JEE Mains (12th)

Sample Paper - I

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(2)
2.	(1)
3.	(1)
4.	(2)
5.	(2)
6.	(2)
7.	(2)
8.	(4)
9.	(1)
10.	(3)
11.	(2)
12.	(4)
13.	(3)
14.	(2)
15.	(3)
16.	(3)
17.	(4)
18.	(3)
19.	(3)
20.	(2)
21.	(16)
22.	(3)
23.	(11)
24.	(2)
25.	(330)
26.	(20)
27.	(19)
28.	(1625)
29.	(10)
30.	(600)

CHEMISTRY

31.	(2)
32.	(4)
33.	(3)
34.	(1)
35.	(3)
36.	(3)
37.	(3)
38.	(2)
39.	(2)
40.	(2)
41.	(2)
42.	(4)
43.	(4)
44.	(4)
45.	(2)
46.	(3)
47.	(3)
48.	(1)
49.	(3)
50.	(3)
51.	(4)
52.	(2)
53.	(2)
54.	(6)
55.	(6)
56.	(6)
57.	(4)
58.	(4)
59.	(6)
60.	(5)

MATHEMATICS

61.	(4)
62.	(1)
63.	(4)
64.	(2)
65.	(3)
66.	(4)
67.	(3)
68.	(1)
69.	(1)
70.	(2)
71.	(2)
72.	(2)
73.	(2)
74.	(3)
75.	(2)
76.	(2)
77.	(1)
78.	(4)
79.	(1)
80.	(3)
81.	(5)
82.	(168)
83.	(4)
84.	(73)
85.	(1)
86.	(2)
87.	(485)
88.	(330)
89.	(11)
90.	(2)

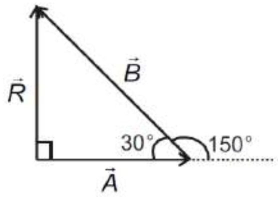
PHYSICS

1. **(2)**

$\Rightarrow \quad R^2 + A^2 = B^2 \qquad$ ...(i)

$$R = 10$$

Also $\tan 30° = \dfrac{\text{Perpendicular}}{\text{Base}}$



$$\frac{1}{\sqrt{3}} = \frac{R}{A}$$

From equation (i) $A = 10\sqrt{3}$

$(10)^2 + (10\sqrt{3})^2 = B^2$

$B = 20$

2. **(1)**

Let the retarding force by one block is F and displacement inside one block is x.

So using work energy theorem for one block

$$-F.x = \frac{1}{2}m\left[\left(\frac{19}{20}v\right)^2 - v^2\right]$$

...(i)

Applying work energy theorem for n blocks

$$-F.nx = \frac{1}{2}m[o - v^2]$$

Using value of Fx from ...(i)

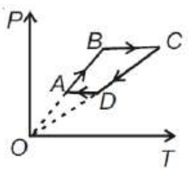
$$\frac{1}{2}m\left[v^2 - \left(\frac{19}{20}v\right)^2\right]n = \frac{1}{2}m[o - v^2]$$

Solving for n

$n = 10.25$ So, 11 Planks.

3. **(1)**

$A \to B \quad V$ = constant



$\because \quad PV = RT$

$$P = \frac{R}{V}T$$

Compare with $y = mx$

$P - T$ graph is a straight line which must passes from origin

$A \to B$ volume constant, P-increasing, T-increasing.

$B \to C$ pressure constant, volume - increasing, temperature - increasing

$B \to C \quad P$ = constant, origin P-T graph is a straight line parallel to v-axis

$C \to D \quad V$ = constant then

$$P = \frac{R}{V}T$$

P-T graph is straight line must passes from origin

$D \to A \quad P$ = constant

P-T graph is a straight line parallel to T-axis.

4. **(2)**

$$v = \sqrt{2gh}$$

$$\frac{h}{3} = \sqrt{2gh}t - \frac{1}{2}gt^2$$

$$\frac{g}{2}t^2 - \sqrt{2gh}t + \frac{h}{3} = 0$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} + \sqrt{2gh - 2gh/3}}{\sqrt{2gh} - \sqrt{2gh - 2gh/3}}$$

$$= \frac{\sqrt{2} + \frac{2}{\sqrt{3}}}{\sqrt{2} - \frac{2}{\sqrt{3}}} = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

5. **(2)**

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} \qquad T_1 = T$$

$$T_2 = T + 6$$

$$\frac{P_2}{P_1} - 1 = \frac{T_2}{T_1} - 1$$

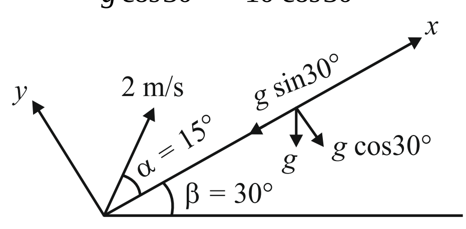
$$\frac{P_2 - P_1}{P_1} \times 100 = \left(\frac{T+6}{T} - 1\right)100$$

$$0.4 = \frac{600}{T} \Rightarrow T = 1500K$$

6. **(2)**

Time of flight $(T) = \dfrac{2u\sin\alpha}{g\cos\beta}$

$$T = \frac{(2)(2\sin 15)}{g\cos 30} = \frac{4}{10}\frac{\sin 15}{\cos 30}$$



Range $(R) = (2\cos 15)\ T - \dfrac{1}{2}g\sin 30(T)^2$

$$= (2\cos 15)\frac{4}{10}\frac{\sin 15}{\cos 30} - \left(\frac{1}{3} \times 10\sin 30\right)\frac{16}{100}\frac{\sin^2 15}{\cos^2 30}$$

