JEE Mains (11^{th})

Sample Paper - IV

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS

1.	(4)
2.	(1)
3.	(3)
4.	(2)
5.	(1)
6.	(4)
7.	(3)
8.	(2)
9.	(4)
10.	(4)
11.	(2)
12.	(2)
13.	(3)
14.	(3)
15.	(1)
16.	(3)
17.	(3)
18.	(1)
19.	(3)
20.	(2)
21.	(14)
22.	(7)
23.	(0.075)
24.	(11.67)
25.	(600)
26.	(100)
27.	(445)
28.	(1.0)
29.	(10)
30.	(112)

CHEMISTRY

31.	(3)
32.	(3)
33.	(4)
34.	(1)
35.	(2)
36.	(4)
37.	(4)
38.	(4)
39.	(3)
40.	(4)
41.	(2)
42.	(2)
43.	(4)
44.	(3)
45.	(4)
46.	(4)
47.	(1)
48.	(2)
49.	(2)
50.	(3)
51.	(8)
52.	(4)
53.	(8)
54.	(2)
55.	(0)
56.	(4)
57.	(4)
58.	(7)
59.	(5)
60.	(6)

MATHEMATICS

61.	(1)
62.	(1)
63.	(2)
64.	(1)
65.	(3)
66.	(2)
67.	(3)
68.	(1)
69.	(1)
70.	(3)
71.	(3)
72.	(2)
73.	(3)
74.	(4)
75.	(3)
76.	(2)
77.	(1)
78.	(1)
79.	(3)
80.	(2)
81.	(16)
82.	(40)
83.	(5)
84.	(27)
85.	(3)
86.	(9)
87.	(3)
88.	(4)
89.	(6)
90.	(17)

PHYSICS

1. **(4)**

Velocity of a particle at maximum height *h* is

$v' = v\cos\theta$

where v = initial velocity of the particle at which it is projected,

θ = angle of projection

Angular momentum, $L = mv'h = mv\cos\theta\, h$

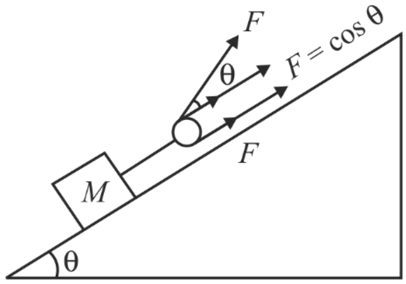
$= mvh\cos 45° = \frac{mvh}{\sqrt{2}}$.

2. **(1)**

$F + F\cos\theta = mg\sin\theta$

$$F = \frac{mg\sin\theta}{1+\cos\theta}$$

$$F = \frac{mg\,2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}$$



$$\left(\because \sin\theta = 2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2} \text{ and } 1+\cos\theta = 2\cos^2\frac{\theta}{2}\right)$$

$$= mg\tan\frac{\theta}{2}$$

3. **(3)**

$$P = Fv = m\frac{dv}{dt}v$$

or $v\frac{dv}{dt} = \frac{P}{m}$ or $v\cdot\frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{P}{m}$

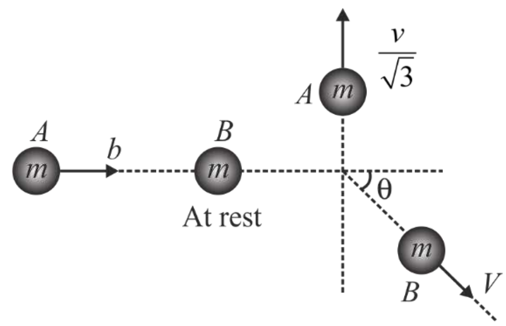
or $v^2\frac{dv}{dx} = \frac{P}{m}$ or $v^2 dv = \frac{P}{m}dx$

On integration, we get;

$$\frac{v^3}{3} = \frac{Px}{m} \text{ or } v = \left(\frac{3xP}{m}\right)^{1/3}$$

4. **(2)**

Let mass *A* moves with velocity *v* and collides inelastically with mass *B*, which is at rest.



According to problem, mass *A* moves in a perpendicular direction and let the mass *B* moves at angle θ with the horizontal with velocity *v*.

