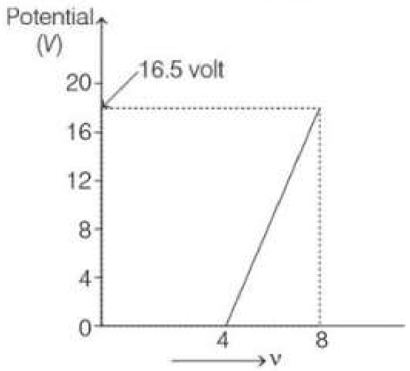
BITSAT - Paper 2022

Solved Paper

Question 1

The stopping potential (V_0) versus frequency (v) of a graph for photoelectric effect in a metal. From the graph, the planck's constant (h) is.



Options:

A. $6.60 \times 10^{-34} \mathrm{J}-\mathrm{s}$

B. $6.69 \times 10^{-34} \mathrm{J}-\mathrm{s}$

C. $6.62 \times 10^{-34} \mathrm{J}-\mathrm{s}$

D. $6.63 \times 10^{-34} \mathrm{J}-\mathrm{s}$

Answer: A

Solution:

Solution:

$eV_0 = h(v - v_0) = hv - hv_0$

$V_0 = \left(\frac{h}{e}\right)v - \left(\frac{h}{v}\right)v_0$ ....(i)

From Eq. (i), it follows that the slope of the graph is $\frac{h}{e}$.

Therefore,

$h = e \times \text{slope} = 1.6 \times 10^{-19} \times \frac{165 - 0}{(8-4) \times 10^{15}}$

$= 16 \times 10^{-19} \times \frac{16.5}{4} \times 10^{15} = 6.6 \times 10^{-34} \mathrm{J} - \mathrm{s}.$

Question 2

In a resonance column first and second resonance are obtained at depths 24 cm and 78 cm the third resonance will be obtained at depth.

Options:

A. 160 cm

B. 132 cm

C. 131 cm

D. 152 cm

Answer: B

Solution:

Solution:
Given, first resonance column at deoth, $l_1 = 24\text{cm}$
Second resonance column at depth, $l_2 = 78\text{cm}$
Third resonance column at depth, $l_3 = ?$

As $l_1 + x = \frac{\lambda}{4} = 24 \ldots\ldots\ldots\ldots\ldots (i)$

$l_2 + x = \frac{3\lambda}{4} = 78 \ldots\ldots\ldots\ldots\ldots (ii)$

$l_3 + x = \frac{5\lambda}{4} \ldots\ldots\ldots\ldots\ldots (iii)$

From Eqs. (i) and (ii),

$x = \frac{l_2 - 3l_1}{2} = \frac{78 - 3(24)}{2} = 3 \ldots\ldots\ldots\ldots\ldots\ldots (iv)$

substituting l_1 and x value, we get

$l_3 = 5(24) + 4(3)$

$\Rightarrow l_3 = 132\text{cm}.$

Question 3

A submarine A travelling at $17\frac{m}{s}$ is being chased along the line of its velocity by another submarine B travelling at $34\frac{m}{s}$. B sends a sonar signal of $600Hz$ to detect A and receives a reflected sound of frequency v. The of v is [Speed of sound in water $= 1500ms^{-1}$]

Options:

A. 613.7 Hz

B. 6137 Hz

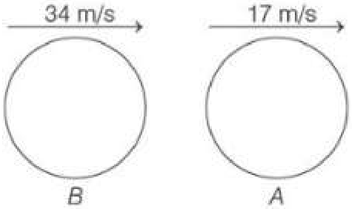
C. 62 Hz

D. 539 Hz

Answer: A

Solution:

Solution:
Given, velocity of submarine A, $v_A 17m / s$
Velocity of Submarine B, $v_B = 34m / s$
Signal sent by submarine B is detected by submarine A can be shown as



Frequency of the signal, $f_0 = 600Hz$
speed of sound in water $v_s = 1500ms^{-1}$
Frequency received by submarine A is

$$f_1 = \left(\frac{v_S - v_A}{v_S - v_B}\right) f_0 = \left[\frac{1500 - 17}{1500 - 34}\right] \times 600hz$$

$$f_1 = \frac{1483}{1466} \times 600 \ldots\ldots\ldots\ldots (i)$$

Frequency received by submarine is

$$f_2 = \left(\frac{v_S + v_B}{v_{svA}}\right) f_1$$

Substituting given values and f_1 value from Eq. (i), we get

$$f_2 = \left(\frac{1500 + 34}{1500 + 17}\right) \times \left[\frac{1483}{1466} \times 600\right]$$

$$f_2 = 1.0112 \times 1.0115 \times 600$$

$$f_2 = 613.7Hz.$$

Question 4

Transverse waves of the same frequency are generated in two steel wires A and B. The diameter of A is twice that of B and the tension in A is half that in B. The ratio of the velocities of the waves in A and B is

Options:

A. 1:2

B. $1 : \sqrt{2}$

C. $1 : 2\sqrt{2}$

D. $3 : 2\sqrt{2}$

Answer: C

Solution:

Solution:
The velocity if transverse waves is given by
$v = \sqrt{\frac{T}{m}}$ where, T = Tension and m = mass per unit length of the wire. If r is the radius if the wire and ρ its density then,

$$m = \pi r^2 \rho$$

$$\therefore v = \sqrt{\frac{T}{m}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\therefore v_A = \frac{\sqrt{T_A}}{r_A\sqrt{\pi\rho}}$$

and $v_B = \frac{\sqrt{T_B}}{r_B\sqrt{\pi\rho}}$

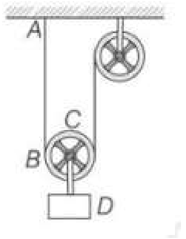
Now, $\frac{v_a}{v_B} = \sqrt{\frac{T_A}{T_B}} \cdot \frac{r_B}{r_A}$

$\because r_A = 2r_B$ and $T_A = \frac{1}{2}T_B$

Hence, $\frac{v_A}{v_B} = \frac{1}{2\sqrt{2}}$

Question 5

In the diagram shown below, both the strings AB and CD are made of same material and have same cross-section. The pulleys are light and fictionless. If the speed of wave in string AB is v_1 and in CD is v_2, then $\frac{v_1}{v_2}$ is



Options:

A. 1

B. $\sqrt{2}$

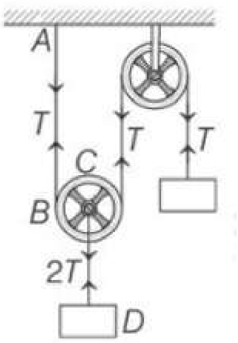
C. 2

D. $\frac{1}{\sqrt{2}}$

Answer: D

Solution:

Solution:



Let the speed of wave in string be v and tension in string b e T.

$v \propto \sqrt{T}$

$\frac{v_{AB}}{v_{CD}} = \frac{\sqrt{T}}{2T} = \frac{1}{\sqrt{2}}$

Question 6

What will be the acceleration due to gravity at a depth d , where g is

acceleration due to gravity on the surface of earth?

Options:

A. $\frac{g}{\left[1+\frac{d}{R}\right]^2}$

B. $g\left[1-\frac{2d}{R}\right]$

C. $\frac{g}{\left[1-\frac{d}{R}\right]^2}$

D. $g\left[1-\frac{d}{R}\right]$

Answer: D

Solution:

Solution:
Acceleration due to gravity at the surface of earth,

$g = G\frac{M}{R^2} = \frac{4}{3}\pi\rho GR \ldots\ldots\ldots\ldots\ldots\ldots (i)$

$\left[\because M = \frac{4}{3}\pi R^3 \cdot \rho\right]$

Where, $\rho =$ density of earth.

Acceleration due to gravity at depth d from the surface of earth,

$g' = \frac{4}{3}\pi\rho G(R-d) \ldots\ldots\ldots\ldots (ii)$

From Eqs. (i) and (ii), we get

$g' = g\left[1-\frac{d}{R}\right]$

Question 7

A direct current of 6A is superimposed on an alternating current $I = 10\sin\omega t$ flowing through a wire. The effective value of the resulting current will be

Options:

A. $5\sqrt{2}$

B. $5\sqrt{3}$

C. 9.27

D. 8.37

Answer: C

Solution:

Solution:
Given, $I = 6 + 10\sin\omega t$

$$I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{\frac{1}{2}} = \left[\frac{1}{T}\int_0^T (6 + 10\sin\omega t)^2 dt\right]^{\frac{1}{2}}$$

$$= \left[\frac{1}{T}\int_0^T (36 + 120\sin\omega t + 100\sin^2\omega t) dt\right]^{\frac{1}{2}}$$

But as, $\frac{1}{T}\int_0^T \sin\omega t\, dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 W t = \frac{1}{2}$

$$\Rightarrow I_{eff} = \left[36 + \frac{1}{2} \times 100\right]^{\frac{1}{2}} = 0.97$$

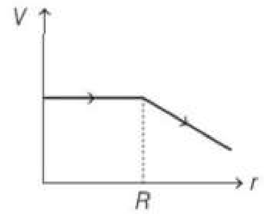
Thus, $I_{eff} = 9.27A$.

Question 8

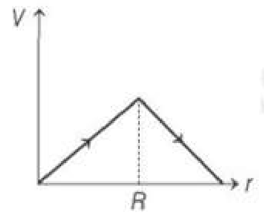
Which one of the following graphs represents the variation of electric potential with distance r from the centre of a non-conducting charged sphere of radius R ?

Options:

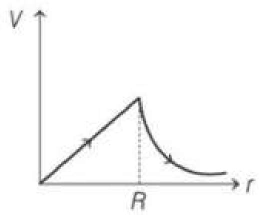
A.



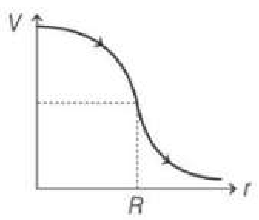
B.



C.



D.



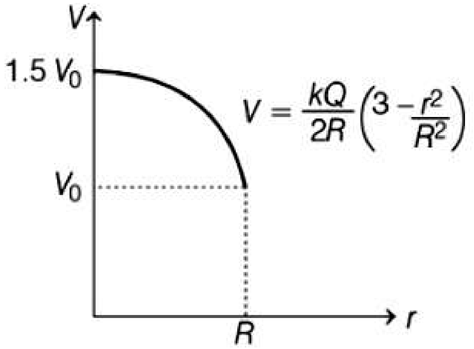
Answer: D

Solution:

Solution:

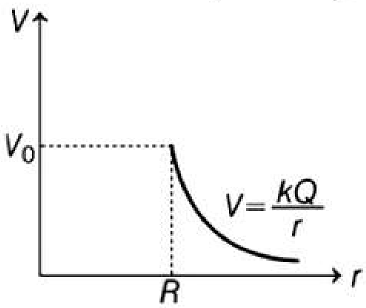
Electric potential at centre $R=0$

$V_{\text{centre}}=1.5V_{\text{surface}}$



where, $V_0=\frac{kQ}{R}$

$V_{\text{inside}}=\frac{kQ}{2R}\left(3-\frac{r^2}{R^2}\right)$



$V_{\text{outside}}=k\frac{Q}{r}\propto\frac{1}{r}$

Question 9

For an insulator, the forbidden energy gap is

Options:

A. Zero

B. 1 eV

C. 2 eV

D. 5 eV

Answer: D

Solution:

Solution:

For an insulator, $E_g > 3\text{eV}$, that is why electron transition from valence band to conduction band is not possible. For semiconductor E_g is 0.2eV to 0.3eV, while for metals $E_g > 5\text{eV}$.

Question 10

A machine gun fires 300 bullets per min if the mass of each bullet is 10g and the velocity of the bullets is 600ms^{-1}, the power (in kW) of the gun is

Options:

A. 43200

B. 9

C. 72

D. 7.2

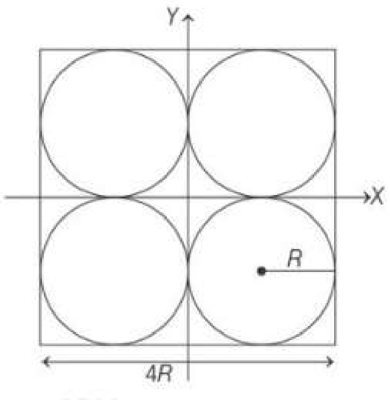
Answer: B

Solution:

Solution:
Work done by the gun = total kinetic energy of the bullets
$= n\frac{1}{2}mv^2 = 300 \times \frac{1}{2} \times 10 \times 10^{-3} \times (600)^2$
$= 150 \times 10 \times 10^{-3} \times 600 \times 600$
Power of gun $= \frac{\text{work done}}{\text{Time taken}}$
$= \frac{150 \times 10 \times 10^{-3} \times 600 \times 600}{60s}$
$= 9kW\ (\because 1\text{ minute} = 60\text{ seconds})$

Question 11

Four holes of radius 5cm are cut from a thin square plate of 20cm and mass 1kg. The moment of inertia of the remaining portion about Z -axis is



Options:

A. $15\,\text{kg} - \text{m}^2$

B. $0.37\,\text{kg} - \text{m}^2$

C. $0.0017\,\text{kg} - \text{m}^2$

D. $0.08\,\mathrm{kg-m^2}$

Answer: C

Solution:

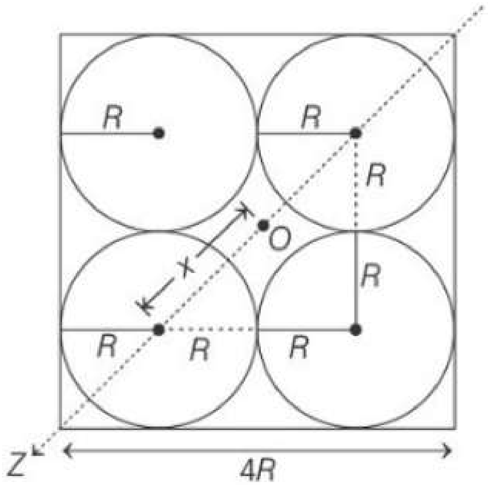
Solution:

Area mass density, $\sigma = \frac{M}{16R^2}(\because \text{Area} = 4R \times 4R = 16R^2)$

Mass of each hole $m_1 = \sigma\pi R^2 = \frac{M}{16R^2}\pi R^2 = \frac{\pi M}{16}$

Distance between centre of plate and centre of hole

$x = \frac{\sqrt{(2R)^2 + (2R)^2}}{2} = \frac{2\sqrt{2}R}{2}$



Moment of inertia of one hole at about Z-axis

$I_1 = \frac{1}{2}m_1R^2 + m_1x^2 = \frac{5\pi}{32}MR^2$

Moment of inertia of whole plate about Z-axis,

$I = \frac{M(4R)^2}{6} = \frac{8}{3}MR^2$

Required moment of Inertia

$I_0 = I - 4I_1, = \left[\frac{8}{3} - 4\left(\frac{5\pi}{32}\right)\right]MR^2 = \left[\frac{8}{3} - \frac{5\pi}{8}\right]MR^2$

Given, R=5cm and M=1kg

So, $I_0\left[\frac{8}{3} - \frac{5\pi}{8}\right]1 \times 25 \times 10^{-4} = 0.0017\mathrm{kg-m^2}$

Question 12

a particle of mass m is projected with velocity v at an angle θ with the horizontal. At its highest point, it explodes into two pieces of equal mass, one of the piece continue to move on the original trajectory, then the velocity of second piece is.