$$= \frac{16\sqrt{3} - 16}{60} \simeq 0.1952\,\text{m} \approx 20\,\text{cm}$$

7. **(2)**

$F\cos\theta - \mu N = ma$

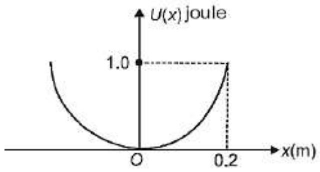
$N = (mg - F\sin\theta)$

$F\cos\theta - \mu mg + \mu F\sin\theta = ma$

8. **(4)**

Mass = 4 kg

Maximum P.E. = $\frac{1}{2}kA^2$



$$1 = \frac{1}{2}\times k\times(0.2)^2$$

$$\frac{2}{0.04} = k$$

k = 50 N/m

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{4}{50}} = \frac{2\sqrt{2}\pi}{5}\text{ s}$$

9. **(1)**

Time period = $2\pi\sqrt{\frac{l}{g}}$

$$T \propto \sqrt{l}$$

$$\frac{T'}{T} \propto \sqrt{\frac{l'}{l}}$$

$$T' = T\sqrt{\frac{l + l \propto \Delta\theta}{l}}$$

$$T' = T\left(1+\frac{1}{2}\propto\Delta\theta\right)[\alpha\Delta\theta = 0.002]$$

$$\Delta T' = T' - T = \frac{1}{2}T\propto\Delta\theta = T\times 0.001$$

Time lost in time t is

$$\Delta T = \frac{1}{2} \quad t = 1 \text{ day} = 24\times 3600\text{ s} = 86400\text{ s}$$

$$\Delta T = \left(\frac{\Delta T}{T}\right)\times t$$

$\Delta T = 0.001 \times 86400$

$\Delta T = 86.4$ s

10. **(3)**

Velocity of sound = $\sqrt{\frac{\gamma RT}{M}}$

RMS speed = $\sqrt{\frac{3RT}{M}}$

$$\frac{v_s}{r_{\text{rms}}} = \sqrt{\frac{\gamma}{3}}$$

$$\frac{v_s}{r_{\text{rms}}} = \sqrt{\frac{1}{2}}$$

$$v_{\text{rms}} = 600\sqrt{2}\text{ m/s}$$

11. **(2)**

Let density of material be ρ

Areas of cross section $A_1 = \pi r_1^2$, $A_2 = \pi r_2^2$ where $r_2 = 2r_1$

$\therefore \quad \mu_1 = \pi r_1^2\rho, \mu_2 = \pi 4r_1^2\rho$

$\therefore \quad v_1 = \sqrt{\frac{T}{\pi r_1^2\rho}}$ and $v_2 = \frac{1}{2}\sqrt{\frac{T}{\pi r_1^2\rho}}$

$\therefore \quad v_1 : v_2 = 2 : 1$

12. **(4)**

$$I = I_{\text{spheres}} + I_{\text{rod}}$$

$$I_{\text{rod}} = \frac{M\times 4R^2}{12} = \frac{MR^2}{3}$$

$$I_{\text{spheres}} = 2\times\left[\frac{2}{5}MR^2 + 4MR^2\right]$$

$$= \frac{44}{5}MR^2$$

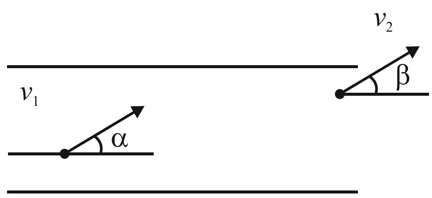
$$I = \left[\frac{44}{5}+\frac{1}{3}\right]MR^2$$

$$= \frac{137}{15}MR^2$$

13. **(3)**

$$v_1\cos\alpha = v_2\cos\beta$$

$$\frac{K_1}{K_2} = \frac{v_1^2}{v_2^2} = \frac{\cos^2\beta}{\cos^2\alpha}$$



14. **(2)**

Let the intensity of each source is I

$$I_0 = 4I$$

$$y = \frac{\beta}{4}$$

(Path difference at y distance from central maxima)

$$\Delta x = \frac{yd}{D} = \frac{\lambda}{4}$$

(Phase difference) $\Delta\phi = \frac{\lambda}{4}\left(\frac{2\pi}{\lambda}\right) = \frac{\pi}{2}$

(Intensity at y distance)

$$I' = 2I(1+\cos\Delta\phi)$$

$$I' = 2\left(\frac{I_0}{4}\right)\left(1+\cos\frac{\pi}{2}\right)$$

$$I' = \frac{I_0}{2}$$

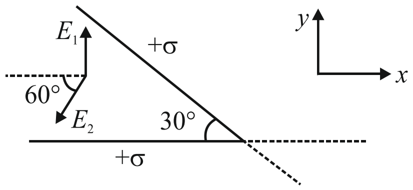
15. **(3)**

$d \sin \theta = \lambda$

$$\sin \theta = \frac{5000\times10^{-10}}{.001\times10^{-3}}$$

$$\sin \theta = \frac{1}{2} \Rightarrow \theta = 30°$$

16. **(3)**



$$\vec{E_1} = \frac{\sigma}{2\varepsilon_0}\hat{y}$$

$$\vec{E_2} = \frac{\sigma}{2\varepsilon_0}\left(-\cos 60^\circ \hat{x} - \sin 60^\circ \hat{y}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

$$\therefore \vec{E_P} = \vec{E_1} + \vec{E_2} = \frac{\sigma}{2\varepsilon_0}\left[-\frac{1}{2}\hat{x} + \left(1-\frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

17. **(4)**

The ray is incident at Brewster's angle so it reflected ray will be plane polarised. When passed through polarises the ray will display intensity according to the law of malus ($I_0 \cos^2 \theta$).