Initial horizontal momentum of system

(before collision) = mv ...(i)

Final horizontal momentum of system

(after collision) = $mV\cos\theta$ ...(ii)

From the conservation of horizontal linear momentum, $mv = mV\cos\theta$

$\Rightarrow v = V\cos\theta$ ...(iii)

Initial vertical momentum of system (before collision) is zero.

Final vertical momentum of system $= \frac{mv}{\sqrt{3}} - mv\sin\theta$

From the conservation of vertical linear momentum

$$\frac{mv}{\sqrt{3}} - mv\sin\theta = 0$$

$$\Rightarrow \frac{v}{\sqrt{3}} = V\sin\theta \quad \text{...(iv)}$$

By solving (iii) and (iv), we get

$$v^2 + \frac{v^2}{3} = V^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$\Rightarrow \frac{4v^2}{3} = V^2 \Rightarrow V = \frac{2}{\sqrt{3}}v.$$

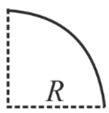
5. **(1)**

$$a = \frac{3m-m}{3m+m}g = \frac{g}{2}$$

Acceleration of centre of mass

$$= \frac{3m\times\frac{g}{2} - \frac{mg}{2}}{3m+m} = \frac{g}{4}$$

6. **(4)**



$$l = \frac{2\pi R}{4}$$

or $R = \frac{2l}{\pi}$

$\therefore I = mR^2$

$$= m\left(\frac{2l}{\pi}\right)^2$$

$0.4\,ml^2$ (as $\pi^2 \simeq 10$)

7. **(3)**

The particles will always remain diametrically opposite so that the force on each particle will be directed along the radius. Considering the circular motion of one particle.

$$\frac{mv^2}{r}=\frac{Gm^2}{(2r)^2} \text{ or } v=\sqrt{\frac{Gm}{4r}}$$

8. **(2)**

Let V be the volume of the load and ρ be its relative density. Then

$$Y=\frac{FL}{Al_a}=\frac{V\rho gL}{Al_a} \qquad \text{...(i)}$$

When load is immersed in liquid, the net weight = weight-upthrust

$$\therefore \quad Y=\frac{F'L}{Al_w}=\frac{(V\rho g-V\times 1\times g)L}{Al_w} \qquad \text{...(ii)}$$

Equating eqns. (i) and (ii),

$$\frac{\rho}{l_a}=\frac{(\rho-1)}{l\omega} \text{ or } \rho=\frac{l_a}{l_a-l_w}$$

9. **(4)**

$$W=T\cdot\Delta A=T\cdot 2\times 4\pi R^2$$

and $V=\frac{4}{3}\pi R^3$

When volume is doubled new radius becomes

$$R'=(2)^{1/3}R$$

$$\therefore \quad W'=T\times 2\times 4\pi R'^2$$

$$=T\times 2\times 4\pi (2)^{2/3}R^2$$

$$=T\times 2\times 4\pi (4)^{1/3}R^2=(4)^{1/3}W$$

10. **(4)**

Let h be the height of liquid surface in the vessel. The velocity of efflux is given by:

$$v_{\text{eff.}}=\sqrt{(2gh)}$$

If H be the height of table, then

$$H=\frac{1}{2}gt^2 \text{ or } t=\sqrt{(2H/g)}$$

$$\therefore \quad R=v_{\text{eff.}}\times t=\sqrt{2gh}\sqrt{2H/g}R^2$$

$$R^2=4hH \text{ or } h=\frac{R^2}{4H}.$$

11. **(2)**

Net pressure $=h_1\rho_1 g+h_2\rho_2 g$.

According to Bernoulli's theorem, this pressure energy will be converted into KE while flowing through the hole A.

$$\therefore \; h_1\rho_1 g+h_2\rho_2 g=\frac{1}{2}\rho_1 v^2$$

$$\therefore \; v=\sqrt{2\left(h_1+h_2\frac{\rho_2}{\rho_1}\right)g}.$$

12. **(2)**

$$\frac{dQ}{dt}=\frac{dQ_1}{dt}+\frac{dQ_2}{dt}$$

$$K\frac{4\pi R^2(\theta_1-\theta_2)}{L}$$

$$=K_1\frac{\pi R^2(\theta_1-\theta_2)}{L}+K_2\frac{3\pi R^2(\theta_1-\theta_2)}{L}$$

or $4K=K_1+3K_2$ or $K=\frac{K_1+3K_2}{4}$.

