JEE Mains (11th)

Sample Paper - II

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS		CHEMISTRY		Mathematics	
1.	(2)	31.	(3)	61.	(3)
2.	(1)	32.	(2)	62.	(1)
3.	(1)	33.	(4)	63.	(4)
4.	(2)	34.	(1)	64.	(1)
5.	(1)	35.	(4)	65.	(2)
6.	(3)	36.	(2)	66.	(3)
7.	(1)	37.	(4)	67.	(3)
8.	(1)	38.	(4)	68.	(3)
9.	(2)	39.	(2)	69.	(4)
10.	(4)	40.	(2)	70.	(2)
11.	(2)	41.	(3)	71.	(1)
12.	(3)	42.	(4)	72.	(1)
13.	(2)	43.	(1)	73.	(4)
14.	(2)	44.	(1)	74.	(2)
15.	(3)	45.	(1)	75.	(2)
16.	(4)	46.	(1)	76.	(3)
17.	(1)	47.	(3)	77.	(1)
18.	(1)	48.	(2)	78.	(3)
19.	(3)	49.	(4)	79.	(1)
20.	(3)	50.	(4)	80.	(3)
21.	(4)	51.	(4)	81.	(20)
22.	(85)	52.	(25)	82.	(8)
23.	(35)	53.	(3)	83.	(1)
24.	(80)	54.	(20)	84.	(2)
25.	(8)	55.	(4)	85.	(0)
26.	(4)	56.	(8)	86.	(2)
27.	(1)	57.	(81)	87.	(4)
28.	(52)	58.	(80)	88.	(3)
29.	(34)	59.	(7)	89.	(2)
30.	(8)	60.	(26)	90.	(1)

PHYSICS

1. **(2)**

$$p = p_0 e^{-\alpha t^3}$$

$$\frac{dp}{dt} = p_0 e^{-\alpha t^3}(-3\alpha t^2)$$

$$\frac{dp}{p} = -3\alpha t^2 dt$$

$= -3 \times 1 \times 1 \times 10^{-2} = -0.03$

% error = 3%

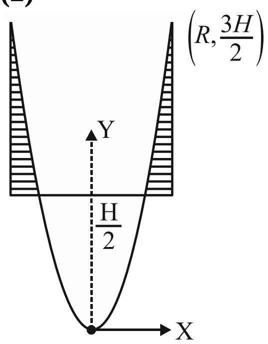
2. **(1)**

$$K = \frac{1}{2}mv^2 \Rightarrow \frac{dK}{dS} = mv\frac{dV}{dS}$$

3. **(1)**

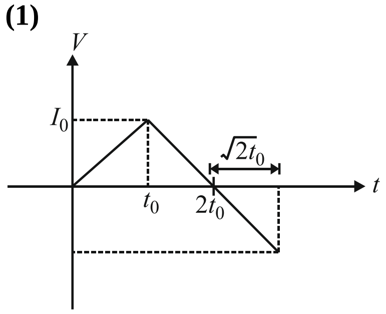
Angular momentum is conserved only about C since torque of friction is zero only about that point.

4. **(2)**



$$y = \frac{\omega^2 x^2}{2g}, \frac{3H}{2} = \frac{\omega^2 R^2}{2g}, \omega = \frac{\sqrt{3gH}}{R}$$

5. **(1)**



$$\therefore t = (2+\sqrt{2})t_0$$

6. **(3)**

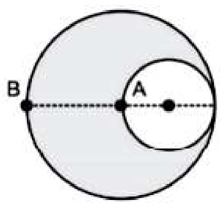
$$T = \frac{2U\sin 30°}{g\cos 30°}$$

7. **(1)**

$W = (P_2 - P_1)(V_2 - V_1) = 24$ J

$P = W \times 25 = 600$ W

8. **(1)**



$$E_A = \frac{\sigma(R/2)}{3\varepsilon_0} = \left(\frac{\sigma R}{6\varepsilon_0}\right)$$

$$E_B = \frac{\sigma R}{3\varepsilon_0} - \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$= \frac{\sigma R}{3\varepsilon_0} - \frac{\sigma R}{54\varepsilon_0} \quad \Rightarrow E_B = \frac{17}{54}\left(\frac{\sigma R}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right| = \frac{1\times 54}{6\times 17} = \left(\frac{9}{17}\right)$$

9. **(2)**

$$l = \frac{\lambda_1}{2} \quad \therefore \lambda_1 \Rightarrow 2l$$

$$\therefore f_1 = \frac{v}{2l}$$

Similarly $f_2 = \dfrac{v}{2l\left(1+\frac{x}{l}\right)} \simeq \dfrac{v}{2l}\left(1-\frac{x}{l}\right)$

$$\therefore \Delta f = f_1 - f_2 = \frac{v}{2l}\left(1-1+\frac{x}{l}\right) = \frac{vx}{2l^2}$$