18. **(3)**

Path difference between the emerging rays

$= \mu_1 L - \mu_2 L$

$$\frac{\phi}{2\pi} = \frac{(\mu_1 - \mu_2)L}{\lambda}$$

19. **(3)**

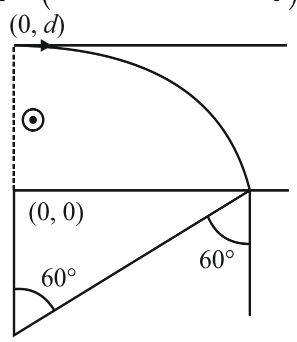
$\varepsilon = Bv\ell$

$$i = \frac{\varepsilon}{R}$$

$$i = \left(\frac{\mu_0 I}{2\pi r}v\ell\right)\frac{1}{R}$$

20. **(2)**

$$F = qvB\left(-\sin 60°\hat{i} - \cos 60°\hat{j}\right)$$



Assuming particle enters from (0, d)

$$F = -\frac{qvB}{2}(\sqrt{3}\hat{i} + \hat{j})$$

$$a = -\frac{qvB}{2m}(\sqrt{3}\hat{i} + \hat{j})$$

21. **(16)**

$$I \propto \frac{1}{r^2}$$

$\therefore$ If at 2 m intensity is I_1 at 4 m, intensity is $\frac{I_1}{4}$

22. **(3)**

$$y = 5\sin\frac{\pi x}{3}\cos 40\pi t$$

At the nodes the values of x is such that $y = 0$.

Taking position 1 at $x = 0$ to make sine function zero; the value of sine function will be zero node for $\frac{\pi x}{3} = \pi$.

$\therefore \quad x = 3$

Hence, shortest distance between two nodes is 3 cm.

23. **(11)**

$$R = \frac{l}{2\pi} = \frac{314}{2\times3.14} = 50\,\text{cm}$$

$$\mu = \pi R^2 i$$

$$= 14\times3.14\times(0.5)^2$$

$$= 11\,\text{A}-\text{m}^2$$

24. **(1.78)**

Let tension be T

$$f_1 = \sqrt{\frac{T}{\mu}} \times \frac{1}{2l} = 256 \quad ...(i)$$

$$f_2 = \sqrt{\frac{T+10}{\mu}} \times \frac{1}{2l} = 320 \quad ...(ii)$$

$$\sqrt{\frac{T}{T+10}} = \frac{256}{320} \quad \text{[Dividing (i) by (ii)]}$$

or $$\frac{T}{T+10} = \frac{(16)^2}{(16)^2\times(20)^2}$$

or $$\frac{T}{T+10} = \frac{(16)^2}{(20)^2}$$

or $400\, T = 256\, T^2 + 2560$

or $144\, T = 2560$

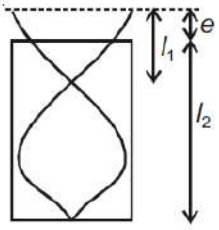
or $T = \frac{2560}{144}$

or $T = \frac{2560}{16\times9}$

or $T = \frac{160}{9}$ Newton

$= \frac{16}{9}$ kg-wt

25. **(330)**



$$I_1 + e = \frac{\lambda}{4}$$

$$I_2 + e = \frac{3\lambda}{4}$$

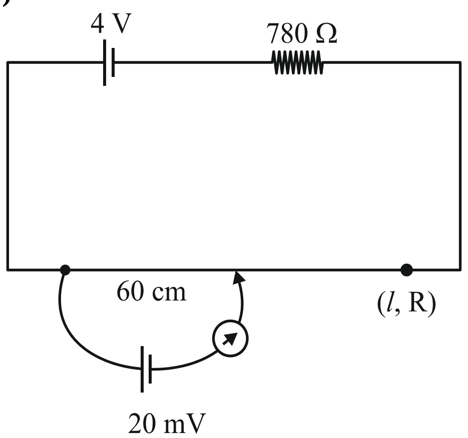
$$I_2 - I_1 = \frac{\lambda}{2}$$

$$0.48 - 0.15 = \frac{\lambda}{2}$$

$\lambda = 0.66$

Velocity = $f\lambda = 500 \times 0.66 = 330$ m/s

26. **(20)**



$\ell = 300\,\text{cm}$

$\varepsilon = Kx$

$$20 \times 10^{-3} = \left(\frac{4 \times R}{780 + R} \times \frac{1}{300}\right)60$$

$$\boxed{R = 20}$$

27. **(19)**

$$f = f_0 \frac{v}{v - v_s}$$

$$f_1 = f_0 \frac{340}{340 - 34}$$

$$f_2 = f_0 \frac{340}{340 - 17}$$

$$f_1 = \frac{10}{9} f_0$$

$$f_2 = \frac{20}{19} \qquad \Rightarrow \qquad \frac{f}{f_2} = \frac{19}{18}$$

28. **(1625)**

$\lambda = 6.5 \times 10^{-7}$ mm

$d = 1$ mm

$D = 1$ m

$\left(\frac{nD\lambda}{d}\right)$ 5^{th} bright fringe = $\frac{5D\lambda}{d}$

$\left((2n-1)\frac{D\lambda}{2d}\right)$ 3^{rd} dark fringe $\frac{5D\lambda}{2d}$