Options:

A. $2v\cos\theta$

B. $v\cos\theta$

C. $3v\cos\theta$

D. $\frac{v}{2}\cos\theta$

Answer: B

Solution:

Solution:
According to the law of conservation of linear momentum at the highest point,

$$mv\cos\theta = \frac{m}{2}(v\cos\theta) + \frac{m}{2}v'$$

$$\Rightarrow mv\cos\theta = \frac{m}{2}v\cos\theta = \frac{m}{2}v'$$

$$\Rightarrow v' = 2\left(v\cos\theta - \frac{v\cos\theta}{2}\right) = v\cos\theta$$

Question 13

In the circuit shown assume the diode to be ideal. When V_i increases from $-2V$ to $6V$, the change in current is (in mA)

Options:

A. Zero

B. 20

C. $\frac{25}{8}$

D. 32

Answer: B

Solution:

Solution:
$I_{initial} = 0 \text{ for } V_i \leq +1V$
This diode conduct only beyond $V_i = +1V$

$$I_{final} = \frac{5}{250} = 0.02A$$

So, change in $I = 0.02A = 20mA$

Question 14

The de-Broglie wavelength of an electron moving with a velocity $\frac{c}{3}(c = 3 \times 10^8 m / s)$ is equal to the wavelength of photon. The ratio of the kinetic energies of electron and photon is

Options:

A. 1 : 4

B. 1 : 3

C. 1 : 2

D. 2 : 1

Answer: B

Solution:

Solution:

The de-Brogile wavelength, $\lambda = \frac{h}{mv}$

Here, $\lambda_e = \frac{h}{m_e \frac{c}{3}}$ and $\lambda_p = \frac{h}{m_p c}$

Given, $\lambda_e = \lambda_p$

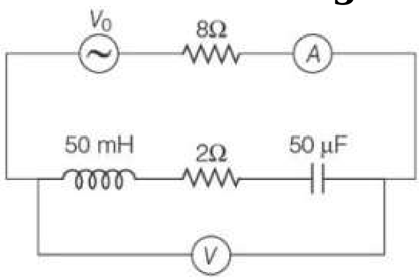
So, $\frac{h}{m_e \frac{c}{3}} = \frac{h}{m_p c}$

$\Rightarrow \frac{m_e}{m_p} = 3$

Ratio of K E $= \frac{K_e}{K_p} = \frac{\frac{1}{2} m_e v_e^{\ 2}}{\frac{1}{2} m_p v_p^{\ 2}} = \frac{1}{3} \left(\because \frac{m_e}{m_p} = 3, v_e = \frac{c}{3} \text{and } v_p = c \right)$

Question 15

In the circuit shown in the figure, the AC source gives a voltage V = 20 cos(2000t) neglecting source resistance, the voltmeter and ammeter reading will be



Options:

A. 0V, 0.47A

B. 2.82V, 1.41A

C. 1.41V, 0.47A

D. 1.5V, 8.37A

Answer: B

Solution:

Solution:
Given,
$R_1 = 8\Omega$
$R_2 = 2\Omega$
L = 5mH

C = 50μF
and $V_0 = 20\cos(2000t)$
Independence, $Z = \sqrt{R^2 + (X_L - X_C)^2}$
As, here, $X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\Omega$
Similarly, $X_L = \frac{1}{\omega}C = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\Omega$
$\because X_L = X_C$, Hence, $i_{max} = \frac{V_0}{Z} = \frac{20}{(8+2)} = 2A$
Hence, $i_{rms} = \frac{i_{max}}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41A$
and $V = R_2 i_{rms} = 1.41A \times 2$
$= 282V$

Question 16

An electromagnetic wave is propagating along X-axis. At $x = 1$ cm and $t = 18$ s, its electric vector $|E| = 8V/m$, then the magnitude of its magnetic vector is

Options:

A. 266×10^{-8}

B. 3×10^{-7}

C. 3.14×10^{-8}

D. 3.16×10^{-7}

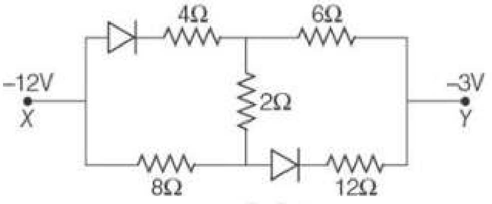
Answer: A

Solution:

Solution:
As we know that, $\frac{E}{B} = c$
so, $B = \frac{E}{c} = \frac{8}{3} \times 10^8 = 266 \times 10^{-8}$

Question 17

In the following circuit the equivalent resistance between X and Y is ...... Ω



Options:

A. 5

B. 12

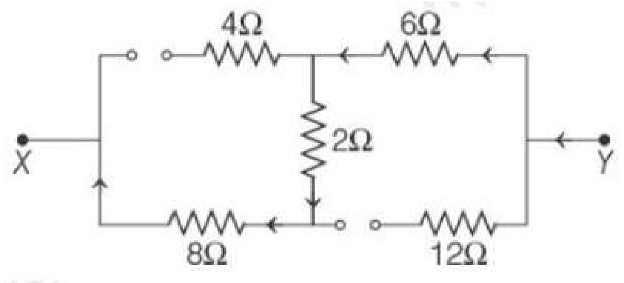
C. 16

D. 20

Answer: C

Solution:

Solution:
According to the given figure X is at lower potential w.r.t γ. Hence, both diodes are in revere biasing, so, equivalent citcuit can be redrawn as follows



Equivalent resistance between X and Y
$R_{eq} = 8 + 2 + 6 = 16\Omega$

Question 18

A monoatomic gas of molar mass m is kept in a insulated container. Container is moving with velocity v. If the container is suddenly stopped, then the change in the temperature of the gas is

Options:

A. $\frac{mv^2}{4R}$

B. $\frac{mv^2}{2R}$

C. $\frac{mv^2}{R}$

D. $\frac{mv^2}{3R}$

Answer: D

Solution:

Solution:
Let the insulated container has n moles of the monoatomic gas. The loss of kinetic energy of the gas,
$\Delta E_K = \frac{1}{2}(mv)v^2$
If the change in temperature of the gas is ΔT , then heat gained by the gas is,
$\Delta Q = \frac{3}{2}nR\Delta T$
Now, according to question,
$\Delta Q = \Delta E_K$
$\frac{3}{2}nR\Delta T = \frac{1}{2}mnv^2$

$\Rightarrow \Delta T = \frac{mv^2}{3R}$

Question 19

A projectile is projected with the velocity of $\left(3\hat{i} + 4\hat{j}\right)$ m / s. The horizontal range of the projectile will be

Options:

A. 1.2m

B. 2.4m

C. 3.6m

D. 4.5m

Answer: B

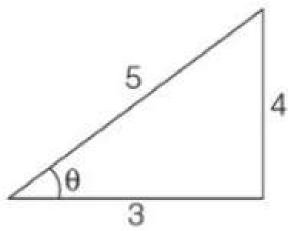
Solution:

Solution:

Given, $v = 3\hat{i} + 4\hat{j}$

$\therefore v = |v| = \sqrt{3^2 + 4^2} \Rightarrow v = 5m / s$

From figure,



$\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

$\because R = \frac{v^2 \sin 2\theta}{g}$

$= \frac{v^2 . 2\sin\theta . \cos\theta}{g}$

$= \frac{5 \times 5 \times 2 \times \frac{4}{5} \times \frac{3}{5}}{10} = 24m$

Question 20

A transistor is connected in common-emitter (CE) configuration. The collector supply is 8V and the voltage drop across a resistor is 500Ω in the collector circuit is 0.6V. If the current gain factor α is 0.96 , find the base current

Options:

A. 25μA

B. 50μA

C. 20μA

D. 35μA

Answer: B

Solution:

Solution:
Given, $V_{CC} = 8V$
Voltage drop, $V_0 = \Delta I_C R_L = 0.6V$
Load Resistor, $R_L = 500\Omega$
Gain factor, $\alpha = 0.96$
$\because \beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$
Now, $V_0 = \Delta I_C R_L$
$0.6 = \Delta I_C \times 500$
$\Delta I_C = \frac{0.6}{500} = 1.2 \times 10^{-3}A$
Now, $\beta = \frac{\Delta I_C}{\Delta I_B}$
$\therefore$ Base current $\Delta I_B = \frac{\Delta I_C}{\beta} = \frac{1.2 \times 10^{-3}}{24} = 5 \times 10^{-5}$
$= 50 \times 10^{-6}$
$\Delta I_B = 50\mu A$

Question 21

A solid sphere of 80 kg and radius 15m moving in a space becomes a circular disc of radius 20m in 1h. The rate of change of moment of Inertia in this process is ......

Options:

A. $\frac{30}{9}kg^{-m^2 s^{-1}}$

B. $\frac{25}{9}kg - m^2 s^{-1}$

C. $\frac{10}{9}kg - m^2 s^{-1}$

D. $\frac{22}{9}kg - m^2 s^{-1}$

Answer: D

Solution:

Solution:
Given, mass of solid sphere $= 80kg$
radius of solid sphere, $R_s = 15m$
radius of circular disc, $R_c = 20m$
and time $= 1\,\text{hour} = 60\,\text{minutes} = 60 \times 60sec$
$\therefore$ Moment if inertia of solid sphere, $I_s = \frac{2}{5}MR^2$

$= \frac{2}{5} \times 80 \times (15)^2$

$= 7200 kg - m^2$

Similarly,

moment of inertia of the disc, $I_c = \frac{1}{2} M R_c^2$

$= \frac{1}{2} \times 80 \times (20)^2$

$= 16000 kg - m^2$

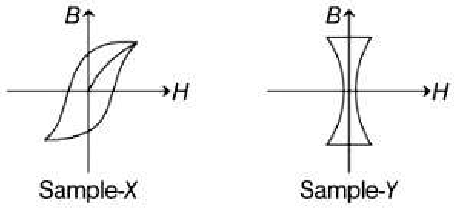
Rate of change of moment of Inertia $= \frac{I_c - I_s}{t}$

$= \frac{16000 - 7200}{60 \times 60}$

$= \frac{22}{9} kg - m^2 s^{-1}$

Question 22

If the $B - H$ curves of two samples of X and Y of iron are as shown below, then which one of the following statement is correct?



Options:

A. Both X and Y are suitable for making electromagnets.

B. Both X and Y are suitable for making permanent magnet.

C. X is suitable for making permanent magnet and Y for making electromagnet.

D. X is suitable for making electromagnet and Y is suitable for permanent magnet.

Answer: C

Solution:

Solution:
For permanent magnet we prefer a material with high retentivity and high coercivity. For electromagnet, we prefer high saturated magnetism, low coercivity and least possible area of hysteris loop.
Therefore, X is suitable for making parmanent magnet and γ for making electromagnet.

Question 23

In a radioactive material the activity at time t_1, is A_1 and at a later time t_2, it is A_2. If the decay constant of the material is λ, then

Options:

A. $A_1 = A_2 e^{-\lambda(t_1 - t_2)}$

B. $A_1 = A_2 e^{\lambda(t_1 - t_2)}$

C. $A_1 = A_2 \left(\frac{t_2}{t_1}\right)$

D. $A_1 = A_2$

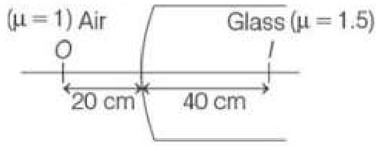
Answer: A

Solution:

Solution:
From radioactive decay law,
$-\frac{dN}{dt} \propto N$ or $-\frac{dN}{dt} = \lambda N$
Thus, $A = -\frac{dN}{dt}$ or $A = \lambda N$
$\Rightarrow A = \lambda N_0 e^{-\lambda t}$ ....(i)
Where, $A_0 = \lambda N_0$ is the activity of the radioactive material at time, $t = 0$
At time, $t_1 \ldots\ldots A_1 = A_0 e^{-\lambda t_1}$. . (ii)
At time, $t_2 \ldots\ldots A_2 = A_0 e^{-\lambda t_2}$ .... (iii)
On didviding Eq. (ii) by Eq. (iii), we have
$\frac{A_1}{A_2} = \frac{e^{-\lambda t_1}}{e^{-\lambda t_2}} = e^{-\lambda(t_1 - t_2)}$
$\Rightarrow A_1 = A_2 e^{-\lambda(t_1 - t_2)}$

Question 24

A mosquito O is sitting infront of a glass rod having spherical end of radius of curvature 40 cm. The image would be formed at



Options:

A. 40 cm left

B. infinity

C. 20 cm to the right

D. 15 cm to the left

Answer: A

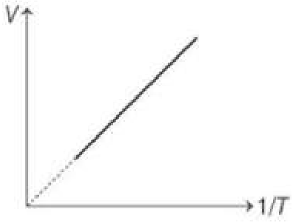
Solution:

Solution:
As, we know, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$
$\Rightarrow \frac{1.5}{v} = \frac{1}{(-20)} = \frac{1.5 - 1}{+40}$
$\Rightarrow v = -40\text{cm}$

Negative sign shows that image is obtained on the same side of object i.e., towards left.

Question 25

One mole of an ideal diatomic gas undergoes a process as shown in the figure. The molar specific heat of the gas in the process is



Options:

A. $\frac{3R}{2}$

B. $\frac{R}{2}$

C. $\frac{5R}{2}$

D. $\frac{7R}{2}$

Answer: A

Solution:

Solution:

In the curve, $V \propto \frac{1}{T}$

$\Rightarrow V = m\frac{1}{T}$ where, $m =$ slope of curve.

$\Rightarrow VT =$ constant

$\therefore TdV + VdT = 0 \ldots\ldots\ldots (i)$

$\Rightarrow dV = -\frac{V}{T}dT$

As, $Q = \Delta U + W$

$CdT = C_V dT + pdV \ldots\ldots (ii)$

$CdT = C_V dT - \frac{p}{T}VdT$

$\Rightarrow C = C_V - p\frac{V}{T}$

$= C_V - R\left(\because p\frac{V}{T} = R\right)$

For diatomic gas, $C_V = \frac{5}{2}R$

$\therefore C = C_V - R = \frac{5}{2}R - R = 3\frac{R}{2}$

Question 26

A capillary tube is attached horizontally to a constant heat arrangement. If the radius of the capillary tube is increased by 25%, then the rate of flow of liquid will change nearly by

Options:

A. 100%

B. 112%

C. 124%

D. 144%

Answer: D

Solution:

Solution:
Liquid volume coming out of the tube per second

$V = \frac{p\pi r^4}{8\eta l}$

$\Rightarrow V\,\alpha r^4$

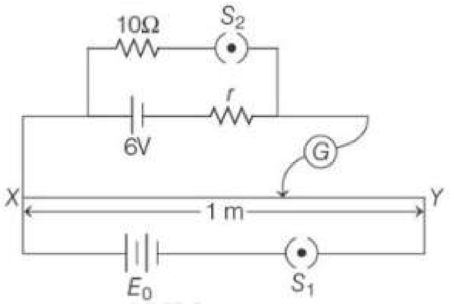
$\Rightarrow V_2 V_1 = \left[\frac{r_2}{r_1}\right]^4 \Rightarrow V_2 = V_1\left[\frac{125}{100}\right]^4 = V_1\left(\frac{5}{4}\right)^4$

$\Rightarrow V_2 = V_1(244)$

so, now $\frac{\Delta V}{V} = \frac{V_2 - V_1}{V_1} = \frac{(244)V_1 - V_1}{v_1} = 144\%$

Question 27

In the arrangement shown in figure, when the switch S_2 is open, the galvanometer, shows no deflection for $l = 50\,cm$ when the switch S_2 is closed, the galvanometer shows no deflection for $l = 0.416m$. The internal resistance (r) of 6V cell is



Options:

A. 2Ω

B. 3Ω

C. 5Ω

D. 9Ω

Answer: A

Solution:

Solution:

When S_2 open.
Assume resistance of $X\gamma = R$
Resistance of wire per unit length, $x = \frac{R}{L} = R\Omega m^{-1}$
$\because I = \frac{E_0}{R}$
Now, the potential drop across 50 cm length is 6V , so
$\frac{E_0}{R} \times R \times \frac{50}{100} = 6$
$\Rightarrow E_0 = 12V$
When S_2 closed, potential drop across $0.416cm$ length,
$V_1 = \frac{E_0}{R} \times R \times 0.416 = 12 \times 12 \times 0.416 \approx 5V$
Hence, $E - Ir = 5V$
$\Rightarrow 6 - Ir = 5$
$\because I = \frac{5}{10}$
$\therefore 6 - 5 = \frac{5}{10} r$
$\Rightarrow r = 2\Omega$