10. **(4)**

$$k(R\theta)\times R = -\left(\frac{MR^2}{2} + mR^2\right)\times\frac{d^2\theta}{dt^2}$$

$$\Rightarrow T = 2\pi\sqrt{\frac{(M+2m)}{2k}}$$

11. **(2)**

Use weight = Buoyant force

12. **(3)**

$$l_1 + e = \frac{V}{4f} \qquad l_2 + e = \frac{3V}{4f}$$

$$l_2 - l_1 = \frac{V}{2f}$$

$$\Delta V = (\Delta l_2 + \Delta l_1)\cdot 2f$$

$= 2 \times 512 \times 0.2 = 204.8$ cm/s

13. **(2)**

$\omega A = 10$

$\omega^2 A = 50$

$\therefore \quad A = 2$ cm, and $\omega = 5$ rad/s

$$\therefore \quad v = v_0\sqrt{1-\frac{x^2}{A^2}} = 10\left(\frac{\sqrt{3}}{2}\right) = 5\sqrt{3} \text{ cm/s}$$

14. **(2)**

C.M. of circular sector from centre is

$$r = \frac{4R\sin\theta/2}{3\theta} \qquad \Rightarrow r = \frac{2R}{\pi}$$

$$T = 2\pi\sqrt{\frac{\frac{MR^2}{2}}{mg\frac{2R}{\pi}}} = \frac{2\pi}{2}\sqrt{\frac{\pi R}{g}}$$

15. **(3)**

Initial phase angle say is ϕ, at $t = 0$

$$\frac{A}{\sqrt{2}} = A\sin\left(\frac{2\pi}{T}\times 0+\phi\right)$$

$$\phi = \frac{\pi}{4}, \frac{3\pi}{4}$$

As particle is returning towards left, $\phi = \frac{3\pi}{4}$

16. **(4)**

$$7mgR = \frac{1}{2}\times\left(\frac{7}{5}mr^2\right)\times\frac{v^2}{r^2}$$

$$\Rightarrow v = \sqrt{10gR}$$

$$\therefore\ N = \frac{mv^2}{(R-r)} = \frac{10mgR}{\frac{4R}{5}} = \frac{50}{4}\text{ mg}$$

17. **(1)**

AB = d

$$\therefore OA = \frac{d}{\sqrt{3}}$$

From parallel axis theorem

$$I_0 = 3\times\left[\frac{2}{5}M\left(\frac{d}{2}\right)^2 + M\left(\frac{d}{\sqrt{3}}\right)^2\right] = \frac{13}{10}Md^2$$

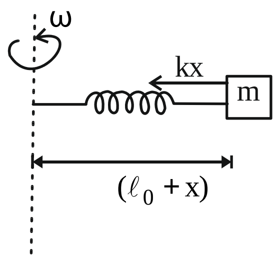
$I_A = I_0 + 3Md^2 + Md^2$

$$= \frac{13}{10}Md^2 + Md^2$$

$$= \frac{23}{10}Md^2$$

$$\therefore \frac{I_0}{I_A} = \frac{13}{23}$$

18. **(1)**



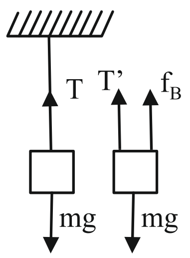
At elongated position (x),

$F_{radial} = mr\omega^2$

$\therefore kx = m\ (l+x)\omega^2$

$$\Rightarrow x = \frac{m\ell\omega^2}{k-m\omega^2}$$

19. **(3)**



$$\frac{F}{A} = y.\frac{\Delta\ell}{\ell}$$

$\Delta\ell \propto F$ ……(i)

T = mg

$$T = mg - f_B = mg - \frac{m}{\rho b}.\rho\ell.g$$

$$= \left(1-\frac{p\ell}{pb}\right)mg$$

$$= \left(1-\frac{2}{8}\right)mg$$

$$T' = \frac{3}{4}mg$$

From (i)

$$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$$

$$\Delta\ell' = \frac{3}{4}\cdot\Delta\ell = 3\text{ mm}$$

20. **(3)**

$$\frac{\Delta T}{\Delta t} = k\left(\frac{T_f+T_i}{2} - T_0\right)$$

$$\Rightarrow \frac{50-40}{300} = K\left(\frac{90}{2}-20\right)$$

$$\Rightarrow \frac{40-T}{300} = k\left(\frac{40+T}{2}-20\right)$$

$$\Rightarrow \frac{10}{40-T} = \frac{25\times 2}{40+T-40}$$

$$\Rightarrow \frac{1}{40-T} = \frac{5}{T} \qquad \Rightarrow T = 200-5T$$

$\Rightarrow 6T = 200$

$\Rightarrow T = 33°C$

21. **(4)**

As in elastic or in elastic collision, momentum is conserved.

$\therefore$ $P_i = P_f$

P_i = Initial momentum

P_f = Final momentum

$Mv = (2m + m)\ V_f$

$$\Rightarrow V_f = \frac{mv}{2m+M}$$

Here due to collision $\frac{5}{6}$ th of kinetic energy is lost.