13. **(3)**

Internal energy of n moles of an ideal gas at temperature T is given by:

$U=\frac{f}{2}nRT$ [f = degree of freedom]

$U_1=U_2$

Here, f_2 = degree of freedom of He = 3

and f_1 = degree of freedom of H_2 = 5

$$\therefore \quad \frac{n_1}{n_2}=\frac{f_2T_2}{f_1T_1}=\frac{3\times 2}{5\times 1}=\frac{6}{5}.$$

14. **(3)**

$PV=RT$ = constant. ...(i)

Also, VP^2 = constant ...(ii)

From eqn. (i), $P=\frac{RT}{V}$

From eqn. (ii), $V\left(\frac{RT}{V}\right)^2$ = constant

$\frac{R^2T^2}{V}$ = constant or $\frac{T^2}{V}$ = constant

$$\therefore \; \frac{T^2}{V}=\frac{T'^2}{2V} \text{ or } T'^2=2T^2$$

$$\therefore \; T'=\sqrt{2}T.$$

15. **(1)**

$$K=\frac{1}{2}m\omega^2\left(A^2-y^2\right)=\frac{1}{2}m\omega^2A^2\left(1-\frac{y^2}{A^2}\right)$$

when $y=\frac{A}{2}$, $K=\frac{1}{2}m\omega^2A^2\left(1-\frac{1}{4}\right)$

$=\frac{3E}{4}$. (Where $E=\frac{1}{2}m\omega^2A^2$)

16. **(3)**

$$I = I_1 + I_2 + 2\left(\sqrt{I_1 I_2}\right)\cos\phi$$

$$= I_0 + 4I_0 + 2\sqrt{(I_0)(4I_0)}\cos\phi$$

$$= 5I_0 + 4I_0\cos\phi$$

As $\phi = 0,$, so $\cos\phi = 1$

$\therefore\ I = 5I_0 + 4I_0 = 9I_0$

17. **(3)**

L = 1 m = 100 cm, m = (0.5/100)

= 5×10^{-3} g/cm and p = 4

In the transverse arrangement the frequency of the vibrating string is equal to the frequency of the tuning fork, i.e., 200 Hz.

Now, $n = \dfrac{p}{2L}\sqrt{\dfrac{T}{m}}$

or $200 = \dfrac{4}{2\times 100}\sqrt{\dfrac{T}{5\times 10^{-3}}}$

$\therefore\ T = 5 \times 10^5$ dyne.

18. **(1)**

$P \propto T^2$

PT^{-2} = constant

compare with $PT^{\left(\frac{\gamma}{1-\gamma}\right)}$ = constant

$$\frac{C_p}{C_v} = \gamma = 2$$

19. **(3)**

For capillary tube

$$h = \frac{2T}{r\rho g}$$

We can say

$h \propto \dfrac{1}{r}$ or $h \propto \dfrac{1}{d}$

So, $\dfrac{h_1}{h_2} = \dfrac{d_2}{d_1}$

$\Rightarrow \dfrac{4}{x} = \dfrac{d}{2d}$

$\Rightarrow x$ = 8 cm

20. **(2)**

$$\theta = 2t^3 = 6t^2$$

$$\omega = \frac{d\theta}{dt} = 6t^2 - 12t$$

$\alpha = 0 \qquad \Rightarrow 12t - 12 = 0 \qquad \Rightarrow t = 1s$.

21. **(14)**

Position time relation of the particle, $s = t^3 + 3$

Speed of the particle, $v = \dfrac{ds}{dt} = \dfrac{d}{dt}\left(t^3 + 3\right) = 3t^2$

Tangential acceleration,

$$a_t = \frac{dv}{dt} = \frac{d}{dt}\left(3t^2\right) = 6t$$

At time $t = 2s$

Speed of the particle, $v = 3(2)^2$ = 12 m/s

Tangential acceleration, a_t = 6(2) = 12 m/s^2

Centripetal acceleration,

$a_c = \dfrac{v^2}{R} = \dfrac{(12)^2}{20} = \dfrac{144}{20}$ = 7.2 m/s^2

Net acceleration, $a = \sqrt{\left(a_c\right)^2 + \left(a_t\right)^2}$

$= \sqrt{(7.2)^2 + (12)^2} \approx$ 14 m/s^2

22. **(7)**

Net external torque is zero. Therefore, angular momentum of system will remain conserved, i.e.,

$L_i = L_f$

Initial angular momentum $L_i = 0$.