Question 28

In a young's double slit arrangement frings are produced using light of wavelength 4000Å. One slit is covered by a thin plate of glass of refractive index 1.4 and the other with another glass plate of same thickness but of refractive index 1.7. By doing so the central bright shifts to original sixth fringe from Centre. Thickness of glass plate is

Options:

A. 2μm

B. 8μm

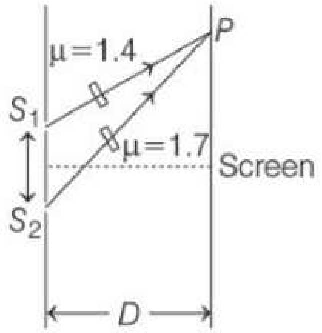
C. 11μm

D. 16μm

Answer: B

Solution:

Solution:
As, shift $\Delta x = \frac{\beta}{\lambda}(\mu - 1)t$



Shift due to one plate $\Delta x_1 (\mu_1 - 1)t$
Shift due to another path $\Delta x_2 = \frac{\beta}{\lambda}(\mu_2 - 1)t$

Net shift $\Delta x = \Delta x_2 - \Delta x_1 = \frac{\beta}{\lambda}(\mu_2 - \mu_1)t \ldots\ldots (i)$

Also, it is given that $\Delta x = 6\beta$

Hence, $6\beta = \frac{\beta}{\lambda}(\mu_1 - \mu_2)t$

$\Rightarrow t = 6\frac{\lambda}{(\mu_1 - \mu_2)} = 6 \times \frac{4000}{(1.7 - 1.4)} - 80000Å$

$\Rightarrow t = 8 \times 10^{-6} = 8\mu m$

Question 29

An electric current I enters and leaves a uniform circular wire of radius r through diametrically opposite points. A charged particle q moves along the axis of circular wire passes through its centre at speed v. The magnetic force on the particle when it passes through the centre has a magnitud

Options:

A. $\frac{qv\mu_0 I}{2\pi r}$

B. $qv\,\frac{\mu_o I}{\pi r}$

C. $\frac{qv\mu_0 I}{r}$

D. 0

Answer: D

Solution:

Solution:

Force on a moving charged particle in uniform magnetic field

$F = Bqv\sin\theta \ldots\ldots (i)$

Since, charge particle moves along the axis odf circular current carrying loop, therefore, $\theta = 0° \text{ or } 180°$

When $\theta = 0°$; $f = Bqv\sin 0°$

$F = 0$

When $\theta = 180°$, $f = Bqv\sin 180°$

$F = 0$

Question 30

An achromatic convergent doublet of two lenses in contact has a power of +5D. The power of converging lens is +6D. The ratio of the dispersive power of the convergent and divergent lenses is

Options:

A. 3 : 7

B. 2 : 3

C. 1 : 5

D. 5 : 3

Answer: C

Solution:

Solution:
The condition of achromatism is $W_1 P_1 + W_2 P_2 = 0$
$\Rightarrow W_1 P_1 = -W_2 P_2$
$\Rightarrow \frac{W_1}{W_2} = \frac{P_2}{P_1} \ldots\ldots\ldots$ (i)
Now, $P_1 + P_2 = 4D \ldots\ldots\ldots$ (ii)
but, Power of converting lens,
$P_1 = 5D$
∵ Power of diverging lens
$P_2 = 4D - P_1$ [From ii]
$= 4D - 5D = -D$
∴ From Eq. (i), we have
$\frac{W_1}{W_2} = \frac{P_2}{P_1} = \frac{-(-D)}{5D} = \frac{1}{5} \Rightarrow \frac{W_1}{W_2} = \frac{1}{5}$

Question 31

Which one of the following is correct order of given isotopes?
I. $T_2 > D_2 > P_2$ (order of boiling point)
II. $T_2 > D_2 > P_2$ (order of bond energy)
III. $T_2 = D_2 = P_2$ (order of bond length)
IV. $T_2 < D_2 < P_2$ (order of reactivity with Cl_2)

Options:

A. I and II

B. III and IV

C. II, III and IV

D. All of these

Answer: D

Solution:

Solution:
Protium (P), deuterium (D) and tritium (T) are the three isotopes of hydrogen. These isotopes follow the order in different contexts as shown below :
$T_2 > D_2 > P_2$ [Order of boiling point (BP)]
$T_2 > D_2 > P_2$ [Order of bond energy (BE)]
$T_2 = D_2 = P_2$ [Order of bond length (BL)]
$T_2 Hence, all the given order are correct.

Question 32

Ninhydrin gives yellow colour in paper chromatography with which amino acid?

Options:

A. Tryptophan

B. Proline

C. Alanine

D. Tyrosine

Answer: B

Solution:

Solution:
The amino acid which gives yellow colour with ninhydrin in paper chromatography is proline.
Ninhydrin + Proline → Yellow-orange product

Question 33

How will raise in temperature affects the viscosity of liquids and gases?

Options:

A. Both increases

B. Both decreases

C. In case of liquids, decreases and in case of gases, increases.

D. In case of liquid, increases and in case of gases, decreases.

Answer: B

Solution:

Solution:
In case of liquids, cohesive forces decrease and hence, their viscosity decreases while increasing the temperature. However, in case of gases, with the increase in temperature kinetic energy of gaseous molecules increases and the molecules become more mobile.

Question 34

Which of the following compounds is thermodynamically is the most stable?

Options:

A. $BaCO_3$

B. $MgCO_3$

C. $SrCO_3$

D. $CaCO_3$

Answer: A

Solution:

Solution:
The thermal stability of carbonates increases down the group. So, the order of thermal stability of carbonates of alkaline earth metals is
$BaCO_3 > CaCO_3 > MgCO_3 > BeCO_3$.
Therefore, $BaCO_3$ is thermodynamically most stable and $BeCO_3$ is least stable.

Question 35

Glucose reacts with X number of molecules of phenyl hydrazine to yield osazone. The value of X is,

Options:

A. three

B. two

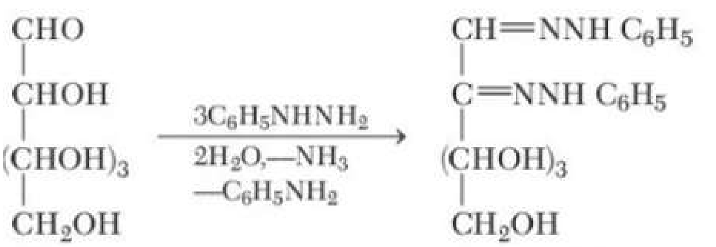
C. one

D. four

Answer: A

Solution:

Solution:



Glucose   Osazone (glucosazone)

Hence, three molecules of phenyl hudrazine (C_6H_5NHNH_2) is used to yield osazone.

Question 36

Nylon- 6,6 is obtained from

Options:

A. adipic acid and hexamethylene diamine

B. tetrafluoroethylene

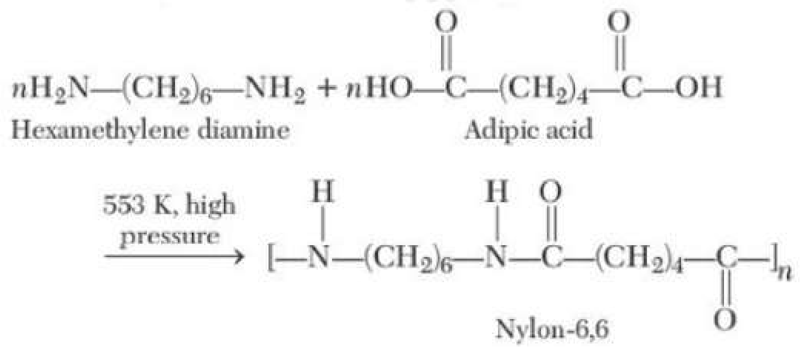
C. vinyl cyanide

D. vinyl benzene

Answer: A

Solution:

Solution:
The monomer units of nylon -6, 6 are hexamethylene diamine and adipic acid.



Question 37

What is the hybridisation of $[CrF_6]^{3-}$?

Options:

A. sp^3d

B. sp^3d^2

C. d^2sp^3

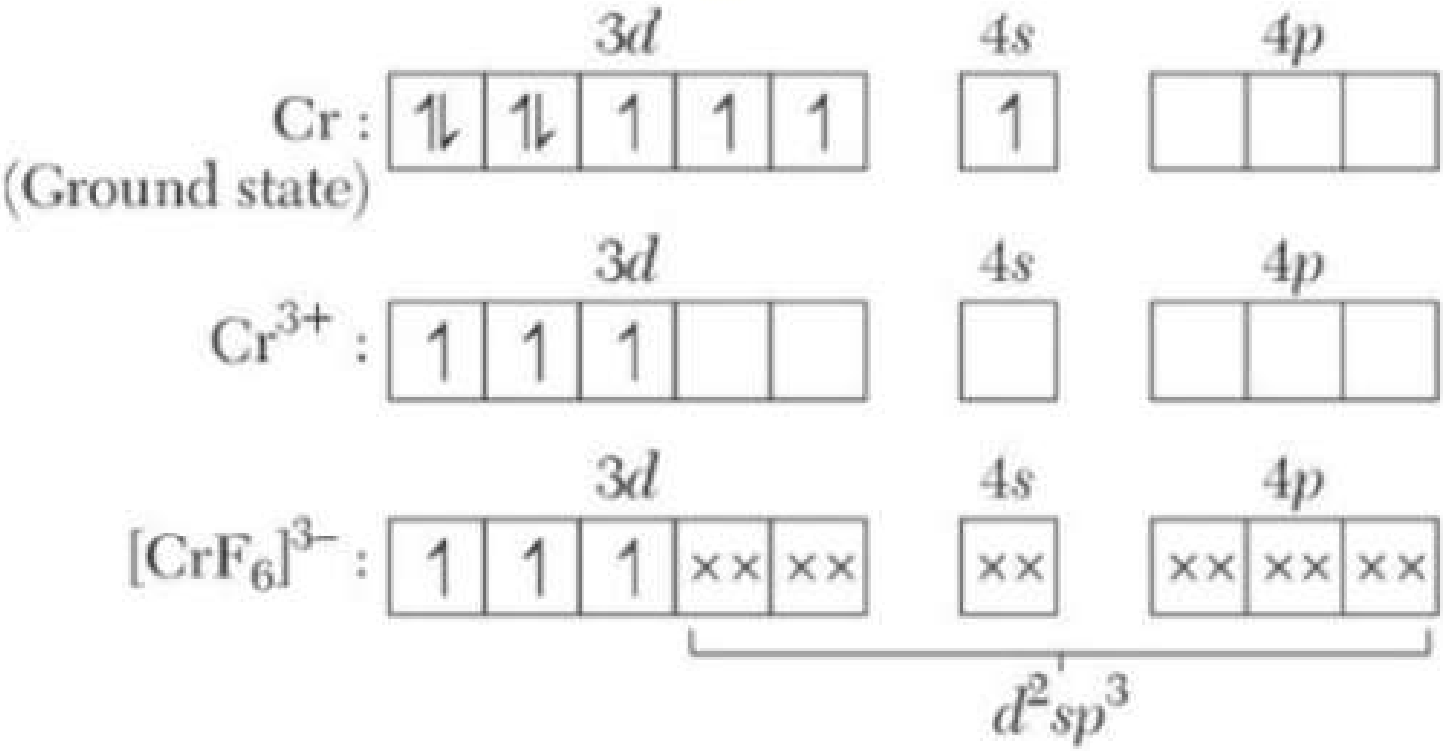
D. d^2sp

Answer: C

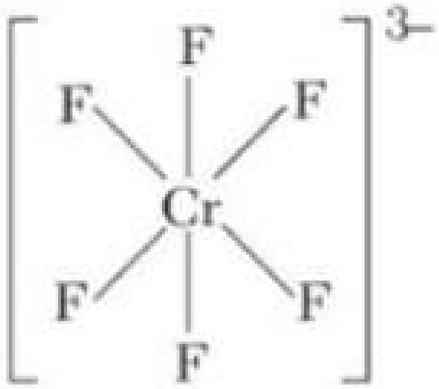
Solution:

Solution:

The electronic configuration of Cr is $[Ar]3d^5 4s^1$.



$[CrF_6]^{3-}$ has d^2sp^3 - hybridization and octahedral geometry.



Question 38

OF and F_2 can be compared in terms of

Options:

A. OF is paramagnetic while F_2 is diamagnetic

B. OF is more stable towards dissociation into atoms

C. Both (a) and (b) are correct

D. None of the above is correct

Answer: C

Solution:

Solution:

Species	Electons	Bond order	Unpaired electron	
$O-F$	17	1.5	1	Paramegnetic
F_2	18	1.0	0	Diamagnetic

This is one unpaired electron on ($\mathrm{O-F}$), hence paramagnetic, thus (a) is correct.
As bond order of $\mathrm{O-F} > \mathrm{F-F}$ thus, bond energy of $\mathrm{O-F} > \mathrm{F-F}$.
Hence, (a) and (b) both correct.

Question 39

ortho and para form of hydrogen have

Options:

A. different physical and chemical properties

B. identical physical properties but different chemical properties

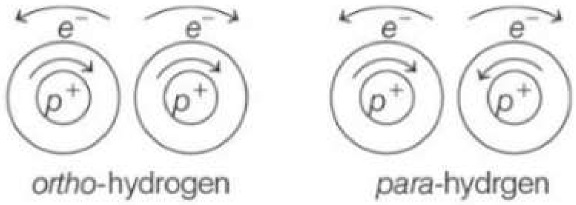
C. identical chemical properties but different physical properties

D. identical chemical and physical properties

Answer: C

Solution:

Solution:
ortho and para hydrogen similar to each other in their chemical properties but they have different physical properties like boiling point, thermal conductivity, due to their difference in overall spins.



Question 40

The structure of H_2O_2 is

Options:

A. planar, linear

B. non-planar, linear

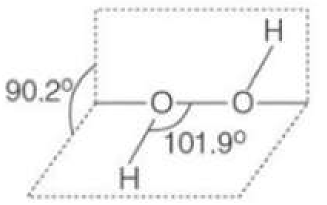
C. planar, non-linear

D. non-planar, non-linear

Answer: D

Solution:

Solution:
Hydrogen peroxide (H_2O_2) is a non-planar molecule with twisted symmetrical structure. It has non-linear structure with an open book structure.



Question 41

Match the species in Column I with their types in Column II.

Column I	Column II
A. DDT	1. Photochemical smog
B. $NaClO_3$	2. Disinfectant
C. Cl_2	3. Herbicides
D. PAN	4. Pesticides

Options:

A. A → 4, B → 3, C → 2, D → 1

B. A → 1, B → 2, C → 3, D → 1

C. A → 2, B → 3, C → 1, D → 4

D. A → 3, B → 1, C → 2, D → 4

Answer: A

Solution:

Solution:
The correct match is
A → (4); B → (3); C → (2); D → (1)
A. DDT → Pesticides
B. $NaClO_3$→ Herbicides
C. Cl_2→ Disinfectants
D. PAN → Photochemical smog

Question 42

In which pair or pairs is the stronger bond found in the first species?
I. O_2^{2-}, O_2;
II. N_2, N_2^+;   III. NO^+, NO^-

Options:

A. I only

B. II only

C. I and II only

D. II and III only

Answer: D

Solution:

Solution:

	Bond order		
I.	O_2^{2-} O_2	1 1	Weaker
II.	N_2 N_2^+	3 2.5	Stronger
III.	NO^+ NO^-	3 2	Stronger

Larger the bond order, stronger the bond formed.

Question 43

Select the correct statement about the complex $[Co(NH_3)_5 SO_4] Br$.