$\therefore$ Remaining kinetic energy,

$$K_f = \frac{1}{6}Ki$$

$$\Rightarrow \frac{1}{2}(2m+M)\times\frac{(mv)^2}{(2m+M)^2} = \frac{1}{6}\times\frac{1}{2}mv^2$$

$$\Rightarrow \frac{m}{2m+M} = \frac{1}{6}$$

$\Rightarrow$ 6m = 2m + M

$\Rightarrow$ M = 4m $\qquad\Rightarrow \frac{M}{m} = 4$

22. **(85)**

$|n-80|=5$

n = 85 or 75

23. **(35)**

$\frac{(2n+1)}{4L}\times V = n_0$

$\Rightarrow L = \frac{(2n+1)V}{4\times n_0}$ = 25 cm (fundamental mode)

Length of water column = 60 – 25 = 35 cm

24. **(80)**

$V = \int a dt$

25. **(8)**

$\alpha = \frac{\tau}{l} = \frac{\mu mgR}{\left(\frac{2}{5}\right)mR^2} = \frac{5\mu g}{2R} = \frac{5\times 0.1\times 10}{2\times 1}$

= 2.5 rad/s^2

$t = \frac{\omega_0}{\alpha} = \frac{20}{2.5} = 8$ s

26. **(4)**

Using momentum conservation at collision,

$mA\omega - m\frac{A}{2}\omega = 2mv'$

$v' = \frac{A}{4}\omega \Rightarrow \text{amplitude} = \left(\frac{A}{4}\right)$

27. **(1)**

$t = \frac{T}{4} = \frac{1}{4}\times 2\pi\sqrt{\frac{m}{k}} = 1$ s

28. **(52)**

9 MSD = 10 VSD

9 × 1 mm = 10 VSD

$\therefore$ 1 VSD = 0.9 mm

LC = 1 MSD – 1 VSD = 0.1 mm

Reading = MSR + VSR × LC

10 + 8 × 0.1 = 10.8 mm

Actual reading = 10.8 – 4 = 10.4 mm

radius $= \frac{d}{2} = \frac{10.4}{2}$ = 5.2 mm

= 52 × 10^{-2} cm

29. **(34)**

Given, radius of sphere = (7.50 ± 0.85) cm

$\therefore$ r = 7.50 and dr = 0.85

We know, volume of a sphere $v = \frac{4}{3}\pi r^3$

Taking log both sides, we get

$\ln v = \ln\frac{4\pi}{3} + 3\ln r$

Differentiating both sides,

$\frac{dv}{v} = 0 + 3\frac{dr}{r}$

$\therefore$ Fractional error in volume $\frac{dv}{v} = 3\frac{dr}{r}$

$\therefore$ % error in volume,

$\frac{dv}{v}\times 100 = 3\frac{dr}{r}\times 100$

$= 3\times\frac{0.85}{7.50}\times 100$ = 34%

30. **(8)**

Given, power of transmitted signal, P_i = 0.1 kW = 0.1 × 10^{3}W = 10^{2}W

Rate of attenuation, R = –5 dB/km

Length of cable, 1 = 20 km

Powder received at receiver, P_X = 10^{-x} W

Total loss,

$\beta = R\times 1 = -5\times 20 = -100 dB$

$\because$ Gain $(\beta) = 10\log_{10}\frac{P_0}{P_i}$

$\therefore \beta = -100 = 10\log_{10}\frac{P_0}{P_i}$

$\Rightarrow -10 = \log_{10}\frac{P_0}{P_i} \Rightarrow 10^{-10} = \frac{P_0}{P_i}$

$\Rightarrow P_0 = 10^{-10}P_i = 10^{-10}\times 10^2 = 10^{-8} \Rightarrow P_0 = 10^{-8}V$

Hence, x = 8

CHEMISTRY

31. **(3)**

$\text{Number of meq} = \left(\frac{\text{mass}}{\text{Equivalent mass}}\right)\times 1000$

$50 = \left(\frac{\text{mass}}{\frac{39+16+1}{1}}\right)\times 1000$

$\text{Mass} = \frac{50\times 56}{1000} = 2.8\text{g}$

32. **(2)**

$O_2 = \pi * 2p_x^1 = \pi^* 2p_y^1$ $O_2^- = \pi * 2p_x^2 = \pi^* 2p_y^1$

$N_2 = \pi 2p_x^2 = \pi 2p_y^1$ $N_2^- = \pi 2p_x^2 = \pi 2p_y^2, \pi 2p_y^1$

[N_2^- have one unpaired electrons i.e. paramagnetic]

33. **(4)**

XeO_2F_2 $\quad\quad$ PCl_5

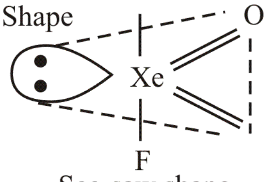
Hybridisation $= \frac{1}{2}[V + M - C + A]$

$H = \frac{1}{2}[V + M - C + A]$

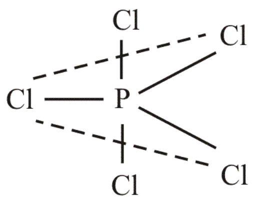
$H = \frac{1}{2}[8 + 2 - 0 + 0]$ $\quad\quad$ $H = \frac{1}{2}[5 + 5 - 0 + 0]$

$\frac{10}{2} = 5[sp^3d]$ $\quad\quad$ H = 5 $\quad$ sp^3d

Shape



See-saw shape



Trigonal bipyramidal

34. **(1)**

Size; Mg < Na

Hence, ionization energy of Na < Mg

Size; Si < Al

Hence, ionization energy of Si > Al

Mg = $1s^2\ 2s^2\ 2p^6\ 3s^2$

Al = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^1$

Mg has stable configuration. hence its ionization energy will be higher than Al.