$\therefore$ Final angular momentum should also be zero, or angular momentum of man = angular momentum of platform in opposite direction,

or $mv_0 r = I\omega$

$\therefore\ \omega = \dfrac{mv_0 r}{I} = \dfrac{70\times 10\times 2}{200}$

$\therefore\ \omega$ = 7 rad/sec.

23. **(0.075)**

Energy stored = $\dfrac{1}{2}$ × work done

$= \dfrac{1}{2} \times F \times \Delta x$

$= \dfrac{1}{2} \times \dfrac{YA}{L}\Delta x \cdot \Delta x \qquad \left[F = \dfrac{YA\Delta x}{b}\right]$

Substituting values

$$E = \frac{1}{2} \times \frac{2\times 10^{11} \times 3\times 10^{-6} \times 1\times 10^{-3} \times 10^{-3}}{4}$$

E = 0.075 J

24. **(11.67)**

$$PV = \frac{m}{M}RT$$

$$20 \times V = \frac{m}{M}R \times 300, P' \times V = \frac{(m/2)}{M}R \times 350$$

$\therefore P' = \dfrac{140}{12}$ = 11.67 atm.

25. **(600)**

Maximum heat supplied by water

$\Delta Q_1 = 500 \times 1 \times (20 - 0)$

$= 10{,}000$ cal

Heat required to raise the temperature of ice upto 0°C

$\Delta Q_2 = 200 \times 0.5 \times 20$

$= 2000$ cal

$\Delta Q_1 - \Delta Q_2 = 8000$ cal

Melts the ice

$8000 = m \times 80$

$m = 100$ g

So, mass of water is 500 +100 = 600 g.

26. **(100)**

$$\Delta\phi = \frac{2\pi}{\lambda}\cdot\Delta x$$

$$\frac{\pi}{4} = \frac{2\pi}{\lambda}\times\frac{1.25}{100}$$

$$\therefore \quad = \frac{1}{10} \text{ m/s}$$

$$v = n\lambda = 1000\times\frac{1}{10} = 100 \text{ m/s.}$$

27. **(445)**

v – 5 = 440 Hz

and v – 8 = 437 Hz

∴ v = 445 Hz (by both the methods)

It could have been 435 Hz. It would have satisfied 440 – v = 5 but this would not have satisfied 437 Hz.

28. **(1.0)**

The frequencies are in the ratio of 5 : 7 : 9. Hence, it is a COP.

Now, $425 = 5\left(\frac{v}{4l}\right)$

$$\therefore\ l = \frac{5v}{4\times 425} = \frac{5\times 340}{4\times 425} = 1.0 \text{ m.}$$

29. **(10)**

When the man is approaching the factory:

$$n' = \left(\frac{v+v_o}{v}\right)n = \left(\frac{320+2}{320}\right)800 = \left(\frac{322}{320}\right)800$$

When the man is going away from the factory,

$$n'' = \left(\frac{v-v_o}{v}\right)n = \left(\frac{320-2}{320}\right)800 = \left(\frac{318}{320}\right)800$$

$$\therefore\quad n' - n'' = \left(\frac{322-318}{320}\right)800 = 10.$$

30. **(112)**

Radiation $\propto T^4$

So $\frac{R_1}{R_2} = \left(\frac{T_1}{T_2}\right)^4$

$$\frac{7}{x} = \left(\frac{500}{1000}\right)^4$$

$x = 112$ cal/cm^{2}s

CHEMISTRY

32. **(3)**

Number of molecules = $\frac{\text{mass}}{\text{M.M.}}\times N_A$

.

32. **(3)**

$r_n \propto n^2$

$r_2 = (2)^2 a_0 = 4a_0$

$$mvr = \frac{nh}{2\pi} = \frac{2h}{2\pi}$$

$$v = \frac{h}{\pi mr}\ (r = 4a_0)$$

$$v = \frac{h}{\pi m(4a_0)}$$

So, K.E. = 1/2 mv^2

$$= \frac{1}{2}m\left(\frac{h}{\pi m(4a_0)}\right)^2$$

$$= \frac{h^2}{32\pi^2 m a_0^2}$$

33. **(4)**

Screening effect is not observed in single electron system.