Options:

A. Its ionisation isomer is $[Co(NH_3)_5 Br]SO_4$.

B. It gives yellow precipitate with $AgNO_3$.

C. Its ionisation isomer give while precipitate with $BaCl_2$.

D. All the above are correct statements.

Answer: D

Solution:

Solution:

(d) $[Co(NH_3)_5SO_4]Br \rightleftharpoons [Co(NH_3)_5SO_4]^+ + Br^-$

$\uparrow$

Yellow ppt.
with $AgNO_3$

$[Co(NH_3)_5Br]SO_4 \rightleftharpoons [Co(NH_3)_5Br]^{2+} + SO_4^{2-}$

$\uparrow$

White ppt. with $BaCl_2$

Question 44

A certain metal sulphide, MS_2, is used extensively as a high temperature lubricant. If MS_2 is 40.06% by mass sulphur, metal M has atomic mass.

Options:

A. 160 u

B. 64 u

C. 40u

D. 96u

Answer: D

Solution:

Solution:

$MS-2 = M + 32 \times 2 = M + 64$

% of sulphur $= \left(\frac{64}{M+64}\right) \times 100 = 40.06$

$M + 64 = \frac{6400}{40.06}$

$M + 64 = 160$

$M = 160 - 64 = 96u$

Question 45

$X \xrightarrow[\text{Boiling}]{Cl_2}$ Benzotrichloride $\xrightarrow{H_3O^+}$ Y

X and Y are

Options:

A. benzene, benzaldehyde

B. toluene, benzaldehyde

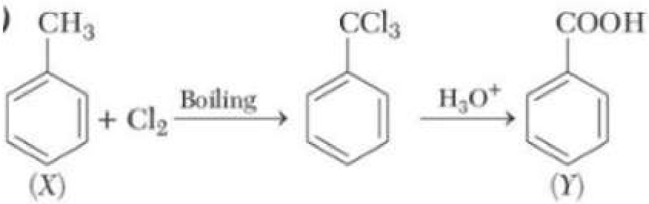
C. toluene, benzoic acid

D. benzene, benzoic acid

Answer: C

Solution:

Solution:



Question 46

Ge (II) compounds are powerful reducing agents whereas Pb (IV) compounds are strong oxidants. It can be because

Options:

A. Pb is more electropositive than G

B. ionisation potential of lead is less than that of Ge.

C. ionic radii of Pb^{2+} and Pb^{4+} are larger than that of Ge^{2+} and Ge^{4+}.

D. more pronounced inert pair effect in lead has.

Answer: D

Solution:

Solution:
Inert pair effect is more pronounced in heavier members like Pb.
Hence, Pb (IV) compounds act as strong oxidising agents and are reduced to more stable Pb (II) compounds.

Question 47

Which compound has antifluorite structure?

Options:

A. MnO_4

B. Na_2O

C. Na_2O_2

D. Li_2O_2

Answer: B

Solution:

Solution:
Anti-fluorite structure refers to an anion array with tetrahedral cations and the compound having A_2B formula have anti-fluorite structure. So, Na_2O has an anti-fluorite structure in which Na^+ ions occupy all the tetrahedral voids and O^{2-} occupy half of the cubic holes.

Question 48

$100\,mL$ of 2M of formic acid($pK_a = 3.74$) is neutralise by NaOH, at the equivalence point pH is

Options:

A. 7

B. 6

C. 9.5

D. 8.87

Answer: D

Solution:

Solution:
Sodium formate is present at the equivalence point. It is the salt of weak acid + strong base. So, final solution will be basic in nature.
As we know
$pH = 7 + \frac{pK_a}{2} + \frac{\log C}{2}$
where, C is concentration of salt.
Total volume of solution $= 100 + 100 = 200mL$
Concentration of salt (C) $= 2 \times \frac{100}{200} = 1M$
$pH = 7 + \frac{3.74}{2} + \frac{\log[1]}{2}$
$pH = 7 + 1.87 + 0$
$pH = 8.87$

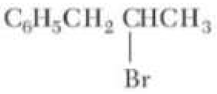
Question 49

The reaction of $C_6H_5CH = CHCH_3$ with HBr produces

Options:

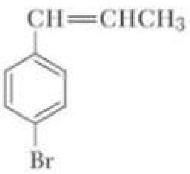
A.

B.



C. $C_6H_5CH_2CH_2Br$

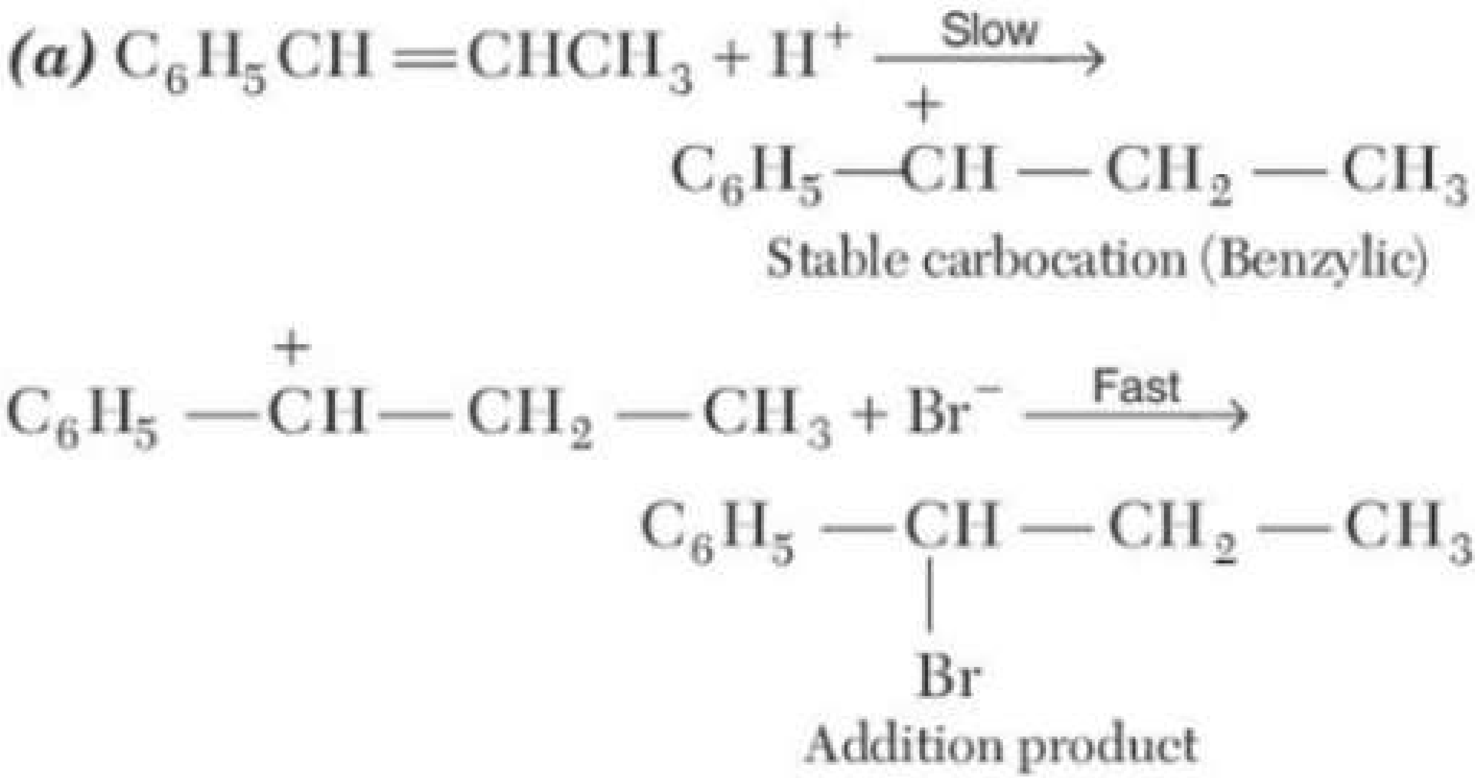
D.



Answer: A

Solution:

Solution:



Electrophile addition reaction takes place via more stable carbocation.

Question 50

The number of $3C - 2e^{-}$ bonds present in diborane is

Options:

A. 1

B. 2

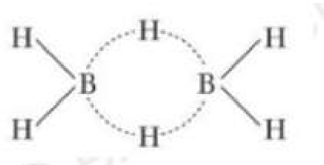
C. 3

D. 4

Answer: C

Solution:

Solution:
The structure of diborane is



In this structure, there are two $3C - 2e^{-}$bonds. 3 atoms, $B - H\ - B$ share two electrons and form an angular geometry, leading bent bond, also known as banana bond.

Question 51

Standard entropy of X_2, Y_2 and XY_2 are 60,40 and $50JK^{-1}mol^{-1}$, respectively. For the reaction, $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightarrow XY_3$, $\Delta H\ = -30\,kJ$, to be at equilibrium, the temperature will be

Options:

A. 1250K

B. 500K

C. 750K

D. 1000K

Answer: C

Solution:

Solution:
$\Delta S = S(XY_3) - \frac{1}{2}S(X_2) - \frac{3}{2}S(Y_2)$
$= 50 - 30 - 60 = -40J\,mol^{-1}K^{-1}$
$\Delta H\ = -30kJ\ = -3000J$
$\Delta G = \Delta H\ - T\,\Delta S$
At equilibrium,
$\Delta G = 0$
$T\ = \frac{\Delta H}{\Delta S} = \frac{-3000}{-40} = 750K$

Question 52

The total number of $P - OH$ bonds for pyrophosphoric acid

Options:

A. 4

B. 5

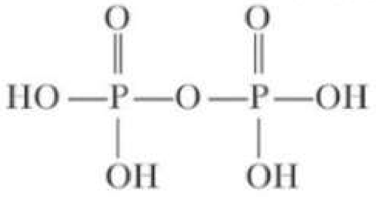
C. 6

D. 8

Answer: A

Solution:

Solution:
The structure of pyrophosphoric acid is



There are $P-OH$ bonds, $1P-O-P$ bond and two $P=O$ bonds.

Question 53

Using the standard electrode potential, find out the pair between which redox reaction is not feasible. $E^{\ominus}$ values

$Fe^{3+}/Fe^{2+} = +0.77; I_2/I^- = +0.54$

$Cu^{2+}/Cu = +0.34; Ag^+/Ag = 0.80V$

Options:

A. Fe^{3+} and I^-

B. $Ag^+$and Cu

C. Fe^{3+} and Cu

D. Ag and Fe^{3+}

Answer: D

Solution:

Solution:
(a) $2Fe^{3+} + 2e^- \rightarrow 2Fe^{2+}; E^\circ = 0.77V$
$2I^- \rightarrow I_2 + 2e^-; E^\circ = -0.54V$ (sign of E° is reversed)

$2Fe^{3+} + 2I^- \rightarrow 2Fe^{2+} + I_2; E^\circ_{cel} = +0.23V$

This reaction is feasible, since E°_{cell} is positive.
(b) $Cu \rightarrow Cu^{2+} + 2e^-; E^\circ = -0.34V$ (sign of E° gas been reversed)
$2Ag^+ + 2e^- \rightarrow 2Ag; E^\circ = +0.80V$

$Cu + 2AG^+ \rightarrow Cu^{2+} + 2Ag; E^\circ = +0.46V$

This reaction is feasible, since $E°_{cell}$ is positive.

(c) $2Fe^{3+} + 2e^{-} \rightarrow 2Fe^{2+}; E° = +0.77V$

$Cu \rightarrow Cu^{2+} + 2e^{-}; E° = -34V$ (sign of $E°$ is reversed)

$2Fe^{3+} + Cu \rightarrow Fe^{2+} + Cu^{2+}; E° = +0.43V$

This reaction is feasible, since $E°_{cell}$ is positive.

(d) $Ag \rightarrow Ag^{+} + e^{-}; E° = -0.80V$ (sign of $E°$ is reversed)

$Fe^{3+} + e^{-} \rightarrow Fe^{2+}; E° = +0.77V$

$Ag + Fe^{3+} \rightarrow Ag^{+} + Fe^{2+}; E° = -0.03V$

This reaction is not feasible, since $E°_{cell}$ is negative.

Question 54

What is $[NH_4^+]$in a solution that is $0.02 M NH_3$ $0.01 M KOH$? $[K_b(NH_3) = 1.8 \times 10^{-5}]$

Options:

A. $3.6 \times 10^{-5} M$

B. $1.8 \times 10^{-5} M$

C. $0.9 \times 10^{-5} M$

D. $7.2 \times 10^{-5} M$

Answer: A

Solution:

Solution:

$NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$K_b = \frac{[NH_4^+][OH^-]}{[NH_4OH]}$

$1.8 \times 10^{-5} = \frac{[NH_4^+][0.01]}{[0.02]}$

$\therefore [NH_4^+] = 3.6 \times 10^{-5} M$

Question 55

For an isomerisation reaction $A \rightleftharpoons B$, the temperature dependence of equilibrium constant is given by $\log_e K = 4.0 - \frac{2000}{T}$ The value of ΔS° at Hook is, therefore

Options:

A. 4R

B. 5R

C. 400R

D. 2000R

Answer: A

Solution:

Solution:
van't Hoff equation giving variation of equilibeium constant, K with temperature T is
$\log_e K = -\frac{\Delta H°}{RT} + \frac{\Delta S°}{R}$
Given, $\log_e K = -\frac{2000}{T} + 4.0$
$\therefore \frac{\Delta S°}{R} = 4$
$\Rightarrow \Delta S° = 4R$

Question 56

In an adiabatic process, no transfer of heat takes place between system and surrounding. Choose the correct option for free expansion of an ideal gas under adiabatic condition from the following.

Options:

A. $q = 0, \Delta T \neq 0, W = 0$

B. $q \neq 0, \Delta T = 0, W = 0$

C. $q = 0, \Delta T = 0, W = 0$

D. $q = 0, \Delta T < 0, W \neq 0$

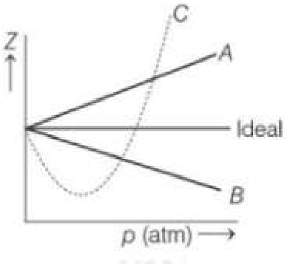
Answer: C

Solution:

Solution:
Free expansion, $W = 0$
Adiabatic process, $q = 0$
$\Delta U = q + W = 0$, this means that internal energy remains constant. Therefore, $\Delta T = 0$ in ideal gas there is no intermolecular attraction.
Hence, when such a gas expands under adiabatic conditions into a vacuum, no heat is absorbed or evolved, since no external work is done to separate the molecules.

Question 57

The given graph represents the variation of compressibility factor $(Z) = \frac{pV}{nRT}$, for three real gases A, B and C. Identify the only incorrect statement.



Options:

A. For the gas A, a = 0 and its dependence on p is linear at all pressure.

B. For the gas B, b = 0 and its dependence on p is linear at all pressure.

C. For the gas C, which is typical real gas for which neither. a nor b = 0. By knowing the minima and point of the intersection, with $Z = 1$, a and b can be calculated.

D. At high pressure the slope is positive for all real gases.

Answer: B

Solution:

Solution:
From the graph it is clear that, the value of ' Z ' decreases with increase of pressure. We can explain as follows on the basis of van der Waals' equation.
At high pressure, when ' p ' is large, V will be small and one cannot ignore ' b ' in comparison to V. However, the team $\frac{a}{V^2}$ may be considered negligible in comparison to ' p ' in van der Waals' equation.

$$\left(p + \frac{a}{V^2}\right)(V - b) = nRT$$

$$p(V - b) = nRT$$

$$\Rightarrow pV - pb = nRT$$

or $p\frac{V}{n}RT = 1 + \frac{pb}{nRT}$

or $Z = 1 + \frac{pb}{nRT}$

Thus, Z is greater than 1. As pressure is increased (at constant T), the factor $\frac{pb}{nRT}$ increases. This explains why after minima in the curves, Z increase continuously with pressure. Hence, the only incorrect statement is (b).

Question 58

Which one of the following statements in relation to the hydrogen atom is correct?

Options:

A. 3s, 3p and 3d -orbitals all have the same energy.

B. 3s and 3p-orbitals are of lower energy than 3d -orbital.

C. 3p-orbital is lower in energy than 3d -orbital.

D. 3s-orbital is lower in energy than 3p-orbital.

Answer: A

Solution:

Solution:
The hydrogen atom has the configuration 1s and 3s, 3p and 3d -orbitals will have same energy with respect to 1s-orbital.

