BITSAT - Paper 2021

Solved Paper

Question 1

The electric potential V is given as a function of distance × (metre) by $V = (5x^2 + 10x - 4)$ volt.
Value of electric field at x = 1m is

Options:

A. $-23 V/m$

B. $11 V/m$

C. $6 V/m$

D. $-20 V/m$

Answer: D

Solution:

Solution:
$V = 5x^2 + 10x - 4$
$E = \frac{-dV}{dx} = -(10x + 10)$.
At $x = 1m$, $E = -20 V/m$.

Question 2

A proton moving with a velocity $3 \times 10^5 m/s$ enters a magnetic field of 0.3 tesla at an angle of 30° with the field. The radius of curvature of its path will be (e/m for proton $= 10^8 C/kg$)

Options:

A. 2 cm

B. 0.5 cm

C. 0.02 cm

D. 1.25 cm

Answer: A

Solution:

Solution:
given that
$v = 3 \times 10^5 m / s$, $B = 0.3T$, and $e /m = 10^8 C / kg$
$F = \frac{mv^2}{r} = qvB \sin\theta$
$r = \frac{mv}{qB \sin\theta} \{\because q / m = 10^8\}$
$r = \frac{3 \times 10^5}{10^8 \times 0.3 \sin 30}$
radius of curvature of its path
$r = 2cm$

Question 3

If force (F), length (L) and time (T) are assumed to be fundamental units, then the dimensional formula of the mass will be

©

Options:

A. $[FL^{-1}T^2]$

B. $[FL^{-1}T^{-2}]$

C. $[FL^{-1}T^{-1}]$

D. $[FL^2T^2]$

Answer: A

Solution:

Solution:
Let $m = KF^aL^bT^c$
Substituting the dimensions of
$[F] = [MLT^{-2}]$, $[L] = [L]$ and $[T] = [T]$ and
comparing both side, we get $m = FL^{-1}T^2$

Question 4

A metal disc of radius $100\,cm$ is rotated at a constant angular speed of $60\,rad / s$ in a plane at right angles to an external field of magnetic induction $0.05\,Wb / m^2$. The emf induced between the centre and a point on the rim will be

©

Options:

A. 3V

B. 1.5V

C. 6V

D. 9V

Answer: B

Solution:

Solution:
Induced emf produced between the centre and a point on the disc is given by
$e = \frac{1}{2}\omega BR^2$
$\omega = 60\,\mathrm{rad/s}$, $B = 0.05\,\mathrm{Wb/m^2}$
and $R = 100\,\mathrm{cm} = 1\mathrm{m}$
We get $e = \frac{1}{2} \times 60 \times 0.05 \times (1)^2 = 1.5\mathrm{V}$

Question 5

A balloon is rising vertically up with a velocity of $29\mathrm{ms}^{-1}$. A stone is dropped from it and it reaches the ground in 10 seconds. The height of the balloon when the stone was dropped from it, was $(g = 9.8\mathrm{ms}^{-2})$

Options:

A. 100m

B. 200m

C. 400m

D. 150m

Answer: B

Solution:

Solution:
For stone to be dropped from rising balloon of velocity $29\mathrm{m/s}$, $u = -29\mathrm{m/s}$, $t = 10\,\mathrm{sec}$.
$\therefore h = -29 \times 10 + \frac{1}{2} \times 9.8 \times 100$
$= -290 + 490 = 200\mathrm{m}$.

Question 6

An alternating voltage $V = V_0 \sin \omega t$ is applied across a circuit. As a result, a current $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$ flows in it. The power consumed per cycle is

Options:

A. zero

B. $0.5V_0I_0$

C. $0.707V_0I_0$

D. $1.414V_0I_0$

Answer: A

Solution:

Solution:
The phase angle between voltage V and current I is $\frac{\pi}{2}$.

Therefore, power factor $\cos\phi = \cos\left(\frac{\pi}{2}\right) = 0$.

Hence the power consumed is zero.

Question 7

A gun fires two bullets at 60° and 30° with horizontal. The bullets strike at some horizontal distance. The ratio of maximum height for the two bullets is in the ratio of

Options:

A. 2 : 1

B. 3 : 1

C. 4 : 1

D. 1 : 1

Answer: B

Solution:

Solution:
The bullets are fired at the same initial speed

$$\frac{H}{H'} = \frac{u^2\sin^2 60°}{2g} \times \frac{2g}{u^2\sin^2 30°} = \frac{\sin^2 60°}{\sin^2 30°}$$

$$= \frac{(\sqrt{3}/2)^2}{(1/2)^2} = \frac{3}{1}$$

Question 8

Green light of wavelength 5460Å is incident on an air-glass interface. If the refractive index of glass is 1.5, the wavelength of light in glass would be ($c = 3 \times 10^8 ms^{-1}$)

Options:

A. 3640Å

B. 5460Å

C. 4861Å

D. None of the above

Answer: A

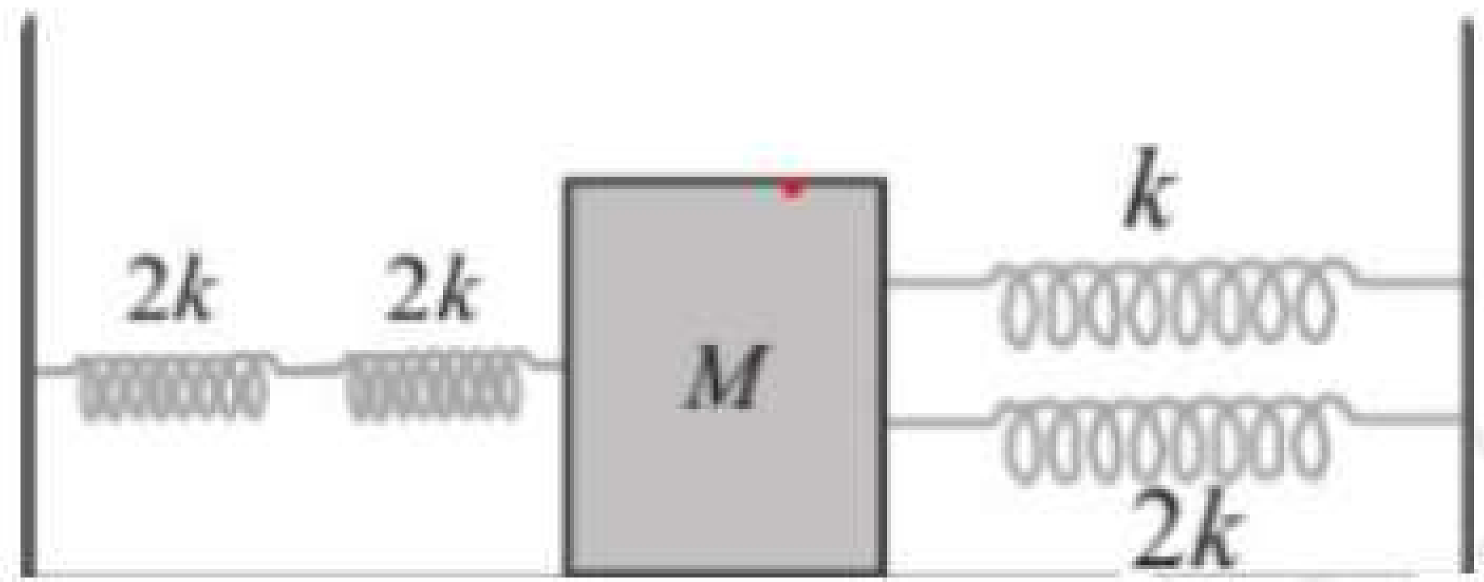
Solution:

Solution:

$a^{\lambda}g = \frac{\lambda_a}{\mu} = \frac{5460}{1.5} = 3640Å$

Question 9

Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system



Options:

A. $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

B. $\frac{1}{\pi}\sqrt{\frac{k}{M}}$

C. $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$

D. $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

Answer: A

Solution:

Solution:
Two springs on the L.H.S. of mass M are in series and two springs on the R.H.S. of mass M are in parallel. These combinations of springs will be considered in parallel to mass M . Thus effective spring constant,

$K = \frac{2k \times 2k}{2k + 2k} + (k + 2k) = 4k$

$\therefore$ Frequency $v = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{4k}{M}}$.

Question 10

Suppose the law of gravitational attraction suddenly changes and becomes an inverse cube law i.e. F $\propto \frac{1}{r^3}$, but still remaining a central force. Then

Options:

A. Kepler's law of area still holds

B. Kepler's law of period still holds

C. Kepler's law of area and period still holds

D. neither the law of area, nor the law of period still holds

Answer: A

Solution:

Solution:
According to kepler's law of area

$\frac{dA}{dt} = \frac{L}{2m}$

For central forces, torque = 0

$\therefore L =$ constant

$\therefore \frac{dA}{dt} =$ constant

Question 11

Three specimens A, B, C of same radioactive element has activities 1 microcurie, 1 rutherford and 1 becquerel respectively. Which specimen has maximum mass?

Options:

A. A

B. B

C. C

D. all have equal masses

Answer: B

Solution:

Solution:
1 becquerel $= 1 \text{ dis / s}$
1 rutherford $= 10^6 \text{ dis / s}$
1 curie $= 3.7 \times 10^{10} \text{ dis / s}$

Question 12

A solenoid is 2m long and 3 cm in diameter. It has 5 layers of winding of 1000 turns each and carries a current of 5A. What is the magnetic field at its centre ? Use the standard value of μ_0.

Options:

A. $1.57 \times 10^{-4}\text{T}$

B. $2.57 \times 10^{-3}\text{T}$

C. $1.57 \times 10^{-2}\text{T}$

D. $3.57 \times 10^{-6}\text{T}$

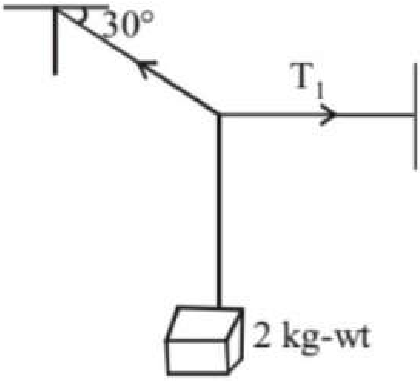
Answer: C

Solution:

Solution:
Length of solenoid, $l = 2\text{m}$
Total number of turns, $N = 5 \times 1000 = 5000$
Number of turns per unit length,
$n = \frac{N}{l} = \frac{5000}{2} = 2500\text{m}^{-1}$
Current, $I = 5\text{A}$
Now, $B = \mu_0 nI = 4\pi \times 10^{-7} \times 2500 \times 5\text{T}$
$= 1.57 \times 10^{-2}\text{T}$

Question 13

A body of weight 2 kg is suspended as shown in the figure. The tension T_1 in the horizontal string (in kg wt) is



Options:

A. $\frac{2}{\sqrt{3}}$

B. $\frac{\sqrt{3}}{2}$

C. $2\sqrt{3}$

D. 2

Answer: C

Solution:

Solution:
The tension in the above string can be divided into two components where the component opposite to weight will balance the weight while the component of this tension opposite to T_1 will be equal to T_1.
$T\ \sin\theta = 2\text{kgwt}$
$T\ \cos\theta = T_1$
The first equation gives
$T\ = 4\text{kgwt}$
So T_1 will be,
$T_1 = 2\sqrt{3}\text{kgwt}$

Question 14

A wire of a certain material is stretched slowly by ten per cent. Its new resistance and specific resistance become respectively:

Options:

A. 1.2 times, 1.3 times

B. 1.21 times, same

C. both remain the same

D. 1.1 times, 1.1 times

Answer: B

Solution:

Solution:
Let we have a wire of length l, cross-sectional area A, and Resistivity ρ
Then Resistance of wire is
$R = \rho \frac{l}{A} - (1)$
The volume of cylindrical wire is the product of its length and cross-sectional area.
$V = I \cdot A$
If length is increased by 10 percent, the new length l' is
$I' = | + 10\% \text{ of } I = 1.1 | - (2)$
After stretching the cross-sectional area will also change, as volume remains constant. Let the new cross-sectional area be A'
$V = I \cdot A = I' A' \ldots (3)$
Putting (2) in (3) we have
$I \cdot A = 1.1 I \cdot A'$
$A' = \frac{A}{1.1} \ldots . (4)$
So, the Resistance of this stretched wire is
$R' = \rho \frac{l'}{A'} \cdots \quad (5)$
Putting (2) and (4) in (5)
$R' = \rho \left(\frac{1.1 l}{A / 1.1} \right)$
$\Rightarrow R' = \rho \left(\frac{1.1 \times 1.1 l}{A} \right)$
$\Rightarrow R' = \rho \left(\frac{1.21 l}{A} \right)$
$\Rightarrow R' = 1.21 \left(\rho \frac{l}{A} \right) - (6)$
Comparing $Eq(6)$ and $Eq(1)$ We get $R' = 1.21R$
So, the resistance increases 1.21 times.

Question 15

The significant result deduced from the Rutherford's scattering experiment is that

Options:

A. whole of the positive charge is concentrated at the centre of atom

B. there are neutrons inside the nucleus

C. α - particles are helium nuclei

D. electrons are embedded in the atom

Answer: A

Solution:

Solution:
The significant result deduced from the Rutherford's scatter ing is that whole of the positive charge is concentrated at the centre of atom i.e. nucleus.

Question 16

A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2m while applying the force and the ball goes

upto 2m height further, find the magnitude of the force. (Consider $g = 10m / s^2$).

Options:

A. 4N

B. 16N

C. 20N

D. 22N

Answer: D

Solution:

Solution:
Let the velocity of the ball just when it leaves the hand is u then applying, $v^2 - u^2 = 2as$ for upward journey
$\Rightarrow -u^2 = 2(-10) \times 2$
$\Rightarrow u^2 = 40$
Again applying $v^2 - u^2 = 2as$ for the upward journey of the ball, when the ball is in the hands of the thrower,
$v^2 - u^2 = 2as$
$\Rightarrow 40 - 0 = 2(a)0.2$
$\Rightarrow a = 100m / s^2$
$\therefore F = ma = 0.2 \times 100 = 20N$
$\Rightarrow N - mg = 20$
$\Rightarrow N = 20 + 2 = 22N$

Question 17

A zener diode of voltage $V_Z(= 6V)$ is used to maintain a constant voltage across a load resistance $R_L(= 1000\Omega)$ by using a series resistance $R_s(= 100\Omega)$. If the e.m.f. of source is E (= 9V), what is the power being dissipated in Zener diode ?

Options:

A. 0.144 watt

B. 0.324 watt

C. 0.244 watt

D. 0.544 watt

Answer: A

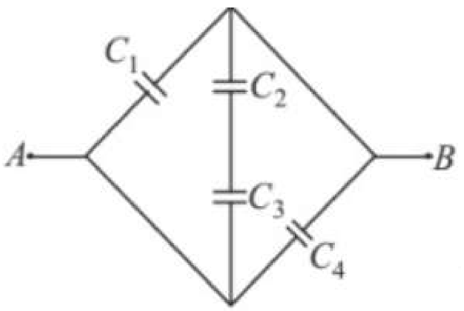
Solution:

Solution:
Here, $E = 9V; V_z = 6; R_L = 1000\Omega$
and $R_s = 100\Omega$,

Potential drop across series resistor
$V = E - V_Z = 9 - 6 = 3V$
Current through series resistance R_S is
$I = \frac{V}{R} = \frac{3}{100} = 0.03A$
Current through load resistance R_L is
$I_L = \frac{V_Z}{R_L} = \frac{6}{1000} = 0.006A$
Current through Zener diode is
$I_Z = I - I_L = 0.03 - 0.006 = 0.024$ amp.
Power dissipated in Zener diode is
$P_Z = V_Z I_Z = 6 \times 0.024 = 0.144$ Watt

Question 18

In a given network the equivalent capacitance between A and B is $C_1 = C_4 = 1\mu F$, $C_2 = C_3 = 2\mu F$.



Options:

A. 3μF

B. 6μF

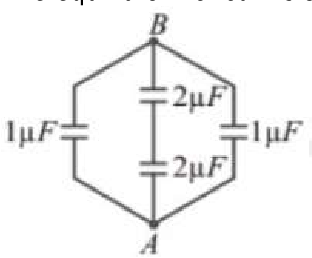
C. 4.5μF

D. 2.5μF

Answer: A

Solution:

Solution:
The equivalent circuit is shown in figure.



$C_{AB} = 1 + \frac{2 \times 2}{2 + 2} + 1$
$C_{AB} = 3\mu F.$

Question 19

In a simple pendulum of length I the bob is pulled aside from its equilibrium position through an angle θ and then released. The bob passes through the equilibrium position with speed

Options:

A. $\sqrt{2gl\,(1+\cos\theta)}$

B. $\sqrt{2gl\ \sin\theta}$

C. $\sqrt{2gl}$

D. $\sqrt{2gl\,(1-\cos\theta)}$

Answer: D

Solution:

Solution:
If l is length of pendulum and θ be angular amplitude then from principle of conservation of energy
(Initial total energy) $0 + mgh = \frac{1}{2}mv^2 + 0$ (Final total energy)
$\Rightarrow v = \sqrt{2gh} = \sqrt{2gl\,(1-\cos\theta)}$

Question 20

A moving coil galvanometer has resistance of 10Ω and full scale deflection of 0.01A. It can be converted into voltmeter of 10V full scale by connecting into resistance of

Options:

A. 9.90Ω in series

B. 10Ω in series

C. 990Ω in series

D. 0.10Ω

Answer: C

Solution:

Solution:
Let G be resistance of galvanometer and i_g the current through it. Let V is maximum potential difference, then from Ohm's law

$i_g = \frac{V}{G+R}$
$\Rightarrow R = \frac{V}{i_g} - G$
Given, $G = 10\Omega$, $i_g = 0.01A$
$V = 10$ volt

$\therefore R = \frac{10}{0.01} - 10 = 990\Omega$

Thus, on connecting a resistance R of 990Ω in series with the galvanometer, the galvanometer will become a voltmeter of range zero to $10V$.

For the voltmeter, a high resistance is series with the galvanometer and so the resistance of a voltmeter is very high compared to that of galvanometer.