34. **(1)**

Bond angle $\propto$ E.N. of central atom

35. **(2)**

SF_6 is sp^3d^2 hydridised

Bond angle = 90°

% d character = $\frac{\text{no. of d orbitals}}{\text{Total no. of orbitals}}\times 100$

36. **(4)**

K.E. $\propto$ T

$$\mu_{avg} = \sqrt{\frac{8RT}{\pi M}}$$

37. **(4)**

$$PV = \frac{m}{M.M.} RT$$

38. **(4)**

$\Delta_r G^o < 0$ describe spontaneous process

$\therefore$ stable oxidation state of Pb is + 2 and Sn is +4.

39. **(3)**

$K_C = [OH^-]^3 [Fe^{3+}]$

$K_C = \left(\frac{1}{4}\right)^3 \left[OH^-\right]^3 \left[Fe^{3+}\right]$

64. $K_C = [OH^-]^3 [Fe^{3+}]$

40. **(4)**

$\left[H^+\right]_{Total} = \left[H^+\right]_{HCl} + \left[H^+\right]_{H_2O}$

$= 10^{-8} + 10^{-7}$

$= 1.1 \times 10^{-7}$

pH = $-\log(1.1 \times 10^{-7})$

pH = 6.95

41. **(2)**

$K_2 \overset{+6}{Cr_2} O_7 \rightarrow Cr^{3+}$

Eq. wt. $= \frac{M.M.}{n - factor} = \frac{M}{6}$

(Change in O.S. of Cr in $K_2Cr_2O_7$ is (2 × 3) = 6)

42. **(2)**

Estimation of hardness of water. The hardness of water due to Ca^{2+} and Mg^{2+} ions is usually estimated volumetrically. A known volume of hard water contining buffer solution of pH 10 is titrated against a standard solution of EDTA (ethylenediamine tetraacetic acid disodium salt) using Eriochrome Black T as indicator. Under these conditions, Ca^{2+} and Mg^{2+} ions form complexes with EDTA. When all the Ca^{2+} and Mg^{2+} ions are consumed, the next drop of EDTA changes the colour of the indicator from wine red to blue.

43. **(4)**

$Al_4C_3 + 12.H_2O \rightarrow 4Al(OH)_3 + 3CH_4$

$Be_2C + 4H_2O \rightarrow 2Be(OH)_2 + CH_4$

44. **(3)**

Emerald has cyclic structure

Asbestos – Chain silicate

Talc – Sheet silicate

Mica – Chain silicate

45. **(4)**

–M and –I effect stablise the carbanion

46. **(4)**

Ketone has higher priority and hence is main functional group.

47. **(1)**

$O\overset{\delta+}{N} - \overset{\delta-}{Cl} + CH_3 - CH = CH_2 \rightarrow$

$CH_3 - \overset{+}{C}H - CH_2 - NO$

$\downarrow$

$CH_3CH(Cl)CH_2NO$

48. **(2)**

$Br_2/FeBr_3$ – Electrophilic aromatic substitution

49. **(2)**

Informative

50. **(3)**

$K_{sp} = [Ag^+] [Cl^-]$

$4 \times 10^{-10} = \left[Ag^+\right](0.08)_{CaCl_2}$

51. **(8)**

Weak electrolytes $\rightarrow$ do not dissociate completely

52. **(4)**

Divide the given equation by (3), we get

$K' = (K)^{1/3}$

53. **(8)**

0.4 MH_2SO_4 = 0.8 M H^+

0.1 M HCl = 0.1 M H+

$$\frac{[H^+]_{H_2SO_4}}{[H^+]_{HCl}} = \frac{8}{1}$$

54. **(2)**

PM = dRT

$$d = \frac{PM}{RT}$$

Now, $d' = \frac{(4P)M}{R(2T)}$

$$d' = \frac{2PM}{RT}$$

55. (0)

BrF_5 is sp^3d^2 hybridized with 1 lone pair

It has square pyramidal shape

But due to repulsion by lone pair on the bond pairs, all the four planar bond angles decreases from 90° to 84.8°. The axial bond also no longer remains 90° with the planer bonds.