Question 59

In the molecules CH_4, NF_3, $NH_4^+$and H_2O

Options:

A. number of lone pairs are same

B. all have same hybridisation of centre of atom

C. the bond angles are same

D. number of bond pairs are same

Answer: B

Solution:

Solution:
CH_4; bp = 4; sp^3- hybridisation
NF_3; bp = 3; lp = 1; sp^3- hybridisation
NH_4^+; bp = 4; lp = 0; sp^3- hybridisation
H_2O; bp = 2; lp = 2; sp^3- hybridisation
Hence, all have the same slope.

Question 60

0.20g of an organic compound gave 0.12g of AgBr By using Carius method, the percentage of bromine in the compound will be

Options:

A. 34.06%

B. 44.04%

C. 54%

D. 25%

Answer: D

Solution:

Solution:
Given,
Mass of an organic compound = 0.20g
Mass of AgBr = 0.12g
Molecular mass of AgBr = $188 g mol^{-1}$
188g of AgBr contains 80g of bromine

$\therefore 0.12g$ of AgBr will contain $= \frac{80}{188} \times 0.12$

$= 0.05g$ of bromine

$\therefore$ Percentage of bromine $= \frac{0.05}{0.20} \times 100$

$= 25\%$

Question 61

Forthrightness in speech may not always be a desirable quality.

Options:

A. Outspokenness

B. Obliqueness

C. Mendacity

D. Equivocation

Answer: A

Solution:

Solution:
Forthrightness and 'outspokenness' both have a same meaning, i.e. straight forward.

Question 62

The inexorable demands of the workers brought the company to a closure.

Options:

A. Unreasonable

B. Relentless

C. Monetary

D. Violent

Answer: B

Solution:

Solution:
'Inexorable' means 'relentless', i.e. without any particular end.

Question 63

Select the one which best expresses the same sentence in Passive/Active voice.
Then her face was bowed.

Options:

A. Then she was being bowed her face.

B. Her face was bowed by them.

C. Then she bowed her face.

D. Then her face has been bowed.

Answer: C

Solution:

Solution:
'Then she bowed her face', is the correct passive form of the given sentence.

Question 64

The complex form of the sentence given below would be
Spare the rod and spoil the child.

Options:

A. The child is spoiled if the rod is spared.

B. The child becomes spoiled when the rod is spared.

C. The child is spoiled whenever the rod is spared.

D. The child is spoiled when the rod is spared.

Answer: D

Solution:

Solution:
The complex form of the sentence would be the child is spoiled when the rod is spared'.

Question 65

The attack on the freedom of the press is a retrograde step.

Options:

A. progressive

B. stubborn

C. punitive

D. aggressive

Answer: A

Solution:

Solution:
Retrograde means declining. Hence, 'progressive' is its correct antonym.

Question 66

The leader might have had some covert reason for the change of his political affiliations.

Options:

A. Unjustifiable

B. Obvious

C. Inexplicable

D. Flimsy

Answer: B

Solution:

Solution:
'Covert' means hidden. So, 'obvious' would be its correct antonym.

Question 67

Regard for others as a principle of action or selflessly.

Options:

A. Gynicism

B. Nepotism

C. Philanthropy

D. Altruism

Answer: D

Solution:

Solution:
Disinterested and selfless concern for the well-being of other is called Altruism.

Question 68

Code of diplomatic etiquette and precedence is

Options:

A. Formalism

B. Statesmanship

C. Protocol

D. Hierarchy

Answer: C

Solution:

Solution:
Code of diplomatic etiquette and precedence is called protocol.

Question 69

(A) Now under liberated economy they are learning to compete domestically and globally.
(B) In India corporations until recently achieved success by avoiding competition, using protected and regulated domestic markets.
(C) The trend is irreversible.
(D) Business leaders are preparing themselves to meet competitive challenges, and to avoid being swept away.

Options:

A. BADC

B. BDCA

C. BDAC

D. CDBA

Answer: A

Solution:

Solution:
BADC is the correct sequence to from a logical paragraph.

Question 70

(A) Recovery was given inadequate attention and consequently some bank branches regularly incurred heavy losses and their parent bodies

had to bale them out.
(B) As a result, banks indulged in extensive lending to borrowers who had little or no potential to make repayments.
(C) To fulfil the social objectives laid down by the masters of nationalisation, banks were asked to lend to identified priority sectors.
(D) 1992-93 results showed that the loss making branches of public sector banks increased from 10,000 to 13,000 and the quantum of losses showed at ₹3, 369 crores.

Options:

A. BACD

B. DABC

C. CBAD

D. BCAD

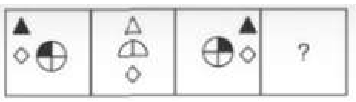
Answer: C

Solution:

Solution:
CBAD is the correct sequence to form a meaningful and logical paragraph.

Question 71

Select the figure that can replace the question mark (?) in the following series.

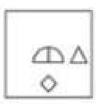


Options:

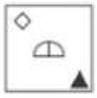
A.

B.

C.



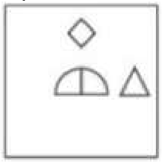
D.



Answer: A

Solution:

Solution:
In the given series triangle is moving $\frac{1}{2}$ place in clockwise direction along the sides and corners of square and also unshaded and shaded alternatively. The rhombus is moving one place in anti-clockwise direction along the sides of square. Hence, we will get the following figure.



Question 72

'A+B' means 'A is the mother of B'.
'A-B' means 'A is the brother of B'.
'A $\times$B ' means ' A is the father of B '.
'A $\div$B ' means ' A is the daughter of B '.
If, $P - K \times Y - J \div S + R$, then which of the following statement is not correct?

Options:

A. K is husband of S

B. Y is son of S

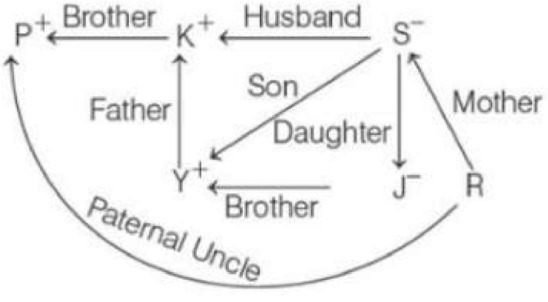
C. J is daughter of P

D. P is paternal uncle of R

Answer: C

Solution:

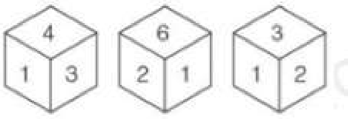
Solution:
Let us draw the family diagram.
$P - K \times Y - J \div S + R \Rightarrow$



Clearly, option (c) is wring because J is the daughter of K and S.

Question 73

Three different positions of the same dice are shown, the six faces of which are numbered from 1 to 6 . Select the number that will be on the face opposite to the one showing ' 6 '.



Options:

A. 2

B. 4

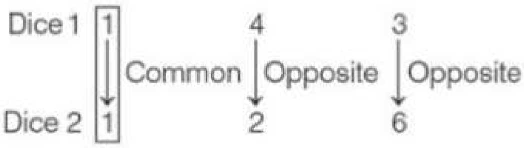
C. 5

D. 3

Answer: D

Solution:

Solution:
Inumber is common in first two position of dices. Now, start with 1 number, writing all the remaining numbers in clockwise direction.



Hence, 3 number will be on the face opposite to the one showing 6.

Question 74

Select the option in which the given figure X is embedded (rotation is not allowed).

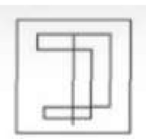
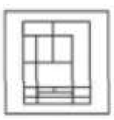


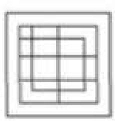
Figure X

Options:

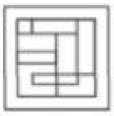
A.



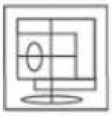
B.



C.



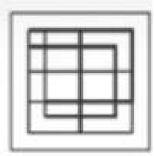
D.



Answer: B

Solution:

Solution:
The question figure is embedded in answer figure (b).



Question 75

Select the letter-cluster that can replace the question mark(?) in the following series. TULG, WRPC, ZOTY, CLXU,?

Options:

A. FIBQ

B. FICR

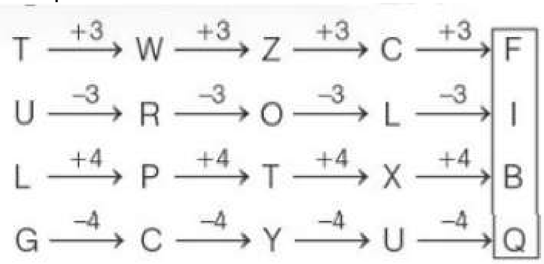
C. FJCQ

D. GIAQ

Answer: A

Solution:

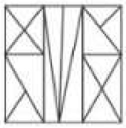
Solution:
The pattern of the series is as follows



∴? = F I BQ

Question 76

How many triangles are there in the given figure?



Options:

A. 33

B. 18

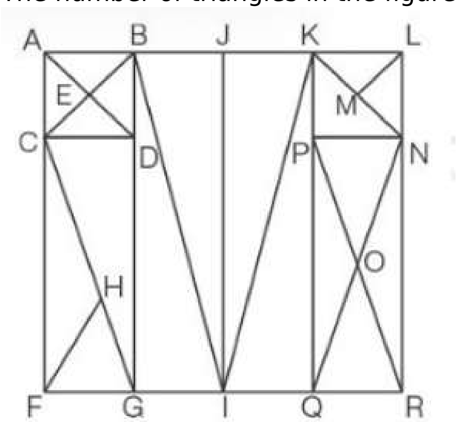
C. 31

D. 29

Answer: C

Solution:

Solution:
The number of triangles in the figure are :



ΔABE , ΔAE C, ΔCE D, ΔBE D, ΔADC, ΔCDB, ΔABC, ΔABD, ΔPON , ΔQOR, ΔPOQ, ΔN OR, ΔPN Q, ΔPQR, ΔN QR, ΔN PR, ΔCF G, ΔCDG, ΔCH F , ΔH F G, ΔK PN , ΔK LN , ΔK LM , ΔLM N , ΔBGI , ΔBI J , ΔJ I K , ΔBI K , ΔK I Q, ΔK N Q and Δ B C G = 31

Question 77

The average marks of 50 students in a class was found to be 64 . If the marks of two students were incorrectly entered as 38 and 42 instead of 83 and 24 , respectively, then what is the correct average?

Options:

A. 64.54

B. 62.32

C. 61.24

D. 61.86

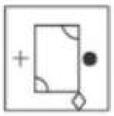
Answer: A

Solution:

Solution:
Average marks of 50 students $= 64$
Sum of marks of 50 students
= Average × number of students
$= 64 \times 50 = 3200$
According to the question,
Correct sum of marks of 50 students
$= 3200 - 38 - 42 + 83 + 24$
$= 3200 + 27$
$= 3227$
∴ Correct average marks of 50 students
$= \frac{3227}{50} = 64.54$

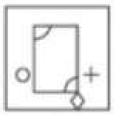
Question 78

Select the correct mirror image of the given figure when the mirror is placed on the right of the figure.

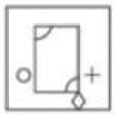


Options:

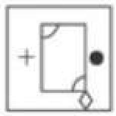
A.



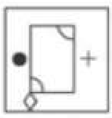
B.



C.



D.



Answer: A

Solution:

Solution:

When a mirror is placed on the right side of the image,
The facing of the figure to the left and right direction changes to the opposite direction in the mirror image. There will be change in top and bottom direction. The mirror image of the given figure is

----

Question 79

Six friends A, B, C, D, E and F are sitting around a round table facing the centre. A sits second to the right of B, E sits second to the left of C. B doesn't sit adjacent to E. D does not sit opposite to E or C. Who sits to the immediate left of E?

Options:

A. A

B. D

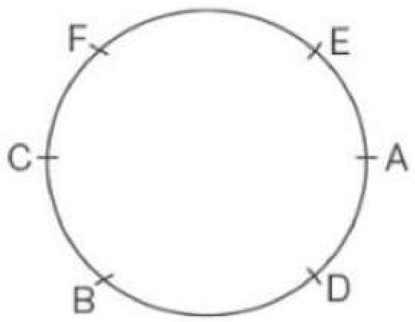
C. B

D. C

Answer: A

Solution:

Solution:
According to the question,



Hence, A sits immediate left of E .

Question 80

Five friends A, B, C, D and E bought cars which were priced differently. B's car was costlier than C's car but was less costly than E's car. A's car was costlier than D's car but less costly than C's car. Whose car was the 2 nd costliest?

Options:

A. E

B. A

C. B

D. C

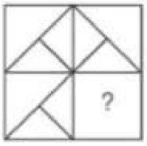
Answer: C

Solution:

Solution:
According to the question,
E > $\boxed{B}$ > C > A > D
Hence, B's car was the 2nd costliest.

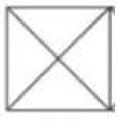
Question 81

In the following question, complete the missing segment by selecting the appropriate figure from the given alternatives, (a), (b), (c) and (d).

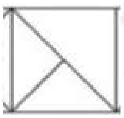


Options:

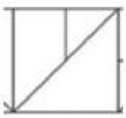
A.



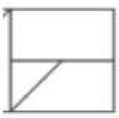
B.



C.



D.



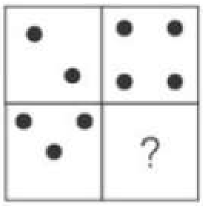
Answer: B

Solution:

Solution:
If figure shown in option (b), is placed in the place of missing portion of the original figure, then it is completed as shown below
E > $\boxed{B}$ > C > A > D

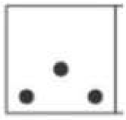
Question 82

In each of the following question, find out which of the answer figures (a), (b), (c) and (d) completes the figure matrix?

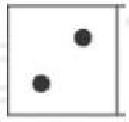


Options:

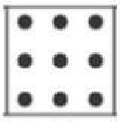
A.



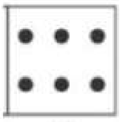
B.



C.



D.



Answer: D

Solution:

Solution:
Clearly, in the first row, the number of dots in the second figure is twice the number of dots in the first figure. Similarly, in the second row, the number of dots in the second figure must be twice the number of dots in the first figure.
So, number of dots in missing segment $= 2 \times 3 = 6$

Question 83

Statements 60% of government employees went on strike.
Mr. Gopal is a government employee.
Conclusions
I. Mr. Gopal went on strike.
II. Mr. Gopal did not participate in the strike.

Options:

A. Only conclusion I follows

B. Only conclusion II follows

C. Both conclusions i and II follows

D. Either conclusion I or II follows

Answer: D

Solution:

Solution:
Either of the situation is possible. If Mr. Gopal was one of the member of 60^{wedge} employees, then he went on strike. If he was not in group of 60%, then he did not participate in the strike. Hence, either conclusion I or II follows.

Question 84

Statements
Lawyers marry only fair girls.
Shobha is very fair.
Conclusions
I. Shobha is married to a lawyer.
II. Shobha is not married to a lawyer.

Options:

A. Only conclusion I follows.

B. Only conclusion II follows.

C. Both conclusion I and II follows

D. Either conclusion I or II follows

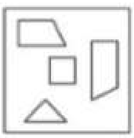
Answer: D

Solution:

Solution:
The statement I is talking about in a condition with the lawyer that they marry only fair girls. But it is not talking about any condition with Shobha. So, Shobha can marry either a lawyer or anyone else.

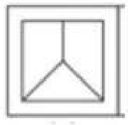
Question 85

In the question given below, find out which of the figures can be formed from the pieces given in the problem figure.

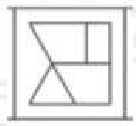


Options:

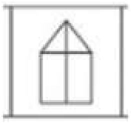
A.



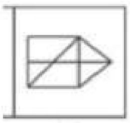
B.



C.



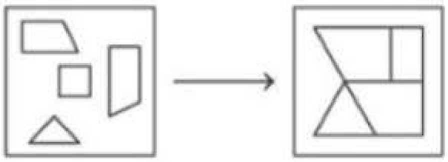
D.



