JEE Mains (11th)

Sample Paper - III

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	31.	(4)	61.	(4)
2.	(2)	32.	(2)	62.	(3)
3.	(1)	33.	(4)	63.	(3)
4.	(1)	34.	(3)	64.	(1)
5.	(2)	35.	(1)	65.	(4)
6.	(4)	36.	(4)	66.	(3)
7.	(4)	37.	(4)	67.	(1)
8.	(1)	38.	(4)	68.	(3)
9.	(2)	39.	(2)	69.	(1)
10.	(2)	40.	(3)	70.	(2)
11.	(3)	41.	(2)	71.	(4)
12.	(3)	42.	(2)	72.	(4)
13.	(3)	43.	(2)	73.	(4)
14.	(3)	44.	(4)	74.	(3)
15.	(1)	45.	(2)	75.	(2)
16.	(3)	46.	(4)	76.	(1)
17.	(1)	47.	(4)	77.	(4)
18.	(3)	48.	(2)	78.	(4)
19.	(1)	49.	(4)	79.	(4)
20.	(3)	50.	(3)	80.	(2)
21.	(68)	51.	(15)	81.	(9)
22.	(8)	52.	(19)	82.	(4)
23.	(10)	53.	(16)	83.	(2)
24.	(2)	54.	(30)	84.	(2)
25.	(2)	55.	(57)	85.	(2)
26.	(7)	56.	(23)	86.	(1)
27.	(30)	57.	(9)	87.	(10)
28.	(5000)	58.	(21)	88.	(3)
29.	(1.0)	59.	(2)	89.	(6)
30.	(10)	60.	(4)	90.	(6)

PHYSICS

1. **(2)**

The horizontal range is the same for the angles of projection θ and (90° – θ)

$$t_1 = \frac{2u\sin\theta}{g}$$

$$t_2 = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

$$\therefore\ t_1 t_2 = \frac{2u\sin\theta}{g} \times \frac{2u\cos\theta}{g}$$

$$= \frac{2}{g}\left[\frac{u^2\sin 2\theta}{g}\right] = \frac{2}{g}R$$

where $R = \frac{u^2\sin 2\theta}{g}$

Hence, $t_1 t_2 \propto R$. (as g is constant)

2. **(2)**

Suppose F = upthrust due to buoyancy

Then while descending, we find

$\mathrm{Mg} - \mathrm{F} = \mathrm{M}\alpha$ ...(i)

when ascending, we have:

$\mathrm{F} - (\mathrm{M} - \mathrm{m})\mathrm{g} = (\mathrm{M} - \mathrm{m})\alpha$ ...(ii)

Solving eqns. (i) and (ii), we get;

$$m = \left[\frac{2\alpha}{\alpha + g}\right]M$$

3. **(1)**

Limiting friction between block and slab

$= \mu_s m_A g$

$= 0.6 \times 10 \times 9.8 = 58.8\mathrm{N}$

But applied force on block A is 100 N. So that block will slip over slab.

Now kinetic friction works between block and slab

$F_k = \mu_k m_A g = 0.4 \times 10 \times 9.8 = 39.2$ N

This kinetic friction helps to move the slab

$\therefore$ Acceleration of slab $= \frac{39.2}{m_B} = \frac{39.2}{40} = 0.98$ m/s^2

4. **(1)**

Fore of friction $= \mu mg = m\omega^2 a$

$= m(2\pi v)^2 a$

$$\Rightarrow v = \frac{1}{2\pi}\sqrt{\frac{\mu g}{a}}$$

5. **(2)**

For parallel combination of first two identical springs of spring constant k_1, effective spring constant

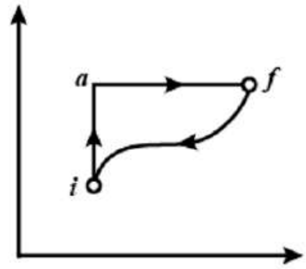
$$k_p = 2k_1$$

Now, springs of spring constants k_p and k_2 are joined in series, so the force constant or the spring constant of the system is,

$$\frac{1}{k_s} = \frac{1}{k_p} + \frac{1}{k_2}$$

$$\therefore\ k_s = \left(\frac{1}{k_p} + \frac{1}{k_2}\right)^{-1} = \left(\frac{1}{2k_1} + \frac{1}{k_2}\right)^{-1}.$$

6. **(4)**



From process iaf

Find ΔU first, $\Delta Q = \Delta W + \Delta U$

$80 = 50 + \Delta U$

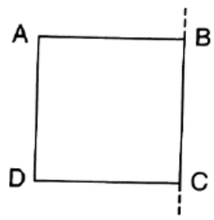
30 cal $= \Delta U$

Use this ΔU for process if

$\Delta Q = \Delta W + \Delta U$

$\Delta Q = -30 + (-30) = -60$ cal

7. **(4)**



$$\frac{AB}{BC} = 2$$

$$\therefore AB = DC = \frac{l}{3}$$

and $BC = AD = \frac{l}{6}$

Similarly, $m_{AB} = m_{DC} = \frac{m}{3}$

and $m_{BC} = m_{AD} = \frac{m}{6}$

Now, $I = 2I_{AB} + I_{AD} + I_{BC}$

$$= 2\left[\frac{m}{3}\left(\frac{l}{3}\right)^2 \times \frac{1}{3}\right] + \left[\left(\frac{m}{6}\right)\left(\frac{l}{3}\right)^2\right] + [0]$$

$$= \frac{2}{81}ml^2 + \frac{1}{54}ml^2 = \frac{7}{162}ml^2$$

8. **(1)**

From conservation of energy

Potential energy = translational KE + rotational KE

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}\right)mR^2\frac{v^2}{R^2}$$

