XI Higher Secondary Exam. March 2024

Mathematics (Science)

1\. i) $2^{3\times 2} = 2^6 =$

ii) $\frac{x}{3} + 1 = -\frac{2}{3} \Rightarrow \frac{x}{3} + \frac{3}{3} = \frac{-2}{3}$

$\Rightarrow x + 3 = -2$

$\therefore x = -2 - 3 = -5$

$y - \frac{2}{3} = \frac{2}{3} \Rightarrow \frac{3y}{3} - \frac{2}{3} = \frac{2}{3}$

$3y = 2 + 2 = 4$

$y = \frac{4}{3}$

2) i) $\cos(x+y) + \cos(x-y) = 2\cos x \cos y$

ii) $LHS = \cos\left(\frac{3\pi}{4} + x\right) + \cos\left(\frac{3\pi}{4} - x\right)$

$= 2\cos\frac{3\pi}{4} \cdot \cos x$

$= 2 \times -\frac{1}{\sqrt{2}} \cos x$

$= -\sqrt{2}\cos x$

$= RHS$

$\cos\frac{3\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$

$= -\frac{1}{\sqrt{2}}$

3) i) $x + \frac{x}{2} + \frac{x}{3} \leq 10 + \frac{x}{6}$

$6x + 3x + 2x \leq 60 + x$

$11x - x \leq 60$

$10x \leq 60$

$x \leq \frac{60}{10}$

$x \leq 6$

$\therefore x = (-\infty, 6]$

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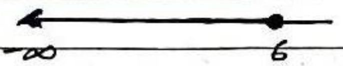
ii)



4) i) $\Rightarrow$ $^nC_r = \dfrac{n!}{r!\,(n-r)!}$

ii) No. of selections $= {}^7C_3 \times {}^5C_2$

$= 35 \times 10 = 350$

5 i) Sum of coe. $= 2^n$

ii) $\left(\dfrac{x}{3} + \dfrac{3}{x}\right)^4 = \left(\dfrac{x}{3}\right)^4 + {}^4C_1 \left(\dfrac{x}{3}\right)^3 \cdot \dfrac{3}{x} +$

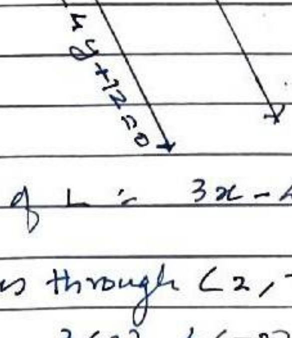
${}^4C_2 \left(\dfrac{x}{3}\right)^2 \left(\dfrac{3}{x}\right)^2 + {}^4C_3 \left(\dfrac{x}{3}\right)\left(\dfrac{3}{x}\right)^3 + \left(\dfrac{3}{x}\right)^4$

$= \dfrac{x^4}{81} + 4 \cdot \dfrac{x^3}{27} \cdot \dfrac{3}{x} + 6\,\dfrac{x^2}{9} \times \dfrac{9}{x^2} +$

$4 \cdot \dfrac{x}{3} \cdot \dfrac{27}{x^3} + \dfrac{81}{x^4}$

$= \dfrac{x^4}{81} + \dfrac{4}{9}x^2 + 6 + \dfrac{36}{x^2} + \dfrac{81}{x^4}$

6) i)



Eqn of L : $3x - 4y + k = 0$ — (1)

(1) passes through $(2,-3)$

$3(2) - 4(-3) + k = 0$

$6 + 12 + k = 0 \Rightarrow k = -18$

in (1) : $3x - 4y - 18 = 0$.

ii) distance $= \left| \dfrac{12}{\sqrt{3^2+(-4)^2}} \right|$

$= \dfrac{12}{5}$ units.

v) $x^2 = 12y$

$\Rightarrow x^2 = 4(3)y$ is an upward parabola.

Focus : $S(0,a) = (0,3)$

Equation of directrix : $y = -a$

$y = -3 \Rightarrow y+3=0$

Length of LR $= 4a = 4\times3 = 12$.

8) i) na^{n-1}

ii) $n.2^{n-1} = 32 = 2^5 = 4 \cdot 2^{4-1}$

$\Rightarrow n = 4$

9) i) $A \cap A' = \phi$

ii) $A \cap B = \{2,3,4\}$

LHS $= (A\cap B)' = \{1,5,6\}$ —— (1)

$A' = \{1,5,6\}$

$B' = \{1,5\}$

RHS $= A' \cup B'$

$= \{1,5,6\}$ —— (2)

From (1) & (2)

LHS = RHS.