Na < Mg > Al < Si

35. **(4)**

Cr (24) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^1\ 3d^5$ (Half filled-full filled rule)

Magnetic quantum number varies from +l to –l through zero for a given value of l.

Ag(47) = $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^1\ 4d^{10}$ (Half filled-full filled rule)

In Ag except $5s^1$, all subshells are fully filled which contains total $46e^-$ out of which $23e^-$ have anticlockwise spin, hence total number of e^- having clockwise spin

= 23 + 1($5s^1$) = 24

Total number of e^- having anticlockwise spin = 23

van der Waal's radii of a molecule is more than its covalent radii.

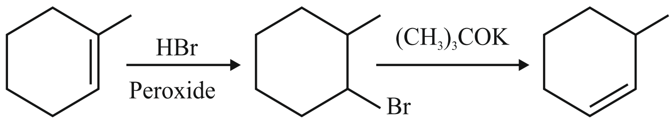
36. **(2)**

37. **(4)**

$CH-\underset{H}{\overset{CH_3}{C}}-CH_2-CH_2-COONa \xrightarrow[CaO,\ \Delta]{NaOH} CH_3-\underset{H}{\overset{CH_3}{C}}-CH_2-CH_3 + Na_2CO_3$ (2-methylbutane)

$CH_3-CH_2-\overset{CH_3}{CH}-CH_2-COONa \xrightarrow[CaO,\ \Delta]{NaOH} CH_3-CH_2-\overset{CH_3}{CH}-CH_3$ (2-methylbutane)

38. **(4)**



39. **(2)**

The solubility of $Ag_2CrO_4 = [CrO_4^{-2}]$

$\therefore [CrO_4^{-2}] = \frac{K_{sp}}{[Ag^+]^2} = \frac{1.1\times10^{-12}}{(0.1)^2} = 1.1\times10^{-10}\ mol\ L^{-1}$

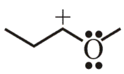
40. **(2)**

41. **(3)**

The metals of 7, 8 and 9 groups do not form hydrides and hence this region of periodic table is known as hydride gap.

42. **(4)**

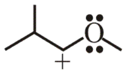
is most stable carbocation because this carbocation is stabilized by the +R effect of –I group



(IV)

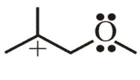
is less stable than carbocation I because it has less hyper conjugative structure than carbocation IV

due to less number of alpha hydrogens



(I)

Carbocation II is more stable than carbocation III because carbocation II is a tertiary carbocation but



(II)

(II) less stable than I and IV because it is not resonance stabilized like carbocation I and IV

43. **(1)**

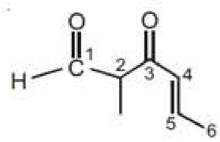
$[OH^-] = 2 \times 10^{-7}$

$\Rightarrow$ pOH = – log [OH] = – log 2×10^{-7}

$\Rightarrow$ pOH = 7 –log 2.0 = 7 – 0.3010

$\Rightarrow$ pOH = 6.7

44. **(1)**



Aldehydes get higher priority over ketone and alkene in numbering of principal C-chain.

$\therefore$ 3-keto-2-methylhex-4-enal

45. **(1)**

46. (1)

Same volume of gas is being diffused.

As per Grahma's law diffusion,

$$\frac{\frac{V_1}{t_1}}{\frac{V_2}{t_2}} = \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

or $$\frac{t_1}{t_2} = \sqrt{\frac{M_2}{M_1}}$$

Let gas 1 be O_2

$$\therefore \quad \frac{\sqrt{M_{O_2}}}{t_{O_2}} = \frac{\sqrt{M_2}}{t_2} \text{ or } \sqrt{\frac{M_{O_2}}{t_{O_2}^2}} = \sqrt{\frac{M_2}{t_2^2}}$$

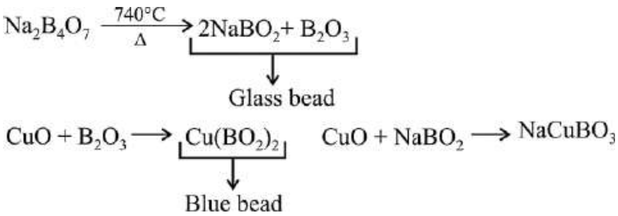
Which is possible when $M_2 = 2 \text{ g mol}^{-1}$ and $t_2 = 2.5$ s

47. (3)

m = represents the orientation of orbital in magnetic field.

m = orbitals

48. (2)



49. (4)

Boron and aluminium halides behave as Lewis acid. Al forms $[AlF_6]^{3-}$ ion but B does not form $[BF_6]^{3-}$ ion. The $p\pi$-$p\pi$ back bonding occurs in the halides of boron and not in those of aluminium.