Question 21

The frequency of whistle of an engine appears to be $(4/5)^{th}$ of initial frequency when it crosses a stationary observer. If the velocity of sound is $330m/s$, then the speed of engine will be

Options:

A. $30m/s$

B. $36.6m/s$

C. $40m/s$

D. $330m/s$

Answer: B

Solution:

Solution:

$n' = \frac{nv}{v - v_s}$ ....(1)

$n'' = \frac{nv}{v + v_s}$ ....(2)

From (1) and (2), $\frac{n'}{n''} = \frac{v + v_s}{v - v_s}$ ....(3)

According to question, $\frac{n'}{n'} = \frac{5}{4}$

$v_s = ?v = 330m/s$ ....(4)

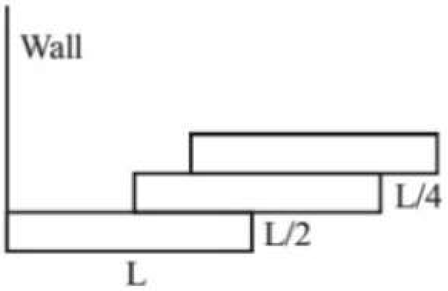
From eq. (3) and (4)

$\frac{5}{4} = \left[\frac{330 + v_s}{330 - v_s}\right] \Rightarrow 9v_s = 330$

$\therefore v_s = 36.6m/s$

Question 22

Three bricks each of length L and mass M are arranged as shown from the wall. The distance of the centre of mass of the system from the wall is



Options:

A. $\frac{L}{4}$

B. $\frac{L}{2}$

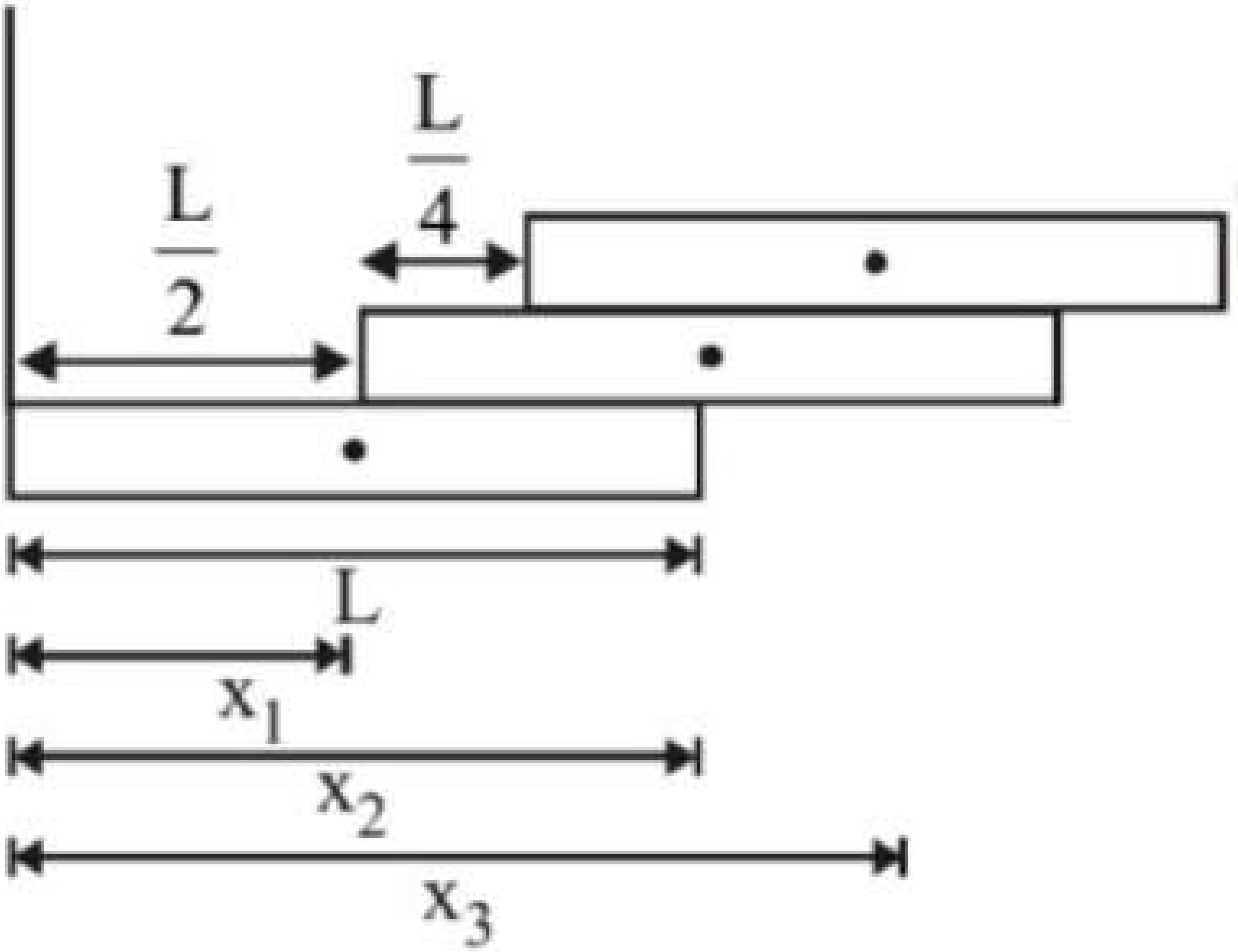
C. $\left(\frac{3}{2}\right)L$

D. $\left(\frac{11}{12}\right)L$

Answer: D

Solution:

Solution:



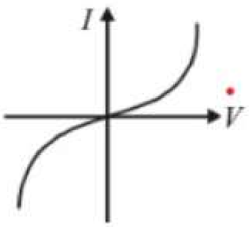
$x_1 = \frac{L}{2}, x_2 = L, x_3 = \frac{5L}{4}$

$\therefore x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$

$= \frac{M \times \frac{L}{2} + M \times L + M \times \frac{5L}{4}}{M + M + M} = \frac{11L}{12}$

Question 23

The I − V characteristics shown in figure represents



Options:

A. ohmic conductors

B. non-ohmic conductors

C. insulators

D. superconductors

Answer: B

Solution:

Solution:
The figure is showing I − V characteristics of non ohmic or non linear conductors.

Question 24

The angle of dip at a place is 37° and the earth's magnetic field is 6×10^{-5}T. The earth's magnetic field at this place is $\left(\tan 37° = \frac{3}{4}\right)$

Options:

A. 7×10^{-5}T

B. 6×10^{-5}T

C. 5×10^{-5}T

D. 10^{-4}T

Answer: D

Solution:

Solution:

$\tan\theta = \frac{V}{H} = \frac{3}{4}\left[\because \tan 37° = \frac{3}{4}\right]$

$\therefore V = \frac{3}{4}H$

$V = 6 \times 10^{-5}T$

$H = \frac{4}{3} \times 6 \times 10^{-5}T = 8 \times 10^{-5}T$

$\therefore B_{total} = \sqrt{V^2 + H^2} = \sqrt{(36 + 64)} \times 10^{-5}$

$= 10 \times 10^{-5} = 10^{-4}T.$

Question 25

A round disc of moment of inertia I_2 about its axis perpendicular to its plane and passing through its centre is placed over another disc of moment of inertia I_1 rotating with an angular velocity ω about the same axis. The final angular velocity of the combination of discs is

Options:

A. $\frac{(I_1 + I_2)\omega}{I_1}$

B. $\frac{I_2\omega}{I_1 + I_2}$

C. ω

D. $\frac{I_1\omega}{I_1 + I_2}$

Answer: D

Solution:

Solution:

Angular momentum will be conserved

i.e., $I_1\omega = I_1\omega' + I_2\omega'$

$\Rightarrow \omega' = I_1\omega I_1 + I_2$

Question 26

For an RLC circuit driven with voltage of amplitude V_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance the quality factor, Q is given by

Options:

A. $\frac{\omega_0}{L}$

B. $\frac{\omega_0 L}{R}$

C. $\frac{R}{\omega_0 C}$

D. $\frac{CR}{\omega_0}$

Answer: B

Solution:

Solution:

Question 27

The two ends of a rod of length L and a uniform cross-sectional area A are kept at two temperatures T_1 and $T_2(T_1 > T_2)$. The rate of heat transfer, $\frac{dQ}{dt}$ through the rod in a steady state is given by:

Options:

A. $\frac{k(T_1 - T_2)}{LA}$

B. $kLA(T_1 - T_2)$

C. $\frac{kA(T_1 - T_2)}{L}$

D. $\frac{kL(T_1 - T_2)}{A}$

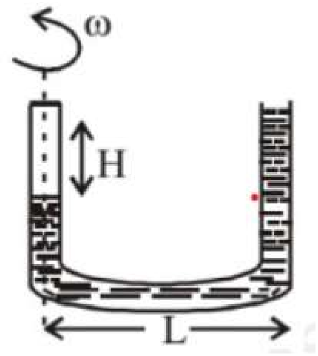
Answer: C

Solution:

Solution:
$\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{L}$
[$(T_1 - T_2)$ is the temperature difference]

Question 28

A U - shaped tube contains a liquid of density rho and it is rotated about the left dotted line as shown in the figure.
Find the difference in the levels of liquid column.



Options:

A. $\frac{\omega^2 L^2}{2g}$

B. $\frac{\omega^2 L^2}{2\sqrt{2}g}$

C. $\frac{2\omega^2 L^2}{g}$

D. $\frac{2\sqrt{2}\omega^2 L^2}{g}$

Answer: A

Solution:

Solution:

Question 29

Monochromatic light of wavelength 667 nm is produced by a helium neon laser. The power emitted is 9 mW. The number of photons arriving per sec on the average at a target irradiated by this beam is:

Options:

A. 3×10^{16}

B. 9×10^{15}

C. 3×10^{19}

D. 9×10^{17}

Answer: A

Solution:

Solution:
$\lambda = 667 \times 10^{-9}\text{m}$, $P = 9 \times 10^{-3}\text{W}$
$P = \frac{Nhc}{\lambda}$, $N =$ No. of photons emitted/sec.

$N = \frac{9 \times 10^{-3} \times 667 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 3 \times 10^{16}$ / sec

----

Question 30

The displacement y of a particle in a medium can be expressed as, $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$ m where t is in second and x in meter. The speed of the wave is

©

Options:

A. 20m / s

B. 5m / s

C. 2000m / s

D. 5πm / s

Answer: B

Solution:

Solution:
From equation given,
$\omega = 100$
and $k = 20$;
$v = \frac{\omega}{k} = \frac{100}{20} = 5$m / s

Question 31

An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of ' γ ' of the gas is :

©

Options:

A. 1.5

B. 1.4

C. 1.3

D. 1.6

Answer: B

Solution:

Solution:

Volume $v = \frac{m}{d}$ and using $PV^{\gamma} =$ constant

$\frac{P'}{P} = \frac{V}{V'} = \left(\frac{d'}{d}\right)^{\gamma}$

or $128 = (32)^{\gamma}$

$\therefore \gamma = \frac{7}{5} = 1.4$

Question 32

A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then $\frac{Q}{q}$ equals:

©

Options:

A. -1

B. 1

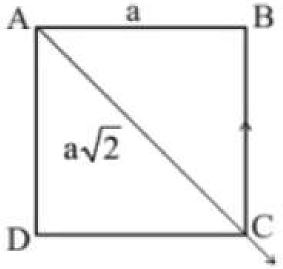
C. $-\frac{1}{\sqrt{2}}$

D. $-2\sqrt{2}$

Answer: D

Solution:

Solution:



It is given that $\vec{F}_A + \vec{F}_B + \vec{F}_D = 0$

Where $\vec{F}_A$, $\vec{F}_B$ and $\vec{F}_D$ are the forces applied by charges placed at A, B and D on the charge placed at C.

$\Rightarrow \vec{F}_B + \vec{F}_D = -\vec{F}_A$

$\left|\vec{F}_B + \vec{F}_D\right| = \sqrt{2}\frac{KqQ}{a^2}$

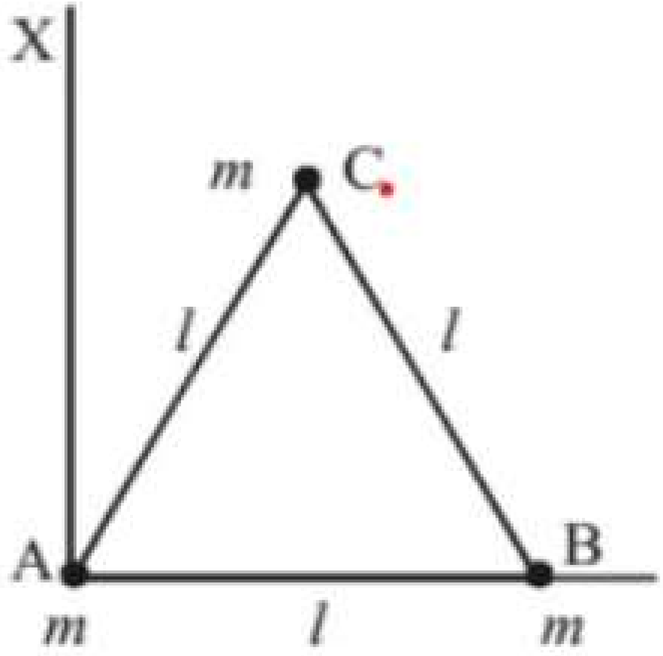
$\Rightarrow \left|\vec{F}_A\right| = \frac{KQ^2}{2a^2}$

$\therefore \sqrt{2}\frac{KqQ}{a^2} = -\frac{KQ^2}{2a^2}$

$\Rightarrow \frac{Q}{q} = -2\sqrt{2}$

Question 33

Three particles, each of mass m gram, are situated at the vertices of an equilateral triangle ABC of side l cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to A B and in the plane of ABC , in gram - cm^2 units will be



Options:

A. $\frac{3}{2}ml^2$

B. $\frac{3}{4}ml^2$

C. $2ml^2$

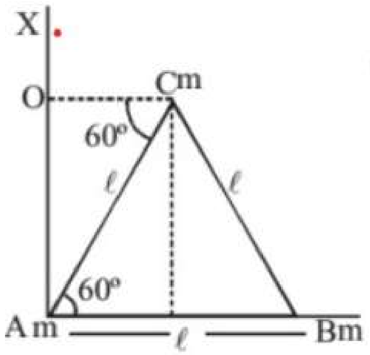
D. $\frac{5}{4}ml^2$

Answer: D

Solution:

Solution:

$I_{AX} = m(AB)^2 + m(OC)^2 = ml^2 + m(l\cos 60°)^2 = ml^2 + \frac{ml^2}{4} = \frac{5}{4}ml^2$



Question 34

Electrons are accelerated through a potential difference V and protons are accelerated through a potential difference 4V. The de - Broglie wavelengths are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by: (given m_e is mass of electron and m_p is mass of proton).

©

Options:

A. $\sqrt{\frac{m_p}{m_e}}$

B. $\sqrt{\frac{m_e}{m_p}}$

C. $\frac{1}{2}\sqrt{\frac{m_e}{m_p}}$

D. $2\sqrt{\frac{m_p}{m_e}}$

Answer: D

Solution:

Solution:
Energy = charge × potential diff.
$E_{electron} = q_e V$ and $E_{proton} = q_p 4V$
de - Broglie wavelength $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2\,mE}}$
$\lambda_e = \frac{h}{\sqrt{2m_e eV}}$ and $\lambda_P = \frac{h}{\sqrt{2m_p e4V}}$ $\because (q_e = q_p)$

$$\therefore \frac{\lambda_e}{\lambda_P} = \frac{\frac{h}{\sqrt{2m_e eV}}}{\frac{h}{\sqrt{2m_P e4V}}} = 2\sqrt{\frac{m_P}{m_e}}$$

Question 35

The temperature of an ideal gas is increased from 120K to 480K. If at 120K, the root mean square velocity of the gas molecules is v , at 480K it becomes

©

Options:

A. 4u

B. 2u

C. $\frac{u}{2}$

D. $\frac{u}{4}$

Answer: B

Solution:

Solution:

Question 36

A point particle is held on the axis of a ring of mass m and radius r at a distance r from its centre C. When released, it reaches C under the gravitational attraction of the ring. Its speed at C will be

©

Options:

A. $\sqrt{\frac{2Gm}{r}(\sqrt{2}-1)}$

B. $\sqrt{\frac{Gm}{r}}$

C. $\sqrt{\frac{2Gm}{r}\left(1-\frac{1}{\sqrt{2}}\right)}$

D. $\sqrt{\frac{2Gm}{r}}$

Answer: C

Solution:

Solution:
Let ' M ' be the mass of the particle
Now, $E_{initial} = E_{final}$
i.e. $\frac{GMm}{\sqrt{2}r} + 0 = \frac{GM\,m}{r} + \frac{1}{2}MV^2$
or, $\frac{1}{2}MV^2 = \frac{GMm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$
$\Rightarrow \frac{1}{2}V^2 = \frac{Gm}{r}\left[1 - \frac{1}{\sqrt{2}}\right]$ or,
$V = \sqrt{\frac{2Gm}{r}\left(1 - \frac{1}{\sqrt{2}}\right)}$

Question 37

A plane electromagnetic wave is incident on a plane surface of area A, normally and is perfectly reflected. If energy E strikes the surface in time t then force exerted on the surface is (c = speed of light)

Options:

A. $\frac{2E}{Atc}$

B. $\frac{E}{2c}$

C. $\frac{2E}{ct}$

D. zero

Answer: C

Solution:

Solution:
Incident momentum, $p = \frac{E}{c}$
For perfectly reflecting surface with normal incidence
$\Delta p = 2p = \frac{2E}{c}$; $F = \frac{\Delta p}{\Delta t} = \frac{2E}{ct}$

Question 38

The compressibility of water is 4×10^{-5} per unit atmospheric pressure. The decrease in volume of $100cm^3$ of water under a pressure of 100 atmosphere will be

Options:

A. $0.4cm^3$

B. $4 \times 10^{-5}cm^3$

C. $0.025 cm^3$

D. $0.004 cm^3$

Answer: A

Solution:

Solution:

Compressibility $= \frac{1}{B} = \frac{\Delta V / V}{P}$.