or $\frac{7}{10}mv^2 = mgh$ or $v \geq \sqrt{\frac{10}{7}gh}.$

9. **(2)**

Given that; $\dfrac{R_A}{R_B}=K_1$ and $\dfrac{g_A}{g_B}=K_2$

$$\frac{(v_e)_A}{(v_e)_B}=\sqrt{\frac{g_A R_A}{g_B R_B}}=\sqrt{K_1 K_2}.$$

10. **(2)**

$$AC=CB=\sqrt{l^2+d^2}$$

Change in length $=AC+CB-AB$

$$=2\sqrt{l^2+d^2}-2l$$

Let T be the tension in the wire, then longitudinal stress $=\dfrac{T}{\pi r^2}$

Longitudinal strain $=\dfrac{\text{change in length}}{\text{original length}}$

$$=\frac{2\sqrt{l^2+d^2}-2l}{2l}$$

$$\therefore Y=\frac{\text{long. stress}}{\text{long. strain}}=\frac{\left(\dfrac{T}{\pi r^2}\right)}{\dfrac{2\sqrt{l^2+d^2}-2l}{2l}}$$

$$=\frac{Tl}{\pi r^2(\sqrt{l^2+d^2}-l)}$$

$$\therefore \frac{Y\pi r^2(\sqrt{l^2+d^2}-l)}{l}=Y\pi r^2\left[1+\frac{d^2}{2l^2}-1\right]$$

$$=\frac{Y\pi r^2 d^2}{2l^2}$$

11. **(3)**

The centre of mass of the 'block plus wedge' must move with speed

$$\frac{mu}{m+\eta m}=\frac{u}{1+\eta}=v_{\text{CM}}$$

$$\therefore \quad \frac{1}{2}mu^2-mgh=\frac{1}{2}(m+\eta m)v_{\text{CM}}^2.$$

$$\frac{1}{2}mu^2-mgh=\frac{1}{2}m(1+\eta)\frac{u^2}{(1+\eta)^2}$$

$$u=\sqrt{2gh\left(1+\frac{1}{\eta}\right)}$$

12. **(3)**

Maximum force of friction = kmg.

$\therefore$ maximum acceleration of insect $=a_1=\dfrac{kmg}{m}=kg$

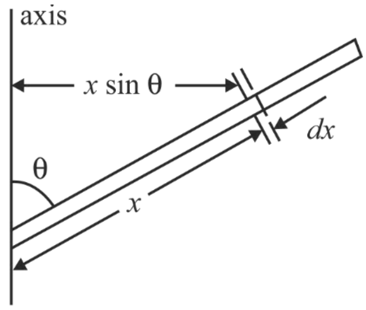
and maximum acceleration of stick $=a_2=\dfrac{kmg}{M}$.

$\therefore$ acceleration of insect with respect to stick

$$=a=a_1-(-a_2)=kg\left(1+\frac{m}{M}\right).$$

$$\therefore \quad L=\frac{1}{2}at^2 \text{ or } t^2=\frac{2L}{a}=\frac{2ML}{kg(M+m)}.$$

13. **(3)**

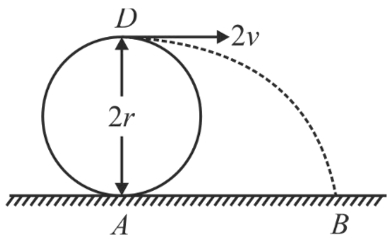


Mass of the element $=\left(\dfrac{m}{x}\right)dx$.

Moment of inertia of the element about the axis $=\left(\dfrac{m}{l}dx\right)(x\sin\theta)^2$

$$I=\frac{m}{l}\sin^2\theta\cdot\int_0^l x^2dx=\frac{ml^2}{3}\sin^2\theta$$

14. **(3)**



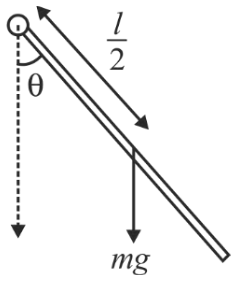
At the point of leaving the wheel, the blob of mud is at a height $2r$ above the road and has a horizontal velocity $2v$

Let t = time of travel from D to B. Then, $2r=\dfrac{1}{2}gt^2$

or $t=2\sqrt{\dfrac{r}{g}}$ and $AB=(2v)t$

$$AB=2v\times 2\sqrt{\frac{r}{g}}=4v\sqrt{\frac{r}{g}}$$

15. **(1)**



$T=2\pi\sqrt{\dfrac{l}{g}}$ where l = length of simple pendulum = length of rod.

$$\tau=(mg)\frac{l}{2}\sin\theta$$

For small $\theta,\tau=\dfrac{1}{2}mgl\theta=-I\alpha=-\left(\dfrac{ml^2}{3}\right)\alpha$

or $\alpha=-\left(\dfrac{3g}{2l}\right)\theta$

Time period $=2\pi\sqrt{\dfrac{2l}{3g}}<T$.

16. (3)

Let M, R be the mass and radius of the planet, and g be the acceleration due to gravity on its surface.

Then, $V=\sqrt{2Rg}$ and $GM = R^2g$.

Gravitational potential at the surface is $-\dfrac{GM}{R}$ and at the centre is $-\dfrac{3GM}{2R}$. In going from the surface to the centre, loss in gravitational PE

$$=m\left[-\frac{GM}{R}-\left(-\frac{3}{2}\frac{GM}{R}\right)\right]=\frac{1}{2}\frac{GMm}{R}=\frac{1}{2}mv^2$$

or $v^2=\dfrac{GM}{R}=Rg=\dfrac{V^2}{2}$ or $\dfrac{V}{\sqrt{2}}$.

