b \Rightarrow a^2 = 3b^2$ $\Rightarrow$ 3 divides a ie, $a = 3c \Rightarrow a^2 = 9c^2$ $\therefore 3b^2 = 9c^2$	1	
	$b^2 = 3c^2 \Rightarrow$ 3 divides b. ie, 3 is a common factor of a and b which contradicts our assumption that a and b have no common factor. $\therefore$ Our assumption is wrong.	1	3
	$\therefore \sqrt{3}$ is not rational $\therefore \sqrt{3}$ is irrational.		

Answer any 6 questions from 9 to 17 Each carries 4 scores.	

9 (i) (a) $[-4, 5]$ 1

(ii) $U = \{1, 2, 3, 4, 5, 6, 7\}$

$A = \{2, 3, 4, 6\}$

$B = \{3, 4, 5\}$

$A \cup B = \{2, 3, 4, 5, 6\}$ 1

$(A \cup B)' = \{1, 7\}$

$A' = \{1, 5, 7\}$, $B' = \{1, 2, 6, 7\}$ 1

$A' \cap B' = \{1, 7\}$ 4

$\therefore (A \cup B)' = A' \cap B'$ verified. 1

10. (i) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$R = \{(x, y) : 2x - y = 0,\ x, y \in A\}$

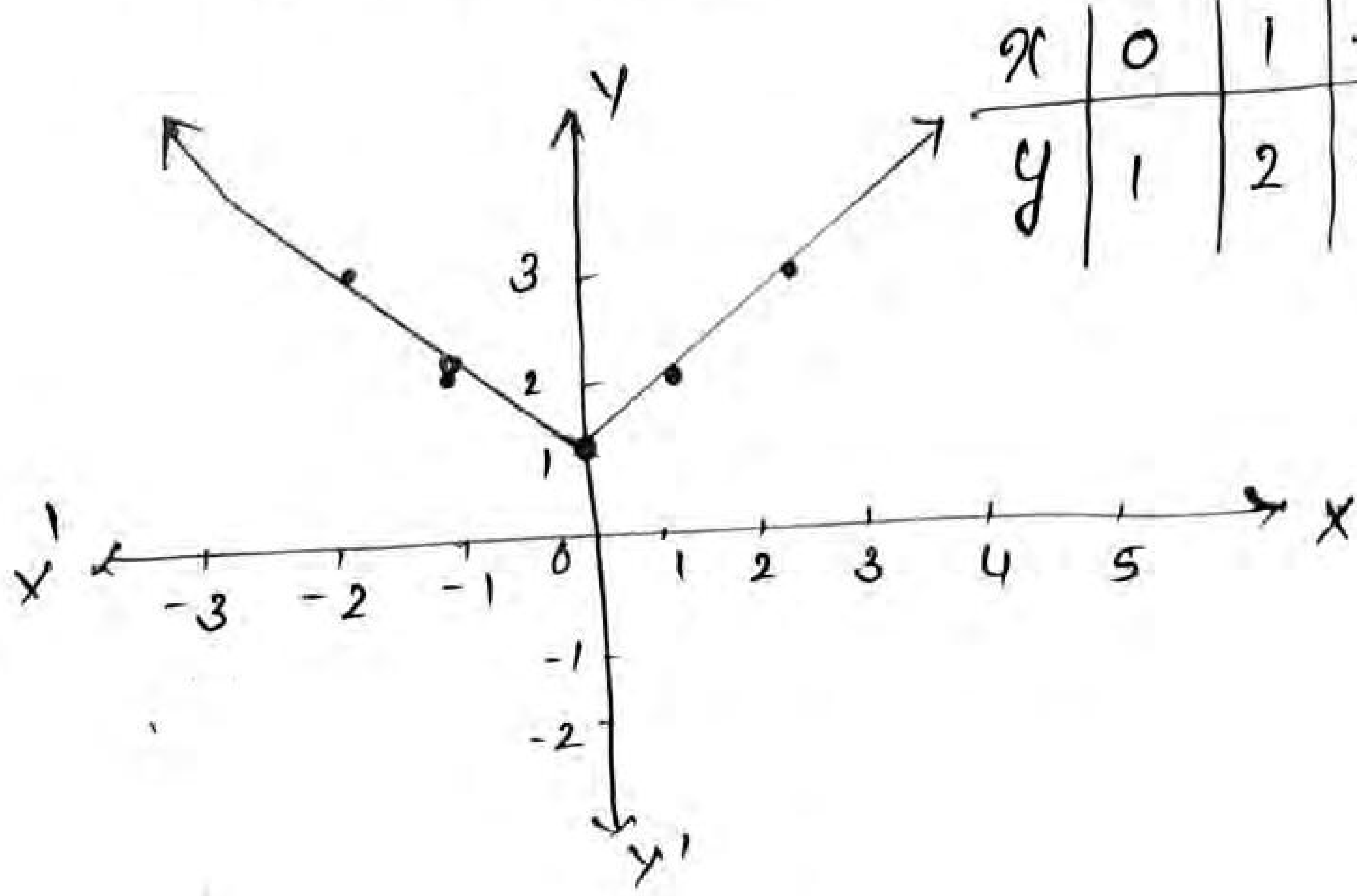
$R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$ 1