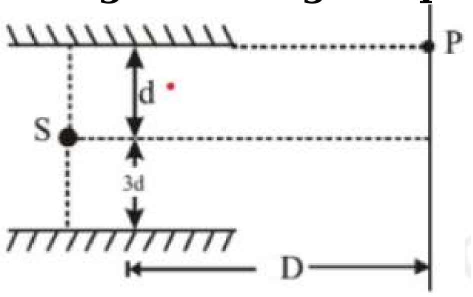
Here, $P = 100\,atm$

Compressibility $= 4 \times 10^{-5}$ and $V = 100 cm^3$

Hence, $\Delta V = 0.4 cm^3$

Question 39

Consider the optical system shown in Fig. The point source of light S is having wavelength equal to λ . The light is reaching screen



only after reflection. For point P to be second maxima, the value of λ would be ($D >> d$ and $d >> \lambda$)

Options:

A. $\frac{12d^2}{D}$

B. $\frac{6d^2}{D}$

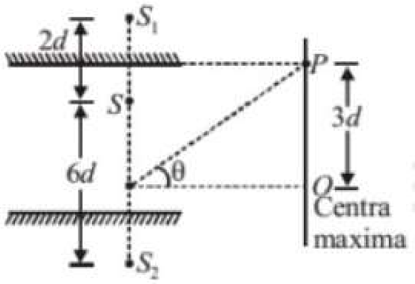
C. $\frac{3d^2}{D}$

D. $\frac{24d^2}{D}$

Answer: A

Solution:

Solution:



At P, $\Delta x = \frac{(8d) \times 3d}{D}$
For 2 nd maxima, $\Delta x = 2\lambda$
$\Rightarrow \frac{24d^2}{D} = 2\lambda$
$\Rightarrow \lambda = \frac{12d^2}{D}$

Question 40

A spherical drop of radius R is divided into eight equal droplets. If surface tension is T , then the work done in this process is

©

Options:

A. $2\pi R^2 T$

B. $3\pi R^2 T$

C. $4\pi R^2 T$

D. $2\pi R T^2$

Answer: C

Solution:

Solution:
Volume of 8 small droplets = Volume of 1 big drop
$\Rightarrow \frac{4}{3}\pi r^3 \times 8 = \frac{4}{3}\pi R^3 \Rightarrow r = \frac{R}{(8)^{1/3}}$
Work done =(Change in area) × surface tension
$= (4\pi r^2 n - 4\pi R^2)T$
$\left\{ 4\pi \left(\frac{R^2}{(8)^{2/3}} \right) \times 8 - 4\pi R^2 \right\} T = 4\pi R^2 T$

Question 41

Which one of the following ions has the maximum magnetic moment?

©

Options:

A. Sc^{3+}

B. Ti^{3+}

C. Cr^{3+}

D. Fe^{3+}

Answer: D

Solution:

Solution:
$Fe^{3+}(d^5)$ has 5 unpaired electrons therefore magneticmoment $= \sqrt{n(n+2)} = \sqrt{5(5+2)} = 5.91$ which is maximum among given options. As Sc^{3+}, Ti^{3+}, Cr^{3+} and V^{3+} contain 0, 1, 3, and 2 number of unpaired electrons respectively.

Question 42

Which of the following represents the correct order of $Cl-O$ bond lengths in ClO^-, ClO_2^-, ClO_3^-, ClO_4^- ?

Options:

A. $ClO_4^- = ClO_3^- = ClO_2^- = ClO^-$

B. $ClO^- < ClO_2^- < ClO_3^- < ClO_4^-$

C. $ClO_4^- < ClO_3^- < ClO_2^- < ClO^-$

D. $ClO_3^- < ClO_4^- < ClO_2^- < ClO^-$

Answer: C

Solution:

Solution:

The b.o in $Cl-O^-$ is 1

The b.o in $O=Cl-O^-$ is 1.5

The b.o. in $O=\underset{\underset{O^-}{|}}{Cl}=O$ is $\frac{5}{3}=1.66$

The b.o. in $O=\overset{\overset{O}{||}}{\underset{\underset{O}{||}}{Cl}}-O^-$ is $\frac{7}{4}=1.75$

The bond length increases as b.o. decreases.

Question 43

Which one of the following coordination compounds is used to inhibit the growth of tumours?

Options:

A. Trans-platin

B. EDTA complex of calcium

C. $[(Ph_3P)_3RhCl]$

D. Cis - platin

Answer: D

Solution:

Solution:
Cisplatin $[Pt(NH_3)_2Cl_2]$ is used in cancer treatment.

Question 44

Aspirin is known as :

Options:

A. acetyl salicylic acid

B. phenyl salicylate

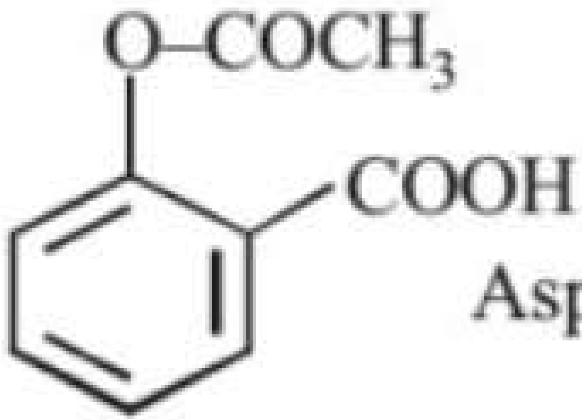
C. acetyl salicylate

D. methyl salicylic acid

Answer: A

Solution:

Solution:



Aspirin (Acetyl salicylic acid)

Question 45

In a 1st order reaction, reactant concentration C varies with time t as :

Options:

A. $\frac{1}{C}$ increases linearly with t

B. $\log C$ decreases linearly with t

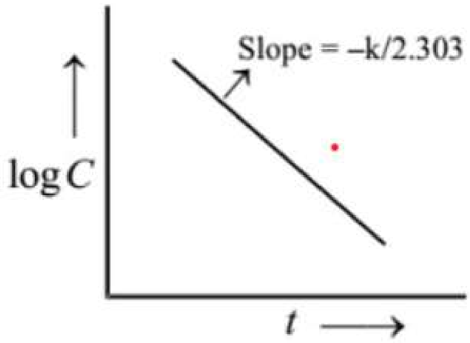
C. C decreases with $\frac{1}{t}$

D. $\log C$ decreases with $\frac{1}{t}$

Answer: B

Solution:

Solution:



Question 46

Which of the following has the highest $p\pi - p\pi$ bonding tendency ?

©

Options:

A. N

B. P

C. As

D. Sb

Answer: A

Solution:

Solution:
Nitrogen due to small size is able to show $p\pi - p\pi$ lateral overlap forming $N \equiv N$, rest elements due to bigger size are not able to show $p\pi - p\pi$ lateral overlap.

Question 47

Identify the incorrect statement :

©

Options:

A. The S - S - S bond angles in the S_8 and S_6 rings are the same.

B. Rhombic and monoclinic sulphur have S_8 molecules.

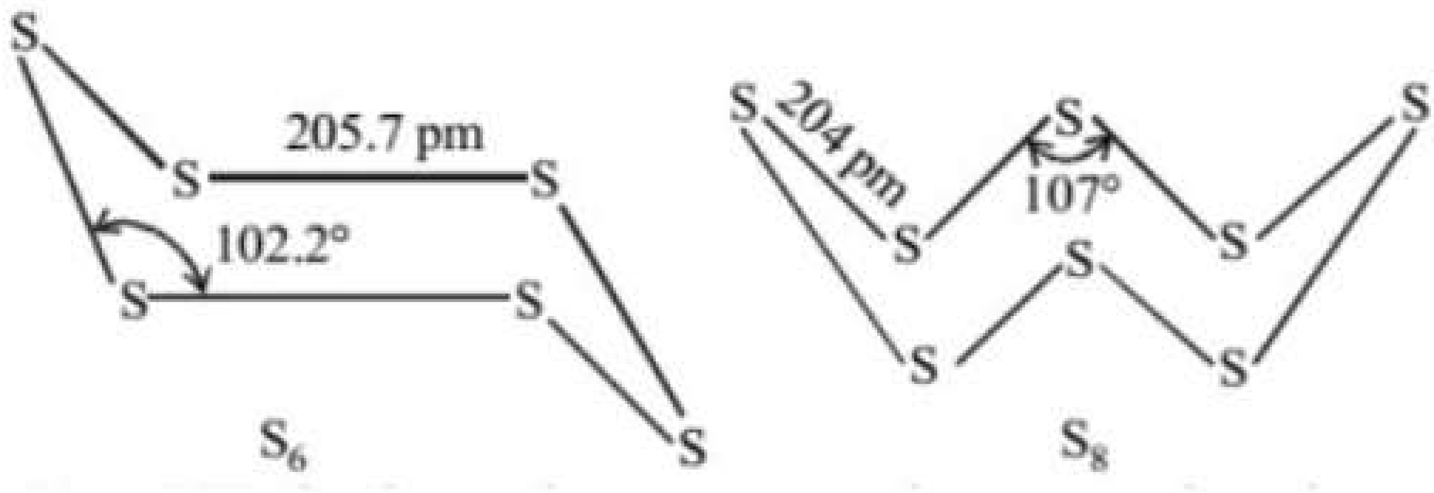
C. S_2 is paramagnetic like oxygen

D. S_8 ring has a crown shape.

Answer: A

Solution:

Solution:
The S_6 molecule has a chair-form hexagon ring with the approx. same bond length as that in S_8 , but with some what smaller bond angles i.e.
bond lengths are approx. same but bond angles are different.



Question 48

Which of the following does not contain any coordinate bond?

Options:

A. H_3O^+

B. BF_4^-

C. HF_2^-

D. SO_2^-

Answer: C

Solution:

Solution:
HF_2 is the only compound among the given options which does not contain any coordinate bond because it has hydrogen bonding.
$[H - F \bullet\bullet \bullet H]^-$

Question 49

Which of the following set contains species having same angle around the central atom?

Options:

A. SF_4, CH_4, NH_3

B. NF_3, BCl_3, NH_3

C. BF_3, NF_3, $AlCl_3$

D. BF_3, BCl_3, BBr_3

Answer: D

Solution:

Solution:
BF_3, BCl_3, BBr_3 are sp^2 hybridised. So, all have same structure and bond angle i.e. 120° .

Question 50

Liquids A and B form an ideal solution in the entire composition range. At 350K, the vapor pressures of pure A and pure B are 7×10^3 Pa and 12×10^3 Pa, respectively. The composition of the vapour is in equilibrium with a solution containing 40 mole percent of A at this temperature is:

©

Options:

A. $x_A = 0.37$; $x_B = 0.63$

B. $x_A = 0.28$; $x_B = 0.72$

C. $x_A = 0.4$; $x_B = 0.6$

D. $x_A = 0.76$; $x_B = 0.24$

Answer: B

Solution:

Solution:
$P°_A = 7 \times 10^3$
$P°_B = 12 \times 10^3$
$x'_A = 0.4$; $x'_B = 1 - 0.4$
$x'_B = 0.6$
$P_{total} = P°_A x'_A + P°_B x'_B$
$= 7 \times 10^3 \times 0.4 + 12 \times 10^3 \times 0.6$
$= (7 \times 0.4 + 12 \times 0.6) \times 10^3 = 10^4$
$x_A = \frac{P°_A x'_A}{P_{total}} = \frac{7 \times 10^3 \times 0.4}{10^4}$
$\therefore x_A = 0.28$, $x_B = 1 - 0.28 = 0.72$

Question 51

Hydrolysis of NCl_3 gives NH_3 and X .
Which of the following is X ?

©

Options:

A. $HClO_4$

B. $HClO_3$

C. HOCl

D. $HClO_2$

Answer: C

Solution:

Solution:
Completing the reaction, we get
$NCl_3 + 3H_2O \rightarrow 3\underset{'X'}{HClO} + NH_3$

Question 52

Element 'B' forms ccp structure and 'A' 'occupies half of the octahedral voids, while oxygen atoms occupy all the tetrahedral voids. The structure of bimetallic oxide is :

©

Options:

A. A_2BO_4

B. AB_2O_4

C. A_2B_2O

D. A_4B_2O

Answer: B

Solution:

Solution:
No. of lattice points = No. of octahedral voids $= \frac{1}{2} \times$ No. of tetrahedral voids in ccp structure
∴ No. of atoms of B = 4
No. of atoms of A = $\frac{1}{2} \times$ No. of octahedral voids
$= \frac{1}{2} \times 4 = 2$
No. of atoms of O = All tetrahedral voids $= 2 \times$ No. of lattice points $= 2 \times 4 = 8$
Hence, $A : B : O = 1 : 2 : 4$
Therefore, the formula of the compound is AB_2O_4

Question 53

$[Fe(en)_2(H_2O)_2]^{2+}$ + en → complex (X). The correct statement about the complex (X) is -

©

Options:

A. it is a low spin complex

B. it is diamagnetic

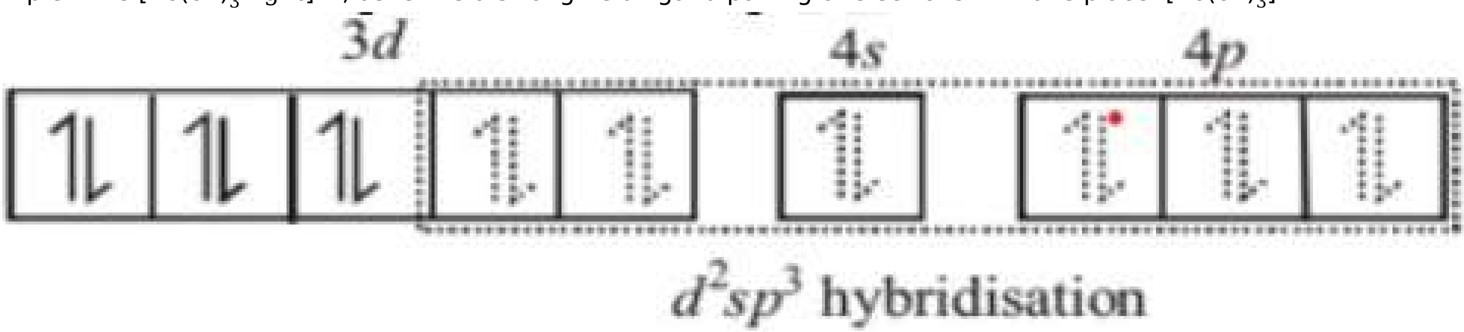
C. it shows geometrical isomerism

D. (a) and (b) both

Answer: D

Solution:

Solution:
Complex X is $[Fe(en)_3 right]^{2+}$; as 'en' is a strong field ligand pairing of electrons will take place. $[Fe(en)_3]^{2+}$:



Hence, hybridisation is d ^{2}sp^3 and complex is diamagnetic. As it has 3 bidentate symmetrical 'en' ligands so it will not show geometrical isomerism.

Question 54

The frequency of radiation emitted when the electron falls from n = 4 to n = 1 in a hydrogen atom will be (Given : ionization energy of H = 2.18×10^{-18}J atom $^{-1}$ and h = 6.625×10^{-34}Js)

©

Options:

A. $1.54 \times 10^{15} s^{-1}$

B. $1.03 \times 10^{15} s^{-1}$

C. $3.08 \times 10^{15} s^{-1}$

D. $2.00 \times 10^{15} s^{-1}$

Answer: C

Solution:

Solution:

$$v = \frac{1}{h} \times IE \times \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

$$= \frac{2.18 \times 10^{-18}}{6.625 \times 10^{-34}} \times \left[\frac{1}{1} - \frac{1}{16} \right] = 3.08 \times 10^{15} s^{-1}$$

Question 55

Cetyltrimethyl ammonium bromide is a popular

Options:

A. anionic detergent

B. cationic detergent

C. non-ionic detergent

D. sweetener

Answer: B

Solution:

Solution:
Cetyltrimethyl ammonium bromide which is a germicide, is a popular cationic detergent.

Question 56

Which of the following chemicals can be added for sweetening of food items at cooking temperature and does not provides calories?

Options:

A. Sucrose

B. Glucose

C. Aspartame

D. Sucralose

Answer: D

Solution:

Solution:
Sucralose is trichloro derivative of sucrose. It is stable at cooking temperture. It does not provide calories.

Question 57

In XeF_2, XeF_4, XeF_6 the number of lone pairs on Xe are respectively

Options:

A. 2, 3, 1

B. 1, 2, 3

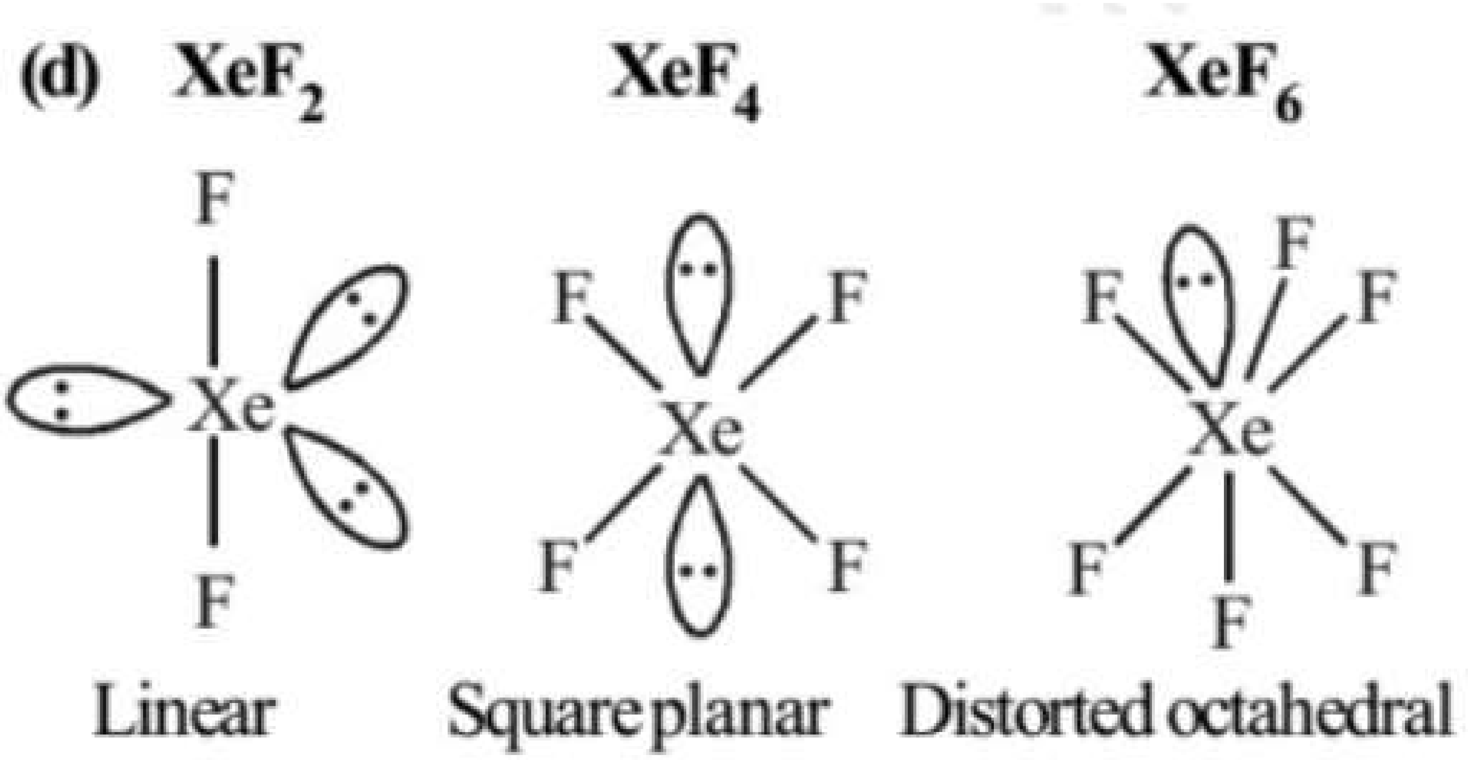
C. 4, 1, 2

D. 3, 2, 1.

Answer: D

Solution:

Solution:



Question 58

Which one of the following molecules will form a linear polymeric structure due to hydrogen bonding?