Answer: B

Solution:

Solution:
Figure given in option (b) can be formed by joining the pieces given in question figure, as shown below.



Question 86

Select the option in which the words share the same relationship as that shared by the given pair of words.
Barometer : Pressure

Options:

A. Ammeter : Current

B. Thermometer : Volume

C. Voltmeter : Heat

D. Scale : Seconds

Answer: A

Solution:

Solution:
As, pressure measure in Borometer. Similarly, current measure in Ammeter.

Question 87

Select the option in which the words share the same relationship as that shared by the given set of words.
Cat : Lion : Jaguar

Options:

A. Shark : Dolphin : Bat

B. Sports : Athlete : Javelin

C. Monkey : Chimpenzee : Gorilla

D. Reptile : Snake : Toad

Answer: C

Solution:

Solution:
As Cat, Lion and Jaguar belongs to the same species called Cat. Similarly, Monkey, Gorilla and Chimpanzee belong to the same species of animal called Apes.

Question 88

'Needle' is related to 'Sew' in the same way as 'Microsope' is related to '...........'.

Options:

A. Laboratory

B. Lens

C. Science

D. Magnify

Answer: D

Solution:

Solution:
As, 'Needle' is used for 'Sewing'. Similarly,'Microscope' is used for 'Magnifying'.

Question 89

Select the option that is related to the fifth number in the same way as the second number is related to the first number and the fourth number is related to the third number.
14 : 289: : 17 : 400: : 21 :   ?

Options:

A. 576

B. 504

C. 570

D. 441

Answer: A

Solution:

Solution:
As, 14 : 289 ⇒ 14 + 3 = 17
⇒17^2 = 289
And 17 : 400 ⇒ 17 + 3 = 20
⇒20^2 = 400
Similarly, 21 : ? ⇒ 21 + 3 = 24
⇒24^2 = 576

Question 90

Select the letter-cluster that can replace the question mark (?) in the following series.
TXB, QWE, NVH, KUK, ?

Options:

A. ITM

B. JTM

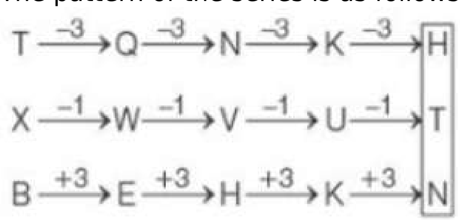
C. HTN

D. HSN

Answer: C

Solution:

Solution:
The pattern of the series is as follows,



∴? = H T M

Question 91

If α be a root of the equation $4x^2 + 2x - 1 = 0$, then the other root of the equation is

Options:

A. $4\alpha^3 + 2\alpha$

B. $4\alpha^2 - 2\alpha$

C. $4\alpha^3 - 3\alpha$

D. $4\alpha^3 + 3\alpha$

Answer: C

Solution:

Solution:

$4x^2 + 2x - 1 = 0$

$\therefore \alpha + \beta = -\frac{1}{2}, \alpha\beta = -\frac{1}{4} \ldots\ldots (i)$

Also, $4\alpha^2 + 2\alpha - 1 = 0$ as α is a root and we have to prove that $\beta = 4\alpha^3 - 3\alpha$

$= \alpha(1 - 2\alpha) - 2\alpha = -2\alpha^2 - 2\alpha$

$= -\frac{1}{2}[4\alpha^2 + 4\alpha] = -\frac{1}{2}[1 - 2\alpha + 4\alpha]$

$= -\frac{1}{2}(1 + 2\alpha) = -\frac{1}{2} - \alpha = \beta$

Now, $\alpha + \beta = -\frac{1}{2}$ [from Eq. (i)]

Hence, $\beta = 4\alpha^3 - 3\alpha$.

Question 92

If A = { x : x is a multiple of 4} and, B = { x : x is a multiple of 6}, then A ∩ B consists of multiples of

Options:

A. 16

B. 12

C. 8

D. 4

Answer: B

Question 93

If |w| = 2, then the set of points z = w − $\frac{1}{w}$ is contained in or equal to the set of points z satisfying

Options:

A. Im(z) = 0

B. |Im(z)| ≤ 1

C. |Re(z)| ≤ 2

D. |z| ≤ 3

Answer: D

Solution:

Solution:

We have, $|w| = 2 \Rightarrow w = 2(cos\theta + isin\theta)$

$\therefore z = w - \frac{1}{w}$

$= 2(cos\theta + isin\theta) - \frac{1}{2}(cos\theta - isin\theta)$

$\Rightarrow z = \frac{3}{2}cos\theta + \frac{5}{2}isin\theta$

$\Rightarrow |z| = \sqrt{\frac{9}{4} + \frac{25}{4}} = \frac{\sqrt{17}}{2} < 3$

Hence, option (d) is correct.

Question 94

The value of $\lim_{x \to 0} \frac{1 - \cos(1 - \cos x)}{x^4}$ is

Options:

A. $\frac{1}{6}$

B. $\frac{1}{8}$

C. $\frac{1}{10}$

D. $\frac{1}{12}$

Answer: B

Solution:

Solution:

$$1 - cos(1 - cosx) = 2sin^2\left(\frac{1 - cosx}{2}\right)$$

$$= 2sin^2\left(sin^2\frac{x}{2}\right)$$

$$\therefore \lim_{x \to 0} \frac{1 - cos(1 - cosx)}{x^4} = \lim_{x \to 0} \frac{2sin^2\left(sin^2\frac{x}{2}\right)}{x^4}$$

$$= \lim_{x \to 0} \frac{2sin^2\left(sin^2\frac{x}{2}\right)}{\left(sin^2\frac{x}{2}\right)^2} \times \frac{sin^4\frac{x}{2}}{\left(\frac{x}{2}\right)^4 \times 16} \left[\because \lim_{x \to 0} \frac{sinx}{x} = 1\right]$$

$$= \frac{1}{8}$$

Question 95

Let $a_1, a_2, \ldots . a_{40}$ be in AP and $h_1, h_2, \ldots . h_{10}$ be in HP. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then a_4h_7 is

Options:

A. 2

B. 3

C. 5

D. 6

Answer: D

Solution:

Solution:

Let d be the common difference of the AP. Then,

$a_{10} = 3 \Rightarrow a_1 + 9d = 3$

$\Rightarrow 2 + 9d = 3 \Rightarrow d = \frac{1}{9}$

$\therefore a_4 = a_1 + 3d = 2 + \frac{1}{3} = \frac{7}{3}$

Let D be the common difference of $\frac{1}{h_1}, \frac{1}{h_2}, \ldots.. - \frac{1}{h_{10}}$.

Then, $h_{10} = 3$

$\Rightarrow \frac{1}{h_{10}} = \frac{1}{3} \Rightarrow \frac{1}{2} + 9D = \frac{1}{3}$

$\Rightarrow 9D = -\frac{1}{6} \Rightarrow D = -\frac{1}{54}$

$\therefore \frac{1}{h_7} = \frac{1}{h_1} + 6D = \frac{1}{2} - \frac{1}{9} = \frac{7}{18}$

$\Rightarrow h_7 = \frac{18}{7}$

$\therefore a_4 h_7 = \frac{7}{3} \times \frac{18}{7} = 6$

Question 96

The number of terms in the expansion of $(1 + 5\sqrt{2}x)^9 + (1 - 5\sqrt{2}x)^9$

Options:

A. 5

B. 7

C. 9

D. 10

Answer: A

Solution:

Solution:

We have,

$(1 + 5\sqrt{2}x)^9 + (1 - 5\sqrt{2}x)^9$

$= 2\{^2C_0 + {}^9C_2(5\sqrt{2}x)^2 + \ldots. + {}^9C_8(5\sqrt{2}x)^8\}$

Clearly, it has 5 terms.

Question 97

The number of different seven-digit numbers that can be written using

only the three digit 1 , 2 and 3 with the condition that the digit 2 occurs twice in each number is

Options:

A. ${}^7C_2 2^5$

B. ${}^7p_2 2^5$

C. ${}^7C_2 5^2$

D. None of these

Answer: A

Solution:

Solution:

Others than 2, remaining five places can be filled by 1 and 3 for each place. The number of ways for five places is $2 \times 2 \times 2 \times 2 \times 2 = 2^5$. For 2, selecting 2 places out of 7 is ${}^7C_2$. Hence, the required number of ways is ${}^7C_2 \times 2^5$.

Question 98

Given $2x - y + 2z = 2$, $x - 2y + z = -4$, $x + y + \lambda z = 4$, then the value of λ such that the given system of equation has no solution is

Options:

A. -3

B. 1

C. 0

D. 3

Answer: D

Solution:

Solution:

Since, the system has no solution.

$$\begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & -1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow 2(-2\lambda + 1) + 1(\lambda + 1) + 2(3) = 0$

$\Rightarrow -4\lambda + 2 + \lambda + 1 + 6 = 0$

$\Rightarrow 3\lambda = 9$

$\Rightarrow \lambda = 3$

Question 99

Let A = $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and 10B = $\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$ If B is the inverse of A, then the value of α is

Options:

A. 4

B. −4

C. 3

D. 5

Answer: D

Solution:

Solution:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1 = 10$$

$$\Rightarrow AdjA = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow B = A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \left[\because A^{-1} = \frac{1}{|A|}AdjA\right]$$

$$\Rightarrow 10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Hence, $\alpha = 5$.

Question 100

If $x \in \left(0,\ \frac{\pi}{2}\right)$, then the value of

$\cos^{-1}\left(\ \frac{7}{2}(1 + \cos 2x) + \sqrt{(\sin^2 x - 48\cos^2 x)\sin x}\ \right)$

is equal to

Options:

A. $x - \cos^{-1}(7\cos x)$

B. $x + \sin^{-1}(7\cos x)$

C. $x + \cos^{-1}(6\cos x)$

D. $x + \cos^{-1}(7\cos x)$

Answer: A

Solution:

Solution:

$$y = cos^{-1}\left(\frac{7}{2}(1 + cos2x) + \sqrt{(sin^2x - 48cos^2x)sinx}\right)$$

$$= cos^{-1}\left((7cosx)(cosx) + \sqrt{1 - 49cos^2x}\ \sqrt{1 - cos^2x}\right)$$

$$= cos^{-1}(cosx) - cos^{-1}(7cosx)[\because cosx < 7cosx]$$

$$= x - cos^{-1}(7cosx)$$

Question 101

A running track of 440 ft is to be laid out enclosing a football field, the shape of which is a rectangle with a semi-circle at each end. If the area of the rectangular portion is to be maximum, then the lengths of its side are

Options:

A. 70 ft and 110 ft

B. 80 ft and 120 ft

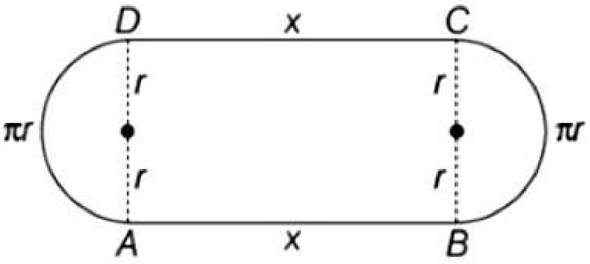
C. 35 ft and 110 ft

D. 35 ft and 120 ft

Answer: C

Solution:

Solution:



Perimeter $= 440$ ft

$\Rightarrow 2x + \pi r + \pi r = 440$

$2x + 2\pi r = 440$

$A =$ Area of the rectangular portion $= x2r$

$A = x\dfrac{(440-2x)}{\pi} = \dfrac{1}{\pi}(440x - 2x^2)$

Let $\dfrac{dA}{dx} = \dfrac{1}{\pi}(440 - 4x) = 0$

$\Rightarrow x = 110$ for which $\dfrac{d^2A}{dx^2} < 0$

$\Rightarrow A$ is maximum when $x = 110$

$\Rightarrow 2r = \dfrac{440-2x}{\pi} = \dfrac{440-220}{\frac{22}{7}} = 70$

$\therefore r = 35$ ft and $x = 110$ ft

Question 102

$\left(\frac{dy}{dx}\right)\tan x = y\sec^2 x + \sin x$, find general solution

Options:

A. $y = \tan x(\log|\operatorname{cosec} x - \cot x| + \cos x + c)$

B. $y = \sec^2 x + \tan x + c$

C. $y = \log|\sec x + \tan x| + \operatorname{cosec} x + c$

D. $y = \tan^2 x + \sin x + c$

Answer: A

Solution:

Solution:

Given that

$$\left(\frac{dy}{dx}\right)\tan x = y\sec^2 x + \sin x$$

$$\tan x\left(\frac{dy}{dx}\right) - y\sec^2 x = \sin x$$

$$\frac{dy}{dx} + \left(\frac{-\sec^2 x}{\tan x}\right)y = \frac{\sin x}{\tan x}$$

Here, $P = \frac{\sec^2 x}{\tan x}$ and $Q = \cos x$

$$IF = e^{\int P dx} = e^{\int \frac{-\sec^2 x}{\tan x} dx}$$

$$IF = e^{-\log(\tan x)} = e^{\log \cot x} = \cot x$$

Now, $y \times IF = \int (Q \times IF) dx + C$

$$y \, . \cot x = \int \cos x \, . \cot x + C$$

$$y \, . \cot x = \int \frac{1 - \sin^2 x}{\sin x} + C$$

$$y \, . \cot x = \log|\operatorname{cosec} x - \cot x| + \cos x + C$$

Question 103

If the straight line y = mx + c touches the parabola $y^2 - 4ax + 4a^3 = 0$, then c is

Options:

A. am + $\frac{a}{m}$

B. am − $\frac{a}{m}$

C. $\frac{a}{m} + a^2m$

D. $\frac{a}{m} - a^2m$

Answer: D

Solution:

Solution:

Solving the given equations,

$(mx+c)^2 = 4ax - a^3$

$\Rightarrow m^2x^2 + 2mc\,.\,x + c^2 = m4ax - 4a^3$

$\Rightarrow m^2x^2 + (2mc - 4a)x + c^2 + 4a^3 = 0$

Since the straight line touches the parabola at a point,

so, the discriminant $= 0$

$\Rightarrow (2mc - 4a)^2 - 4m^2(c^2 + 4a^3) = 0$

$\Rightarrow 4m^2c^2 - 16amc + 16a^2 - 4m^2c^2 - 16a^3m^2 = 0$

$\Rightarrow -mc + a - a^2m^2 = 0$

$\Rightarrow mc = a - a^2m^2 \Rightarrow c = \frac{a}{m} - a^2m$

Question 104

A normal is drawn at the point P to the parabola $y^2 = 8x$, which is inclined at 60° with the straight line y = 8. Then the point P lies on the straight line

Options:

A. $2x + y - 12 - 4\sqrt{3} = 0$

B. $2x - y - 12 + 4\sqrt{3} = 0$

C. $2x - y - 12 - 4\sqrt{3} = 0$

D. None of these

Answer: C

Solution:

Solution:

For the parabola $y^2 = 4ax$, the equation of normal at $P(am^2, -2am)$ is $y = mx - 2am - am^3$.