17. (1)

$AB \rightarrow$ constant p, increasing V; $\therefore$ increasing T

$BC \rightarrow$ constant T, increasing V, decreasing p

$CD \rightarrow$ constant V, decreasing p; $\therefore$ decreasing T

$DA \rightarrow$ constant T, decreasing V, increasing p

Also, BC is at a higher temperature than AD.

18. (3)

Let p_A, p_B be the initial pressures in A and B respectively. When the gases double their volumes at constant temperature, their pressures fall to $\dfrac{p_A}{2}$ and $\dfrac{p_B}{2}$

$\therefore$ for $A, p_A-\dfrac{p_A}{2}=\Delta p$ or

$p_A=2\Delta p$

for $B, p_B-\dfrac{p_B}{2}=1.5\Delta p$

or $p_B=3\Delta p$

$\therefore \quad \dfrac{p_A}{p_B}=\dfrac{2}{3}$

Also, $p_AV=\dfrac{m_A}{M}RT$

and $p_BV=\dfrac{m_B}{M}RT$.

$\therefore \quad \dfrac{p_A}{p_B}=\dfrac{m_A}{m_B}$

$\therefore \quad \dfrac{m_A}{m_B}=\dfrac{2}{3}$

or $3m_A=2m_B$.

19. (1)

Area of spherical shell = $4\pi R^2$

Rate of heat flow $=P=k(4\pi R^2)\dfrac{T}{d}$, here d = thickness of shell.

20. (3)

Let a = initial amplitude due to S_1 and S_2 each.

$I_0 = k(4a^2)$, where k is a constant.

After reduction of power of S_1, amplitude due to S_1 = $0.6a$.

Due to superposition,

$a_{max}=a+0.6a=1.6a$, and

$a_{min}=a-0.6a=0.4a$

$$\frac{I_{max}}{I_{min}}=\left(\frac{a_{max}}{a_{min}}\right)^2=(1.6a/0.4a)=16.$$

21. (68)

$$\frac{T-20}{100-20}=\frac{60-0}{100-0}$$

$T = 48 + 20 = 68°C$

22. (8)

Weight on earth = mg

$=m\times\dfrac{GM}{R^2}=72\ N$

Weight at height, $h = 2R$will be

$$mg'=m\left(\frac{GM}{r^2}\right)=m\times\frac{GM}{(R+2R)^2}$$

$$=\frac{GMm}{9R^2}=\frac{72}{9}=8\text{ N}$$

23. (10)

Spring constant,

k = 1960 N/m = 1960000 dyne/cm

Let x cm be the maximum compression of the spring.

Decrease in potential energy of the block = increase in potential energy of the spring

$$mg[h+x]=\frac{1}{2}kx^2$$

$$2000\times980[40+x]=\frac{1}{2}\times1960000x^2$$

or $40+x=\dfrac{x^2}{2}$ or x = 10 cm.

24. (2)

From work-energy theorem,

$\Delta_{KE}=W_{net}$

or $K_f-K_i=\int Pdt$

or $\dfrac{1}{2}mv^2=\int_0^2\left(\dfrac{3}{2}t^2\right)dt$

or $v^2=\left[\dfrac{t^3}{2}\right]_0^2$

$\therefore$ v = 2 m/s.

25. (2)

Force constant, $K=\dfrac{YA}{L}$ or $K\propto Y$

$\therefore \dfrac{K_A}{K_B}=\dfrac{Y_A}{Y_B}=2$

26. (7)

$\eta = 0.07$ kg m^{-1} s^{-1}

$dv = 1$ m/s, $dx = 1$ mm $= 1 \times 10^{-3}$m

$A = 0.1$ m^2

$$\therefore\ F = \eta A\frac{dv}{dx} = 0.07 \times 0.1 \times \frac{1}{1\times 10^{-3}} = 7N.$$

27. (30)

Since, the block of ice at 0°C is large, the whole of ice will not melt, hence final temperature is 0°C.

$\therefore\ Q_1$ = heat given by water in cooling upto 0°C $= ms\Delta\theta = 80 \times 1 \times (30 - 0)$

= 2400 cal

If m gm be the mass of ice melted, then

$Q_2 = mL_F = m \times 80$

Now, $Q_2 = Q_1$

$m \times 80 = 2400$ or $m = 30$ gm.

28. (5000)

$P = \frac{2}{3}E$ or $E = \frac{3}{2}P$

$\therefore$ Total energy $= EV = \frac{3}{2}PV$

For H_e : $1500 = \frac{3}{2}PV, PV = 1000$

For N_2 : $E'V = \frac{5}{2} \times 2PV = 5PV = 5 \times 1000$

= 5000 J

29. (1.0)

The frequencies are in the ratio of 5 : 7 : 9. Hence, it is a COP.

Now, $425 = 5\left(\frac{v}{4l}\right)$

$\therefore\ l = \frac{5v}{4\times 425} = \frac{5\times 340}{4 \times 425}$ = 1.0 m.