Domain $= \{1, 2, 3, 4\}$

Range $= \{2, 4, 6, 8\}$ 1

(ii) $f(x) = |x| + 1$

x	0	1	-1	2	-2
y	1	2	2	3	3



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4

11 (i) $P(n): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n} = 1-\frac{1}{2^n}$

$P(1)$: $\frac{1}{2} = 1-\frac{1}{2}$

$\frac{1}{2} = \frac{1}{2}$

$\therefore$ P (1) is true 1

(ii) Assume that P (k) is true.

$P(k): \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^k} = 1-\frac{1}{2^k}$.

To P. T P (k+1) is true.

P (k+1) : 1

$LHS = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^k}+\frac{1}{2^{k+1}}$

$= 1-\frac{1}{2^k}+\frac{1}{2^{k+1}}$ ½

$= 1-\frac{1}{2^k}\left(1-\frac{1}{2}\right)$

$= 1-\frac{1}{2^k}\cdot\frac{1}{2}$ ½

$= 1-\frac{1}{2^{k+1}} = RHS.$

$\therefore$ P (k+1) is true.

Hence by principle of mathematical induction P (n) is true for all $n \in N$. 1 4

12.
(i) $^nC_9 = {^nC_8}$, then $n = 17$ 1

(ii) $^nP_r = \frac{n!}{(n-r)!}$ 1

(iii) MATHEMATICS

$n = 11$

M repeats 2 times

A repeats 2 times 1

T repeats 2 times

Number of permutations $= \dfrac{11!}{2!\,2!\,2!}$ 1 4

$= 4989600.$

13. $(x + 9y)^{10}$

(i) number of terms $= 11$ 1

(ii) General term,

$T_{r+1} = {}^{n}C_r \, a^{n-r} \, b^{r}$ $\quad n = 10$ 1

$= {}^{10}C_r \, (x)^{10-r} \, (9y)^{r}$ $\quad a = x$, $\quad b = 9y.$

$= {}^{10}C_r \, 9^{r} \, (x)^{10-r} \, y^{r}.$ 1

(iii) 5^{th} term,

$T_5 = T_{4+1} = {}^{10}C_4 \, 9^{4} \, (x)^{10-4} \, y^{4}$

$= {}^{10}C_4 \, 9^{4} \, x^{6} \, y^{4}$ 1 4

$= \underline{1377810 \, x^{6} y^{4}}.$

14. $8 + 88 + 888 + \ldots$ to n terms

$= 8\,(1 + 11 + 111 + \ldots$ to n terms$)$ 1

$= \dfrac{8}{9}\,(9 + 99 + 999 + \ldots$ to n terms$)$ 1

$= \dfrac{8}{9}\left((10-1) + (100-1) + (1000-1) + \ldots\right.$ to n terms$\left.\right)$