Here, $m = tan 60^\circ = \sqrt{3}$

$\therefore P \equiv (a(\sqrt{3})^2, -2a(\sqrt{3})) \equiv (6, -4\sqrt{3}) (\because a = 2)$

Thus, P satisfies $2x - y - 12 - 4\sqrt{3} = 0$

Question 105

The value of $\int \frac{1}{x1}$ dx, is $[(x-1)^3(x+2)^5]^{\frac{1}{4}}$

Options:

A. $\frac{4}{3}\left(\frac{x+1}{x-2}\right)^{\frac{1}{4}}+C$

B. $\frac{3}{4}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

C. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

D. $\frac{1}{3}\left(\frac{2x-1}{4x-3}\right)^{\frac{1}{4}}+C$

Answer: C

Solution:

Solution:

$$I=\int \frac{1}{\left(\frac{x-1}{x+2}\right)^{\frac{3}{4}}(x+2)^2}dx$$

$$\text{Let } \frac{x-1}{x+2}=t \Rightarrow \frac{3dx}{(x+2)^2}=dt$$

$$\Rightarrow I=\frac{1}{3}\int \frac{1}{t^{\frac{3}{4}}}dt=\frac{1}{3}\left(\frac{t^{\frac{1}{4}}}{\frac{1}{4}}\right)+C$$

$$=\frac{4t^{\frac{1}{4}}}{3}+C=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$$

Question 106

The area of the region bounded by the parabola $(y-2)^2=(x-1)$, the tangent to the parabola at the point (2, 3) and the X -axis is

Options:

A. 3

B. 6

C. 9

D. 12

Answer: C

Solution:

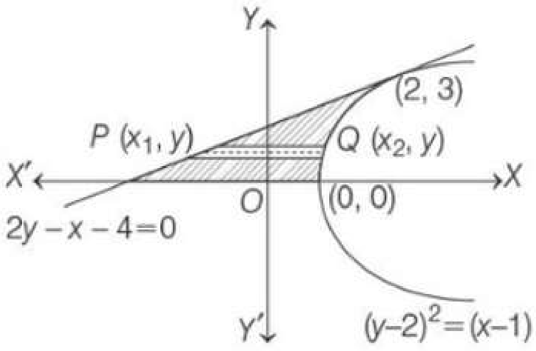
Solution:

The equation of the parabola is $y^2 - 4y - x + 5 = 0$

The equation of tangent at $(2, 3)$ is

$3y - 2(y+3) - \frac{x+2}{2} + 5 = 0$

$2y - x - 4 = 0$



∴ Required area A is given by

$A = \int_0^3 (x_2 - x_1) dy$

$\Rightarrow A = \int_0^3 [\{(y-2)^2 + 1\} - \{2y - 4\}] dy$

$\Rightarrow A = \int_0^3 (y^2 - 6y + 9) dy = \int_0^3 (3-y)^2 dy$

$= -\left[\frac{(3-y)^3}{3}\right]_0^3 = 9$

Question 107

$\hat{u}$ and $\hat{v}$ are two non-collinear unit vectors such that $\left|\frac{\hat{u}+\hat{v}}{2} + \hat{u} \times \hat{v}\right| = 1$. Then the value of $|\hat{u} \times \hat{v}|$ is equal to

Options:

A. $\left|\frac{\hat{u}+\hat{v}}{2}\right|$

B. $|\hat{u} + \hat{v}|$

C. $|\hat{u} - \hat{v}|$

D. $\left|\frac{\hat{u}-\hat{v}}{2}\right|$

Answer: D

Solution:

Solution:

Given that, $\left|\frac{\hat{u}+\hat{v}}{2}+\hat{u}\times\hat{v}\right|=1$

$\Rightarrow\left|\frac{\hat{u}+\hat{v}}{2}+\hat{u}\times\hat{v}\right|^2=1$

$\Rightarrow\frac{2+2cos\theta}{4}+sin^2\theta=1\left[\because\hat{u}.\left(\hat{u}\times\hat{v}\right)=\hat{v}\left(\hat{u}\times\hat{v}\right)=0\right]$

$\Rightarrow cos^2\frac{\theta}{2}=cos2\theta$

$\Rightarrow\theta=n\pi\pm\frac{\theta}{2}, n\in z$

$\Rightarrow\theta=\frac{2\pi}{3}$

$\Rightarrow\left|\hat{u}\times\hat{v}\right|=sin\frac{2\pi}{3}=sin\frac{\pi}{3}=\left|\frac{\hat{u}-\hat{v}}{2}\right|$

Question 108

A six faced die is a biased one. It is thrice more likely to show an odd numbers than show an even number. It is thrown twice. The probability that the sum of the numbers in two throws is even, is

Options:

A. $\frac{5}{9}$

B. $\frac{5}{8}$

C. $\frac{1}{2}$

D. None of these

Answer: B

Solution:

Solution:

Let P be the probability of getting an even number. Then by hypothesis the probability of getting an odd number is 3P. Since the events of getting an even number and an odd number are mutually exclusive and exhaustive.

∴ P + 3P = 1 ⇒ P - 1/4

Thus, the probability of getting an odd number in a single throw is 3/4 and that of an even number is 1/4.

If the die is thrown twice, then the sum of the numbers in two throws is even if both the numbers are even or both are odd.

∴ Required probability $=\frac{3}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}=\frac{10}{16}=\frac{5}{8}$.

Question 109

The sum of all the solution of the equation
$\cos\theta \cos\left(\frac{\pi}{3} + \theta\right) \cos\left(\frac{\pi}{3} - \theta\right) = \frac{1}{4}, \theta \in [0, 6\pi]$

Options:

A. 15π

B. 30π

C. $\frac{100\pi}{3}$

D. None of these

Answer: B

Solution:

Solution:

$2cos\theta[cos120° + cos2\theta] = 1$

$\Rightarrow 2cos\theta\left(-\frac{1}{2} + 2cos^2\theta - 1\right) = 1$

$\Rightarrow 4cos^3\theta - 3cos\theta - 1 = 0$

$\Rightarrow 3\theta = 2n\pi \text{ or } \theta = \frac{2n\pi}{3}, n \in z$

Given the values so that 2n does not exceed 18.

Hence, the sum $= \frac{2\pi}{3}\Sigma_1^9 n = \frac{2\pi}{3} \times \frac{9(9+1)}{2} = 30\pi$

Question 110

Let α be the solution of $16^{\sin^2\theta} + 16^{\cos^2\theta} = 10$ in $\left(0, \frac{\pi}{4}\right)$. If the shadow of a vertical pole is $\frac{1}{\sqrt{3}}$ of its height, then the altitude of the sun is

Options:

A. α

B. $\frac{\alpha}{2}$

C. 2α

D. $\frac{\alpha}{3}$

Answer: C

Solution:

Solution:

We have,

$16^{sin^2\theta} + 16^{cos^2\theta} = 10$

$\Rightarrow 16^{sin^2\theta} + 16^{1-sin^2\theta} = 10$

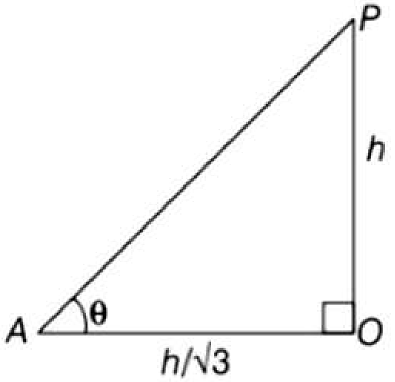
$x + \dfrac{16}{x} = 10$, where $x = 16^{sin^2\theta}$

$\Rightarrow x^2 - 10x + 16 = 0 \Rightarrow x = 2, 8$

$\therefore 16^{sin^2\theta} = 2, 8 \Rightarrow 2^{4sin^2\theta} = 2, 2^3$

$\Rightarrow 4sin^2\theta = 1, 3 \Rightarrow sin^2\theta = \frac{1}{4}, \left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow sin\theta = \frac{1}{2}, \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$



Let the altitude of the sum be θ. Then,

$tan\theta = \dfrac{h}{\frac{h}{\sqrt{3}}} \Rightarrow tan\theta = \sqrt{3}$

$\Rightarrow \theta = \frac{\pi}{3} \Rightarrow \theta = 2\alpha$

Question 111

For each parabola y = x^2 + px + q, meeting coordinate axes at 3-distinct points, if circles are drawn through these points, then the family of circles must pass through

Options:

A. (1, 0)

B. (0, 1)

C. (1, 1)

D. (p, q)

Answer: B

Solution:

Solution:

Suppose the parabola $y=x^2+px+q$ cuts X-axis at $A(\alpha, 0)$and $B(\beta, 0)$.

Then, α, β are roots of the equation $x^2+px+q=0$

$\therefore \alpha+\beta=-p$ and $\alpha\beta=q$

The parabola $y=x^2+px+q$ cuts Y-axis at $(0, q)$.

Let the equation of the circle passing through A, B and C be

$x^2+y^2+2gx+2fy+c=0.....(i)$

$\therefore \alpha^2+2g\alpha+c=0......(ii)$

$\beta^2+2g\beta+c=0....(iii)$

and $q^2+2fq+c=0...(iv)$

Subtracting Eq. (iii) from Eq. (ii), we get

$\alpha+\beta+2g=0$

$\Rightarrow g=\frac{p}{2}$

Adding Eqs. (ii) and (iii), we get

$\alpha^2+\beta^2+2g(\alpha+\beta)+2c=0$

$(\alpha+\beta)^2-2\alpha\beta+2g(\alpha+\beta)+2c=0$

$p^2-2q-p^2+2c=0\left[\because \alpha+\beta=p and\, g=\frac{p}{2}\right]$

Putting $c=q$ in Eq. (iv), we get $f=-\left(\frac{q+1}{2}\right)$

Substituting the values of g, f and c in Eq. (i), we obtain the equation of family of circles passing through A, B and C as

$x^2+y^2+px-(q+1)y+q=0$

$x^2+y^2+px-(q+1)y+q=0$

Clearly, it passes through $(0, 1)$.

Question 112

The number of ways of arranging letters of the word HAVANA so that V and N do not appear together is

Options:

A. 40

B. 60

C. 80

D. 100

Answer: C

Solution:

Solution:

We can arrange the letters H, A, A, A in 4!/3! = 4 WAYS

If one possible arrangement is XXXX
Then, we can arrange, V,N at any of the two places marked with O in the following arrangement
O × O × O × O × O
Thus, we can arrange V and N in $^5P_2$ = 20 ways. Thus, the number of ways in which letters can be arranged is 4 × 20 = 80

Question 113

Let a_1, a_2, a_3... be a harmonic progression with a_1 = 5 and a_{20} = 25. The least positive integer n for which a_n < 0, is

Options:

A. 22

B. 23

C. 24

D. 25

Answer: D

Solution:

Solution:

It is given that $a_1, a_2, a_3.....$ are in HP.

Therefore, $\frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}, ...$ are in AP

Let d be the common difference of the AP.

$$\therefore \frac{1}{a_n} = \frac{1}{a_1} + (n-1)d \text{ and } \frac{1}{a_{20}} = \frac{1}{a_1} + 19d$$

$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} + (n-1)d \text{ and } \frac{1}{25} = \frac{1}{5} + 19d$$

$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} + (n-1)d \text{ and } d = \frac{-4}{19 \times 25}$$

$$\Rightarrow \frac{1}{a_n} = \frac{1}{5} - \frac{4(n-1)}{19 \times 25}$$

$$\Rightarrow \frac{1}{a_n} = \frac{95 - 4n + 4}{19 \times 25}$$

$$\Rightarrow a_n = \frac{99 - 4n}{19 \times 25}$$

Now, $a_n < 0$

$$\Rightarrow \frac{99 - 4n}{19 \times 25} < 0$$

$$\Rightarrow 99 - 4n < 0$$

$$\Rightarrow 4n > 99$$

$$\Rightarrow n > 24\frac{3}{4} \Rightarrow n \geq 25$$

Question 114

If the plane 3x + y + 2z + 6 = 0 is parallel to the line $\frac{3x-1}{2b} = 3 - y = \frac{z-1}{a}$, then the value of 3a + 3b is

Options:

A. $\frac{1}{2}$

B. $\frac{3}{2}$

C. 3

D. 4

Answer: B

Solution:

Solution:

Given plane $3x+y+2z+6=0$

and line $\dfrac{x-\frac{1}{3}}{\frac{2b}{3}}=\dfrac{y-3}{-1}=\dfrac{z-1}{a}$

Since, plane is parallel to line, then

$3\left(\dfrac{2b}{3}\right)+(1)(-1)+2(a)=0$

$2b-1+2a=0$

$\Rightarrow a+b=\dfrac{1}{2}$

Now, $3a+3b=\dfrac{3}{2}$

Question 115

Let a, b be the solutions of $x^2+px+1=0$ and c, d be the solution of $x^2+qx+1=0$. If $(a-c)(b-c)$ and $(a+d)(b+d)$ are the solution of $x^2+ax+\beta=0$, then β is equal to

Options:

A. $p+q$

B. $p-q$

C. p^2+q^2

D. q^2-p^2

Answer: D

Solution:

Solution:

Since, $a+b=-p, ab=1....(i)$

and $c+d=-q, cd=1$

Now $(a-c)(b-c)$ and $(a+d)(b+d)$ are

the roots of $x^2+ax+\beta=0$

$(a-c)(b_c)(a+d)(b+d)=\beta$

$\Rightarrow(ab-ac-bc+c^2)(ab+ad+bd+d^2)=\beta$

$\Rightarrow\{1-c(a+b)+c^2\}\{1+d(a+b)+d^2\}=\beta$

$\Rightarrow(1+pc+c^2)(1-pd+d^2)=\beta$

$\Rightarrow 1-pd+d^2+pc-p^2cd+pcd^2+c^2-pc^2d+c^2d^2=\beta$

$\Rightarrow 1-pd+d^2+pc-p^2+pd+c^2-pc+1=\beta[\because cd=1]$

$\Rightarrow 2+d^2+c^2-p^2=\beta$

$\Rightarrow 2cd+c^2+d^2-p^2=\beta$

$(c+d)^2-p^2=\beta$

$q^2-p^2=\beta[\because(c+d)=-q]$

Question 116

If $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$**, then**

Options:

A. a = 1, b = 1

B. a = sin 2θ, b = cos 2θ

C. a = cos 2θ, b = sin 2θ

D. None of these

Answer: C

Solution:

Solution:

Given,

$$\begin{bmatrix} 1 & -tan\theta \\ tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & tan\theta \\ -tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -tan\theta \\ tan\theta & 1 \end{bmatrix} \cdot \frac{1}{1+tan^2\theta}\begin{bmatrix} 1 & -tan\theta \\ tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \frac{1}{1}+tan^2\theta \begin{bmatrix} 1-tan^2\theta & -2tan\theta \\ 2tan\theta & 1-tan^2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{1-tan^2\theta}{1+tan^2\theta} & \frac{2tan\theta}{1+tan^2\theta} \\ \frac{2tan\theta}{1+tan^2\theta} & \frac{1-tan^2\theta}{1+tan^2\theta} \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} cos2\theta & -sin2\theta \\ sin2\theta & cos2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$\Rightarrow a = cos2\theta,\ b = sin2\theta$$

Question 117

The value of $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e + \frac{1}{2}ex}{x^2}$ **is**

Options:

A. $\frac{11}{24}e$

B. $-\frac{11}{24}e$

C. $\frac{e}{24}$

D. None of these

Answer: A

Solution:

Solution:

Let $y=(1+x)^{\frac{1}{x}}$

$\Rightarrow l\,ogy=\frac{1}{x}\log(1+x)=\frac{1}{x}\left[x=\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+....\right]$

$=1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}+.....$

$y=e^{\left(1-\frac{x}{2}+\frac{x^2}{3}+..\right)}=ee^{\left(-\frac{x}{2}+\frac{x^2}{3.}..\right)}$

$=e\left[1+\left(-\frac{x}{2}+\frac{x^2}{3}-....\right)+\frac{1}{2!}\left(-\frac{x}{2}+\frac{x^2}{3}-...\right)^2+...\right]$

$y-e+\frac{1}{2}ex=ex^2\left[\frac{1}{3}+0(x)+\frac{1}{2}\left(-\frac{1}{2}+0(x)\right)^2+...\right]$

$[\because 0(x)$ in terms containing $x]$

$\lim\frac{y-e+\frac{1}{2}ex}{x^2}=e\left[\frac{1}{3}+\frac{1}{8}\right]=\frac{11}{24}e$

Question 118

The locus of the mid-point of the chord if contact of tangents drawn from points lying on the straight line $4x-5y=20$ to the circle $x^2+y^2=9$ is

Options:

A. $20(x^2+y^2)-36x+45y=0$

B. $20(x^2+y^2)+36x-45y=0$

C. $36(x^2+y^2)-20x+45y=0$

D. $36(x^2+y^2)+20x-45y=0$

Answer: A

Solution:

Solution:

Let $P\left(t, 4t - \frac{20}{5}\right)$ be a point on the line

$4x - 5y = 20$. Then the chord of contact of tangents drawn from P to the circle $x^2 + y^2 = 9$ is

$tx\left(4t - \frac{20}{5}\right)y = 9 \ldots (i)$

Let $\theta(h, k)$ be the mid-point of this chord of contact, then its equation is also $hx + ky = h^2 + k^2 \ldots (ii)$

Clearly Eqs. (i) and (ii) represent the same line

$$\therefore \frac{t}{h} = \frac{4t - 20}{5k} = \frac{9}{h^2 + k^2}$$

$$\Rightarrow \frac{t}{h} = \frac{9}{h^2 + k^2} \text{ and } \frac{t}{h} = \frac{4t - 20}{5k}$$

$$\Rightarrow t = 9\frac{h}{h^2 + k^2} \text{ and } t = \frac{20h}{4h - 5k}$$

$$\Rightarrow 9\frac{h}{h^2 + k^2} = \frac{20h}{4h - 5k}$$

$$\Rightarrow h\{20(h^2 + k^2) - 36h + 45k\} = 0$$

$$x = 0, \text{ or}[20(x^2 + y^2) - 36x + 45y] = 0$$

Question 119

Let $f(x) = \int \frac{x^2\,dx}{(1 + x^2)\left(1 + \sqrt{1 + x^2}\right)}$ and $f(0) = 0$, then the value of $f(1)$ be

Options:

A. $\log(1 + \sqrt{2})$

B. $\log(1 + \sqrt{2}) - \frac{\pi}{4}$

C. $\log(1 + \sqrt{2}) + \frac{\pi}{2}$

D. None of these

Answer: B

Solution:

Solution:

$f(x)=\int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $x=tan\theta$

$\Rightarrow dx=sec^2\theta d\theta=(1+x^2)d\theta$

$f(x)=\int \frac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

$=\int \frac{tan^2\theta sec^2\theta d\theta}{sec^2\theta(1+sec\theta)}=\int \frac{tan^2\theta d\theta}{1+sec\theta}$

$=\int \frac{sin^2\theta d\theta}{cos\theta(1+cos\theta)}$

$=\int \frac{1-cos^2\theta d\theta}{cos\theta(1+cos\theta)}$

$=\int \frac{1-cos^{\theta}d\theta}{cos\theta(1+cos\theta)}$

$=\int \frac{(1-cos\theta)d\theta}{cos\theta}$

$=\int sec\theta d\theta-\int d\theta$

$=\log\left(x+\sqrt{1+x^2}\right)-tan^{-1}x+C$

$\therefore f(0)=\log(0+\sqrt{1+0})-tan^{-1}(0)+C$

$0=\log 1-0+C$

$\Rightarrow C=0$

$\therefore f(1)=\log\left(1+\sqrt{1+1^2}\right)-tan^{-1}(1)$

$=\log(1+\sqrt{2})-\frac{\pi}{4}$

Question 120

The mean of five observations is 4 and their variance is 5.2. If three of these observations are 1,2 and 6 , then the other two are

Options:

A. 2 and 9

B. 3 and 8

C. 4 and 7

D. 5 and 6

Answer: C

Solution:

Solution:

Let the two unknown items be x and y. Then,

Mean $= 4 \Rightarrow \frac{1+2+6+x+y}{5} = 4$

$\Rightarrow x + y = 11 \ldots (i)$

and variance $= 5.2$

$\Rightarrow \frac{1^2+2^2+6^2+x^2+y^2}{5} - (\text{mean})^2 = 5.2$

$\Rightarrow 41 + x^2 + y^2 = 5(5.2 + 16)$

$\Rightarrow 41 + x^2 + y^2 = 106$

$\Rightarrow x^2 + y^2 = 65 \ldots (ii)$

On solving Eqs. (i) and (ii), we get

$x = 4, y = 7$ or $x = 7, y = 4$

Question 121

In a sequence of 21 terms, the first 11 terms are in AP with common difference 2 and the last 11 terms are in GP with common ratio 2 . If the middle term of AP be equal to the middle term of the GP, then the middle term of the entire sequence is

Options:

A. $-\frac{10}{31}$

B. $\frac{10}{31}$

C. $\frac{32}{31}$

D. $-\frac{31}{32}$

Answer: A

Solution:

Solution:

Since, the first 11 terms are Ap, $d = 2$

$\therefore a_{11} = a + 10d = a + 20$

The middle term of AP is

$T_6 = a + 5d = a + 10$

For the next 11 terms in GP

$r = 2$

$\therefore$ The middle term of GP is $b(2)^5$ where, b is the first term of a GP which is the last term of AP

$b(2)^5 = (a + 20)32$

According to the given condition,

$\Rightarrow a + 10 = (a + 20)32$

$\Rightarrow 32a = 10 - 640$

$a = -\frac{630}{31}$

$\therefore$ Middle term of entire sequence is $11th$ term

$\therefore T_{11} = \frac{-630}{31} + 10 \times d$

$= \frac{-630}{31} + 10 \times 2 = \frac{-10}{31}$

Question 122

If p ≠ a, q ≠ b, r ≠ c and the system of equations
px + ay + az = 0
bx + qy + bz = 0
cx + cy + rz = 0
has a non-trivial solution, then the value of $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c}$ **is**

Options:

A. 1

B. 2

C. $\frac{1}{2}$

D. 0

Answer: B

Solution:

Solution:

As the given system of equations has a non-trivial solution.

$$\Delta = \begin{vmatrix} p & a & a \\ b & q & b \\ c & c & r \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Delta = \begin{vmatrix} p & a-p & a-p \\ b & q-b & 0 \\ c & 0 & r-c \end{vmatrix} = 0$$

Expanding along C_3, we get

$$(a-p)\begin{vmatrix} b & q-b \\ c & 0 \end{vmatrix} + (r-c)\begin{vmatrix} p & a-p \\ b & q-b \end{vmatrix} = 0$$

$\Rightarrow (a-p)(-c)(q-b) + (r-c)$

$\{p(q-b) - b(a-p)\} = 0$

$\Rightarrow (p-a)(q-b)c + p(r-c)(q-b) + b(r-c)(p-a) = 0$

Dividing by $(p-a)(q-b)(r-c)$, we get

$$\frac{c}{r-c} + \frac{p}{p-a} + \frac{b}{q-b} = 0$$

$$\Rightarrow \frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = \frac{q-b}{q-b} + \frac{r-c}{r-c} = 2$$

Question 123

If $g(x) = x^2 + x - 2$ and $\frac{1}{2} g \circ f(x) = 2x^2 - 5x + 2$, then $f(x)$ is equal to

Options:

A. $2x - 3$

B. $2x + 3$

C. $2x^2 + 3x + 1$

D. $2x^2 - 3x + 1$

Answer: A

Solution:

Solution:

We have, $\frac{1}{2}gof(x) = 2x^2 - 5x + 2$

$\Rightarrow g(f(x)) = 4x^2 - 10x + 4$

$\Rightarrow (f(x))^2 + f(x) - 2 = 4x^2 - 10x + 4$

$\Rightarrow (f(x))^2 + f(x) - (4x^2 - 10x + 6) = 0$

$\Rightarrow f(x) = \frac{-1 \pm \sqrt{1 + 4(4x^2 - 10x + 6)}}{2}$

$\Rightarrow f(x) = \frac{-1 \pm \sqrt{16x^2 - 40x + 25}}{2}$

$\Rightarrow f(x) = \frac{-1 \pm (4x - 5)}{2} = 2x - 3, -2x + 2$

Hence, $f(x) = 2x - 3$

Question 124

The smallest positive integral value of n such that $\left[\frac{1 + \sin\frac{\pi}{8} + i\cos\frac{\pi}{8}}{1 + \sin\frac{\pi}{8} - i\cos\frac{\pi}{8}}\right]^{n}$ **is purely imaginary, is equal to**

Options:

A. 4

B. 3

C. 2

D. 8

Answer: A

Solution:

Solution:

$$\left[\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right]^n = \left[\frac{1+\cos\alpha+i\sin\alpha}{1+\cos\alpha-i\sin\alpha}\right]^n$$

$$\left[\because \alpha = \frac{\pi}{2}-\frac{\pi}{8}\right]$$

$$= \left[\frac{2\cos^2\frac{\alpha}{2}+2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}-2i\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}\right]^n$$

$$= \left[\frac{\cos\frac{\alpha}{2}+i\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-i\sin\frac{\alpha}{2}}\right]^n = \left(e^{2i\frac{\alpha}{2}}\right)^n = e^{in\alpha}$$

$$= e^{in\left(\frac{3\pi}{8}\right)} = \cos\frac{3n\pi}{8}+i\sin\frac{3n\pi}{8}$$

For $n = 4$ we get imaginary part.

Question 125

A house subtends a right angle at the window of a opposite house and the angle of elevation of the window from the bottom of the first house is 60. If the distance between two houses be 6m, then the height of the first house is

Options:

A. $8\sqrt{3}$m

B. $6\sqrt{3}$m

C. $4\sqrt{3}$m

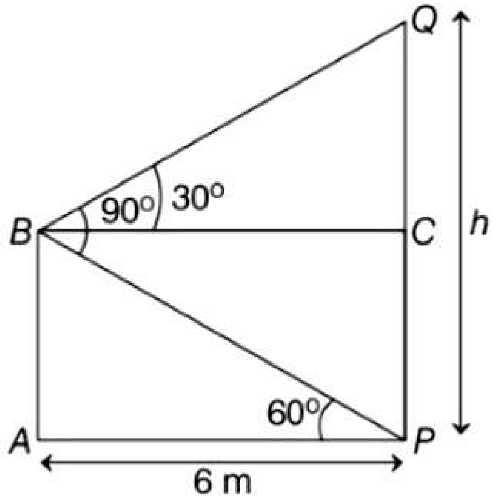
D. None of these

Answer: A

Solution:

Solution:

Let PQ be the house subtending a right angle at the window B of opposite house AB



In ΔPAB, we have

$tan60° = \frac{AB}{6} = AB = 6\sqrt{3}m$

In ΔCBQ, we have

$tan30° = \frac{h-CP}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h-6\sqrt{3}}{6} [\because AB = CP, BC = AP]$

$\Rightarrow h = 6\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

$\Rightarrow h = 8\sqrt{3}m$

Question 126

A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute then the rate (in meters per minute) at which the radius of the balloon decreases 49 min after the leakage began is

Options:

A. $\frac{9}{7}$

B. $\frac{7}{9}$

C. $\frac{2}{9}$

D. 9

Answer: C

Solution:

Solution:

$V = \frac{4}{2}\pi r^3 \Rightarrow 4500\pi = \frac{4\pi r^3}{3}$

$\Rightarrow r = 15m$

After $49min = (4500 - 49.72)\pi$

$= 972\pi m^3$

$\Rightarrow 952\pi = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 3 \times 243 = 3 \times 3^5$

$r = 9$

$\frac{dv}{dt} = 4\pi r^2 \left(\frac{dr}{dt}\right)$

$72\pi = 4\pi \times 9 \times 9 \left(\frac{dr}{dt}\right)$

$\frac{dr}{dt} = \left(\frac{2}{9}\right)$

Question 127

If in a $\triangle ABC$, $2b^2 = a^2 + c^2$, then $\frac{\sin 3B}{\sin B}$ is equal to

Options:

A. $\frac{c^2 - a^2}{2ca}$

B. $\frac{c^2 - a^2}{ca}$

C. $\frac{(c^2 - a^2)^2}{(ca)^2}$

D. $\left[\frac{c^2 - a^2}{2ca}\right]^2$

Answer: D

Solution:

Solution:

$$\frac{\sin 3B}{\sin B} = \frac{3\sin B - 4\sin^3 B}{\sin B}$$

$$= 3 - 4\sin^2 B$$

$$= 3 - 4(1 - \cos^2 B)$$

$$= -1 + \frac{4(a^2 + c^2 - b^2)^2}{4(ac)^2}$$

$$= -1 + \frac{\left(\frac{a^2 + c^2}{2}\right)^2}{(ac)^2}$$

$$= -1 + \frac{(a^2 + c^2)^2}{4(ac)^2}$$

$$= \left(\frac{c^2 - a^2}{2ac}\right)^2$$

Question 128

If the sum of the coefficients in the expansion of $(x + y)^n$ is 1024 , then the value of the greatest coefficient in the expansion is

Options:

A. 356

B. 252

C. 210

D. 120

Answer: B

Solution:

Solution:

Given, sum of the coefficient $= 1024$

i.e. $2^n = 1024 = 2^{10}$

$\Rightarrow n = 10$

Since, n is even, so greatest coefficient

$= {}^nC_{\frac{n}{2}} = {}^{10}C_5 = 252$

Question 129

The area enclosed by the curves $y = \sin x + \cos x$ and $y = |\cos x - \sin x|$ over the interval $\left[0, \frac{\pi}{2}\right]$ is

Options:

A. $4(\sqrt{2}-1)$

B. $2\sqrt{2}(\sqrt{2}-1)$

C. $2(\sqrt{2}+1)$

D. $2\sqrt{2}(\sqrt{2}+1)$

Answer: B

Solution:

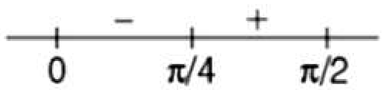
Solution:

Given, $y = sinx + cosx, x \in \left[0, \frac{\pi}{2}\right]$

$$\frac{dy}{dx} = cosx - sinx$$

$$y = |cosx - sinx|$$

$$= \begin{bmatrix} cosx - sinx & x \in \left[0, \frac{\pi}{4}\right] \\ sinx - cosx & x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right] \end{bmatrix}$$

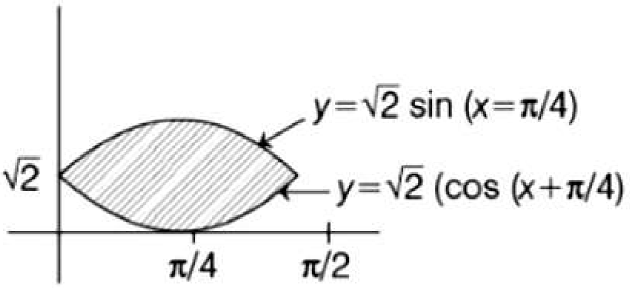


∴ Required area

$$= \int_0^{\frac{\pi}{4}} \left|(sinx + cosx) - (cosx - sinx)\right| dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left|2cosx\right| dx$$

$$= \int_0^{\frac{\pi}{4}} \left|2sinx\right| dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left|2cosx\right| dx$$

$$= 2\left[-\frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}}\right]$$



$$= 2\left(2 - \frac{2}{\sqrt{2}}\right)$$

$$= 2(2 - \sqrt{2})$$

$$= 4 - 2\sqrt{2}$$

$$= 2\sqrt{2}(\sqrt{2} - 1)$$

Question 130

If α, β, γ ∈ [0, π] and if α, β, γ are in AP, then $\frac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$ is equal to

Options:

A. $\sin\beta$

B. $\cos\beta$

C. $\cot\beta$

D. $2\cos\beta$

Answer: C

Solution:

Solution:

$$\frac{sin\alpha - sin\gamma}{cos\gamma - cos\alpha} = \frac{2cos\frac{\alpha+\gamma}{2} sin\alpha - \frac{\gamma}{2}}{2sin\frac{\alpha+\gamma}{2} . sin\frac{\alpha-\gamma}{2}}$$

$$= cot\alpha + \frac{\gamma}{2}$$

$\left[\because sinA - sinB = 2cos\left(\frac{A+B}{2}\right) sin\left(\frac{A-B}{2}\right)\right.$

and $\left. cosA - cosB = 2sin\left(\frac{A+B}{2}\right) sin\left(\frac{B-A}{2}\right) \right]$

But α, β, γ are in AP

$\Rightarrow \frac{\alpha+\gamma}{2} = \beta$

So, required value is $cot\beta$.