30. (10)

When the man is approaching the factory:

$$n' = \left(\frac{v+v_o}{v}\right)n = \left(\frac{320+2}{320}\right)800 = \left(\frac{322}{320}\right)800$$

When the man is going away from the factory,

$$n'' = \left(\frac{v-v_o}{v}\right)n = \left(\frac{320-2}{320}\right)800 = \left(\frac{318}{320}\right)800$$

$$\therefore\ n' - n'' = \left(\frac{322-318}{320}\right)800 = 10.$$

CHEMISTRY

31. (4)

$$\underset{n = 3 \times 2 = 6}{\underbrace{0 \qquad\qquad -3}}$$

Eq.wt = mol.wt/n factor $= \frac{28}{6} = 4.67$

32. (2)

Percentage of $C = \frac{12}{44} \times \frac{0.44}{0.30} \times 100 = 40\%$

Percentage of $H = \frac{2}{18} \times \frac{0.18}{0.30} \times 100 = 6.6\%$

Percentage of O = 100 – (40 + 6.6) = 53.4%

Element	%	Molar ratio	Simplest ratio
C	40	$\frac{40}{10} = 3.3$	$\frac{3.3}{3.3} = 1$
H	6.6	$\frac{6.6}{1} = 6.6$	$\frac{6.6}{3.3} = 2$
O	53.4	$\frac{53.4}{16} = 3.3$	$\frac{3.3}{3.3} = 1$

Hence, empirical formula = CH_2O

$$n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}} = \frac{60}{30} = 2$$

$\Rightarrow$ Molecular formula of the compound = $(CH_2O)_2$

= $C_2H_4O_2$

33. (4)

$$\underset{10\text{ g}}{MgCO_3} \longrightarrow \underset{4\text{ g}}{MgO} + \underset{0.1\text{ mol}}{CO_2}$$

Molar mass of $MgCO_3$ = 24 + 12 + 3 × 16 = 84 g mol^{-1}

Molar mass of MgO = 24 + 16 = 40 g mol^{-1}

Molar mass of CO2 = 12 + 2 × 16 = 44 g mol^{-1}

40 g of MgO will be obtained from

$\frac{84}{40}$ × 4 g of $MgCO_3$ = 8.4 g of $MgCO_3$

% purity of $MgCO_3$ $= \frac{8.4}{10} \times 100 = 84\%$

34. (3)

K.E. = hυ – hυ$_0$ = 6.2 – 5.0 = 12 eV

1 eV = 1.6 × 10^{-19} J

then 1.2 eV = 1.2 × 1.6 × 10^{-19} J = 1.92 × 10^{-19} J

35. (1)

36. (4)

$$\lambda_p = \frac{h}{\sqrt{2eVm_p}}$$

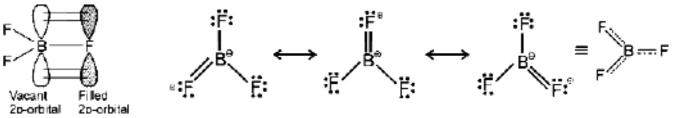
$$\lambda_{Be^{3+}} = \frac{h}{\sqrt{2 \times 3eVm_{Be^{3+}}}} = \frac{h}{\sqrt{2\times 3eV \times 9m_p}}$$

Hence, $\frac{\lambda_{Be^{3+}}}{\lambda_p} = \sqrt{\frac{2eVm_p}{2\times 3eV\times 9m_p}} = \frac{1}{3\sqrt{3}}$

37. **(4)**

Large jump between I.E.$_3$ and I.E.$_4$ suggests that the element has three valence electrons.

38. **(4)**



Decrease in B–F bond length is due to delocalised pπ–pπ bonding between filled p-orbital of F atom and vacant p-orbital of B atom.

39. **(2)**

$$r_1 \propto \frac{1}{\sqrt{M_1}} \text{ or } \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

Given $r_1 = 3\sqrt{3}r_2 \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}} = 3\sqrt{3}$

or $3\sqrt{3} = \sqrt{\frac{M_2}{2}} \Rightarrow 27 = \frac{M_2}{2}$

$M_2 = 27 \times 2 = 54$

Now, $12 \times n + 2n - 2 = 54 \Rightarrow 14n = 56 \Rightarrow n = 4$

40. **(3)**

Work done by the gas in the cyclic process = Area bounded (ABCA) = $5P_1V_1$

41. **(2)**

$2SO_2(g) + O_2 \rightleftharpoons 2SO_3(g)$

Kp = 4.0 atm^{-2}

$$Kp = \frac{(SO_3)^3}{(SO_2)^2(O_2)}$$

Give that at equilibrium the amount of SO_2 and SO_3 is the same so

$$\frac{(SO_3)^3}{(SO_2)^2(O_2)} = 4 \;\Rightarrow\; [O_2] = \frac{1}{4} = 0.25 \text{ atm}$$

42. **(2)**

Alkali metal carbonates except Li_2CO_3 are stable towards heat because they most basic in nature and basic character increase down the group and thermal stability increases down the group.

Bigger CO_3^{2-} anion is polarised by smaller Li+ and thus readily decomposes to give CO_2 gas.

$$Li_2CO_3 \xrightarrow{\Delta} Li_2O + CO_2$$

43. **(2)**

$$4B + 3O_2 \xrightarrow{1173\text{ K}} 2B_2O_3$$

$$2B(s) + N_2(g) \xrightarrow{1173\text{ K}} 2BN(s)$$

44. **(4)**

$$\overset{+1}{Ag_2}O + H_2O_2 \longrightarrow 2\overset{0}{Ag} + H_2O + O_2$$

In this reaction Ag_2O is reduced to metallic silver by hydrogen peroxide.