Distance (D) = $\frac{5D\lambda}{d} - \frac{5}{2}\frac{D\lambda}{d}$

$= \frac{5}{2}\frac{D\lambda}{d}$

29. **(10)**

$$\vec{L} = \vec{r} \times m\vec{v}$$

$$= \left(10\alpha t^2 \hat{i} + 5\beta(t-5)\hat{j}\right) \times m(20\alpha t\hat{i} + 5\beta\hat{j})$$

$$= 50\alpha\beta mt(10-t)\hat{k}$$

$$\vec{L}_{t=0} = 0$$

$$\vec{L}_t = 0 \Rightarrow t = 10\text{sec.}$$

30. **(600)**

Shift of fringes is given by $\frac{(\mu-1)tD}{d}$

This is equal to position of 10^{th} bright fringe $\frac{10\lambda D}{d}$

$$\frac{0.6 \times 1 \times 10^{-5} D}{d} = \frac{10\lambda D}{d}$$

$\lambda = 0.6 \times 10^{-6}$

$\therefore$ $\lambda = 6 \times 10^{-7}$

or $\lambda = 6000$Å

CHEMISTRY

31. **(2)**

$$\frac{(\bar{v})_{He^+}}{(\bar{v})_{Be^{3+}}} = \frac{R_H \cdot (2)^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)}{R_H \cdot (4)^2 \cdot \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} = \frac{4}{16} = \frac{1}{4}$$

or, $\frac{x}{(\bar{v})_{Be^{3+}}} = \frac{1}{4}$ or, $(\bar{v})_{Be^{3+}} = 4x\,\text{cm}^{-1}$

32. **(4)**

Highest negative electron gain enthalpy (Cl)
Most electropositive element (Cs)
Strongest reducing agent (Li)
Highest ionization energy (He)

33. **(3)**

In aluminothermite process Al acts as a reducing agent for Cr_2O_3, Mn_3O_4 etc.

34. **(1)**
Order of thermal stability :
$MgCO_3 < CaCO_3 < SrCO_3 < BaCO_3$

35. **(3)**
$K_2\overset{+6}{Cr_2}O_7$ (Cr^{+6} does not have any unpaired e^-) is coloured but it is diamagnetic.
$\Rightarrow$ Ti^{+4} and Cu^+ do not have any unpaired e^-
$\Rightarrow$ $\overset{+4}{V}OSO_4$ (V^{+4} has one unpaired e^-) is coloured and paramagnetic.

36. **(3)**
H_2O has greater dielectric constant than D_2O

37. **(3)**
O_3 is pale blue gas.

38. **(2)**

Species	Bond order
N_2, CN^-	3
O_2	2
NO, O_2^+, N_2^+, N_2^-	2.5
O_2^-	1.5

39. **(2)**

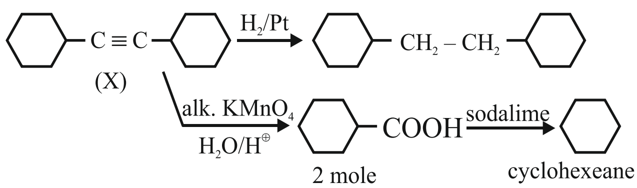
40. **(2)**
$$\frac{r_{H_2}}{r} = \sqrt{\frac{M}{2}} \quad \frac{x/5}{x/t} = \sqrt{\frac{M}{2}} \quad \frac{t}{5} = \sqrt{\frac{M}{2}}$$
Check every option to satisfy the relation option (2) satisfied

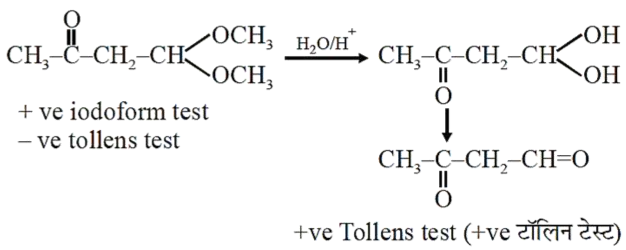
41. **(2)**
$H_2(g) + S(s) \rightleftharpoons H_2S(g)$
$$\log\frac{K_2}{K_1} = \frac{\Delta H}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$
$$\log\left(\frac{9.25}{18.5} = \frac{1}{2}\right) = \frac{\Delta H}{2.303 \times 8.314}\left(\frac{1}{925} - \frac{1}{1000}\right)$$
$$= 0.3010 = \frac{\Delta H}{2.30 \times 8.314}\left(\frac{75}{925 \times 1000}\right)$$
$\Delta H = -71080.57$ J/mole

42. **(4)**
$$A \rightarrow 8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$$
$$B \rightarrow 4 \times \frac{50}{100} = 2$$
$$C \rightarrow 8 \times \frac{50}{100} = 4$$
Formula of compound A_2BC_2