Options:

A. NH_3

B. H_2O

C. HCl

D. HF

Answer: D

Solution:

Solution:
HF form linear polymeric structure due to hydrogen bonding.

Question 59

Arsenic containing medicine used for the treatment of syphilis, is

Options:

A. erythromycin

B. ofloxacin

C. tetracycline

D. salvarsan

Answer: D

Solution:

Solution:

Question 60

The difference in the number of unpaired electrons of a metal ion in its high-spin and low-spin octahedral complexes is two. The metal ion is:

Options:

A. Ni^{2+}

B. Fe^{2+}

C. Co^{2+}

D. Mn^{2+}

Answer: C

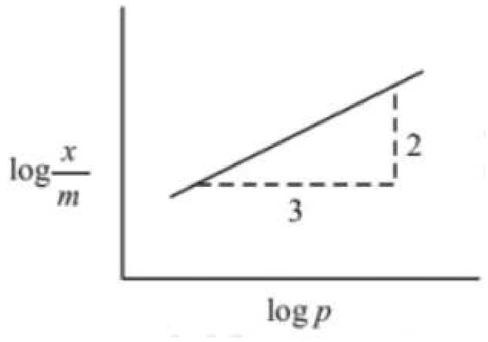
Solution:

Solution:

Metel ion	No. of unpaired elec.		Difference in the unpaired electrons
	High spin	Low spin	
$Ni^{2+}\left(3d^8\right)$	2	2	0
$Mn^{2+}\left(3d^5\right)$	5	1	4
$Fe\left(3d^6\right)$	4	0	4
$Co^{2+}\left(3d^7\right)$	3	1	2

Question 61

Adsorption of a gas follows Freundlich adsorption isotherm. x is the mass of the gas adsorbed on mass m of the adsorbent. The plot of $\frac{x}{m}$ versus $\log p$ is shown in the given graph.
$\frac{x}{m}$ is proportional to :



Options:

A. $p^{2/3}$

B. $p^{3/2}$

C. p^3

D. p^2

Answer: A

Solution:

Solution:
According to Freundlich adsorption isotherm

$\frac{x}{m} \propto p^{\frac{1}{n}}; \frac{x}{m} = kp^{\frac{1}{n}}$

Slope $= \frac{2}{3}$

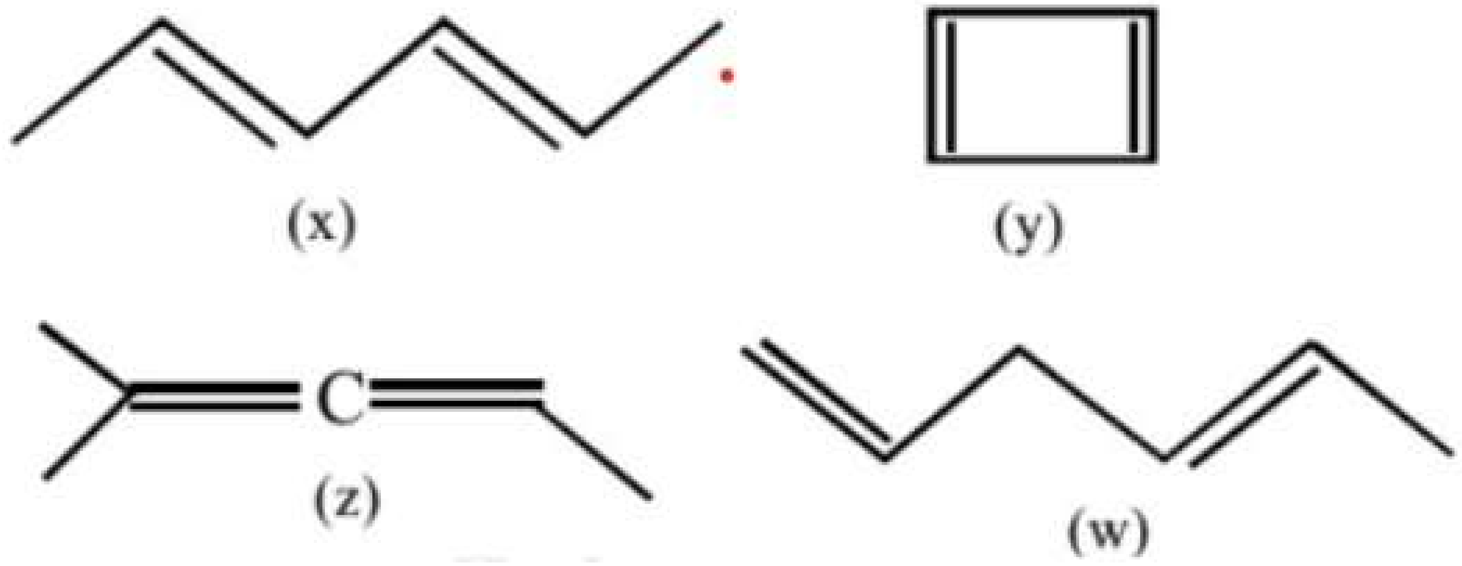
$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$

Slope $= \frac{1}{n} = \frac{2}{3}$

$\frac{x}{m} \propto p^{\frac{2}{3}}$

Question 62

The correct stability order of following species is -



Options:

A. x > y > w > z

B. y > x > w > z

C. x > w > z > y

D. z > x > y > w

Answer: C

Solution:

Solution:
x has a conjugated diene system, w an isolated diene system, z a cumulated diene system, and y an antiaromatic system.

Question 63

Nitrogen is obtained by the thermal decomposition of:

Options:

A. NH_4NO_2

B. NH_4NO_3

C. $AgNO_3$

D. $Pb(NO_3)_2$

Answer: A

Solution:

Solution:

$NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

Question 64

Calculate the wavelength (in nanometer) associated with a proton moving at $1.0 \times 10^3 ms^{-1}$. (Mass of proton $= 1.67 \times 10^{-27}$ kg and h $= 6.63 \times 10^{-34}$ Js)

Options:

A. 0.40 nm

B. 2.5 nm

C. 14.0 nm

D. 0.32 nm

Answer: A

Solution:

Solution:

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{1.67 \times 10^{-27} \times 1 \times 10^3}$$

$= 3.97 \times 10^{-10}$ meter $= 0.397$ nanometer ≈ 0.40 nm

Question 65

The solubility of silver bromide in hypo solution involves the formation of:

Options:

A. Ag_2SO_3

B. $Ag_2S_2O_3$

C. Ag^+

D. NH_4^+

Answer: B

Solution:

Solution:
Hypo solution is $Na_2S_2O_3$ solution which is used in photography for fixing films & prints. Photographic emulsions are made of AgBr. After developing, the film is put into hypo solution. This forms soluble complex with Ag.

$Na_2S_2O_3 + AgBr \rightarrow Ag_2S_2O_3 \xrightarrow{Na_2S_2O_3} Na_5[Ag(S_2O_3)_3]$

Question 66

Fructose reduces Tollen's reagent due to:

©

Options:

A. enolisation of fructose followed by conversion to glucose (having aldehydic group) by the base present in Tollen's reagent

B. asymmetric carbons

C. primary alcoholic group

D. secondary alcoholic group

Answer: A

Solution:

Solution:

Question 67

When C_2H_2, CH_4 and C_2H_4 pass through a test tube which has ammonical Cu_2Cl_2, find out which gas comes out unaffected from test tube?

©

Options:

A. C_2H_2 and CH_4

B. C_2H_2 and C_2H_4

C. C_2H_4 and CH_4

D. C_2H_2

Answer: C

Solution:

Solution:
C_2H_2 forms copper acetylide with ammonical Cu_2Cl_2
$Cu_2Cl_2 + 2NH_4OH \rightarrow Cu_2O + H_2O + 2NH_4Cl$
$Cu_2O + C_2H_2 \rightarrow \underset{\text{(copper acetylide)}}{Cu_2C_2} + H_2O$

Question 68

Which of the following acts as an antioxidant in edible oils

Options:

A. Vitamin B

B. Vitamin C

C. Vitamin D

D. Vitamin E

Answer: D

Solution:

Solution:
Vitamin E is an antioxidant present in edible oils.

Question 69

$[A] \xrightarrow{\text{reduction}} [B] \xrightarrow{CHCl_3 + KOH} [C] \xrightarrow{\text{reduction}} N-\text{Methylaniline, A is}$

Options:

A. Formaldehyde

B. Trichloromethane

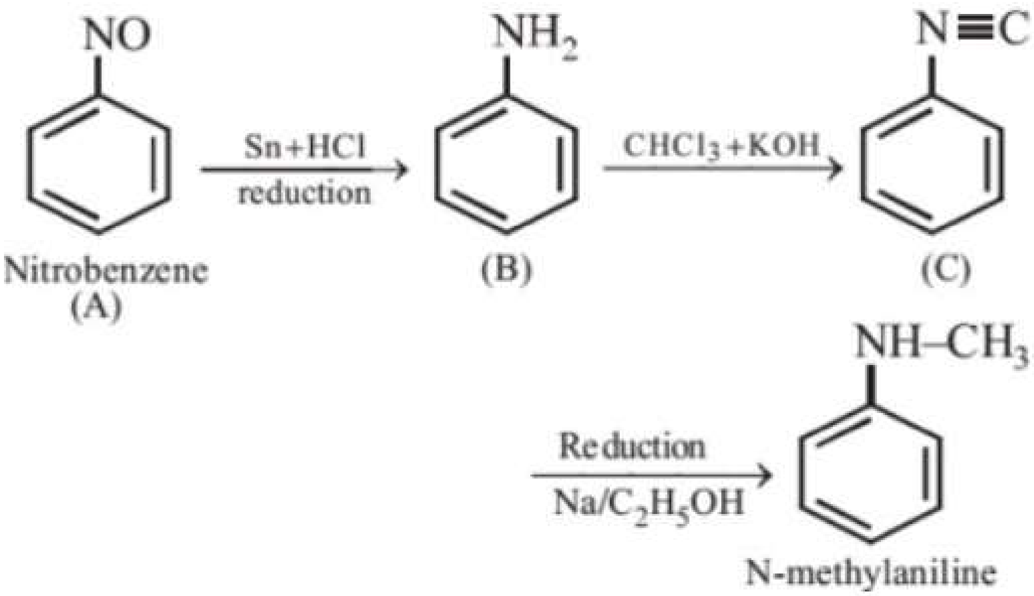
C. Nitrobenzene

D. Toluene

Answer: C

Solution:

Solution:



Question 70

In the reaction:

$$CH_3-\underset{\displaystyle |}{\overset{\displaystyle CH_3}{CH}}-CH_2-O-CH_2-CH_3+HI\xrightarrow{\text{Heated}}$$

Which of the following compounds will be formed?

Options:

A.

$$CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_3+CH_3CH_2OH$$

B.

$$CH_3-\underset{\displaystyle I}{\underset{|}{CH}}-CH_2CH_3+CH_3CH_2OH$$

C.

$$CH_3-\overset{\overset{\displaystyle CH_3}{|}}{CH}-CH_2OH+CH_3-CH_2-I$$

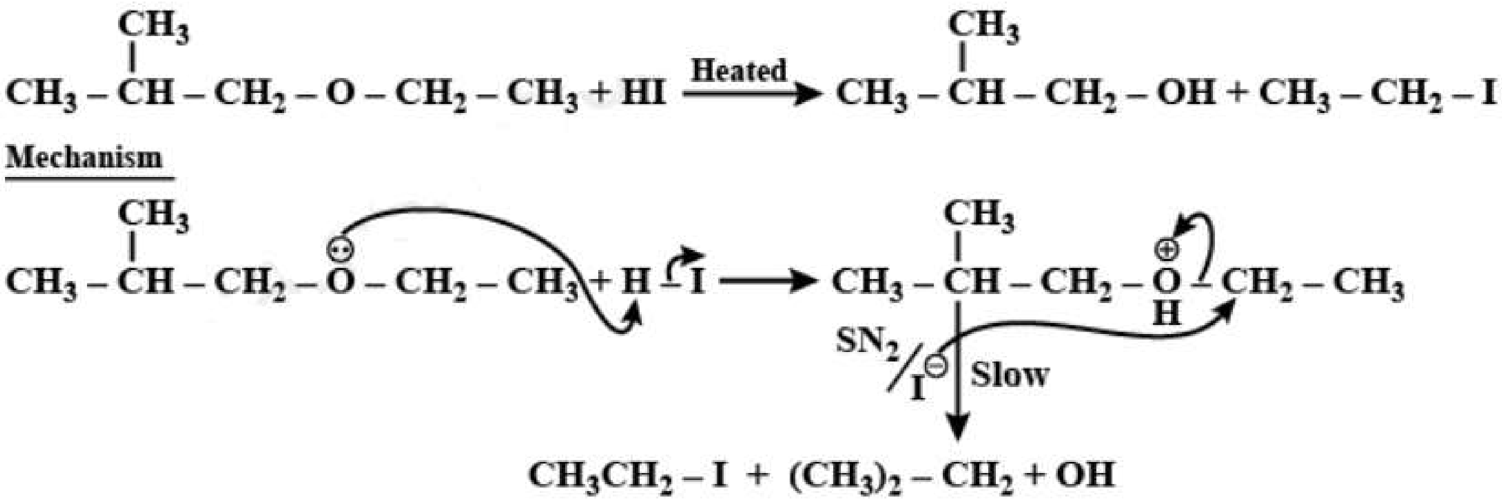
D.

$$CH_3-\overset{\overset{\displaystyle CH_3}{|}}{CH}-CH_2-I+CH_3CH_2OH$$

Answer: C

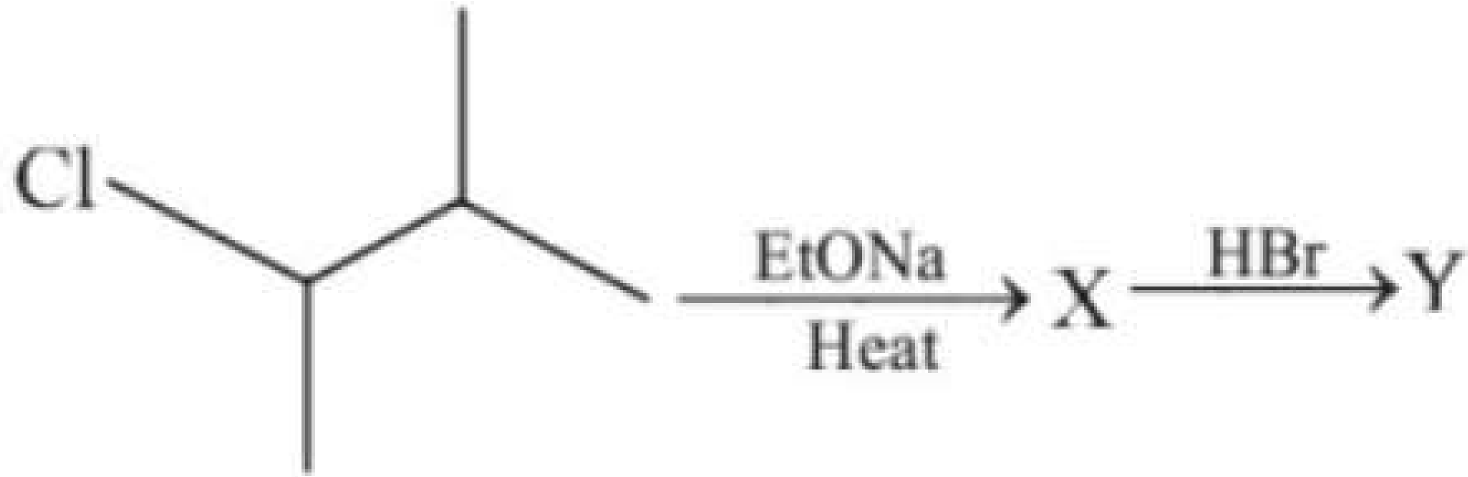
Solution:

Solution:



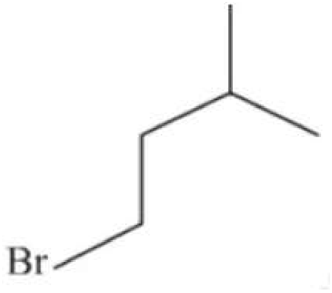
Question 71

The major product ' Y ' in the following reaction is :

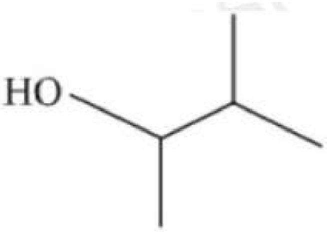


Options:

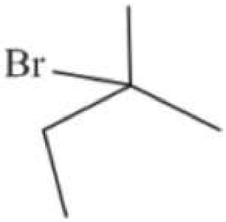
A.



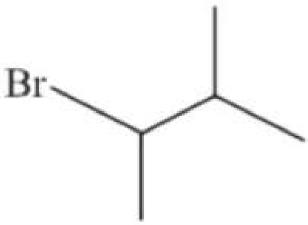
B.



C.



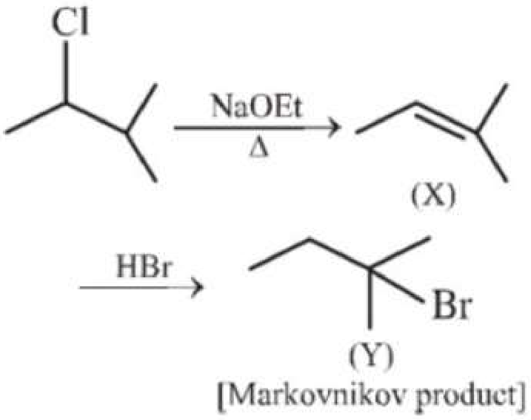
D.



Answer: C

Solution:

Solution:



Question 72

Biochemical oxygen demand(BOD) value can be a measure of water pollution caused by the organic matter.
Which of the following statements is correct?