45. **(2)**

46. **(4)**

(I) sp^3 'N', (III) sp^3 'N' and –I effect, (II) sp^2 'N', (IV) Aromaticity (lp delocalised)

47. **(4)**

48. **(2)**

49. **(4)**

50. **(3)**

$2Fe(NO_3)_3 + 3Na_2CO_3 \rightarrow Fe_2(CO_3)_3 + 6NaNO_3$

mole 2.5 3.6

mole/stoichiometric coefficient 1.25 1.2

Limiting reagent is Na_2CO_3 so moles of $NaNO_3$ should be formed = 3.6 × 2 = 7.2

% yield $= \frac{6.3}{7.2} \times 100 = 87.5$

51. **(15)**

Given $E_{metal} = 2 \times 8 = 16$

$$\frac{Weight_{oxide}}{Weight_{metal}} = ?$$

$eq_{metal} = eq_{oxide}$

$$\frac{W_{metal}}{16} = \frac{W_{oxide}}{16+8}$$

$$\therefore \quad \frac{W_{oxide}}{W_{metal}} = \frac{24}{16} = \frac{3}{2} = 1.5$$

52. **(19)**

x = 4	Period
y = 11	Group

8 + 11 = 19

53. **(16)**

	No. of π bond
$XeOF_2$	1
XeO_2F_4	2
XeO_3	3
XeO_4	4
XeO_3F_2	3
$XeOF_4$	1
XeO_2F_2	2

54. **(30)**

The *rms* velocity of a gas = $\sqrt{\frac{3P}{d}}$

$$c_{rms} = \sqrt{\frac{3 \times 1.2 \times 10^5}{4}} = 0.9 \times 10^5$$

$$= \sqrt{9 \times 10^4} = 3 \times 10^2 = 300 \text{ m s}^{-1}$$

55. **(57)**

When both P and V are changing

$\Delta H = \Delta U + \Delta(PV) = \Delta U + (P_2V_2 - P_1V_1)$

$= 40 + (20 - 30) = 57$ L-atm

56. **(23)**

$$[H^+] = \frac{10^{-2} + 10^{-4}}{2} = \frac{0.01010}{2} = 0.00505 \text{ M}$$

$\therefore$ pH$=3 - \log 5.05 \approx 2.3$

(Taking $\log\ 5.05 \approx \log\ 5 \approx 0.7$)

57. **(9)**

9 (x = 3, y = 4, z = 2)

x = MOH; y = H_2O_2; z = O_2

58. **(21)**

x = 4

B, A*l*, In & T*l* are solid at 40°C. Melting point for Gallium is 30°C.

y = 4

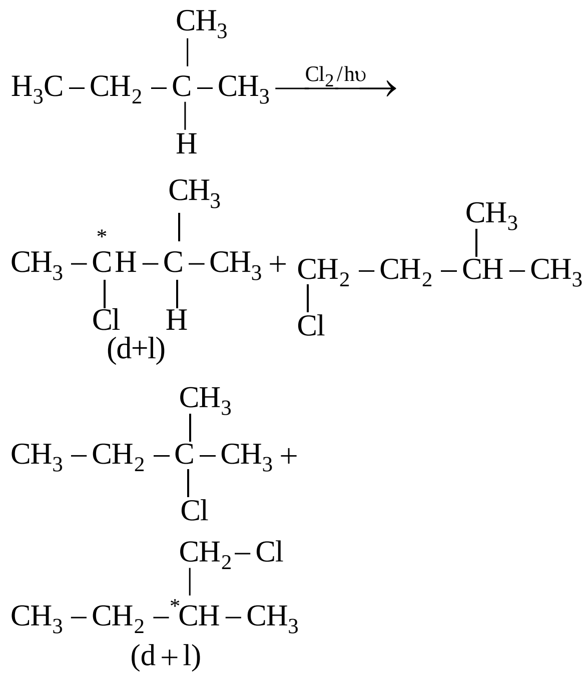
I.E. : B > A*l* < Ga < In < T*l*

z = 3

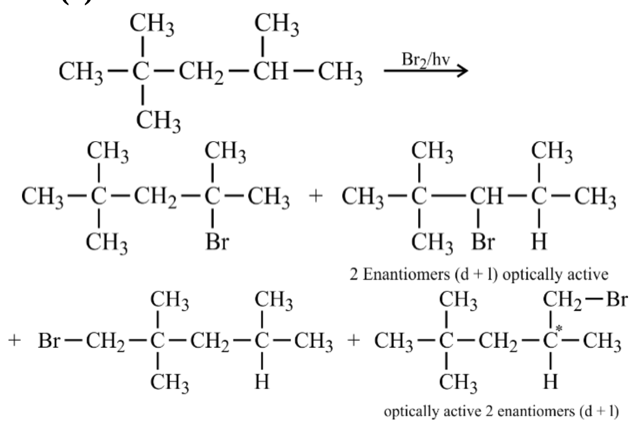
Al is third most abundant element after oxygen and silicon. So it has to be most abundant element in the family.

$\Rightarrow$ x + 2y + 3z = 4 + (2 × 4) + (3 × 3) = 21.

59. **(2)**



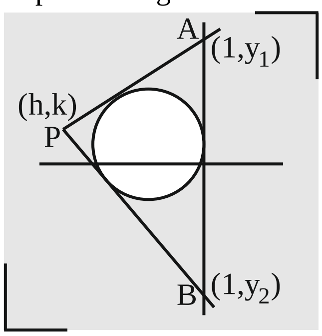
60. **(4)**



MATHEMATICS

61. **(4)**

Equation of pair of tangents *PA* and *PB* is



$(xh + yk - 1)^2 = (x^2 + y^2 - 1)(h^2 + k^2 - 1)$

Put $x = 1$, $(h - 1)^2 + 2ky(h - 1) = y^2(h^2 - 1)$

$y^2(h + 1) - 2ky - (h - 1) = 0$

$AB = |y_1 - y_2| = 2$

$$\Rightarrow\ 4 = \frac{4k^2}{(h+1)^2} + \frac{4(h-1)}{(h+1)}$$

$(h + 1) = k^2 + (h^2 - 1)$

$\Rightarrow k^2 = 2(h + 1) \Rightarrow\ y^2 = 2(x + 1)$

62. **(3)**

We have

$$\sum_{k=1}^{10}\left(\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$$

$$= \sum_{k=1}^{10}\left(-i^2\sin\frac{2\pi k}{11} - i\cos\frac{2\pi k}{11}\right)$$

$$= -i\sum_{k=1}^{10}\left(\cos\frac{2\pi k}{11} + i\sin\frac{2\pi k}{11}\right) = -i\ \sum_{k=1}^{10} e^{i\frac{2\pi k}{11}}$$

$$= -i\left[\sum_{k=0}^{10} e^{i\frac{2\pi k}{11}} - 1\right]$$

$= -i$(sum of 11$^{\text{th}}$ roots of unity -1)