Equilibrium constant for the reaction

$A_3(g)+3B_2(g) \rightleftharpoons 3AB_2(g)$ is 64.0. Then the equilibrium constant for the reaction

$\frac{1}{3}A_3(g)+B_2(g) \rightleftharpoons AB_2(g)$ will be

56. (4)

$E.N._F$ on pauling scale = 4

57. (4)

Total no. of nodes = n – 1

58. (7)

Volume of 1 mole of Ag atoms = $\frac{108}{10.5}cm^3$

$\therefore$ Volume of 1 Ag atom = $\frac{108}{10.5}\times\frac{1}{6.02\times10^{23}}$

If r is the radius of Ag atom, volume of 1 Ag atom

$= \frac{4}{3}\pi r^3$

Thus, $\frac{4}{3}\pi r^3 = \frac{108}{10.5\times6.02\times10^{23}}$

This gives r = 1.6×10^{-8} cm = 1.6×10^{-10} m

Area occupied by 1 Ag atom = πr^2

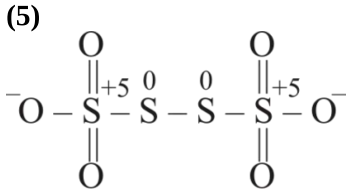
$= 3.14 \times (1.6 \times 10^{-10})^2\ m^2$

Hence, no. of Ag atoms on a surface area of

$10^{-12}\ m^2 = \frac{10^{-12}}{3.14\times(1.6\times10^{-10})^2} = 1.25\times10^7$

$\therefore$ x = 7

59. (5)



60. (6)

Hydrides of group 15, 16 and 17 have more electrons than required to form normal covalent bonds and hence are electron rich hydrides.

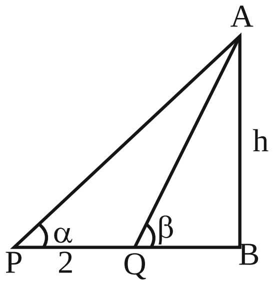
MATHEMATICS

61. (1)

$h\cot\alpha - h\cot\beta = 2$

$\Rightarrow h = \frac{2}{\cot\alpha-\cot\beta}$

$= \frac{2\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$



62. (1)

The given equation is

$(a-2b+c)x^2+(b-2c+a)x+(c-2a+b)=0$

$\because \sum(a-2b+c)=0$

$\therefore$ One root of this equation is 1.

Now,

$\sec\theta+\tan\theta=1$ ....(i)

We know that

$\sec^2\theta-\tan^2\theta=1$

$\sec\theta-\tan\theta=1$ ....(ii)

$\Rightarrow$ On solving eqs. (i) and (ii), we get $\sec\theta=1$

$\therefore$ One root of given equation is $\sec\theta$

63. (2)

Let $r+1=7 \Rightarrow r=6$

Given expansion is $\left(\frac{3}{\sqrt[3]{84}}+\sqrt{3}\ell n\, x\right)^9$, $x>0$

We have $T_{r+1} = {}^nC_r\,(x)^{n-r}\,a^r$ for $(x+a)^n$.

$\therefore$ According to the equation.

$729 = {}^9C_6\left(\frac{3}{\sqrt[3]{84}}\right)^3.\left(\sqrt{3}\,l\,n\,x\right)^6$

$\Rightarrow 3^6 = 84\times\frac{3^3}{84}\times3^3\times(\ell n\,x)^6$

$\Rightarrow (\ell n\,x)^6 = 1 \Rightarrow x=e$

64. (1)

$$\mathrm{Re}\left\{\frac{\tan\alpha-i[\sin(\alpha/2)+\cos(\alpha/2)]}{1+2i\sin(\alpha/2)}\right\}=0$$

$$\mathrm{Re}\left\{\frac{\left[\tan\alpha-i\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)\right]\left(1-2i\sin\frac{\alpha}{2}\right)}{1+4\sin^2\frac{\alpha}{2}}\right\}=0$$

$$\Rightarrow \left\{\begin{array}{l} \tan\alpha - 2\sin\frac{\alpha}{2}\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right) \\ \dfrac{-i\left(\tan\alpha.2\sin\frac{\alpha}{2}+\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}{1+4\sin^2\frac{\alpha}{2}} \end{array}\right\} = 0$$

$$\Rightarrow \tan\alpha = 2\sin^2\frac{\alpha}{2} + \sin\alpha$$

$$\Rightarrow \frac{\sin\alpha}{\cos\alpha} = \sin\alpha + 1 - \cos\alpha$$

$$\Rightarrow \sin\alpha = \sin\alpha.\cos\alpha + \cos\alpha - \cos^2\alpha$$

$$\Rightarrow \sin\alpha(1-\cos\alpha) = \cos\alpha(1-\cos\alpha)$$

$$\Rightarrow \sin\alpha = \cos\alpha,\ \cos\alpha = 1$$

$$\Rightarrow \alpha = n\pi + \frac{\pi}{4},\ \alpha = 2n\pi \text{ where } n \in Z$$