50. (4)

$$M = \frac{\text{Volume strength}}{11.2}$$

Volume strength = 3.57×11.2 = 40 volume

51. (4)

Functional groups present are

$-CONH_2$, $-OH$, $-COON$ and $-CN$

52. (25)

53. (3)

$$\boxed{E_n = \frac{-313.6}{n^2}} \qquad E = -34.84$$

$$\therefore -34.84 = \frac{-313.6}{n^2} \qquad n^2 = \frac{-313.6}{-34.84} = 9$$

$$n = \sqrt{9} = 3$$

$$\boxed{n = 3}$$

54. (20)

$H_2C_2O_4 + Ca(OH)_2 \rightarrow CaC_2O_4 + 2H_2O$

Number of milli equivalents of H_2SO_4

= Number of milliequivalents of $Ca(OH)_2$

$N_1 \times V_1 = N_2 \times V_2$

$M_1 \times$ n-factor $\times V_1 = M_2 \times$ n-factor $\times V_2$

$0.05 \times 2 \times 40 = 0.1 \times 2 \times V_2$

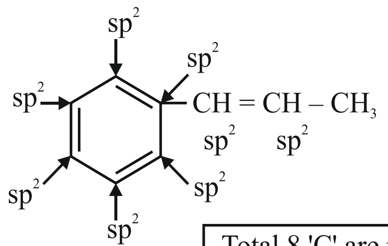
$V_2 = 20$ ml

55. (4)

	C_2H_5OH	$+ CH_3COOH$	$\rightleftharpoons CH_3COOC_2H_5$	$+ H_2O$
t = 0	1	1	0	0
Equilibrium	$1-\frac{2}{3}$	$1-\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$

$$K_c = \frac{\frac{2}{3}\times\frac{2}{3}}{\frac{1}{3}\times\frac{1}{3}} = 4$$ [considering 1L container]

56. (8)



Total 8 'C' are present and have sp^2 hybridisation

57. (81)

Let melting temperature = T

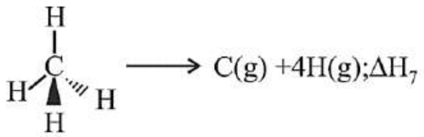
$$\therefore \quad \Delta S_{fusion} = \frac{\Delta H_{fusion}}{T}$$

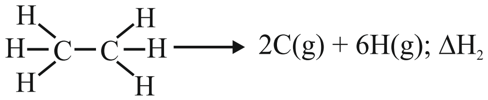
$$\Rightarrow \quad T = \frac{\Delta H_{fusion}}{\Delta S_{fusion}}$$

$$= \frac{7.26\ K\ cal\ mol^{-1}}{6.73\ cal\ mol^{-1}\ k^{-1}} = 1078.75\ K$$

$\therefore$ T = 1078.75 K or (1078.75 – 273)°C = 805.75°C

58. (80)





Given: $\Delta H_1 = 360$ kcal mol^{-1}; $\Delta H_2 = 620$ kcal mol^{-1}

Also, $\Delta H_1 = 4 \times$ Bond energy of C – H = 360 kcal mol^{-1}

$\Rightarrow$ Bond energy of C – H = 90 kcal mol^{-1}

Now, $\Delta H_2 = 6 \times$ Bond energy of C – H + Bond energy of C – C

$\Delta H_2 = 6 \times 90$ kcal mol^{-1} + Bond energy of C – C

$\Delta H_2 = 620$ kcal mol^{-1}

$\Rightarrow$ 620 kcal mol^{-1} = 540 kcal mol^{-1} + Bond energy of C – C or, Bond energy C – C = 80 kcal mol^{-1}

59. (7)

7 (All except Be and Sr)

60. (26)

a = 12, b= 20, c = 30

$3 \times 12 - 2 \times 20 + 30 = 26$

MATHEMATICS

61. (3)

First arrange 15 boys in 15! ways.

16 gaps will be created.

Select any 3 gaps out of 16 gaps in $^{16}C_3$ ways.

Arrange 3 boys to be separated in these selected 3 gaps in 3! ways.

Number of ways $= 15! \times {}^{16}C_3 \times 3!$

$= 15! \times {}^{16}P_3$

62. (1)

We have, $|z - (1 + i)|^2 = 2$

$\Rightarrow (x - 1)^2 + (y - 1)^2 = 2$ (Putting $z = x + iy$)

$\Rightarrow x^2 + y^2 = 2(x + y)$ …... (i)

Let, $\omega = h + ik = \dfrac{2}{z} = \dfrac{2}{x+iy} = \dfrac{2(x-iy)}{x^2+y^2}$

So, $h = \dfrac{2x}{x^2+y^2}, k = \dfrac{-2y}{x^2+y^2}$

$\Rightarrow h - k = \dfrac{2(x+y)}{x^2+y^2} = 1$ (from equation (i))

$\therefore$ Locus of the point $\omega(h, k)$ will be $x - y = 1$.