$= -i(0 - 1) = i$

63. **(3)**

Let, e be the eccentricity of the hyperbola

Now, $2ae = 10 \Rightarrow a^2e^2 = 25$

$\Rightarrow a^2 + b^2 = 25$

Also $(2, \sqrt{3})$ lies on the director circle

$x^2 + y^2 = a^2 - b^2$

$\Rightarrow 7 = a^2 - b^2 \Rightarrow a^2 = 16, b^2 = 9$

$\Rightarrow \left|\frac{b}{a}\right| = \frac{3}{4} = 0.75$

64. **(1)**

p is true and $(q \vee r)$ is false

$\Rightarrow$ p is true, q is false and r is false.

65. **(4)**

Given equation,

$$4\left(x^2 + \frac{1}{x^2}\right) + 16\left(x + \frac{1}{x}\right) - 57 = 0$$

Let, $x + \frac{1}{x} = y;\ x^2 + \frac{1}{x^2} = y^2 - 2$

$\Rightarrow 4y^2 + 16y - 65 = 0$

$\Rightarrow y = -\frac{13}{2}$ or $\frac{5}{2}$

When, $y = \frac{5}{2}$

$x + \frac{1}{x} = \frac{5}{2} \Rightarrow x = 2$ or $\frac{1}{2}$

When, $y = -\frac{13}{2}$

$\Rightarrow x + \frac{1}{x} = -\frac{13}{2}$

$\Rightarrow 2x^2 + 13x + 2 = 0$

$\Rightarrow x = \frac{-13 \pm \sqrt{153}}{4}$

Since x is rational, $x = 2$ or $\frac{1}{2}$

Hence, their product is 1.

66. **(3)**

Let $x_1, x_2 \ldots, x_6$ are the observations and $x_1 = 28$

$\Rightarrow 28.\ x_2 \ldots x_6 = 13^6$

$\Rightarrow x_2 \ldots x_6 = \frac{13^6}{28}$

Now correct observations is 36

$\Rightarrow 36.x_2 \ldots x_6 = \frac{13^6}{28} \times 36$

So, now correct geometric mean = $13\left(\frac{9}{7}\right)^{1/6}$

67. **(1)**

Here, we see that 10 is added in each observation of the first data.

Since we know that SD does not depend on change of origin.

Hence, SD of second data is k.

68. **(3)**

Let the required point be (h, k).

Now from this point, the equation of chord of contact to the ellipse is $T = 0$

$\Rightarrow 4hx + ky = 5$ which is same as $2x + y = 3$

$\Rightarrow \frac{4h}{2} = \frac{k}{1} = \frac{5}{3} \Rightarrow h = \frac{5}{6}, k = \frac{5}{3}$

$\Rightarrow$ Point of intersection of the tangents is

$\left(\frac{5}{6}, \frac{5}{3}\right)$

69. **(1)**

$P = \left(\frac{2a}{\sqrt{3}}, \frac{b}{\sqrt{3}}\right)$

Equation of the tangent at P is

$\frac{2x}{\sqrt{3}a} - \frac{y}{\sqrt{3}b} = 1$

x intercept & y intercept of the tangent are $\frac{\sqrt{3}a}{2}$ & $-\sqrt{3}b$ respectively

Area of the triangle formed by the tangent with the coordinate axes is

$\frac{1}{2} \times \frac{\sqrt{3}a}{2} \times \sqrt{3}b = 3a^2$

$\Rightarrow \frac{b}{a} = 4$

The eccentricity of the hyperbola equals to

$\sqrt{1+16} = \sqrt{17}$

70. **(2)**

Let the two numbers be α, β

$\therefore \frac{\alpha+\beta}{2} = 9$ and

$\sqrt{\alpha\beta} = 4 \Rightarrow \alpha + \beta = 18, \alpha\beta = 16$

$\therefore$ Required equation is

$x^2 - (\alpha+\beta)x + \alpha\beta = 0 \Rightarrow x^2 - 18x + 16 = 0$

71. **(4)**

Let $x = 1.1$

$S = 1 + 2x + 3x^2 + \ldots\ldots 10x^9$ and

$xS = x + 2x^2 + \ldots\ldots x^9 + 10x^{10}$

Subtracting, we get,

$S(1 - x) = (1 + x + x^2 + \ldots\ldots + x^9) - 10x^{10}$

$= \left(\frac{x^{10} - 1}{x - 1}\right) - 10x^{10}$

$\Rightarrow -S = \frac{x^{10} - 1}{(x-1)^2} - \frac{10x^{10}}{(x-1)}$

$\Rightarrow -S = \frac{(1.1)^{10} - 1}{(0.1)^2} - \frac{10(1.1)^{10}}{0.1}$

$= 100 \times (1.1)^{10} - 100 - 100 \times (1.1)^{10} = -100$

$\Rightarrow S = 100.$

72. **(4)**

Let P be (x_1, y_1)