Options:

A. Polluted water has BOD value higher than 10 ppm

B. Aerobic bacteria decreases the BOD value

C. Anaerobic bacteria increases the BOD value

D. Clean water has BOD value higher than 10 ppm

Answer: A

Solution:

Solution:
Clean water has BOD value less than 5 ppm. Polluted water has BOD value higher than 10 ppm.

Question 73

Which of the following has minimum melting point?

Options:

A. CsF

B. HCl

C. HF

D. LiF

Answer: B

Solution:

Solution:
Ionic compounds have high melting point. Greater the ionic character, more is melting point. HCl has least ionic character because of maximum electronegativity difference between the two constituent elements, (H and Cl) among CsF, HCl,HF and LiF
$\therefore$ HCl has minimum melting point.

Question 74

X mL of H_2 gas effuses through a hole in a container in 5 seconds. The time taken for the effusion of the same volume of the gas specified below under identical conditions is :

Options:

A. 10 seconds : He

B. 20 seconds: O_2

C. 25 seconds: CO

D. 55 seconds : CO_2

Answer: B

Solution:

Solution:

Under identical conditions, $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$

As rate of diffusion is also inversely proportional to time, we will have, $\frac{t_2}{t_1} = \sqrt{\frac{M_2}{M_1}}$

(a) Thus, For He, $t_2 = \sqrt{\frac{4}{2}}(5s) = 5\sqrt{2}s \neq 10s$;

(b) For O_2, $t_2 = \sqrt{\frac{32}{2}}(5s) = 20s$

(c) For CO, $t_2 = \sqrt{\frac{28}{2}}(5s) \neq 25s$;

(d) For CO_2, $t_2 = \sqrt{\frac{44}{2}}(5s) \neq 55s$

Question 75

Work out the heat change (cal) when 40g of He gas at 27°C undergoes isothermal and reversible compression from initial pressure of 1 atm to 10 atm($R = 2\,cal\,K^{-1}\,mol^{-1}$) .

Options:

A. 13.818 kcal

B. −13.818 kcal

C. 55.272 kcal

D. −55.272k cal

Answer: B

Solution:

Solution:

$q = -w_{rev} = -\left(-2.303\,nRT\log\frac{P_1}{P_2}\right)$

$= 2.303 \times \frac{40}{4} \times 2 \times 300\log\frac{1}{10} = -13.82\,kcal$

Question 76

PCl_5 is dissociating 50% at 250°C at a total pressure of P atm. If equilibrium constant is K_p , then which of the following relation is numerically correct -

Options:

A. $K_p = 3P$

B. $P = 3K_p$

C. $P = \frac{2K_P}{3}$

D. $K_p = \frac{2P}{3}$

Answer: B

Solution:

Solution:

	PCl_5	→	PCl_3	+	Cl_2
Moles at equilibrium	$\frac{1}{2}$		$\frac{1}{2}$		$\frac{1}{2}$
Mole fraction at equilibrium	$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$
Partial pressure at equilibrium	$\frac{P}{3}$		$\frac{P}{3}$		$\frac{P}{3}$

$$K_p = \frac{\frac{P}{3} \times \frac{P}{3}}{\frac{P}{3}} = \frac{P}{3}$$

Question 77

For the reaction: $NH_3 + OCl^- \rightarrow N_2H_4 + Cl^-$ in basic medium, the coefficients of NH_3, OCl^- and N_2H_4 for the balanced equation are respectively

Options:

A. 2, 2, 2

B. 2, 2, 1

C. 2, 1, 1

D. 4, 4, 2

Answer: C

Solution:

Solution:
The balanced equation :
$2NH_3 + OCl^- \rightarrow N_2H_4 + Cl^- + H_2O$

Question 78

Two elements A and B have similar chemical properties. They don't form solid hydrogencarbonates, but react with nitrogen to form nitrides. A and B, respectively, are :

©

Options:

A. Na and Rb

B. Na and Ca

C. Cs and Ba

D. Liand Mg

Answer: D

Solution:

Solution:
Li and Mg do not form solid bicarbonate, but react with N_2 to give nitrides.

$6\,\mathrm{Li} + N_2 \xrightarrow{\Delta} 2Li_3N$

$3\,\mathrm{Mg} + N_2 \xrightarrow{\Delta} Mg_3N_2$

Question 79

Diborane (B_2H_6) reacts independently with O_2 and H_2O to produce, respectively;

©

Options:

A. B_2O_3 and H_3BO_3

B. B_2O_3 and $[BH_4]^-$

C. H_3BO_3 and B_2O_3

D. HBO_2 and H_3BO_3

Answer: A

Solution:

Solution:
$B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$
$B_2H_6 + 6H_2O \rightarrow 2H_3BO_3 + 6H_2$

Question 80

At 25°C , the molar conductance at infinite dilution for the strong electrolytes NaOH, NaCl and $BaCl_2$ are 248×10^{-4}, 126×10^{-4} and $280 \times 10^{-4} Sm^2 mol^{-1}$ respectively. $\Delta_m{}^0 Ba(OH)_2$ in $Sm^2 mol^{-1}$ is

Options:

A. 52.4×10^{-4}

B. 524×10^{-4}

C. 402×10^{-4}

D. 262×10^{-4}

Answer: B

Solution:

Solution:
$\Delta°_{Na^+} + \Delta°_{OH^-} = 248 \times 10^{-4} Sm^2 mol^{-1}$
$\Delta°_{Na^+} + \Delta°_{Cl^-} = 126 \times 10^{-4} Sm^2 mol^{-1}$
$\Delta°_{Ba}{}^{2+} + \Delta°_{2Cl^-} = 280 \times 10^{-4} Sm^2 mol^{-1}$
Now,
$\Delta°_{Ba(OH)_2} = \Delta°_{BaCl_2} + 2\Delta°_{NaOH} - 2\Delta°_{NaCl}$
$\Delta°_{Ba(OH)_2} = 280 \times 10^{-4} + 2 \times 248 \times 10^{-4} - 2 \times 126 \times 10^{-4}$
$\Delta°_{Ba(OH)_2} = 524 \times 10^{-4} Sm^2 mol^{-1}$

Question 81

1. If Socrates
P : innocent St. joan was at
Q : the age of seventy
R : it may be imagined how
S : was innocent at
6. the age of seventeen
The correct sequence should be

Options:

A. QRSP

B. RPSQ

C. PRSQ

D. SQRP

Answer: D

Solution:

Solution:
The correct sequence 'is 'If Socrates was innocent at the age of seventy it may be imagined now innocent St. John was at the age of seventeen'.
Therefore correct combination is d.

Question 82

1. It was all
P : that seemed
Q : and glamorous here
R: very wonderful
S: in the old places
6. so ordinary
The correct sequence should be

Options:

A. QSPR

B. SPRQ

C. RQSP

D. PRSQ

Answer: C

Solution:

Solution:
The correct sequence is 'It was all very wonderful and glamorous here in the old places that seemed so ordinary'.
Therefore correct combination is c.

Question 83

1. If you feed a dog or tame a bear by hand,
P : or until they succeed in getting it
Q : and tear and pull at it.

R : they get their teeth into the meat
S : until they bite a piece off,
6. all out of your hand.
The correct sequence should be

Options:

A. PSRQ

B. SQRP

C. RPSQ

D. RQSP

Answer: D

Solution:

Solution:
The correct sequence is If you feed a dog or tame a bear by hand, they get their teeth into the meet and tear and pull at it until they bite a piece of or until they succeed in getting it all out of our hand'.
Therefore correct combination is (d).

Question 84

the earth............round the Sun.

Options:

A. move

B. moves

C. moved

D. moving

Answer: B

Solution:

Solution:
To express general truths we use simple present tense,
therefore, (b) is the correct choice.

Question 85

I............... him only one letter up to now.

Options:

A. sent

B. have sent

C. have been sending

D. shall send

Answer: B

Solution:

Solution:
The simple past is used to indicate an action completed in the past.
Therefore option(b) is correct.

Question 86

I shall telephone you when he................back.

Options:

A. comes

B. coming

C. shall come

D. came

Answer: A

Solution:

Solution:
Simple present tense is also used, instead to the simple future tense,
therefore option (a) is correct.

Question 87

Amenities

Options:

A. pageantries

B. privileges

C. facilities

D. courtesies

Answer: D

Solution:

Solution:
'amenities' means 'pleasing acts' and 'courtesies' means 'polite behaviour'. Therefore correct synonym is (d).

Question 88

Deflect

Options:

A. cheat

B. frustrate

C. pervert

D. depress

Answer: C

Solution:

Solution:
'deflect' means 'turn aside' and 'pervert' means 'twist'.
Therefore correct synonym is (c).

Question 89

Exorbitant

Options:

A. odd

B. ridiculous

C. excessive

D. threatening

Answer: C

Solution:

Solution:
'exorbitant' means 'beyond the limit' and 'excessive' means 'extreme'.
Therefore correct synonym is (c).

Question 90

He is not in the office...............the moment.

©

Options:

A. from

B. with

C. for

D. at

Answer: D

Solution:

Solution:
'at' is used for things at rest.

Question 91

She is very affectionate.............. her children.

©

Options:

A. upon

B. with

C. towards

D. over

Answer: C

Solution:

Solution:
'towards' is used to show the sense of direction.

Question 92

People were alarmed................the tiger.

©

Options:

A. at

B. in

C. on

D. for

Answer: A

Solution:

Solution:
'at' is used for things at rest.

Question 93

Our world has heritage of culture means

©

Options:

A. a richness of tradition

B. an inherited cultural tradition

C. too many cultural

D. cultures which are now extinct

Answer: B

Solution:

Solution:
According to the passage, our world has a heritage of culture means, we have an inherited cultural tradition. Therefore option (b) is correct.

Question 94

'The less active and important people' are the

©

Options:

A. lazy people

B. uncommitted people

C. common people

D. people who are in position of power

Answer: C

Solution:

Solution:
According to the passage common people may be described as the less active and important people.
Therefore option (c) is close to correct answer.

Question 95

"Men ignorant of the past" means' the persons who

Options:

A. do not learn from experience

B. ignore the lessons of history

C. are not aware of history.

D. have no respect for the past

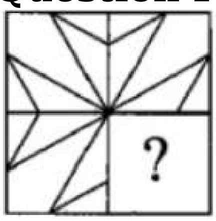
Answer: C

Solution:

Solution:
According to the passage the persons who are not aware of history are the men ignorant of the past.
Therefore option (c) is correct.

Question 96

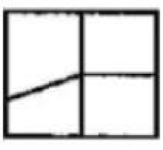
Question Figure:



Answer Figures:

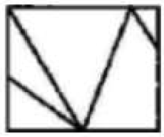
Options:

A.

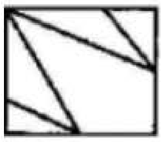


B.

C.



D.



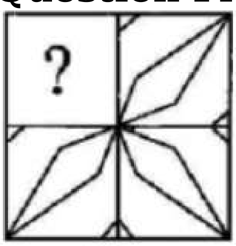
Answer: D

Solution:

Solution:

Question 97

Question Figure:

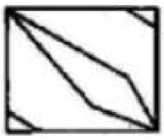


X

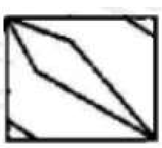
Answer Figures:

Options:

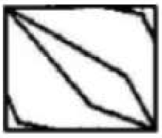
A.



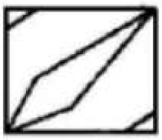
B.



C.



D.



Answer: A

Solution:

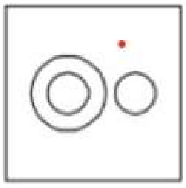
Solution:

Question 98

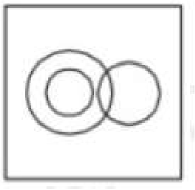
Identify the diagram that best represents the relationship among the given classes.
Rose, Flower, Stone

Options:

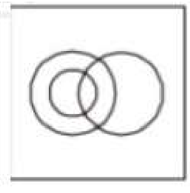
A.



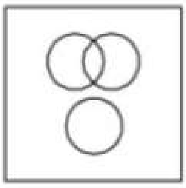
B.



C.



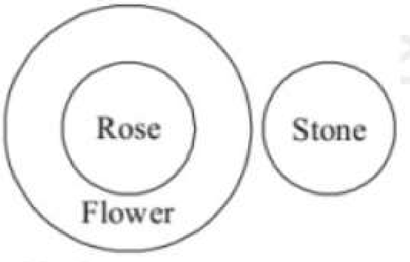
D.



Answer: A

Solution:

Solution:

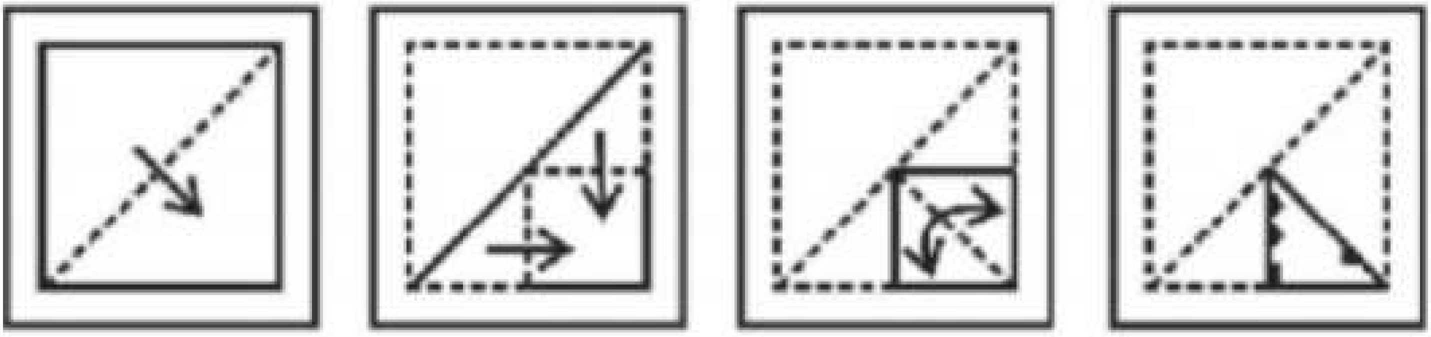


All roses are flower. Stone is neither Flower nor Rose.

Question 99

A piece of paper is folded and punched as shown below in the question figures. From the given answer figures, indicate how it will appear when opened.

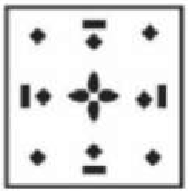
Question Figure:



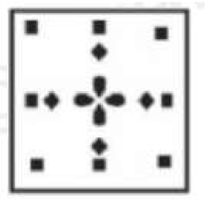
Answer Figures:

Options:

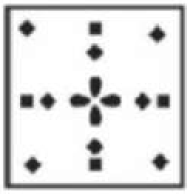
A.



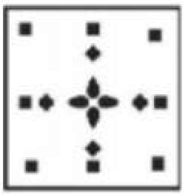
B.



C.



D.



Answer: C

Solution:

Solution:

Question 100

Which of the following indicates similar relationship as PNLJ : 2468, then QOKL: ?

Options:

A. 3591

B. 1367

C. 1276

D. 1376

Answer: D

Solution:

Solution:
$P \Rightarrow 2$
$N \Rightarrow 4$
$L \Rightarrow 6$
$J \Rightarrow 8$
$P \xrightarrow{+1} Q \Rightarrow Q = 2 - 1 = 1$
$N \xrightarrow{+1} O \Rightarrow O = 4 - 1 = 3$

$J \xrightarrow{+1} K \Rightarrow K = 8 - 1 = 7$
L = 2 (Given)

Question 101

A series is given, with one term missing. Choose the correct alternative from the given ones that will complete the series.
B - 1, D - 2, F - 4, H - 8, J - 16,?

Options:

A. K – 64

B. L - 32

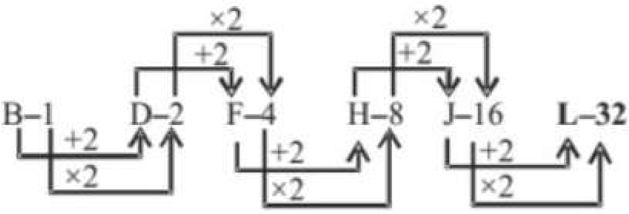
C. M – 32

D. L – 64

Answer: B

Solution:

Solution:



Question 102

In the word FLOURISH, all the vowels are first arranged alphabetically and then all the consonants are arranged alphabetically and then all the vowels are replaced by the previous letter and all the consonants are replaced by the next letter from the English alphabet. Which letter will be third from the right end?

Options:

A. I

B. S

C. M

D. V

Answer: C

Solution:

Solution:
FLOURISH
IOUFHLRS (Vowels arranged alphabetically)
HNT FHLRS (Vowels replaced by previous letter)
HNTGIMST (Consonants replaced by Next Letter)
So M is the answer

Question 103

In a joint family, there are father, mother, 3 married sons and one unmarried daughter. Of the sons two married son have 2 daughters each, and one has a son. How many female members are there in the family?

Options:

A. 2

B. 3

C. 6

D. 9

Answer: D

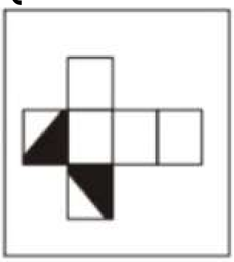
Solution:

Solution:
The female members in the family are mother, wives of 3 married son's unmarried daughter and 2 daughters of each of the two sons. $(1 + 3 + 1 + 2 \times 2) = 9$

Question 104

Choose the cube which will be formed on folding the given question figure.

Question Figures:



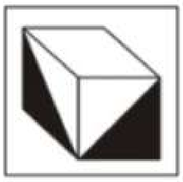
Answer Figures:

Options:

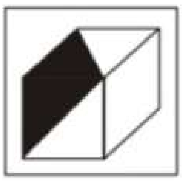
A.

B.

C.



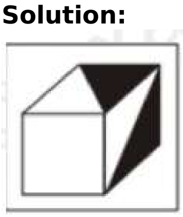
D.



Answer: B

Solution:

Solution:



Question 105

In the following question, select the number which can be placed at the sign of question mark (?) from the given alternatives.