63. (4)

$p \Rightarrow (\sim p \vee q)$ is false means p is true and $\sim p \vee q$ is false.

$\Rightarrow$ p is true and both $\sim p$ and q are false.

$\Rightarrow$ p is true and q is false.

64. (1)

Given, $(\pm ae, 0) = (\pm 3, 0)$

$\Rightarrow ae = 3$

$\Rightarrow a^2 e^2 = 9$

$\Rightarrow b^2 + a^2 = 9$ …...(i)

$\because 2x + y - 4 = 0$

$\Rightarrow y = -2x + 4$

is the tangent to the hyperbola

$\therefore (4)^2 = a^2 (-2)^2 - b^2$ $(\because c^2 = a^2 m^2 - b^2)$

$\Rightarrow 4a^2 - b^2 = 16$ ...…. (ii)

On solving equations (i) & (ii), we get,

$a^2 = 5, b^2 = 4$

$\therefore$ Equation of hyperbola is $\dfrac{x^2}{5} - \dfrac{y^2}{4} = 1$

$\Rightarrow 4x^2 - 5y^2 = 20$

65. (2)

As we know that $p \to q \equiv \sim p \vee q$

Hence $p \vee \sim (p \to \sim q) \equiv p \vee \sim (\sim p \vee \sim q)$

as we know that $\sim (p \vee q) \equiv \sim p \wedge \sim q$

Hence $p \vee \sim (\sim p \vee \sim q) \equiv p \vee (p \wedge q)$

Therefore $p \vee (p \wedge q) \equiv p$

66. (3)

Let, the number are

$k, k + 2, k + 4, k + 6, k + 8$

$\dfrac{k + (k+2) + (k+4) + (k+6) + (k+8)}{5} = 61$

$\Rightarrow 5k + 20 = 61 \times 5 = 305 \Rightarrow k = 57$

So, the difference between highest & lowest = 8

67. (3)

Let, $\alpha = \beta - d$ and $\gamma = \beta + d$

$\beta - d, \beta, \beta + d$ are the roots

$\beta - d + \beta + \beta + d = -6 \Rightarrow \beta = -2$

$(\beta - d)\beta(\beta + d) = 42$

$\Rightarrow (-2 - d)(-2)(-2 + d) = 42$

$\Rightarrow (2 + d)2(2 - d) = -42$

$\Rightarrow 4 - d^2 = -21 \Rightarrow d^2 = 25 \Rightarrow d = \pm 5$

$\Rightarrow$ The roots are –7, –2, 3

$\Rightarrow |\alpha| + |\beta| + |\gamma| = 12$

68. (3)

Here, $a = 10,\ d = -\frac{3}{7}$

Then $t_n = 10 + (n-1)\left(-\frac{3}{7}\right)$

t_n is positive, if $10 + (n-1)\left(-\frac{3}{7}\right) > 0$

or $70 - 3(n-1) > 0$

or $73 > 3n$ or $24\frac{1}{3} > n$

$\therefore$ First 24 terms are positive.

$\therefore$ Sum of the positive terms

$= S_{24} = \frac{24}{2}\left[2 \times 10 + 23 \times \frac{-3}{7}\right]$

$= 12\left[20 - \frac{69}{7}\right] = \frac{852}{7}$

69. (4)

Let $a - d$, a & a + d are three numbers in A.P.

Given, $(a - d) + (1) + (a + d) = 9$

$\Rightarrow a = 3$ and $(a - d)^2 + a^2 + (a + d)^2 = 35$

$\Rightarrow 3a^2 + 2d^2 = 35$

$\Rightarrow d = \pm 2$, if $a = 3$, $d = 2$ $\Rightarrow 1, 3, 5, \ldots\ldots$

$S_n = \frac{n}{2}(2 + (n-1)2) = n^2$

If $a = 3$, $d = -2$ $\Rightarrow 5, 3, 1, \ldots\ldots$

$S_n = \frac{n}{2}[10 + (n-1)(-2)]$

$= n(5 - n + 1) = n(6 - n)$

70. (2)

Given the sum

$S = (1!)^2 + (2!)^2 + (3!)^2 + \ldots\ldots + (2008!)^2$

Since the digit at unit place is zero in $n!$ for $n \geq 5$

Hence,

$S = (1)^2 + (2)^2 + (6)^2 + (24)^2$ + numbers having zero at unit place

= 617 + all other numbers having zero at unit place

71. (1)