Equation of normal at P is $\frac{x}{2x_1} - \frac{y}{y_1} = -\frac{1}{2}$

Since, it passes through $\left(-\frac{1}{3\sqrt{2}}, 0\right)$

$\therefore \frac{-1}{6\sqrt{2}x_1} = -\frac{1}{2} \Rightarrow x_1 = \frac{1}{3\sqrt{2}}$

$\Rightarrow y_1 = \frac{2\sqrt{2}}{3}$ (As P lies in 1st quadrant)

So, $\beta = \frac{y_1}{2} = \frac{\sqrt{2}}{3}$

73. **(4)**

$\sim(p \Rightarrow q) \equiv \sim(\sim p \vee q) \equiv (p \wedge \sim q)$

Hence, the equivalent statement of $\sim(p \Rightarrow q)$ is 4 is an odd number and 4^3 is not an even number.

74. **(3)**

Focus of $y^2 = 4(x - 1)$ is (2, 0) which satisfies the equation $y = \sqrt{3}x - 2\sqrt{3}$.

Hence, line $y = \sqrt{3}x - 2\sqrt{3}$ is a focal chord.

Now, the length of the focal chord equals to $4a\,\text{cosec}^2\theta$ where a =1 and

$\tan\theta = \sqrt{3}$ (or θ = 60°)

$\Rightarrow$ length of chord = 4 × (cosec² 60°)

$= 4 \times \frac{4}{3} = \frac{16}{3}$ units

75. **(2)**

Let the equation of a circle touching both the axes be

$(x - r)^2 + (y - r)^2 = r^2$

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$

If the two given circles are orthogonal, then

$2(-r_1)(-r_2) + 2(-r_1)(-r_2) = r_1^2 + r_2^2$

$\Rightarrow r_1^2 - 4r_1r_2 + r_2^2 = 0$

$\Rightarrow \frac{r_1}{r_2} = 2 + \sqrt{3}$

76. **(1)**

Since, $x^y \cdot y^x = 16$

$\therefore \log_e x^y + \log_e y^x = \log_e 16$

$\Rightarrow y\log_e x + x\log_e y = 4\log_e 2$

Now, on differentiating both sides w.r.t. x, we get

$\frac{y}{x} + \log_e x\frac{dy}{dx} + \frac{x}{y}\frac{dy}{dx} + \log_e y.1 = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{\left(\log_e y + \frac{y}{x}\right)}{\left(\log_e x + \frac{x}{y}\right)}$

$\therefore \left.\frac{dy}{dx}\right|_{(2,2)} = -\frac{(\log_e 2 + 1)}{(\log_e 2 + 1)} = -1$

77. **(4)**

$\frac{x}{3} = \cos t + \sin t$ and $\frac{y}{4} = \cos t - \sin t$

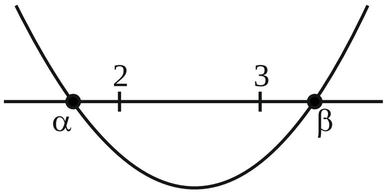
$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 2$

$\frac{x^2}{(3\sqrt{2})^2} + \frac{y^2}{(4\sqrt{2})^2} = 1$

Hence, the length of the latus rectum

$\frac{2(3\sqrt{2})^2}{4\sqrt{2}} = \frac{9}{\sqrt{2}}$ units

78. **(4)**



$D > 0$, $f(2) < 0$ and $f(3) < 0$

$D = (1 - 2\lambda)^2 - 4(\lambda^2 - \lambda - 2)$

$= 1 + 4\lambda^2 - 4\lambda - 4\lambda^2 + 4\lambda + 8$

= 9 > 0 (always true)

$f(2) < 0$

$\Rightarrow 4 + 2(1 - 2\lambda) + (\lambda^2 - \lambda - 2) < 0$

$\Rightarrow 4 + 2 - 4\lambda + \lambda^2 - \lambda - 2 < 0$

$\Rightarrow \lambda^2 - 5\lambda + 4 < 0$

$\lambda \in (1, 4)$ ......(i)

$f(3) < 0$

$\Rightarrow 9 + 3(1 - 2\lambda) + \lambda^2 - \lambda - 2 < 0$

$\Rightarrow 9 + 3 - 6\lambda + \lambda^2 - \lambda - 2 < 0$

$\Rightarrow \lambda^2 - 7\lambda + 10 < 0$

$\Rightarrow \lambda \in (2, 5)$ ......(ii)

Taking intersection of (i) and (ii), we get

$\lambda \in (2, 4)$

79. **(4)**

Let the A.P. be

$a - 3d, a - d, a + d, a + 3d$

The sum of the terms

$= 48 = 4a \Rightarrow a = 12$

Given, $\frac{(12 - 3d)(12 + 3d)}{(12 - d)(12 + d)} = \frac{27}{35}$

$\Rightarrow \frac{9(4 - d)(4 + d)}{(12 - d)(12 + d)} = \frac{27}{35}$

$\Rightarrow (16 - d^2)35 = (144 - d^2)3$

$\Rightarrow 35d^2 - 3d^2 = 16 \times 35 - 144 \times 3$

$\Rightarrow 32d^2 = 16(35 - 27) = 16 \times 8$

$\Rightarrow d^2 = 4 \Rightarrow d = \pm 2$

So the numbers are 6, 10, 14, 18

80. (2)

We have, $|a| < 1$, $|b| < 1$

$\therefore\ |ab| = |a|\,|b| < 1$

Now,

$a(a + b) + a^2(a^2 + b^2) + a^3(a^3 + b^3) + ...$

$= [(a^2 + a^4 + a^6 + ...)] + [\{ab + (ab)^2 + (ab)^3 + ...\}]$

$= \dfrac{a^2}{1-a^2} + \dfrac{ab}{1-ab}$

81. (9)

We have, $\left(2^{1/3} + \dfrac{1}{3^{1/3}}\right)^n$

7^{th} term from the beginning

$= {}^nC_6 \dfrac{1}{\left(3^{1/3}\right)^6}(2^{1/3})^{n-6}$

7^{th} term from the end

$= (n - 7 + 2)^{th}$ term from the beginning

i.e. $(n - 5)^{th}$ term.