$= \frac{8}{9}\left(10+10^2+10^3+\ldots \text{ to } n \text{ terms } - n\right)$

$= \frac{8}{9}\left[10\left(\frac{10^n-1}{10-1}\right) - n\right]$ 1

$= \frac{8}{9}\left(\frac{10^{n+1}-10-9n}{9}\right)$ 1

$= \frac{8}{81}\left(10^{n+1} - 9n - 10\right)$ 4

15. (i) $x - 7y + 5 = 0$. $a = 1,\ b = -7$

Slope $= -\frac{a}{b} = -\frac{1}{-7} = \frac{1}{7}$ 1

(ii) Lines are perpendicular.

$\therefore$ Slope of line $= \frac{-1}{1/7} = -7$ 1

line passes through $(3,0)$

$\therefore$ Equation of line passing through $(3,0)$ with slope -7 is 1

$y - y_1 = m(x - x_1)$

$y - 0 = -7(x - 3)$

$y = -7x + 21$ 1 4

$7x + y - 21 = 0$

16. $\frac{x^2}{36} + \frac{y^2}{16} = 1$

Ellipse is of the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 = 36 \Rightarrow a = 6$
$b^2 = 16 \Rightarrow b = 4.$
$a^2 = b^2 + c^2 \Rightarrow 36 = 16 + c^2$
$\Rightarrow c^2 = 36 - 16 = 20$
$\Rightarrow c = \sqrt{20} = 2\sqrt{5}.$ 1

foci $= (\pm c, 0) = (\pm 2\sqrt{5}, 0)$
vertices $= (\pm a, 0) = (\pm 6, 0)$ 1
length of major axis $= 2a = 2 \times 6 = 12$ 1
length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 16}{6} = \frac{16}{3}$ 1 4

17. $P(A) = 0.54$
$P(B) = 0.69$
$P(A \cap B) = 0.35$
(i) $P(A \text{ or } B) = P(A \cup B)$
$= P(A) + P(B) - P(A \cap B)$ 1
$= 0.54 + 0.69 - 0.35$
$= 0.88.$ 1
(ii) $P(\text{not } A \text{ and not } B) = P(A' \cap B')$
$= P(A \cup B)'$ 1
$= 1 - P(A \cup B)$
$= 1 - 0.88 = \underline{\underline{0.12}}$ 1 4

Answer any 3 questions from 18 to 22
Each carries 6 score.

18. (i) $\frac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}.$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$$ 2

$$= -\frac{\sin(7x)\sin(2x)}{\cos(10x)\sin(7x)}$$ 1

$$= \frac{-\sin 2x}{\cos 10x}$$

(ii) $\sin x = \frac{-\sqrt{3}}{2}$ $\quad\quad \sin \pi/3 = \frac{\sqrt{3}}{2}$.

$\sin x$ is $-ve$

$\therefore$ principal solutions lies in III and IV quadrant.

$\sin(\pi + \pi/3) = -\sin \pi/3 = \frac{-\sqrt{3}}{2}$

ie, $\sin 4\pi/3 = -\sqrt{3}/2$ 1

$\sin(2\pi - \pi/3) = -\sin \pi/3 = -\sqrt{3}/2$

$\sin 5\pi/3 = -\sqrt{3}/2$ 1

$\therefore$ Principal solutions are $4\pi/3$ and $5\pi/3$

~~sin 4π/3~~ $\sin x = \sin 4\pi/3$

$\cancel{x = (-1)^n n\pi}$. $x = n\pi + (-1)^n y$. 1

General Solution is $x = n\pi + (-1)^n \frac{4\pi}{3}$ 6

19 (i) $z = -1 + i\sqrt{3}$ $\quad a = -1, \quad b = \sqrt{3}$

$r = |z| = \sqrt{a^2+b^2}$

$= \sqrt{1+3} = 2.$ 1

~~arg z, θ =~~

$-1 + i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$= 2(\cos\theta + i\sin\theta)$

$-1 = 2\cos\theta, \quad \sqrt{3} = 2\sin\theta$

$\cos\theta = -1/2 \qquad \sin\theta = \sqrt{3}/2$ 1

θ lies in II quadrant.