$x^2 + x + 1 = 0$

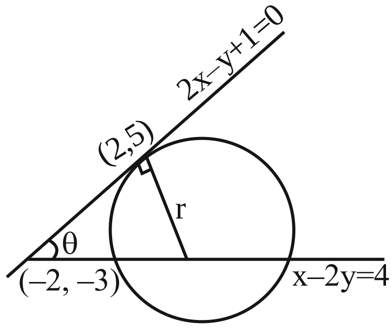
$\Rightarrow x = \omega, \omega^2$

$\Rightarrow \alpha = \omega$ and

$\Rightarrow \alpha^{2020} = \omega^{2020} = (\omega^3)^{673}.\omega = \omega$

$\Rightarrow \beta^{2020} = (\omega^2)^{2020} = (\omega^3)^{2 \times 673} \times \omega^2 = \omega^2$

72. (1)



$\tan\theta = \frac{2 - \frac{1}{2}}{1 + 2\left(\frac{1}{2}\right)} = \frac{3}{4}$

$\frac{r}{\sqrt{8^2 + 4^2}} = \tan\theta \Rightarrow \frac{3}{4} = \frac{r}{4\sqrt{5}}$

$\Rightarrow r = 3\sqrt{5}$ $\Rightarrow$ diameter $= 6\sqrt{5}$ units

73. (4)

We know that, $p \to q \equiv \sim p \vee q$

$\therefore$ Equivalent statement will be

$9 \not> 10$ or $3^2 = 5$

or $9 \leq 10$ or $3^2 = 5$

74. (2)

$\because$ $n(1) = 7$, $n(2) = 13$

$\because$ $(A - B) = n(A) - n(A \cap B) = 7 - 5 = 2$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 7 + 13 - 5 = 15$

$n(A \times B) = n(1) \times n(2) = 7 \times 13 = 91$

$n\{(A \cup B) \times (A \cap B)\} = 15 \times 5 = 75$

75. (2)

Sum of 16 observations $= 16 \times 16 = 256$

Sum of resultant 18 observations

$= 256 - 16 + (3 + 4 + 5) = 252$

Mean of observations $= \frac{252}{18} = 14$

76. (3)

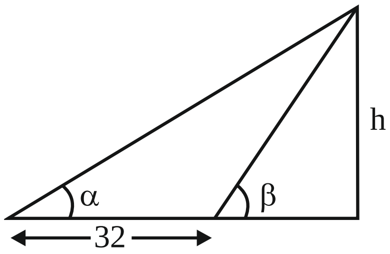
From triangle inequality we know that

$|z_1 + z_2| \geq ||z_1| - |z_2||$

Hence

$\left|z + \frac{1}{z}\right| = \left|z - \left(-\frac{1}{z}\right)\right| \geq |z| - \left|-\frac{1}{z}\right| \geq 3 - \frac{1}{3} = \frac{8}{3}$

77. (1)



$\cot\alpha = \frac{3}{5}, \cot\beta = \frac{2}{5}, 32 = h\cot\alpha - h\cot\beta$

$h = \left(\frac{32}{\cot\alpha - \cot\beta}\right) = \frac{32}{1/5} = 160\,\text{m}$

78. (3)

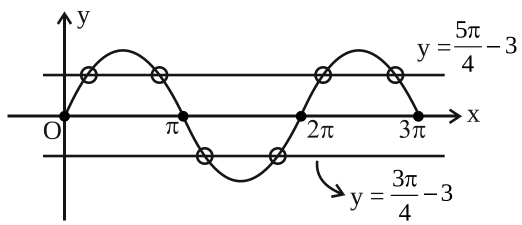
$\cot^2(\sin x + 3) = 1 = \cot^2\frac{\pi}{4}$

$\Rightarrow \sin x + 3 = n\pi \pm \frac{\pi}{4}$

Also, $2 \le \sin x + 3 \le 4$

$\Rightarrow \sin x + 3 = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$\Rightarrow \sin x = \frac{3\pi}{4} - 3$ or $\frac{5\pi}{4} - 3$



Total six solutions

79. (1)

Tangent to $y^2 = 32x$ is $y = mx + \frac{8}{m}$ and

Tangent to $x^2 = 108y$ is $y = mx - 27m^2$

$\therefore \frac{8}{m} = -27m^2$

$\Rightarrow m^3 = \frac{-8}{27}$

$\Rightarrow m = \frac{-2}{3}$

$\therefore$ x-intercepts is $27m = 27\left(\frac{-2}{3}\right) = -18$

80. (3)

We have, $x^2 + 18x + 30 = 2\sqrt{x^2 + 18x + 45}$

Let $x^2 + 18x + 30 = t$

Therefore,

$t = 2\sqrt{t + 15}$

$\Rightarrow t^2 - 4t - 60 = 0$

$\Rightarrow t^2 - 10t + 6t - 60 = 0$

$\Rightarrow (t - 10)(t + 6) = 0$

$\Rightarrow t = 10, -6$

Put $x^2 + 18x + 30 = 10$

$\Rightarrow x^2 + 18x + 20 = 0$

So, product $P = 20$

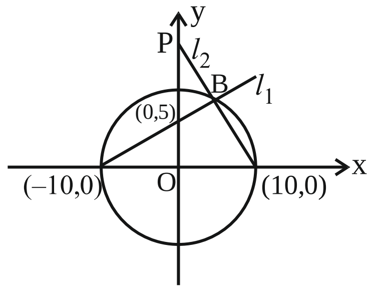
and $t = x^2 + 18x + 30 = -6 < 0$ (not possible)

Hence, product of roots is 20.