1	$\frac{1}{2}$	$\frac{3}{2}$
2	$\frac{2}{3}$	$\frac{8}{3}$
3	?	$\frac{19}{5}$

Options:

A. $\frac{1}{2}$

B. $\frac{2}{3}$

C. $\frac{3}{4}$

D. $\frac{4}{5}$

Answer: D

Solution:

Solution:

$\left(\frac{3}{2} - 1\right) = \frac{1}{2} \Rightarrow 1^{\text{st}}$ row

$\left(\frac{8}{3} - 2\right) = \frac{2}{3} \Rightarrow 2^{\text{nd}}$ row

$\left(\frac{19}{5} - 3\right) = \frac{4}{5} \Rightarrow 3^{\text{rd}}$ row

----

Question 106

If the sum of a certain number of terms of the A.P. 25, 22, 19, ... is 116 then the last term is

Options:

A. 0

B. 2

C. 4

D. 6

Answer: C

Solution:

Solution:

$a = 25, d = 22 - 25 = -3$.
Let n be the no. of terms
Sum $= 116;$
Sum $= \frac{n}{2}[2a + (n-1)d]$
$116 = \frac{n}{2}[50 + (n-1)(-3)]$
or $232 = n[50 - 3n + 3] = n[53 - 3n] = -3n^2 + 53n$
$\Rightarrow 3n^2 - 53 + 232 = 0$
$\Rightarrow (n-8)(3n-29) = 0$
$\Rightarrow n = 8$ or $n = \frac{29}{3}$,
$n \neq \frac{29}{3}$
$\therefore n = 8$
$\therefore$ Now, $T_8 = a + (8-1)d = 25 + 7 \times (-3) = 25 - 21 = 4$
$\therefore$ Last term $= 4$

Question 107

The distance between the parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$ is $\frac{a}{b}$. Value of $a + b$ is

Options:

A. 2

B. 5

C. 7

D. 3

Answer: C

Solution:

Solution:
Given parallel lines are
$3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$
Required distance $= \frac{|7-5|}{\sqrt{(3)^2 + (-4)^2}} = \frac{2}{5}$
$\Rightarrow a = 2, b = 5$

Question 108

Value of $\lim_{x \to 0} \frac{a^{\sin x} - 1}{\sin x}$ is

Options:

A. $\log a$

B. $\sin x$

C. $\log(\sin x)$

D. $\cos x$

Answer: A

Solution:

Solution:

We have, $\lim\limits_{x \to 0} \dfrac{a^{\sin x} - 1}{\sin x} = \lim\limits_{y \to 0} \dfrac{a^{y} - 1}{y}$

$= \log a$, where $y = \sin x$

$\because x \to 0 \Rightarrow y = \sin x \to 0$

Question 109

If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \log \sin(x-\alpha) + C$, then value of (A, B) is

©

Options:

A. $(-\cos\alpha, \sin\alpha)$

B. $(\cos\alpha, \sin\alpha)$

C. $(-\sin\alpha, \cos\alpha)$

D. $(\sin\alpha, \cos\alpha)$

Answer: B

Solution:

Solution:

$\int \dfrac{\sin x}{\sin(x-\alpha)} dx = \int \dfrac{\sin(x-\alpha+\alpha}{\sin(x-\alpha)} dx$

$= \int \dfrac{\sin(x-\alpha)\cos\alpha + \cos(x-\alpha)\sin\alpha}{\sin(x-\alpha)} dx$

$= \int \{\cos\alpha + \sin\alpha\cot(x-\alpha)\} dx$

$= (\cos\alpha)x + (\sin\alpha)\log\sin(x-\alpha) + C$

$\therefore A = \cos\alpha,\ B = \sin\alpha$

Question 110

The domain of the function $f(x) = \frac{1}{\sqrt{x^{12} - x^{9} + x^{4} - x + 1}}$ is given by

©

Options:

A. $(-\infty, -1)$

B. $(1, \infty)$

C. $(-1, 1)$

D. $(-\infty, \infty)$

Answer: D

Solution:

Solution:
f(x) is defined for $x^{12} - x^9 + x^4 - x + 1 > 0$
$\Rightarrow x^4(x^8 + 1) - x(x^8 + 1) + 1 > 0$
$\Rightarrow (x^8 + 1)x(x^3 - 1) + 1 > 0$
If $x \geq 1$ or $x \leq -1$, then the above expres - sion is positive.
If $-1 < x \leq 0$, the above inequality still holds.
If $0 < x < 1$, then
$x^{12} - x(x^8 +) + (x^4 + 1) > 0$
$\because [\, x^4 + 1 > x^8 + 1$ and so $x^4 + 1 > x(x^8 + 1) \,]$
The domain of $f = (-\infty, \infty)$

Question 111

The eccentricity of the ellipse whose major axis is three times the minor axis is:

Options:

A. $\frac{\sqrt{2}}{3}$

B. $\frac{\sqrt{3}}{2}$

C. $\frac{2\sqrt{2}}{3}$

D. $\frac{2}{\sqrt{3}}$

Answer: C

Solution:

Solution:
Let a be the major axis and b, the minor axis of the ellipse, then 3 minor axis = major axis.
⇒ 3b = a
Eccentricity is given by:
$b^2 = a^2(1 - e^2)$
$\Rightarrow b^2 = 9b^2(1 - e^2)$
$\Rightarrow \frac{1}{9} = (1 - e^2)$
$\Rightarrow e^2 = \frac{8}{9}$
$\Rightarrow e = \frac{2\sqrt{2}}{3}$

Question 112

The sum of all odd numbers between 1 and 1000 which are divisible by 3 is

Options:

A. 83667

B. 90000

C. 83660

D. None of these

Answer: A

Solution:

Solution:
Sum of odd numbers between 1 and 1000 , which is divisible by
$3 = 3 + 9 + 15 + 21 + 27 + \ldots. + 999 = S$ (let)
$\therefore$ Let n be the number of terms in series and a is first term.
$\therefore l = a + (n-1)d$,
where l is last term and d is is common difference.
$999 = 3 + (n-1) \times 6$
$n - 1 = \frac{999 - 3}{6} = \frac{996}{6}$
$\Rightarrow n - 1 = 166$
$\Rightarrow n = 167$
$\therefore S = \frac{n}{2}[2a + (n-1)d]$
$= \frac{167}{2}[2 \times 3 + (167 - 1) \times 6]$
$= \frac{167}{2}[1002] = 167 \times 501 = 83667$

Question 113

How many numbers lying between 999 and 10000 can be formed with the help of the digits 0, 2, 3, 6, 7, 8, when the digits are not repeated?

Options:

A. 100

B. 200

C. 300

D. 400

Answer: C

Solution:

Solution:
The numbers between 999 and 10000 are all 4-digit numbers. The number of 4 - digit numbers formed by digits
0, 2, 3, 6, 7, 8 is ${}^6P_4 = 360$.
But here those numbers are also involved which begin from 0. So, we take those numbers as three-digit numbers.
Taking initial digit 0 , the number of ways to fill remaining 3 places from five digits 2, 3, 6, 7, 8 are ${}^5P_3 = 60$
So, the required numbers $= 360 - 60 = 300$.

Question 114

The points (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) form

©

Options:

A. a right angled isosceles triangle

B. a scalene triangle

C. a right angled triangle

D. an equilateral triangle

Answer: A

Solution:

Solution:
Let $P(0, 7, 10)$, $Q(-1, 6, 6)$ and $R(-4, 9, 6)$ be the vertices of a triangle
Here, $PQ = \sqrt{1 + 1 + 16} = 3\sqrt{2}$
$QR = \sqrt{9 + 9 + 0} = 3\sqrt{2}$
$PR = \sqrt{16 + 4 + 16} = 6$
Now, $PQ^2 + QR^2 = (3\sqrt{2})^2 + (3\sqrt{2})^2 = 36 = (PR)^2$
Therefore,ΔPQR is a right angled triangle at Q. Also, $OQ = QR$.
Hence, ΔPQR is a right angled isosceles triangle.

Question 115

Find the value of a such that the sum of the squares of the roots of the equation $x^2 - (a - 2)x - (a + 1) = 0$ is least.

©

Options:

A. 4

B. 2

C. 1

D. 3

Answer: C

Solution:

Solution:
Let α, β be the roots of the equation.
$\therefore \alpha + \beta = a - 2$ and $\alpha\beta = -(a + 1)$
Now, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
$= (a - 2)^2 + 2(a + 1) = (a - 1)^2 + 5$
$\therefore \alpha^2 + \beta^2$ will be minimum if $(a - 1)^2 = 0$,
i.e. $a = 1$.

Question 116

The value of $\int \cos(\log x)\, dx$ is :

Options:

A. $\frac{1}{2}[\sin(\log x) + \cos(\log x)] + C$

B. $\frac{x}{2}[\sin(\log x) + \cos(\log x)] + C$

C. $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + C$

D. $\frac{1}{2}[\sin(\log x) - \cos(\log x)] + C$

Answer: B

Solution:

Solution:

Question 117

The length of the perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is :

Options:

A. $\sqrt{29}$

B. $\sqrt{33}$

C. $\sqrt{53}$

D. $\sqrt{66}$

Answer: C

Solution:

Solution:
Let feet of perpendicular is
$(2\alpha, 3\alpha + 2, 4\alpha + 3)$
$\Rightarrow$ Direction ratio of the $\perp$ line is
$2\alpha - 3, 3\alpha + 3, 4\alpha - 8$. and
$\Rightarrow$ Direction ratio of the line are 2, 3, 4
$\Rightarrow 2(2\alpha - 3) + 3(3\alpha + 3) + 4(4\alpha - 8) = 0$
$\Rightarrow \alpha = 1$
$\Rightarrow$ Feet of $\perp$ is (2, 5, 7)
$\Rightarrow$ Length $\perp$ is $\sqrt{1^2 + 6^2 + 4^2} = \sqrt{53}$

Question 118

If the slope of the tangent to the curve at any point P(x, y) is $\frac{y}{x} - \cos^2 \frac{y}{x}$, then the equation of a curve passing through $\left(1, \frac{\pi}{4}\right)$ is

Options:

A. $\tan\left(\frac{y}{x}\right) + \log x = 1$

B. $\tan\left(\frac{y}{x}\right) + \log y = 1$

C. $\tan\left(\frac{x}{y}\right) + \log x = 1$

D. $\tan\left(\frac{x}{y}\right) + \log y = 1$

Answer: A

Solution:

Solution:
According to the condition,
$\frac{dy}{dx} = \frac{y}{x} - \cos^2 \frac{y}{x}$ ...(i)
This is a homogeneous differential equation Substituting y = vx, we get
$v + x\frac{dv}{dx} = v - \cos^2 v$
$\Rightarrow x\frac{dv}{dx} = -\cos^2 v$
$\Rightarrow \int \sec^2 v dv = -\int \frac{dx}{x}$
$\Rightarrow \tan v = -\log x + C$
$\Rightarrow \tan\frac{y}{x} + \log x = C$
Substituting $x = 1$, $y = \frac{\pi}{4}$, we get $C = 1$
Thus, we get
$\tan\left(\frac{y}{x}\right) + \log x = 1$
which is the required solution,

Question 119

If $y = \frac{2\sin\alpha}{1 + \cos\alpha + \sin\alpha}$, then value of $\frac{1 - \cos\alpha + \sin\alpha}{1 + \sin\alpha}$ is

Options:

A. $\frac{y}{3}$

B. y

C. 2y

D. $\frac{3}{2}$y

Answer: B

Solution:

Solution:

$$\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$$

$$=\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}\cdot\frac{1+\cos\alpha+\sin\alpha}{1+\cos\alpha+\sin\alpha}$$

$$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$=\frac{(1+\sin^2\alpha+2\sin\alpha)-(1-\sin^2\alpha)}{(1+\sin\alpha)(1+\cos\alpha+\sin\alpha)}$$

$$=\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$$

Question 120

The coefficient of the term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is

Options:

A. $\frac{5}{4}$

B. $\frac{7}{4}$

C. $\frac{9}{4}$

D. None of these

Answer: A

Solution:

Solution:

The (r + 1) th term in the expansion of

$\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ is given by

$$T_{r+1}={}^{10}C_r\left(\sqrt{\frac{x}{3}}\right)^{10-r}\left(\frac{3}{2x^2}\right)^r$$

$$={}^{10}C_r\frac{x^{5-\left(\frac{r}{2}\right)}}{3^{5-\left(\frac{r}{2}\right)}}\cdot\frac{3^r}{2^rx^{2r}}$$

$= {}^{10}C_r \dfrac{3^{\left(\frac{3r}{2}\right)-5}}{2^r} x^{5-\left(\frac{5r}{2}\right)}$

For T_{r+1} to be independent of x , we must have $5 - \left(\frac{5r}{2}\right) = 0$
or $r = 2$.
Thus, the 3rd term is independent of x and is equal to
${}^{10}C_2 \dfrac{3^{3-5}}{2^2} = \dfrac{10 \times 9}{2} \times \dfrac{3^{-2}}{4} = \dfrac{5}{4}$

Question 121

If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ =

©

Options:

A. $2^{x-y} \dfrac{2^y - 1}{2^x - 1}$

B. $2^{x-y} \left(\dfrac{2^y - 1}{1 - 2^x} \right)$

C. $\dfrac{2^x + 2^y}{2^x - 2^y}$

D. None of these

Answer: B

Solution:

Solution:
$2^x + 2^y = 2^{x+y}$
Differentiating both sides
$\ln 2.2^x + \ln 2.2^y \dfrac{dy}{dx} = \ln 2.2^{x+y}\left(1 + \dfrac{dy}{dx}\right)$
$2^x + 2y \dfrac{dy}{dx} = 2^{x+y}\left(1 + \dfrac{dy}{dx}\right)$
$2^x + 2^y \dfrac{dy}{dx} = 2^{x+y} + 2^{x+y} \dfrac{dy}{dx}$
$(2^y - 2^{x+y}) \dfrac{dy}{dx} = (2^{x+y} - 2^x)$
$\dfrac{dy}{dx} = \dfrac{2^{x+y} - 2^x}{2^y - 2^{x+y}}$
$\dfrac{dy}{dx} = \dfrac{2^x(2^y - 1)}{2^y(1 - 2^x)}$
$\dfrac{dy}{dx} = 2^{x-y}\left(\dfrac{2^y - 1}{1 - 2^x}\right)$

Question 122

If $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$, then $\frac{dy}{dx}$ is equal to :

©

Options:

A. $-\frac{y}{x}$

B. $\frac{y}{x}$

C. $-\frac{x}{y}$

D. $\frac{x}{y}$

Answer: C

Solution:

Solution:

Let $x = \frac{1-t^2}{1+t^2}$ and $y = \frac{2t}{1+t^2}$

Put $t = \tan\theta$, we get

$x = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ and $y = \frac{2\tan\theta}{1+\tan^2\theta}$

$\Rightarrow x = \cos 2\theta$ and $y = \sin 2\theta$

$\therefore \frac{dx}{d\theta} = -2\sin 2\theta$ and $\frac{dy}{d\theta} = 2\cos 2\theta$

Now, $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = -\frac{\cos 2\theta}{\sin 2\theta} = -\frac{x}{y}$

Question 123

The points at which the tangent passes through the origin for the curve $y = 4x^3 - 2x^5$ are

©

Options:

A. (0, 0), (2, 1) and (−1, −2)

B. (0, 0), (2, 1) and (−2, −1)

C. (2, 0), (2, 1) and (−3, 1)

D. (0, 0), (1, 2) and (−1, −2)

Answer: D

Solution:

Solution:

The equation of the given curve is

$y = 4x^3 - 2x^5$

$\frac{dy}{dx} = 12x^2 - 10x^4$

Therefore, the slope of the tangent at point (x, y) is $12x^2 - 10x^4$.

The equation of the tangent at (x, y) is given by $Y - y = (12x^2 - 10x^4)(X - x)\ldots$ (i)

When, the tangent passes through the origin (0, 0), then $X = Y = 0$

Therefore, eq. (i) reduce to

$-y = (12x^2 - 10x^4)(-x)$

$\Rightarrow y = 12x^3 - 10x^5$

Also, we have $y = 4x^3 - 2x^5$

$\therefore 12x^3 - 10x^5$
$= 4x^3 - 2x^5$
$\Rightarrow 8x^5 - 8x^3 = 0$
$\Rightarrow x^5 - x^3 = 0$
$\Rightarrow x^3(x^2 - 1) = 0$
$\Rightarrow x = 0, \pm 1$
When, $x = 0$,
$y = 4(0)^3 - 2(0)^5 = 0$
When, $x = 1$,
$y = 4(1)^3 - 2(1)^5 = 2$
When, $x = -1$,
$y = 4(-1)^3 - 2(-1)^5 = -2$
Hence, the require points are (0, 0), (1, 2) and (−1, −2).

Question 124

If the radius of a sphere is measured as $9\,\text{cm}$ with an error of $0.03\,\text{cm}$, then find the approximating error in calculating its volume.

Options:

A. $2.46\pi\text{cm}^3$

B. $8.62\pi\text{cm}^3$

C. $9.72\pi\text{cm}^3$

D. $7.6\pi\text{cm}^3$

Answer: C

Solution:

Solution:
Let r be the radius of the sphere and Δr be the error in measuring the radius. Then, $r = 9\,\text{cm}$ and $\Delta r = 0.03\,\text{cm}$
Let V be the volume of the sphere. Then,
$V = \frac{4}{3}\pi r^3$
$\Rightarrow \frac{dV}{dr} = 4\pi r^2$
$\Rightarrow \left(\frac{dV}{dr}\right)_{r=9} = 4\pi \times 9^2 = 324\pi$
Let ΔV be the error in V due to error Δr in r.
Then,
$\Delta V = \frac{dV}{dr}\Delta r$
$\Rightarrow \Delta V = 324\pi \times 0.03 = 9.72\pi\text{cm}^3$

Question 125

If z is a complex number such that $z^2 = (\bar{z})^2$, then

Options:

A. z is purely real

B. z is purely imaginary

C. either z is purely real or purely imaginary

D. None of these

Answer: C

Solution:

Solution:
Let $z = x + iy$, then its conjugate $\bar{z} = x - iy$
Given that $z^2 = (\bar{z})^2$
$\Rightarrow x^2 - y^2 + 2ixy = x^2 - y^2 - 2ixy$
$\Rightarrow 4ixy = 0$

Question 126

$$\int_{-a}^{4} (x^8 - x^4 + x^2 + 1)dx = 2\int_{0}^{4} (x^8 - x^4 + x^2 + 1)dx \text{ then } a =$$

©

Options:

A. 3

B. 4

C. 6

D. 8

Answer: B

Solution:

Solution:
If f(x) is an even function then
$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$
Here $f(x) = x^8 - x^4 + x^2 + 1$ is an even function, therefore $a = 4$.