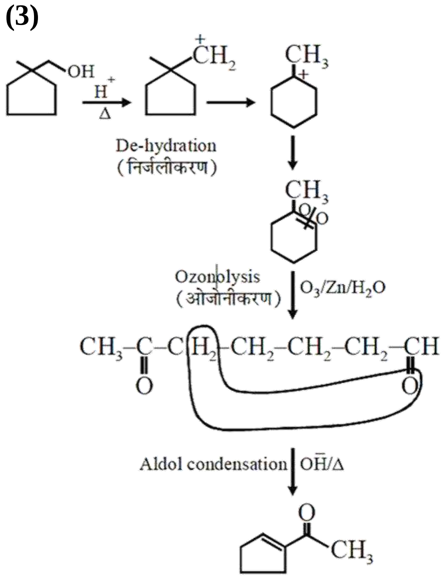
43. **(4)**
$P = P_A^0 \times 0.5 + P_B^0 \times 0.5$
$P_A^0 + P_B^0 \; \frac{1}{0.5} = 2$ ....(i)
$1 + x = P_A^0 \times \frac{1}{(1+3)} + P_B^0 \times \frac{3}{4}$
$0.25\, P_A^0 + 0.75\, P_B^0 = 1 + x$
$P_A^0 + 3P_B^0 = \frac{1+x}{0.25}$ ....(ii)
$1 = P_A^0 \cdot \frac{1}{8} + P_B^0 \cdot \frac{3}{8} + P_C^0 \cdot \frac{4}{8}$
$1 = P_A^0 \times \frac{1}{8} + P_B^0 \cdot \frac{3}{8} + 0.8 \times \frac{4}{8}$
$\frac{3P_B^0}{8} + \frac{P_A^0}{8} = 0.6$
$P_A^0 + 3P_B^0 = 4.8$ ....(iii)
From (ii) & (iii)
$\frac{1+x}{0.25} = 4.8$
$1 + x = 1.2$
$\boxed{x = 0.2}$
So from (i) & (iii) ; (iii) – (i)
$2P_B^0 = 4.8 - 2 = 2.8$
$P_B^0 = 1.4$ atm
$P_A^0 = 2 - P_B^0 = 2 - 1.4 = 0.6$ atm

44. **(4)**
& efficiency of a cell $= \frac{\Delta G}{\Delta H} \times 100$
$$84 = \frac{-nFE_{cell}}{\Delta H} \times 100$$
$$84 = \frac{-2 \times 96500 \times E_{cell}}{-256 \times 10^3} \times 100$$
$E_{cell} = 1.24$ volt

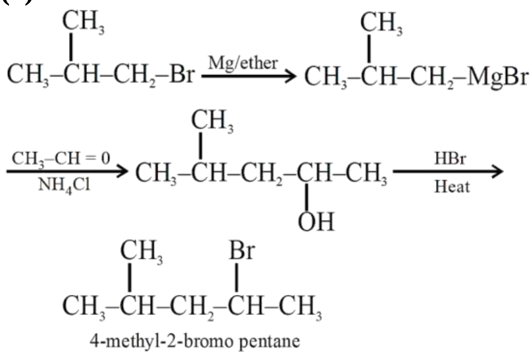
45. **(2)**


46. **(3)**


47. **(3)**

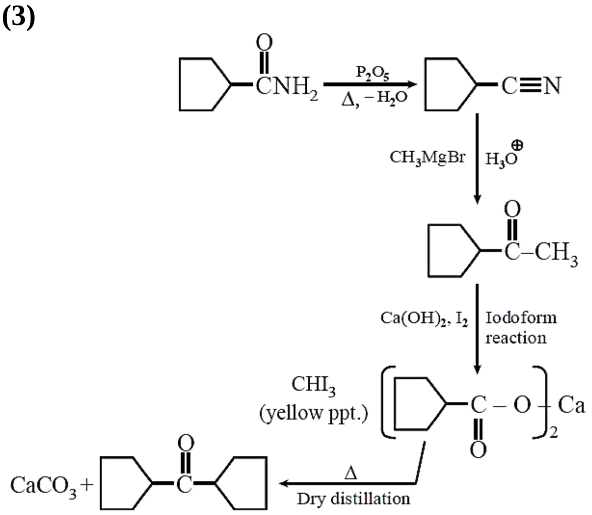


48. **(1)**

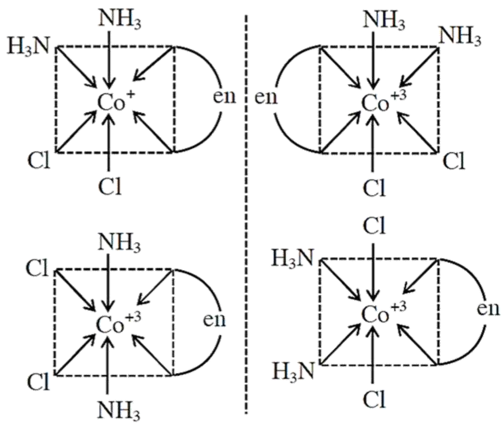


49. **(3)**

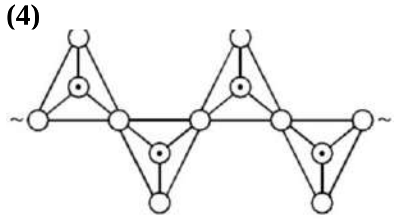
50. **(3)**



51. **(3)**



52. **(4)**



In this, per unit two shared oxygen atoms.

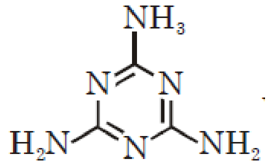
53. **(2)**

⇒ O_2 is paramagnetic (Using M.O.T.)