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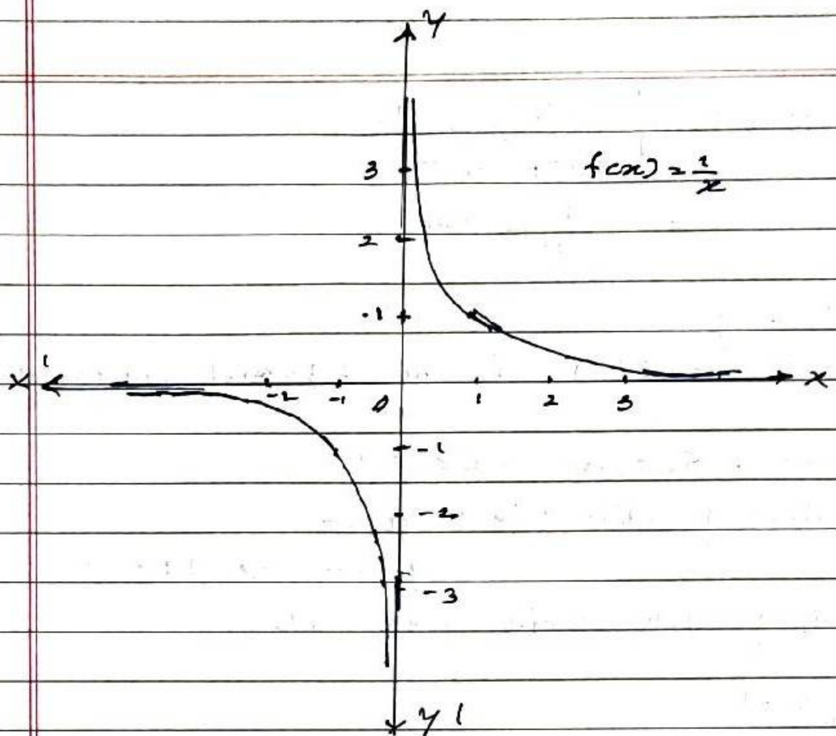
(b)



ii) $f(x) = \sqrt{9-x^2}$

To find the domain:

$9 - x^2 \geq 0$

$9 \geq x^2 \Rightarrow x^2 \leq 9 \Rightarrow x \leq \pm 3$

Domain $= [-3, 3]$

To find the range:

$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 = 9 - y^2$

$x = \sqrt{9-x^2}$

$\Rightarrow 9 - x^2 \geq 0$

$9 \geq x^2$

$\Rightarrow x^2 \leq 9$

$\Rightarrow x \leq \pm 3$

But, $y \geq 0$

$\therefore$ Range $= [0, 3]$

11 i) $(1-i)^6 = 1 - {}^6C_1\, i + {}^6C_2\, i^2 - {}^6C_3\, i^3 + {}^6C_4 \cdot i^4 - {}^6C_5\, i^5 + i^6$

$= 1 - 6(i) + 15(-1) - 20(-i) + 15(1) - 6(i) + (-1)$

$= 1 - 6i - \cancel{15} + 20i + \cancel{15} - 6i - 1$

$= 0 + 20i - 12i$

$= 8i$

$= 0 + i8$

ii) Let $z = \dfrac{1-i}{1+i}$ (Deleted)