65. (3)

30 marks to be alloted to 8 questions. Each question has to be given $\geq$ 2 marks

Let marks of questions be a, b, c, d, e, f, g, h

and $a + b + c + d + e + f + g + h = 30$

Let $a = a_1 + 2$ so, $a_1 \geq 0$

$b = a_2 + 2$ so, $a_2 \geq 0, \ldots\ldots, a_8 \geq 0$

So, $\left.\begin{array}{l} a_1 + a_2 + \ldots. + a_8 \\ +2+2+\ldots.+2 \end{array}\right\} = 30$

$$\Rightarrow a_1 + a_2 + \ldots.. + a_8 = 30 - 16 = 14$$

So, this is a problem of distributing 14 articles in 8 groups.

Number of ways $= {}^{14+8-1}C_{8-1} = {}^{21}C_7$

66. (2)

The centre of given circle is $(-g, -f)$.

If the given line $ax + bx + c = 0$ is normal to the circle, then it passes through the centre of circle

$\therefore\ a(-g) + b(-f) + c = 0$

$\Rightarrow ag + bf - c = 0$

67. (3)

The coordinates of the vertices of the rectangle are $A(1, 4)$, $B(6, 4)$, $C(6, 10)$ and $D(1, 10)$.

The equation of diagonal AC is

$$y - 4 = \frac{10-4}{6-1}(x-1) \Rightarrow 6x - 5y + 14 = 0$$

68. (1)

If the line $y = mx + 2$ cuts the circle $x^2 + y^2 = 1$ at distinct or coincident points, then

Length of perpendicular from centre $\leq$ Radius

$$\Rightarrow \left|\frac{2}{\sqrt{m^2+1}}\right| \leq 1$$

$$\Rightarrow 4 \leq m^2 + 1$$

$$\Rightarrow m^2 - 3 \geq 0$$

$$\Rightarrow (m-\sqrt{3})(m+\sqrt{3}) \geq 0$$

$$\Rightarrow m \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$$

69. (1)

$({}^7C_0 + {}^7C_1) + ({}^7C_1 + {}^7C_2) + \ldots. + ({}^7C_6 + {}^7C_7)$

$= {}^8C_1 + {}^8C_2 + \ldots.. + {}^8C_7$

$= {}^8C_0 + {}^8C_1 + {}^8C_2 + \ldots.. + {}^8C_7 + {}^8C_8 - ({}^8C_0 + {}^8C_8)$

$= 2^8 - (1 + 1) = 2^8 - 2$

70. (3)

a^2, b^2, c^2 are in A.P.

Adding $ab + bc + ca$ to each of these terms.

$a^2 + ab + bc + ca,\ b^2 + ab + bc + ca,\ c^2 + ab + bc + ca$ are in A.P.

$(a + b)(a + c),\ (b + c)(b + a),\ (c + a)(c + b)$ are in A.P.

Dividing each term by $(b + c)(c + a)(a + b)$,

$\dfrac{1}{b+c}, \dfrac{1}{c+a}, \dfrac{1}{a+b}$ are in A.P.

$b + c,\ c + a,\ a + b$ are in H.P.

71. (3)

Given (a, a^2) falls inside the angle made by

$y = \dfrac{x}{2},\ x > 0$ and $y = 3x,\ x > 0$

$$\Rightarrow a^2 - 3a < 0 \text{ and } a^2 - \frac{a}{2} > 0$$

$$\Rightarrow \frac{1}{2} < a < 3 \quad \Rightarrow a \in \left(\frac{1}{2}, 3\right)$$

72. (2)

Let us make the truth table for the given statements, as follows:

p	q	$p \vee q$	$q \rightarrow p$	$p \rightarrow (q \rightarrow p)$	$p \rightarrow (p \vee q)$
T	T	T	T	T	T
T	F	T	T	T	T
F	T	T	F	T	T
F	F	F	T	T	T