Now, $\dfrac{{}^nC_6 \dfrac{1}{\left(3^{1/3}\right)^6}(2^{1/3})^{n-6}}{{}^nC_{n-6}\dfrac{1}{\left(3^{1/3}\right)^{n-6}}(2^{1/3})^6} = \dfrac{1}{6}$

$\Rightarrow \left(3^{1/3}\right)^{n-12} \times \left(2^{1/3}\right)^{n-12} = 6^{-1}$

$\Rightarrow \left(6^{1/3}\right)^{n-12} = 6^{-1}$

$\Rightarrow (6)^{\frac{n-12}{3}} = 6^{-1}$

$\Rightarrow \dfrac{n-12}{3} = -1$

$\Rightarrow n - 12 = -3 \Rightarrow n = 9$

82. (4)

Given, $f(\theta) = 12\sin\theta - 9\sin^2\theta$

$= -9\left(\sin^2\theta - \dfrac{4}{3}\sin\theta\right)$

$= -9\left(\left[\sin\theta - \dfrac{2}{3}\right]^2 - \dfrac{4}{9}\right)$

When $\sin\theta = \dfrac{2}{3}$

$\Rightarrow f(\theta) = 4$

So, maximum value is 4

83. (2)

$2^{301} = 2.2^{300} = 2.4^{150} = 2(5 - 1)^{150}$

Here all terms, except last term are divisible by 5

$\therefore$ Remainder = 2(last term) = $2(-1)^{150} = 2$

84. (2)

We have, $\dfrac{\cos 20° + 8\sin 70°\sin 50°\sin 10°}{\sin^2 80°}$

$= \dfrac{\cos 20° + 8\sin 10°\sin(60° - 10°)\sin(60° + 10°)}{\sin^2 80°}$

$\because\ \dfrac{\sin 3\theta}{4} = \sin(60° - \theta)\sin\theta\sin(60° + \theta)$

Then, $\dfrac{\cos 20° + \dfrac{8\sin 30°}{4}}{\sin^2 80°} = \dfrac{\cos 20° + \dfrac{8}{4}.\dfrac{1}{2}}{\sin^2 80°}$

$= \dfrac{1 + \cos 20°}{\dfrac{1 - \cos 160°}{2}} = \dfrac{1 + \cos 20°}{\dfrac{1 - \cos(180° - 20°)}{2}}$

$\dfrac{2(1 + \cos 20°)}{1 + \cos 20°} = 2$

85. (2)

$(1 + x)^{101}\,(1 + x^2 - x)^{100}$

$= (1 + x)\,((1 + x)\,(1 - x + x^2))^{100}$

$= (1 + x)\,(1 + x^3)^{100}$

$= 1 \times (1 + x^3)^{100} + x \times (1 + x^3)^{100}$

So, the number of terms = (101 terms of the form x^{3k}) + (101 terms of the form x^{3k+1})

= 202 terms

$\Rightarrow n = 202$

86. (1)

We have, $f(x) = \sqrt{\log_{(0.5)}\left(\dfrac{5-2x}{x}\right)}$

Now, $\log_{(0.5)}\left(\dfrac{5-2x}{x}\right) \geq 0$

$\Rightarrow 0 < \dfrac{5-2x}{x} \leq 1$

$\Rightarrow x \in \left[\dfrac{5}{3}, \dfrac{5}{2}\right)$

87. (10)

${}^nC_3 - {}^{n-1}C_2 = 84$

$\dfrac{(n-1)(n-2)}{6}[n-3] = 84$

$\Rightarrow (n-1)(n-2)(n-3) = 9 \times 8 \times 7$

$\Rightarrow n = 10$

88. (3)

Equation of the normal in slope form is

$y = mx - 2m - m^3$ which passes $(c, 0)$

$\Rightarrow 0 = mc - 2m - m^3$

$\Rightarrow m^3 + (2 - c)m = 0$

$\Rightarrow m = 0$ or $m^2 + (2 - c) = 0$

$\Rightarrow m_1 m_2 = \dfrac{2-c}{1} = -1$

$\Rightarrow c = 3$

89. **(6)**

We have

$f(x) = 6^x + 3^x + 6^{-x} + 3^{-x} + 2$

Since, $6^x + 6^{-x} \geq 2$ and $3^x + 3^{-x} \geq 2$

Therefore, $f(x) \geq 2 + 2 + 2$

$\Rightarrow f(x) \geq 6$

Thus, $f(x) \in [6, \infty)$

Hence, the value of *k* is 6.

90. **(6)**

We have, $\dfrac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta}$

$= \dfrac{2\sin 8\theta \cos\theta + 2\sin 4\theta \cos\theta}{2\cos 8\theta \cos\theta + 2\cos 4\theta \cos\theta}$

$= \dfrac{2\cos\theta(\sin 8\theta + \sin 4\theta)}{2\cos\theta(\cos 8\theta + \cos 4\theta)}$

$= \dfrac{2\sin 6\theta \cos 2\theta}{2\cos 6\theta \cos 2\theta} = \tan 6\theta$

$\therefore k = 6$