⇒ In KO_2, O_2^- (Superoxide ion) is present O_2^- is paramagnetic

54. **(6)**

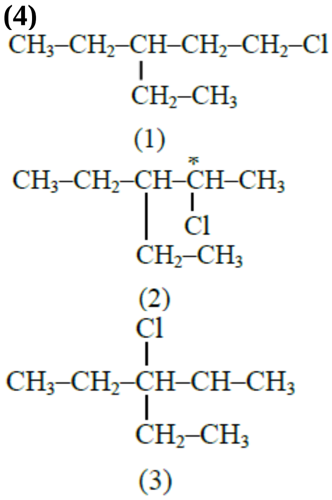
Structure of melamine is



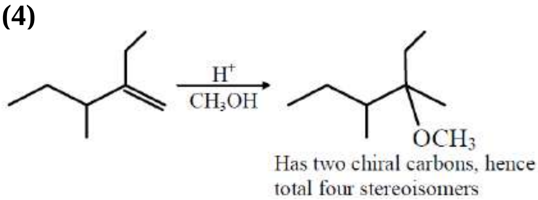
55. **(6)**

56. **(6)**

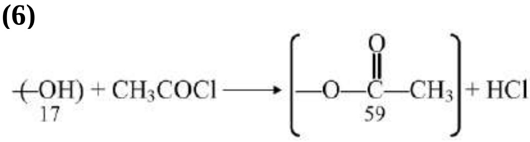
57. **(4)**



58. **(4)**



59. **(6)**



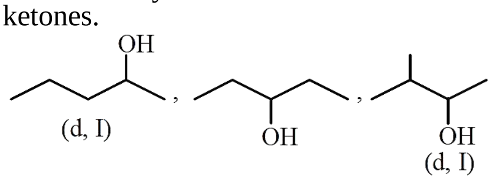
Mass gain due to incorporation of one acetyl group = 59 – 17 = 42

Net mass gain due to acetylation = 518 – 266 = 252

Hence, six hydroxyl groups (6 × 42 = 252) were present.

60. **(5)**

All secondary alcohol isomers can be oxidised to ketones.



MATHEMATICS

61. **(4)**

Here, $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0 \Rightarrow a = 1$

For $a = 1$, the equations become

$x + y + z = 1$

$x + y + z = 1$

$x + by + z = 0$

These equations give no solution for $b = 1$

$\Rightarrow S$ is singleton set

62. **(1)**

$(p \to q) \to ((\sim p \to q) \to q)$

$= (p \to q) \to ((p \vee q) \to q)$

$= (p \to q) \to ((\sim p \wedge \sim q) \to q)$

$= (p \to q) \to ((\sim p \vee q) \wedge (\sim q \vee q))$

$= (p \to q) \to (\sim p \vee q)$

$= (p \to q) \to (p \to q)$

$= T$

63. **(4)**

$5(\sec^2 x - 1 - \cos^2 x) = 2(2\cos^2 x - 1) + 9$

$5\left(\frac{1}{t} - 1 - t\right) = 2(2t - 1) + 9$

$5(1 - t - t^2) = 4t^2 + 7t$

$\Rightarrow 9t^2 + 12t - 5 = 0$

$t = \frac{1}{3}, -\frac{5}{3}$

$\Rightarrow \cos^2 x = \frac{1}{3}$

$\Rightarrow \cos 2x = \frac{2}{3} - 1 = -\frac{1}{3}$

$\Rightarrow \cos 4x = -\frac{7}{9}$

64. **(2)**

$P(A) + P(B) - 2P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - 2P(B \cap C) = \frac{1}{4}$

$P(A) + P(C) - 2P(A \cap C) = \frac{1}{4}$

$\Rightarrow \sum P(A) - \sum P(A \cap B) = \frac{3}{8}$

$\Rightarrow P(A \cup B \cup C)$

$= \sum P(A) - \sum P(A \cap B) + P(A \cap B \cap C)$

$= \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

65. **(3)**

Determinant simplifies to $3k$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = 3\begin{vmatrix} 1 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$

$= -3 \Rightarrow \quad k = -1$

66. **(4)**

$\Delta = \frac{1}{2}\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = 28$

$\Rightarrow k = 2$ (since $k \in I$)

$\Rightarrow$ Orthocentre is $\left(2, \frac{1}{2}\right)$

67. **(3)**

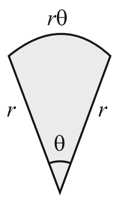
Length of wire = $r(\theta + 2) = 20$ m

Area $A = \frac{\theta}{2} r^2$

$\Rightarrow A(r) = 10r - r^2$

$\Rightarrow$ Area is maximum if $r = 5$.

Maximum area $A = 25$ sq. m



68. **(1)**

Here, $y = 2\tan^{-1}\left(3x^{\frac{3}{2}}\right)$

$\frac{dy}{dx} = \frac{9x^{\frac{1}{2}}}{1 + 9x^3} = \sqrt{x}g(x)$

$\Rightarrow g(x) = \frac{9}{1 + 9x^3}$

69. **(1)**

$(2 + \sin x)dy + \cos x(y + 1)dx = 0$

$(y + 1)(2 + \sin x) = C$

$\Rightarrow (1 + 1)(2 + 0) = C = 4$

$(y + 1) \cdot (2 + \sin x) = 4$

Put $x = \frac{\pi}{2}$

$y = \frac{1}{3}$

70. (2)

$$A^2 = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & -9 \\ -12 & 13 \end{bmatrix}$$

$$(3A^2 + 12A) = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{Adj } (3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

71. (2)

$(15a - 3b)^2 + (15a - 5c)^2 + (3b - 5c)^2 = 0$

Let, $15a = 3b = 5c = 45\lambda$

$\Rightarrow a = 3\lambda;\ b = 15\lambda;\ c = 9\lambda$

$\Rightarrow 2c = a + b$

b, c, a are in A.P.