Question 127

The area bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is

©

Options:

A. 1 sq. unit

B. $\frac{1}{3}$ sq. units

C. $\frac{2}{3}$ sq. units

D. $\frac{4}{3}$ sq. units

Answer: D

Solution:

Solution:

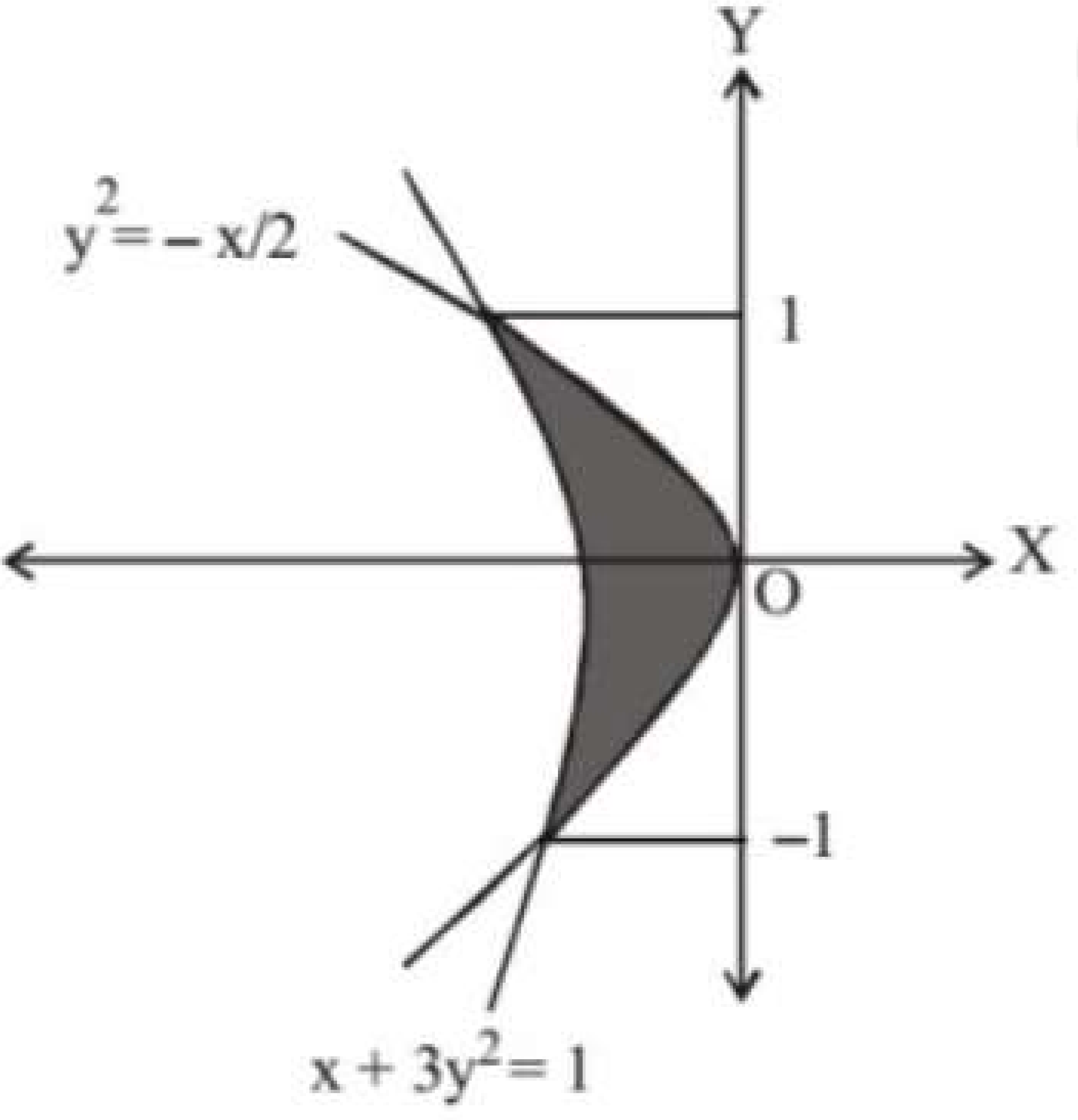
We have, $x + 2y^2 = 0$

$\Rightarrow y^2 = -\frac{x}{2}$ ...(i)

a parabola with vertex (0, 0) and $x + 3y^2 = 1$

$\Rightarrow y^2 = \frac{1-x}{3} = -\left(\frac{x-1}{3}\right)$ ... (ii), a parabola with vertex (1, 0)

Solving (i) and (ii), we get $y = \pm 1$



$$A = \int_{-1}^{1} ((1 - 3y^2) - (-2y^2))d\,y$$
$$= 2\int_{0}^{1} (1 - y^2)d\,y = 2\left[y - \frac{y^3}{3}\right]_0^1$$
$$= \frac{4}{3} \text{ sq.units}$$

Question 128

$$\frac{d}{d\,x}\left(\tan^{-1}\sqrt{\frac{1 + \cos\frac{x}{2}}{1 - \cos\frac{x}{2}}}\right)$$ **is equal to**

Options:

A. $-\frac{1}{4}$

B. $\frac{1}{4}$

C. $-\frac{1}{2}$

D. $\frac{1}{2}$

Answer: A

Solution:

Solution:

We have $\frac{d}{d\,x}\left(\tan^{-1}\sqrt{\frac{1 + \cos\frac{x}{2}}{1 - \cos\frac{x}{2}}}\right)$

$$= \frac{d}{d\,x}\left[\tan^{-1}\sqrt{\frac{1 + \left(2\cos^2\frac{x}{4} - 1\right)}{1 - \left(1 - 2\sin^2\frac{x}{4}\right)}}\right]$$

$$= \frac{d}{d\,x}\left(\tan^{-1}\sqrt{\frac{2\cos^2\frac{x}{4}}{2\sin^2\frac{x}{4}}}\right)$$

$$= \frac{d}{d\,x}\left(\tan^{-1}\sqrt{\cot^2\frac{x}{4}}\right) = \frac{d}{d\,x}\left(\tan^{-1}\cot\frac{x}{4}\right)$$

$$= \frac{d}{d\,x}\left(\tan^{-1}\tan\left(\frac{\pi}{2} - \frac{x}{4}\right)\right) = \frac{d}{d\,x}\left(\frac{\pi}{2} - \frac{x}{4}\right) = 0 - \frac{1}{4} = -\frac{1}{4}$$

Question 129

f(x) = $\left(\frac{e^{2x}-1}{e^{2x}+1}\right)$ is

Options:

A. an increasing function

B. a decreasing function

C. an even function

D. None of these

Answer: A

Solution:

Solution:

$\because f(x) = \left(\frac{e^{2x}-1}{e^{2x}+1}\right)$

$\therefore f(-x) = \frac{e^{-2x}-1}{e^{-2x}+1} = \frac{1-e^{2x}}{1+e^{2x}}$

$\Rightarrow f(-x) = \frac{-(e^{2x}-1)}{e^{2x}+1} = -f(x)$

$\therefore$ f(x) is an odd function.

Again, $f'(x) = \frac{4e^{2x}}{(1+e^{2x})^2} > 0,\ \forall x \in R$

$\Rightarrow$ f(x) is an increasing function.

Question 130

Evaluate: $\int_0^{\pi} \frac{1}{5+4\cos x}\,dx$

Options:

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{6}$

Answer: A

Solution:

Solution:

We have, $\int_0^{\pi} \frac{1}{5+4\cos x}\,dx$

$$= \int_0^{\pi} \frac{1}{5 + 4\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)} dx$$

$$= \int_0^{\pi} \frac{1 + \tan^2 \frac{x}{2}}{5\left(1 + \tan^2 \frac{x}{2}\right) + 4\left(1 - \tan^2 \frac{x}{2}\right)} dx$$

$$= \int_0^{\pi} \frac{1 + \tan^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx = \int_0^{\pi} \frac{\sec^2 \frac{x}{2}}{9 + \tan^2 \frac{x}{2}} dx$$

Let $\tan \frac{x}{2} = t \Rightarrow \frac{1}{2} \sec^2 \frac{x}{2} dx = dt$

Also, $x = 0 \Rightarrow t = 0$ and $x = \pi \Rightarrow t = \infty$

$\therefore I = \int_0^{\infty} \frac{dt}{9 + t^2}$

$\therefore I = 2\int_0^{\infty} \frac{dt}{9 + t^2}$

$\therefore I = \frac{2}{3}\left[\tan^{-1}\frac{t}{3}\right]_0^{\infty} = \frac{2}{3}[\tan^{-1}\infty - \tan^{-1}0]$

$= \frac{2}{3}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{3}$

Question 131

The equation of the circle, which touches the line y = 5 and passes through (−1, 2) and (1, 2) is

Options:

A. $9x^2 + 9y^2 - 60y + 75 = 0$

B. $9x^2 + 9y^2 - 60x - 75 = 0$

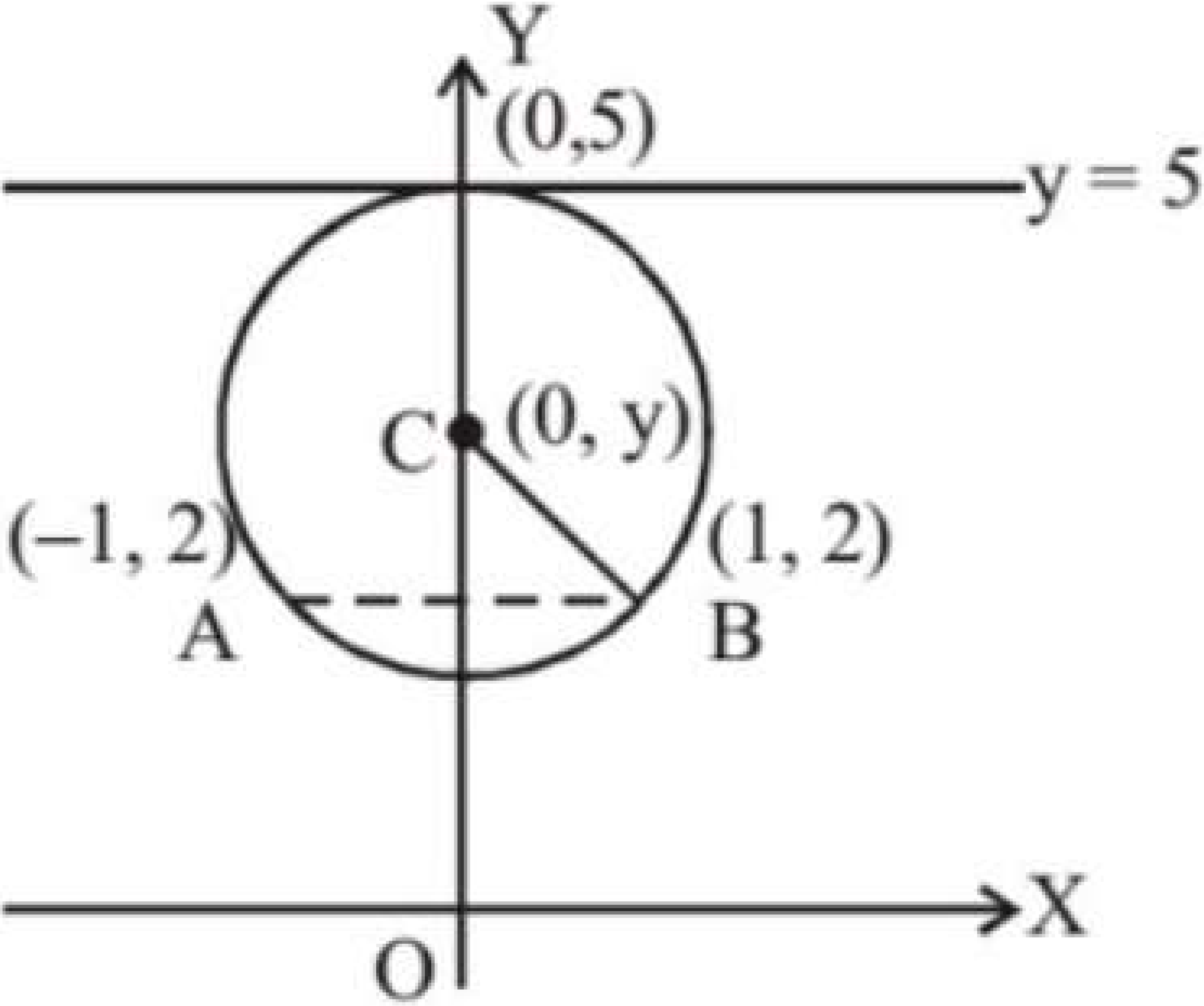
C. $9x^2 + 9y^2 + 60y - 75 = 0$

D. $9x^2 + 9y^2 + 60x + 75 = 0$

Answer: A

Solution:

Solution:



The centre of the circle is on the perpendicular bisector of the line joining $(-1, 2)$ and $(1, 2)$, which is the y - axis. The ordinate of the centre is given by

$(5 - y)^2 = 1 + (y - 2)^2$

$\Rightarrow y = \frac{10}{3}$

Hence, eq. of the circle is :

$x^2 + \left(y - \frac{10}{3}\right)^2 = \left(\frac{5}{3}\right)^2$

$\Rightarrow 9x^2 + 9y^2 - 60y + 75 = 0$

Question 132

Coefficient of x^{13} in the expansion of $(1 - x)^5(1 + x + x^2 + x^3)^4$ is

Options:

A. 4

B. 6

C. 32

D. 5

Answer: A

Solution:

Solution:
Expression $= (1-x)^5 . (1+x)^4(1+x^2)^4$
$= (1-x)(1-x^2)^4(1+x^2)^4$
$= (1-x)(1-x^4)^4$
$\therefore$ Coefficient of $x^{13} = -{}^4C_3(-1)^3 = 4$

Question 133

The general solution of the differential equation $(\tan^{-1}y - x)dy = (1+y^2)dx$ is

Options:

A. $x = (\tan^{-1}y + 1) + Ce^{-\tan^{-1}y}$

B. $x = (\tan^{-1}y - 1) + Ce^{-\tan^{-1}y}$

C. $x = (\tan^{-1}x - 1) + Ce^{-\tan^{-1}x}$

D. $x = (\tan^{-1}x + 1) + Ce^{-\tan^{-1}x}$

Answer: B

Solution:

Solution:
The given differential equation can be written as
$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ... (i)
Now, eq. (i) is a linear differential equation of the form
$\frac{dx}{dy} + P_1x = Q_1$
where $P_1 = \frac{1}{1+y^2}$ and $Q_1 = \frac{\tan^{-1}y}{1+y^2}$
Therefore, I.F $= e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$
Thus, the solution of the given differential equation is given by
$xe^{\tan^{-1}y} = \int \left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy + C$ ...(ii)
Let $I = \int \left(\frac{\tan^{-1}y}{1+y^2}\right)e^{\tan^{-1}y}dy$
On substituting $\tan^{-1}y = t$, so that
$\left(\frac{1}{1+y^2}\right)dy = dt$, we get
$I = \int te^t dt = te^t - \int 1 . e^t dt = te^t - e^t = e^t(t-1)$
or $I = e^{\tan^{-1}y}(\tan^{-1}y - 1)$
On substituting the value of 1 in equation (i1), we get
$x . e^{\tan^{-1}y} = .e^{\tan^{-1}y}(\tan^{-1}y - 1) + C$
or $x = (\tan^{-1}y - 1) + Ce^{\tan^{-1}y}$
which is the general solution of the given differntial equation.

Question 134

The area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$ is

Options:

A. $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$

B. $\frac{2\pi}{3}$

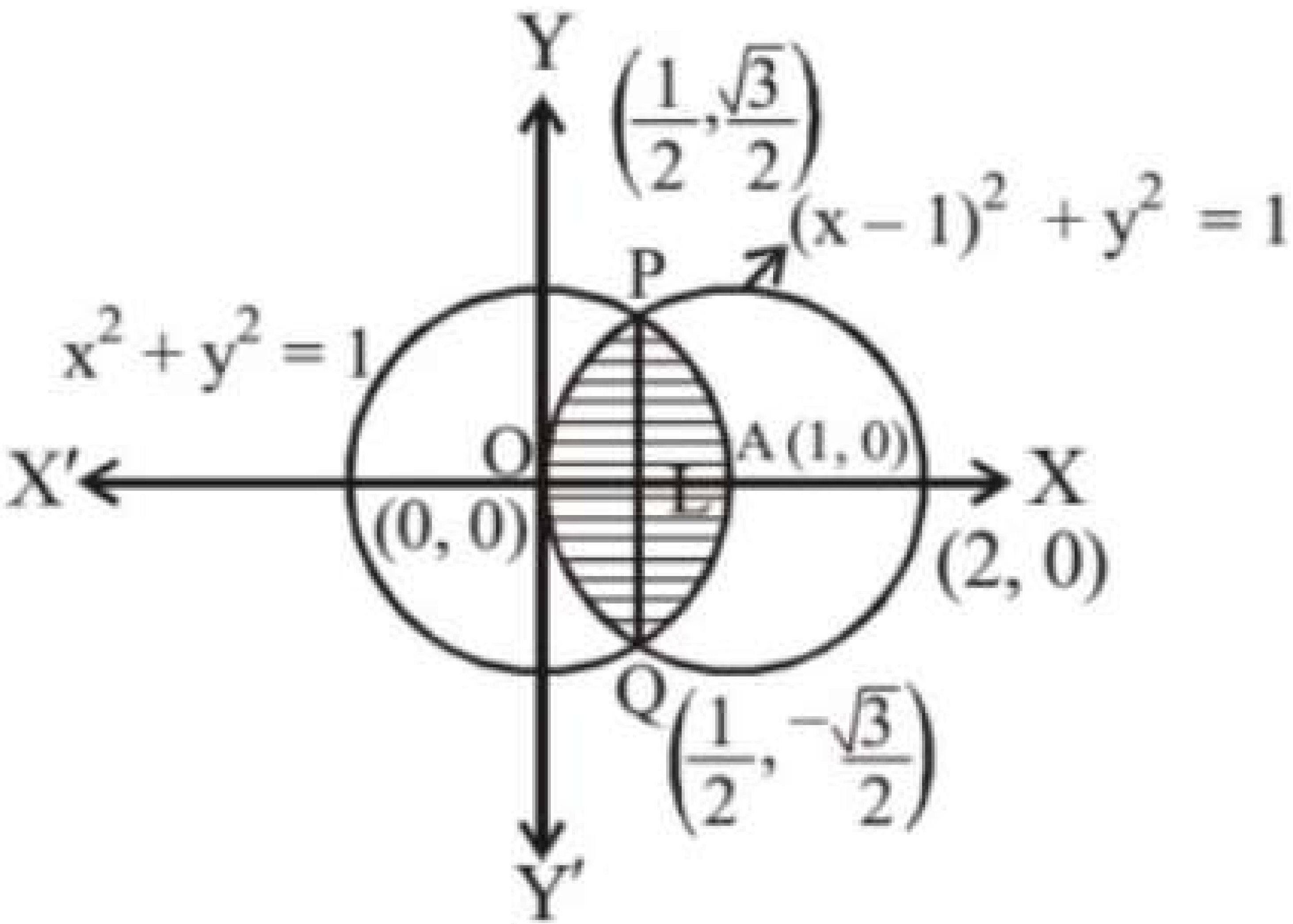
C. $\frac{\sqrt{3}}{2}$

D. $\frac{2\pi}{3} + \frac{\sqrt{3}}{2}$

Answer: A

Solution:

Solution:
Given circles are $x^2 + y^2 = 1$ ...(i)
and $(x-1)^2 + y^2 = 1$ ...(ii)
Centre of (i) is O(0, 0) and radius $= 1$



Both these circle are symmetrical about x-axis solving (i) and (ii), we get, $-2x + 1 = 0$
$\Rightarrow x = \frac{1}{2}$

then $y^2 = 1 - \left(\frac{1}{2}\right)^2 = 34$

$\Rightarrow y = \frac{\sqrt{3}}{2}$

∴ The points of intersection are

$P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ and $Q\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

It is clear from the figure that the shaded portion in region whose area is required.