$\theta = \pi - \pi/3 = 2\pi/3$

Polar form of $-1+i\sqrt{3} = 2\left(\cos 2\pi/3 + i\sin 2\pi/3\right)$ 1

(ii) $\sqrt{5}x^2 + x + \sqrt{5} = 0$

$a = \sqrt{5}, \quad b = 1, \quad c = \sqrt{5}.$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ 1

$= \dfrac{-1 \pm \sqrt{1-20}}{2\sqrt{5}}$ 1

$= \dfrac{-1 \pm \sqrt{-19}}{2\sqrt{5}} = \dfrac{-1 \pm \sqrt{19}\, i}{2\sqrt{5}}$ 1 6

20.

(i) $3\frac{(x-2)}{5} \le 5\left(\frac{2-x}{3}\right)$

$9x-18 \le 50-25x$

$25x+9x \le 50+18$ 1

$34x \le 68$

$x \le 2$

$x \in (-\infty, 2]$ 1

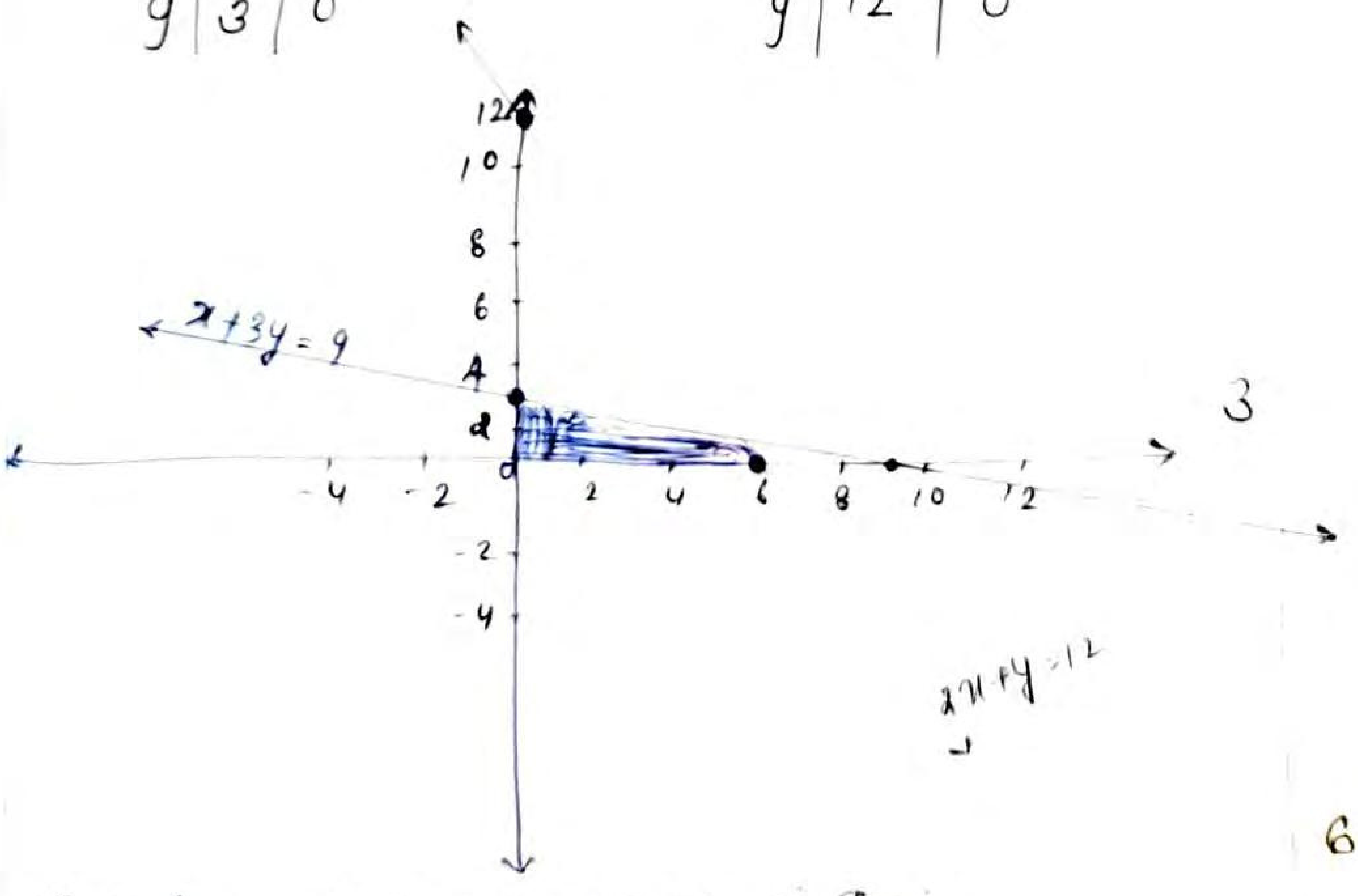
(ii) $x+3y \le 9,\ 2x+y \le 12,\ x \ge 0,\ y \ge 0$