$= \dfrac{1-i}{1+i} \times \dfrac{1-i}{1-i}$

$= \dfrac{(1-i)^2}{(1+i)(1-i)}$

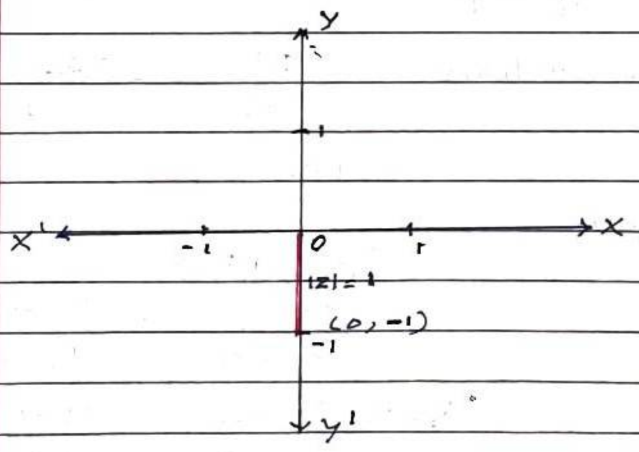
$= \dfrac{1-2i+i^2}{1-i^2}$

$= \dfrac{1-2i-1}{1+1}$

$= \dfrac{-2i}{2} = -i$

$= 0 + i(-1)$

$|z| = (-1)^2 = 1$



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12 i) $^nP_r = 840$

$^nC_r = 35$

$\frac{n!}{r!\,(n-r)!} = 35$

$\frac{n!}{(n-r)!} = 35 \times r!$

$^nP_r = 35 \times r!$

$\frac{840}{35} = r!$

$24 = r!$

$\Rightarrow r! = 24 = 1 \times 2 \times 3 \times 4 = 4!$

$\Rightarrow r = 4$

ii) The word ATTITTUDE has

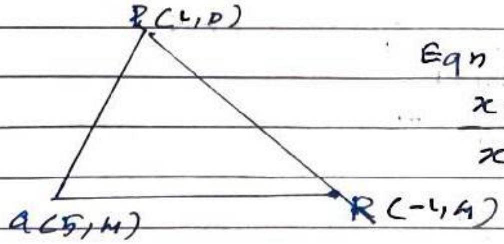
A — 1
T — 4
I — 1
U — 1
D — 1
E — 1

Total — 9

$\therefore$ No of Permutations $= \frac{9!}{4!} = \frac{362880}{24}$

$= 15120.$

13) i)



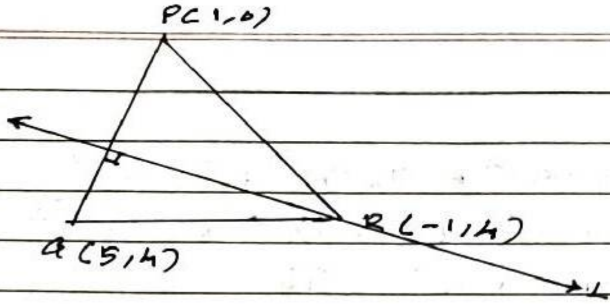
Eqn of PQ is

$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}$

$\frac{x-1}{5-1} = \frac{y-0}{4-0}$

$\frac{x-1}{4} = \frac{y}{4}$

$x-y-1=0.$



Slope of L $= \dfrac{-1}{\text{Slope of PQ}}$

$= \dfrac{-1}{\left(\frac{4-0}{5-1}\right)} = \dfrac{-1}{1} = -1 \; (= m)$

Eq of L is $y - y_1 = m(x - x_1)$

$y - 4 = -1(x - -1)$

$y - 4 = -x - 1$

$y - 4 + x + 1 = 0$

$x + y - 3 = 0.$

14) $c = 4 \quad a = 5$

$c^2 = a^2 - b^2$

$4^2 = 5^2 - b^2$

$b^2 = 5^2 - 4^2 = 9$

$b = \pm 3.$

a) Length of minor axis $= 2b = 2(3) = 6$

b) Length of LR $= \dfrac{2b^2}{a}$

$= \dfrac{2 \times 3^2}{5} = \dfrac{18}{5}$

$e = \dfrac{c}{a} = \dfrac{4}{5}$

ii) Eqn is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

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15) i) $(-1, 2, 3)$

$$AB = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$
$$= \sqrt{(-1-0)^2 + (6-7)^2 + (6-10)^2}$$
$$= \sqrt{1+1+16} = \sqrt{18}$$

$$BC = \sqrt{(-3)^2 + 3^2 + 0^2}$$
$$= \sqrt{9+9} = \sqrt{18}$$

$$AC = \sqrt{(-4)^2 + 2^2 + (-4)^2}$$
$$= \sqrt{16+4+16} = \sqrt{36}$$

$$AB^2 + BC^2 = (\sqrt{18})^2 + (\sqrt{18})^2$$
$$= 18 + 18$$
$$= 36$$
$$= AC^2.$$

$\therefore \triangle ABC$ is a right $\triangle^{le}$.

16) i) $P(\text{all 3 are white}) = \frac{{}^5C_3}{{}^{13}C_3} = \frac{10}{286} = \frac{5}{143}$

ii) $P(\text{all 3 are red}) = \frac{{}^8C_3}{{}^{13}C_3} = \frac{56}{286} = \frac{28}{143}$

iii) P(one is red and two balls are white)

$$= \frac{{}^8C_1 \times {}^5C_2}{{}^{13}C_3} = \frac{8 \times 10}{286}$$
$$= \frac{40}{143}$$

17) i) $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

$\therefore \frac{1-\tan^2 15}{1+\tan^2 15} = \cos 2(15) = \cos 30 = \frac{\sqrt{3}}{2}$

Ans: (b).

ii) $LHS = \dfrac{\sin 3x - \sin x}{\cos^2 x - \sin^2 x}$

$= \dfrac{2\cos\left(\frac{3x+x}{2}\right)\cdot \sin\left(\frac{3x-x}{2}\right)}{\cos 2x}$