∴ Required area = area OQAPO

= 2× area of the region OLAP

= 2 ×(area of the region OLPO + area of LAPL)

$$= 2\left[\int_0^{\frac{1}{2}} \sqrt{1-(x-1)^2}\,dx + \int_{\frac{1}{2}}^{1} \sqrt{1-x^2}\,dx\right]$$

$$= 2\left[\frac{(x-1)\sqrt{1-(x-1)^2}}{2} + \frac{1}{2}\sin^{-1}(x-1)\right]_0^{\frac{1}{2}} + 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_{\frac{1}{2}}^{1}$$

$$= -\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{-1}{2}\right) - \sin^{-1}(-1) + 0 + \sin^{-1}(1) - \left(\frac{1}{2}\cdot\frac{\sqrt{3}}{2} + \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ sq. units.

Question 135

Consider an infinite geometric series with first term a and common ratio r . If its sum is 4 and the second term is $\frac{3}{4}$, then :

Options:

A. $a = \frac{4}{7}, r = \frac{3}{7}$

B. $a = 2, r = \frac{3}{8}$

C. $a = \frac{3}{2}, r = \frac{1}{2}$

D. $a = 3, r = \frac{1}{4}$

Answer: D

Solution:

Solution:

Since, sum $= 4$ and second term $= \frac{3}{4}$

$\Rightarrow \frac{a}{1-r} = 4$, and $ar = \frac{3}{4}$

$\Rightarrow \frac{a}{1-\frac{3}{4a}} = 4$

$\Rightarrow (a-1)(a-3) = 0$

$\Rightarrow a = 1$ or $a = 3$

Question 136

$\int \frac{e^x(1+x)}{\cos^2(e^x x)}$ **dx equals**

Options:

A. $-\cot(ex^x)+C$

B. $\tan(xe^x)+C$

C. $\tan(e^x)+C$

D. $\cot(e^x)+C$

Answer: B

Solution:

Solution:

$\int \frac{e^x(1+x)}{\cos^2(e^x x)} dx$

Let $xe^x = t$

$\Rightarrow (xe^x + e^x) = \frac{dt}{dx}$

$\Rightarrow d\,x = \frac{dt}{e^x(x+1)}$

$\therefore \int \frac{e^x(1+x)}{\cos^2(e^x x)} dx = \int \frac{e^x(1+x)}{\cos^2 t} \times \frac{d\,t}{e^x(1+x)} = \int \frac{1}{\cos^2 t} dt = \int \sec^2 t\, dt = \tan t + C = \tan(xe^x) + C$

Question 137

If $A+B+C=\frac{\pi}{2}$ **then**

Options:

A. $\tan A \tan B + \tan B \tan C + \tan C \tan A \ = 1$

B. $\cot A + \cot B + \cot C \ = \cot A \cot B \cot C$

C. $\cos 2A + \cos 2B + \cos 2C \ = 1 + 4 \sin A \sin B \sin C$

D. All three are correct

Answer: D

Solution:

Solution:

$B + C = \frac{\pi}{2} - A \Rightarrow \tan(B+C) = \cot A$

$\Rightarrow \frac{\tan B + \tan C}{1 - \tan B \tan C} = \cot A$

$\Rightarrow \tan A \tan B + \tan B \tan C + \tan C \tan A = 1$

$\Rightarrow \tan A \tan B \tan C(\cot C + \cot A + \cot B) = 1$

$\Rightarrow \cot A + \cot B + \cot C = \cot A \cot B \cot C$

Again $\cos 2A + \cos 2B + \cos 2C$
$= 2\cos(A+B)\cos(A-B) + \cos 2C$
$= 2\cos\left(\frac{\pi}{2} - C\right)\cos(A-B) + 1 - 2\sin^2 C$
$= 2\sin C[\cos(A-B) - \sin C] + 1$
$= 2\sin C\left[\cos(A-B) - \sin\frac{\pi}{2} - (A+B)\right] + 1$

$= 2\sin C[\cos(A-B) - \cos(A+B)] + 1$
$= 4\sin A\sin B\sin C + 1$

Question 138

$\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}$ is equal to :

Options:

A. 12

B. 18

C. 0

D. 6

Answer: B

Solution:

Solution:

Consider $\lim_{x\to 0}\frac{2\sin^2 3x}{x^2}$

$= 2\,.\lim_{x\to 0}\left[\frac{\sin 3x}{x}\right]^2 = 2\,.\lim_{x\to 0}\left[3\frac{\sin 3x}{3x}\right]^2$

$= 2.9\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)^2 = 18\times 1 = 18$

Question 139

If the harmonic mean between a and b be H, then the value of $\frac{1}{H-a} + \frac{1}{H-b}$ is

Options:

A. $a+b$

B. ab

C. $\frac{1}{a} + \frac{1}{b}$

D. $\frac{1}{a} - \frac{1}{b}$

Answer: C

Solution:

Solution:

Putting $H = \frac{2ab}{a+b}$, we have

$$\frac{1}{H-a} + \frac{1}{H-b}$$

$$= \frac{1}{\left(\frac{2ab}{a+b} - a\right)} + \frac{1}{\left(\frac{2ab}{a+b} - b\right)} = \frac{a+b}{ab-a^2} + \frac{a+b}{ab-b^2}$$

$$= \left(\frac{a+b}{b-a}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \left(\frac{a+b}{b-a}\right)\left(\frac{b-a}{ab}\right) = \frac{a+b}{ab} = \frac{1}{a} + \frac{1}{b}$$

Question 140

Girl students constitute 10% of I year and 5% of II year at Roorkee University. During summer holidays 70% of the I year and 30% of II year students are given a project. The girls take turns on duty in canteen. The chance that I year girl student is on duty in a randomly selected day is

Options:

A. $\frac{3}{17}$

B. $\frac{14}{17}$

C. $\frac{3}{10}$

D. $\frac{7}{10}$

Answer: B

Solution:

Solution:

The desired probability

$$= \frac{\frac{10}{100} \times \frac{70}{100}}{\frac{10}{100} \times \frac{70}{100} + \frac{5}{100} \times \frac{30}{100}} = \frac{14}{17}$$

Question 141

If the tangent at P(1, 1) on $y^2 = x(2-x)^2$ meets the curve again at Q, then Q is

Options:

A. (2, 2)

B. (−1, −2)

C. $\left(\frac{9}{4}, \frac{3}{8}\right)$

D. None of these

Answer: C

Solution:

Solution:

$y^2 = x(2-x)^2 \Rightarrow y^2 = x^3 - 4x^2 + 4x$ ...(i)

$\Rightarrow 2y\frac{dy}{dx} = 3x^2 - 8x + 4$

$\Rightarrow \frac{dy}{dx} = \frac{3x^2 - 8x + 4}{2y}$

$\Rightarrow \left[\frac{dy}{dx}\right]_P = \frac{3-8+4}{2} = -\frac{1}{2}$

∴ Equation of tangent at P is: $y - 1 = -\frac{1}{2}(x-1)$

$\Rightarrow x + 2y - 3 = 0$

Using $y = \frac{3-x}{2}$ in (i), we get: $\left(\frac{3-x}{2}\right)^2$

$= x^3 - 4x^2 + 4x$

$\Rightarrow 4x^3 - 17x^2 + 22x - 9 = 0$ ..(ii)

which has two roots 1,1(Because of (ii) being tangent at (1, 1)).

Sum of 3 roots $= \frac{17}{4}$

∴$3\,\text{rd}$ root $= \frac{17}{4} - 2 = \frac{9}{4}$

Then, $y = \frac{3-\frac{9}{4}}{2} = \frac{3}{8}$

∴ Q is $\left(\frac{9}{4}, \frac{3}{8}\right)$

Question 142

A line makes angles of 45° and 60° with the positive axes of X and Y respectively. The angle made by the same line with the positive axis of Z , is.

Options:

A. 30° or 60°

B. 60° or 90°

C. 90° or 120°

D. 60° or 120°

Answer: D

Solution:

Solution:
Given $\alpha = 45°$, $\beta = 60°$, $\gamma = ?$
$\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
$\therefore \cos^2\gamma = 1 - \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
$\Rightarrow \gamma = 60°$ or $120°$

Question 143

An ellipse has OB as semi minor axis, F and F' its focii and the angle FBF' is a right angle. Then the eccentricity of the ellipse is

©

Options:

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{2}$

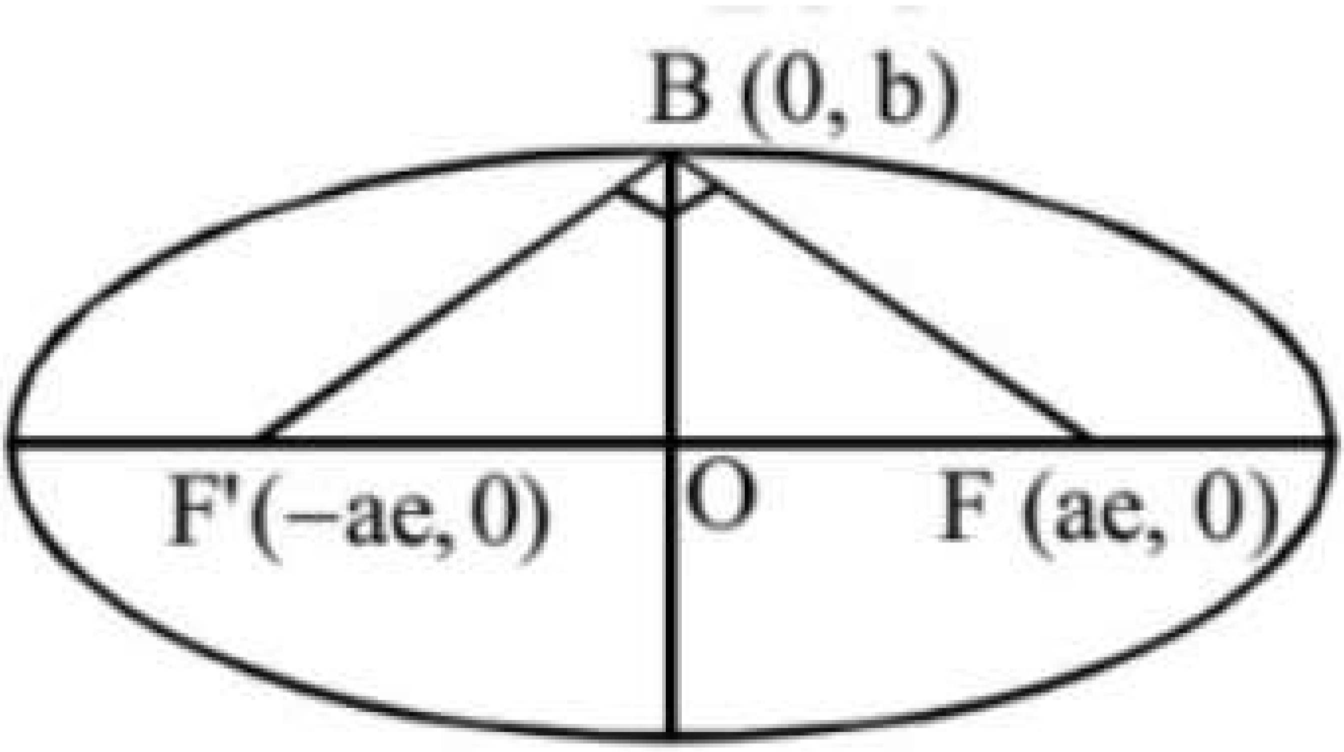
C. $\frac{1}{4}$

D. $\frac{1}{\sqrt{3}}$

Answer: A

Solution:

Solution:
$\because \angle FBF' = 90°$
$\Rightarrow FB^2 + F'B^2 = FF'^2$
$\therefore \left(\sqrt{a^2e^2 + b^2}\right)^2 + \left(\sqrt{a^2e^2 + b^2}\right)^2 = (2ae)^2$
$\Rightarrow 2(a^2e^2 + b^2) = 4a^2e^2$
$\Rightarrow e^2 = \frac{b^2}{a^2}$



Also $e^2 = 1 - \frac{b^2}{a^2} = 1 - e^2$

$\Rightarrow 2e^2 = 1$

$\Rightarrow e = \frac{1}{\sqrt{2}}$.

Question 144

The value of $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$ is

©

Options:

A. $-\frac{2\sqrt{6}}{5}$

B. $-2\sqrt{6}$

C. $-\frac{\sqrt{6}}{5}$

D. None of these

Answer: A

Solution:

Solution:

$\cos[2\cos^{-1}x + \sin^{-1}x]$

$= \cos[\cos^{-1}x + \cos^{-1}x + \sin^{-1}x]$

$= \cos\left[\cos^{-1}x + \frac{\pi}{2}\right] = -\sin[\cos^{-1}x]$

$= -\sin\left[\sin^{-1}\sqrt{1-x^2}\right] = -\sqrt{1-x^2}$

$= -\sqrt{1-\left(\frac{1}{5}\right)^2} = -\sqrt{\frac{24}{25}} = -\frac{2\sqrt{6}}{5}$

Question 145

If $\vec{a}$ is a vector of magnitude 50 , collinear with the vector $\vec{b} = 6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}$ and makes an acute angle with the positive direction of Z - axis, then $\vec{a}$ is equal to

©

Options:

A. $-24\hat{i} + 32\hat{j} + 30\hat{k}$

B. $24\hat{i} - 32\hat{j} - 30\hat{k}$

C. $-12\hat{i} + 16\hat{j} - 15\hat{k}$

D. $12\hat{i} - 16\hat{j} - 15\hat{k}$

Answer: A

Solution:

Solution:

Since $\vec{a} = m\vec{b}$ for some scalar m i.e.,

$\vec{a} = m\left(6\hat{i} - 8\hat{j} - \frac{15}{2}\hat{k}\right)$

$\Rightarrow |a| = \ |m| \sqrt{36 + 64 + \frac{225}{4}}$

$\Rightarrow 50 = \frac{25}{2}\left|m\right|$

$\Rightarrow |m| = 4$

$\Rightarrow m = \pm 4$

Since, a makes an acute angle with the positive direction of Z - axis, so its z component must be positive and hence, m must be -4.

$\therefore a = -4\left(6\hat{i} + 8\hat{j} - \frac{15}{2}\hat{k}\right) \ = -24\hat{i} + 32\hat{j} + 30\hat{k}$

Question 146

The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then P(X = 1) is

Options:

A. $\frac{1}{4}$

B. $\frac{1}{32}$

C. $\frac{1}{16}$

D. $\frac{1}{8}$

Answer: B

Solution:

Solution:

$np = 4$

$npq = 2$

$\Rightarrow q = \frac{1}{2}, p = \frac{1}{2}, n = 8$

$P(X = 1) = {}^8C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^7 = 8 \cdot \frac{1}{2^8} = \frac{1}{2^5} = \frac{1}{32}$

Question 147

The statement p : For any real numbers x, y if x = y , then 2x + a = 2y + a when a ∈ Z .

Options:

A. is true

B. is false

C. its contrapositive is not true

D. None of these

Answer: A

Solution:

Solution:
We prove the statement p is true by contrapositive method and by direct method.
Direct method: For any real number x and y,
$x = y$
$\Rightarrow 2x = 2y$
$\Rightarrow 2x + a = 2y + a$ a for some $a \in Z$
Contrapositive method: The contrapositive statement of p is "For any real numbers x, y if $2x + a \neq 2y + a$, where $a \in Z$, then $x \neq y$."
Given , $2x + a \neq 2y + a$
$\Rightarrow 2x \neq 2y$
$\Rightarrow x \neq y$
Hence, the given statement is true.

Question 148

If $z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$ and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]$, then $|z_1z_2|$ is equal to $\sqrt{m}$. Value of m is

Options:

A. 6

B. 3

C. 2

D. 5

Answer: A

Solution:

Solution:
$z_1 = \sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right] = \sqrt{2}\left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right] = 1 + i$
$|z_1| = \sqrt{2}$
and $z_2 = \sqrt{3}\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right] = \sqrt{3}\left[\frac{1}{2} + i\frac{\sqrt{3}}{2}\right]$
$|z_2| = \sqrt{\frac{3}{4} + \frac{9}{4}} = \sqrt{3}$
$|z_1z_2| = |z_1||z_2| = \sqrt{2}.\sqrt{3} = 6$

Question 149

$\lim_{x \to 0} \sqrt{\frac{x - \sin x}{x + \sin^2 x}}$ **is equal to**

Options:

A. 1

B. 0

C. ∞

D. None of these

Answer: B

Solution:

Solution:

$$\lim_{x \to 0} \sqrt{\frac{x - \sin x}{x + \sin^2 x}} = \lim_{x \to 0} \sqrt{\frac{1 - \frac{\sin x}{x}}{1 + \frac{\sin^2 x}{x}}}$$

$$= \lim_{x \to 0} \sqrt{\frac{1 - \frac{\sin x}{x}}{1 + \left(\frac{\sin x}{x}\right) \sin x}} = \sqrt{\frac{1 - 1}{1 + 1 \times 0}} = 0$$

Question 150

If $\int \frac{3x + 1}{(x - 3)(x - 5)} \, dx = \int \frac{-5}{(x - 3)} \, dx + \int \frac{B}{(x - 5)} \, dx$

then the value of B is

Options:

A. 3

B. 4

C. 6

D. 8

Answer: D

Solution:

Solution:

We have ,

$$\frac{3x+1}{(x-3)(x-5)} = \frac{-5}{x-3} + \frac{B}{x-5}$$

$3x + 1 = -5(x-5) + B(x-3)$

Put $x = 5$

$3(5) + 1 = B(5-3)$

$16 = 2B$ or $B = 8$