Consider the lines

$x+3y=9$ and $2x+y=12$

x	0	9
y	3	0

x	0	6
y	12	0



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Take $(0,0)$, $x+3y \le 9 \Rightarrow 0 \le 9$ is True

$2x+y \le 12 \Rightarrow 0 \le 12$ is True 1

Shaded region is the solution region

21. (i) $f(x) = \cos x \qquad f(x+h) = \cos(x+h)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$ 1

$$= \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)\right]$$

$$= \lim_{h\to 0} \frac{1}{h}\left[-2\sin\left(\frac{2x+h}{2}\right)\sin h/2\right]$$ 1

$$= \lim_{h\to 0} -2\sin\left(\frac{2x+h}{2}\right)\frac{\sin h/2}{2 \cdot h/2}$$

$$= -\sin\left(\frac{2x+0}{2}\right) \cdot 1$$

$$= -\sin x$$ 1

$$\therefore \frac{d}{dx}(\cos x) = -\sin x.$$

(ii) $$\frac{d}{dx}\left(\frac{x^2}{3x-1}\right)$$

$$= \frac{(3x-1)\frac{d}{dx}(x^2) - x^2 \cdot \frac{d}{dx}(3x-1)}{(3x-1)^2}$$ 1

$$= \frac{(3x-1)\,2x - x^2 \cdot 3}{(3x-1)^2}$$ 1

$$= \frac{6x^2 - 2x - 3x^2}{(3x-1)^2}$$

$$= \frac{3x^2 - 2x}{(3x-1)^2} = \frac{x(3x-2)}{(3x-1)^2}$$ 1 6

22.

x_i	f_i	$x_i f_i$	$f_i x_i^2$
5	6	30	150
15	8	120	1800
25	14	350	8750
35	16	560	19600
45	4	180	8100
55	2	110	6050
	$\Sigma f_i = 50$	$\Sigma x_i f_i = 1350$	$\Sigma f_i x_i^2 = 44450$

(3)

(i) Mean, $\bar{x} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{1350}{50} = \underline{\underline{27}}$ 1

(ii) Variance, $\sigma^2 = \frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2$

$= \frac{44450}{50} - (27)^2$ 1

$= 889 - 729$

$= \underline{\underline{160}}$

6

(iii) S.D, $\sigma = \sqrt{var} = \sqrt{160}$

$= \underline{\underline{12.65}}$ 1