$= \dfrac{2\cancel{\cos 2x}\cdot \sin x}{\cancel{\cos 2x}}$

$= 2\sin x$

$= RHS.$

iii) $\tan\theta = \frac{1}{2}$; $\tan\phi = \frac{1}{3}$

Let $\theta + \phi = \frac{\pi}{4}$

$\tan(\theta+\phi) = \tan\frac{\pi}{4}$

$\dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} = 1$

$LHS = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\cdot\tan\phi}$

$= \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\times\frac{1}{3}}$

$= \dfrac{3+2}{6-1} = \dfrac{5}{5} = 1 = RHS$

Hence proved.

18 i) $a = 2$; $r = 4$

$a_n = 32768$

$ar^{n-1} = 32768$

$2\times 4^{n-1} = 32768$

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$$4^{n-1} = 16384$$
$$= 4^7$$
$$4^{n-1} = 4^{8-1}$$
$$\therefore \quad n = 8$$

$$\begin{array}{r|l} 2 & 16384 \\ 2 & 8192 \\ 2 & 4096 \\ 2 & 2048 \\ 2 & 1024 \\ 2 & 512 \\ 2 & 256 \\ 2 & 128 \\ 2 & 64 \\ 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array}$$

ii) $a + ar + ar^2 = 14$ ——— (1)

$ar^3 + ar^4 + ar^5 = 112$ ——— (2)

$(2) \div (1) \Rightarrow$

$$\frac{r^3(a + ar + ar^2)}{(a + ar + ar^2)} = \frac{112}{14}$$

$$r^3 = 8 = 2^3$$
$$\therefore r = 2$$

in (1) $a + a(2) + a(2^2) = 14$

$$a + 2a + 4a = 14$$
$$7a = 14$$
$$a = 2$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r > 1$$
$$= \frac{2(2^n - 1)}{2 - 1}$$
$$= \frac{2(2^n - 1)}{1}$$
$$= \underline{\underline{2(2^n - 1)}}$$

(9)

i) $f(x) = \frac{1}{x}$

$$f(x+h) = \frac{1}{x+h}$$

$$f(x+h) - f(x) = \frac{1}{x+h} - \frac{1}{x}$$

$$= \frac{x-(x+h)}{x(x+h)}$$

$$= \frac{-h}{x(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h \to 0} \left[\frac{1}{\cancel{h}} \times \frac{-\cancel{h}}{x(x+h)}\right]$$

$$= \frac{-1}{x(x+0)} = \frac{-1}{x^2}.$$

ii) $$f(x) = \frac{x^2+1}{x^2-1}$$

$$f'(x) = \frac{(x^2-1)(2x) - (x^2+1)(2x)}{(x^2-1)^2}$$

$$= \frac{2x[\cancel{x^2}-1-\cancel{x^2}-1]}{(x^2-1)^2}$$

$$= \frac{2x(-2)}{(x^2-1)^2}$$

$$= \frac{-4x}{(x^2-1)^2}.$$

20 i)

x_i	$\lvert x_i - \bar{x} \rvert$ (10)
4	6
7	3
8	2
9	1
10	0
12	2
13	3
17	7
$\Sigma x_i = 80$	$\Sigma \lvert x_i - \bar{x} \rvert = 24$

$$* \quad \bar{x} = \frac{\Sigma x_i}{n}$$

$$= \frac{80}{8} = 10$$

$$* \; MD(\bar{x}) = \frac{\Sigma \lvert x_i - \bar{x} \rvert}{n}$$

$$= \frac{24}{8} = 3$$

ii)

class	x_i	f_i	u_i	f_iu_i	u_i^2	$f_iu_i^2$
4 – 8	4	3	−2	−6	4	12
8 – 12	8	6	−1	−6	1	6
12 – 16	$\boxed{12}$	4	0	0	0	0
16 – 20	16	7	1	7	1	7

$N = 20$ $\qquad \Sigma f_iu_i = -5$ $\qquad \Sigma f_iu_i^2 = 25$

$$\text{Variance} = \left[\frac{\Sigma f_iu_i^2}{N} - \left(\frac{\Sigma f_iu_i}{N}\right)^2\right] \times h^2$$

$$= \left[\frac{25}{20} - \left(\frac{-5}{20}\right)^2\right] 4^2$$

$$= \left[\frac{5}{4} - \frac{1}{16}\right] \times 16$$

$$= \left(\frac{20}{16} - \frac{1}{16}\right) \times 16$$

$$= \frac{19}{16} \times 16$$

$$= 19.$$