72. (2)

Eccentricity, $e = \frac{1}{2}$

Let $2a$ be the length of major axis and $2b$ be the length of minor axis

$\Rightarrow \frac{a}{e} = 4$

$\Rightarrow a = 2$

Also, $b = \sqrt{3}$, as $e = \frac{1}{2}$

$\Rightarrow$ Equation of ellipse is $\frac{x^2}{4} + \frac{y^2}{3} = 1$

$\Rightarrow$ Equation of normal at $\left(1, \frac{3}{2}\right)$ is $4x - 2y = 1$

73. (2)

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$\frac{x^2}{a^2} - \frac{y^2}{4 - a^2} = 1$

$\Rightarrow a^2 = 8, 1, (a^2 \neq 8)$

$\Rightarrow \frac{x^2}{1} - \frac{y^2}{3} = 1$

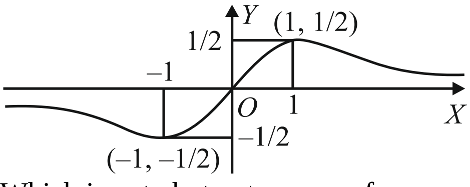
Hence equation of tangent at $P\left(\sqrt{2}, \sqrt{3}\right)$ is

$\frac{x\sqrt{2}}{1} - \frac{y\sqrt{3}}{3} = 1$

$\Rightarrow \sqrt{6}x - y = \sqrt{3}$

74. (3)

For, $f(x) = \frac{x}{1 + x^2}$ the curve has graph as shown



Which is onto but not one-one for,

$f : R \rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$

75. (2)

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x - \cot x}{8\left(x - \frac{\pi}{2}\right)^3}$$

Put $x - \frac{\pi}{2} = t;\ x = t + \frac{\pi}{2}$

$$\lim_{t \to 0} \frac{1}{8} \cdot \frac{-\sin t + \tan t}{t^3}$$

$$\lim_{t \to 0} \frac{1}{8} \cdot \frac{\sin t(1 - \cos t)}{t \cdot \cos t \cdot t^2}$$

$$= \frac{1}{8} \cdot 1 \cdot \frac{1}{2} = \frac{1}{16}$$

76. (2)

$\frac{dy}{dx}$ at $x = 0$ is 1

$\Rightarrow$ Slope of normal at (0, 1) is –1

$\Rightarrow$ Equation of normal is $x + y = 1$

77. (1)

Consider two sequences : 0, 4, 8 and 2, 6, 10

Take both numbers from either of these sequences.

Hence, probability $= \frac{{}^3C_2 + {}^3C_2}{{}^{11}C_2} = \frac{6}{55}$.

78. (4)

Let $S = ({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + \ldots$
$+ ({}^{21}C_{10} - {}^{10}C_{10})$

$\Rightarrow S = ({}^{21}C_0 - {}^{21}C_1 + \ldots + {}^{21}C_{10})$
$- ({}^{10}C_0 + {}^{10}C_1 + \ldots + {}^{10}C_{10})$

$\Rightarrow S = 2^{20} - 2^{10}$

79. (1)

$p = \frac{15}{25},\ q = \frac{10}{25},\ n = 10$

$\sigma^2 = npq = 10 \cdot \frac{3}{5} \cdot \frac{2}{5} = \frac{12}{5}$

80. (3)

Let $P\left(\frac{t}{2}, 4 - \frac{t^2}{4}\right)$ be point where circle touches the parabola $y = 4 - x^2$

$\Rightarrow$ Normal at P: $ty - x + \frac{t^3}{4} - \frac{7t}{2} = 0$ to the parabola passes through centre (3) of the circle
(0, β).

$\Rightarrow t^3 - 14t + 4\beta t = 0$ …(1)

Also, radius $r = \frac{|\beta|}{\sqrt{2}}$

$\Rightarrow t^4 + 4t^2 + 8\beta t^2 - 32t^2 - 128\beta + 256 + 16\beta^2 = 16r^2$

$\Rightarrow t^4 + (8\beta - 28)t^2 - 128\beta + 256 + 8\beta^2 = 0$ …(2)

From equation (1) and (2), we get

Either $\beta = 8 \pm 4\sqrt{2}$ for $t = 0$

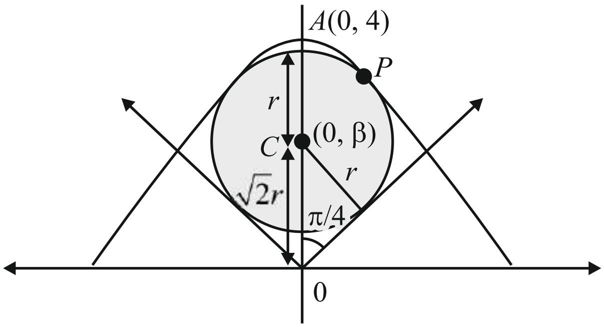
or $\beta = \dfrac{-\sqrt{2} \pm \sqrt{17}}{\sqrt{2}}$ for $t^2 = 14 - 4\beta$

As, $r = \dfrac{|\beta|}{\sqrt{2}} \Rightarrow r = 4\sqrt{2} \pm 4, \dfrac{\sqrt{17}-\sqrt{2}}{2}$

$\Rightarrow$ Minimum possible radius,

$r = \dfrac{\sqrt{17}-\sqrt{2}}{2}$

[But of the given options r = $4\left(\sqrt{2}-1\right)$ is minimum]