81. (20)

Slope of $l_1 = \frac{1}{2}$

Slope of $l_2 = -2$



Equation of l_2

$y = -2(x - 10) \Rightarrow y + 2x = 20$

Hence, $t = 20$

82. (8)

$4200 = 42 \times 100 = (2 \times 3 \times 7)(2^2 \times 5^2)$

$\Rightarrow 4200 = 2^3 \cdot 3^1 \cdot 5^2 \cdot 7^1$

If divisor is neither divisible by 3 nor by 5, then it should not have any power of 3 as well as 5.

Number of divisors

$= (3 + 1)(1)(1 + 1) = 8$

83. (1)

$\because$ Angles A, B, C are in arithmetic progression and

$\angle B = \frac{\pi}{4}$

Then $A = \frac{\pi}{4} - \theta,\ C = \frac{\pi}{4} + \theta$

Hence $\tan\left(\frac{\pi}{4} - \theta\right)\tan\frac{\pi}{4}\tan\left(\frac{\pi}{4} + \theta\right)$

$= \frac{1 - \tan\theta}{1 + \tan\theta}.1.\frac{1 + \tan\theta}{1 - \tan\theta} = 1$

84. (2)

The centres and radii of given circles are

$C_1(-1, -4), C_2(2, 5)$

and $r_1 = \sqrt{1 + 16 + 23} = \sqrt{40}$,

$r_2 = \sqrt{4 + 25 - 9} = \sqrt{20}$

Now,

$C_1C_2 = \sqrt{(2+1)^2 + (5+4)^2} = \sqrt{90}$

And $r_1 + r_2 = \sqrt{40} + \sqrt{20}$

Here, $C_1C_2 < r_1 + r_2$

$\therefore$ Two common tangents can be drawn

85. **(0)**

Equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$

Equation of tangent is $y=mx+\sqrt{9m^2-16}$

$\Rightarrow \sqrt{9m^2-16}=2\sqrt{5} \Rightarrow m=\pm 2$

$\Rightarrow a+b$ = sum of roots = 0

86. **(2)**

$200 < {}^nC_0 + {}^nC_1 + \ldots + {}^nC_n < 400$

$\Rightarrow 200 < 2^n < 400$

$\Rightarrow n = 8$

$T_{(r+1)} = {}^8C_r \left(\sqrt[4]{x^{-3}}\right)^{8-r} \left(a\sqrt[4]{x^5}\right)^r$

$\Rightarrow T_{(r+1)} = {}^8C_r\, a^r\, x^{2r-6}$

For this term to be independent of x,

$2r - 6 = 0 \Rightarrow r = 3$

$T_4 = {}^8C_3\, a^3 \Rightarrow 448 = 56a^3 \Rightarrow a^3 = 8$

$\Rightarrow a = 2$

87. **(4)**

We know that if $A + B + C = \pi$

$\sin 2A + \sin 2B + \sin 2C$

$= 4\sin A \sin B \sin C$

Here $A = 27°$, $B = 98°$, $C = 55°$

So, $\frac{\sin 54° + \sin 196° + \sin 110°}{\sin 27° \sin 98° \sin 55°} = 4$

88. **(3)**

We have, $2^{1+|\cos x|+\cos^2 x+|\cos x|^3+\ldots\infty} = 4$

$\Rightarrow 2^{\frac{1}{1-|\cos x|}} = 2^2$

$\Rightarrow \frac{1}{1-|\cos x|} = 2$

$\Rightarrow 2-2|\cos x| = 1$

$\Rightarrow |\cos x| = \frac{1}{2}$

$\Rightarrow \cos x = \pm\frac{1}{2}$

$\therefore x = n\pi \pm \frac{\pi}{3}$

Then, $k = 3$

89. **(2)**

Number of ways to select 8 out of 10 things

${}^{10}C_8 = {}^{10}C_2 = \frac{10\times 9}{2}$

Number of ways of grouping = $\frac{8!}{1!2!5!}$

Total ways $\left(\frac{10\times 9}{2}\right)\left(\frac{8!}{1!2!5!}\right) = \frac{1}{2}\left(\frac{10!}{1!2!5!}\right)$

Hence, $\lambda = 0.5 \Rightarrow 4\lambda = 2$

90. **(1)**

$(ax + b)^{2020} = (b + ax)^{2020}$

Given, coefficient of x^2 = coefficient of x^3

$\Rightarrow {}^{2020}C_2\, b^{2018}\, a^2 = {}^{2020}C_3\, b^{2017}\, a^3$

$\Rightarrow \frac{b}{a} = \frac{{}^{2020}C_3}{{}^{2020}C_2} = \frac{2020!}{3!2017!}\frac{2!2018!}{2020!} = \frac{2018}{3}$

$\Rightarrow \frac{3b}{2018a} = 1$