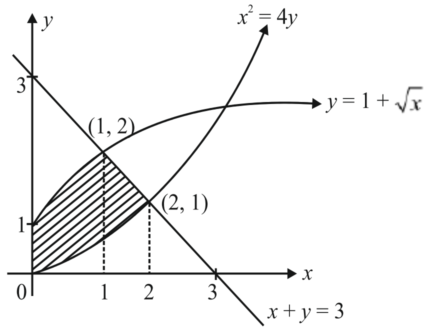


81. **(5)**

Required area

$= \int_0^1 \left(1+\sqrt{x}\right)dx + \frac{1}{2}(3\times 1) - \int_0^2 \frac{x^2}{4}dx$

$= \frac{5}{2}$ sq. units



82. **(168)**

$\dfrac{x-1}{1} = \dfrac{y+2}{4} = \dfrac{z-3}{5} = \lambda$

Let midpoint of PQ be M which lines on the plane

$\Rightarrow M(x, y, z) = (1 + \lambda, 4\lambda - 2, 5\lambda + 3)$

$2(1 + \lambda) + 3(4\lambda - 2) - 4(5\lambda + 3) + 22 = 0$

$\Rightarrow -6\lambda + 6 = 0 \Rightarrow \lambda = 1$

$\Rightarrow M(2, 2, 8), P(1, -2, 3)$

$PM = \sqrt{1+16+25} = \sqrt{42}$

$PQ = 2\sqrt{42}$

83. **(4)**

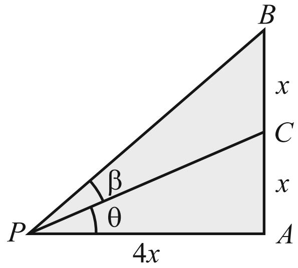
$\tan(\theta + \beta) = \dfrac{1}{2}$

and $\tan\theta = \dfrac{1}{4}$

$\Rightarrow \dfrac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} = \dfrac{1}{2}$

$\Rightarrow 9\tan\beta = 2$

$\Rightarrow \tan\beta = \dfrac{2}{9}$



84. **(73)**

Equation of plane is $\begin{vmatrix} x-1 & y+1 & z+1 \\ 1 & -2 & 3 \\ 2 & -1 & -1 \end{vmatrix} = 0$

$5x + 7y + 3z + 5 = 0$

Distance from $(1, 3, -7)$ $= \dfrac{|5+21-21+5|}{\sqrt{83}}$

$= \dfrac{10}{\sqrt{83}}$

85. **(1)**

$I_n = \int \tan^n x\,dx,\ n > 1$

$= \int \tan^{n-2} x(\sec^2 x - 1)dx$

$= \dfrac{\tan^{n-1} x}{n-1} - I_{n-2} + C$

$\Rightarrow I_n + I_{n-2} = \dfrac{\tan^{n-1} x}{n-1} + C$

$\Rightarrow I_6 + I_4 = \dfrac{\tan^5 x}{5} + C$

Given, $I_4 + I_6 = a\tan^5 x + bx^3 + C$

$\Rightarrow a = \dfrac{1}{5},\ b = 0$

86. **(2)**

$|\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}\cdot\vec{c} = 9$

$\Rightarrow |\vec{c}|^2 - 2\vec{a}\cdot\vec{c} = 0$

and $|\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\sin 30° = 3$

$\Rightarrow 3\times|\vec{c}|\times\dfrac{1}{2} = 3$

$\Rightarrow |\vec{c}| = 2$

$\vec{a}\cdot\vec{c} = 2$

87. (485)

$X : 4L, 3M$; $Y : 3L, 4M$

Possible combinations

	(1)	(2)	(3)	(4)
X	$3L$	$2L, 1M$	$1L, 2M$	$3M$
Y	$3M$	$1L, 2M$	$2L, 1M$	$3L$

$\therefore$ Number of ways $= {}^4C_3 \cdot {}^4C_3 + {}^4C_2 \cdot {}^3C_1 \cdot {}^3C_1 \cdot {}^4C_2 + {}^4C_1 \cdot {}^3C_2 \cdot {}^3C_2 \cdot {}^4C_1 + {}^3C_3 \cdot {}^3C_3$

$= 485$

88. (330)

Partially differentiating, we get

$f'(x) - x = \text{constant} = \lambda$

$\Rightarrow f(x) = \dfrac{x^2}{2} + \lambda x + k$

$f(0) = 0 \Rightarrow k = 0$

$\dfrac{1}{2} + \lambda = 3 \quad \Rightarrow \quad \lambda = \dfrac{5}{2}$

$$\sum_{n=1}^{10} f(n) = a\sum_{n=1}^{10} n^2 + b\sum_{n=1}^{10} n$$

$$= \frac{1}{2}\frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \times \frac{n(n+1)}{2}$$

$$= \frac{1}{2} \cdot \frac{10 \times 11 \times 21}{6} + \frac{5}{2} \times \frac{10 \times 11}{2} = 330$$

89. (11)

$$x^2 + nx + \frac{n^2 - 31}{3} = 0$$

Let I and $I + 1$ be the roots of the equation

$2I + 1 = -n$ … (1)

$I(I + 1) = \dfrac{n^2 - 31}{3}$ …(2)

Eliminating I from (1) and (2), we get

$$\frac{n^2 - 1}{4} = \frac{n^2 - 31}{3}$$

$\Rightarrow n^2 = 121$

$\Rightarrow n = 11$

90. (2)

$$I = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} (\text{cosec}^2 x - \text{cosec}\ x \cdot \cot x)dx = 2$$