From table ne observe

$p \rightarrow (q \rightarrow p)$ is equivalent to $p \rightarrow (p \vee q)$

73. **(3)**

$$\bar{x} = \frac{\text{Sum of quantities}}{n} = \frac{n/2(a+l)}{n}$$

$$= \frac{1}{2}(1+1+100d) = 1+50d$$

M.D. = $\frac{1}{n}\sum|x_i - \bar{x}|$

$$\Rightarrow 255 = \frac{1}{101}(50d + 49d + ... + d + 0 + d + ... + 50d)$$

$$= \frac{2d}{101}\left(\frac{50\times 51}{2}\right) \Rightarrow d = \frac{255\times 101}{50\times 51} = 10.1$$

74. **(4)**

$$a_1 + a_2(2\cos^2 x - 1) + a_3(1-\cos^2 x) = 1$$

or $(2a_2 - a_3)\cos^2 x + (a_1 - a_2 + a_3 - 1) = 0$

This can hold for all x if

$2a_2 - a_3 = 0$ and $a_1 - a_2 + a_3 - 1 = 0$

As there are two equations in three unknowns, the number of solutions is infinite.

75. **(3)**

Let d be the common difference

$\because\ a_7 = 9$

$\therefore\ a_1 + 6d = 9$

Let $D = a_1 a_2 a_7 = (9-6d)(9-5d)9$

$$= 270\left\{\left(d - \frac{33}{20}\right)^2 - \frac{9}{400}\right\}$$

For least value of D,

$$d - \frac{33}{20} = 0$$

$$\therefore\ d = \frac{33}{20}$$

76. **(2)**

Given that sin x + cosec x + tan y + cot y = 4

$\Rightarrow\ x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$

$\Rightarrow\ \tan y = 1$

$\Rightarrow\ \frac{2\tan y/2}{1-\tan^2 y/2} = 1 \quad \Rightarrow\ \tan^2\frac{y}{2} + 2\tan\frac{y}{2} - 1 = 0$

77. **(1)**

$$\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)(\sqrt{x}+1)}$$

$$= \frac{-1}{5.2} = \frac{-1}{10}$$

78. **(1)**

Any tangent to the hyperbola at $P(a\sec\theta, a\tan\theta)$ is

$x\sec\theta - y\tan\theta = a$ ......(i)

Also $x - y = 0$ ......(ii)

$x + y = 0$ .... (iii)

Solving the above three lines in pairs, we get the point A, B, C as

$$\left(\frac{a}{\sec\theta - \tan\theta}, \frac{a}{\sec\theta - \tan\theta}\right),$$

$$\left(\frac{a}{\sec\theta + \tan\theta}, \frac{-a}{\sec\theta + \tan\theta}\right) \text{ and } (0, 0)$$

Since the one vertex is the origin therefore the area of the triangle ABC is

$$\frac{1}{2}|(x_1y_2 - x_2y_1)|$$

$$= \frac{a^2}{2}\left|\left(\frac{-1}{\sec^2\theta - \tan^2\theta} - \frac{1}{\sec^2\theta - \tan^2\theta}\right)\right|$$

$$= \frac{a^2}{2}|(-2)| = |-a^2| = a^2 \text{ sq. unit}$$

79. **(3)**

We have $\alpha + \beta = -b$ and $\alpha\beta = 1$

Let $S = -\left(\alpha + \frac{1}{\beta}\right) + \left(-\left(\beta + \frac{1}{\alpha}\right)\right)$

$$= -(\alpha+\beta) - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= b - \left(\frac{\alpha+\beta}{\alpha\beta}\right) = b - \left(-\frac{b}{1}\right) = 2b$$

And $P = \left[-\left(\alpha + \frac{1}{\beta}\right)\right]\left[-\left(\beta + \frac{1}{\alpha}\right)\right]$

$$= \left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right)$$

$$= \alpha\beta + \frac{1}{\alpha\beta} + 2 = 1 + \frac{1}{1} + 2 = 4$$

$\therefore$ The required equation is $x^2 - Sx + P = 0$

$\Rightarrow\ x^2 - 2bx + 4 = 0$

80. **(2)**

Number of elements in $A \times B = 2 \times 4 = 8$

Required number of subsets

$= {}^8C_3 + {}^8C_4 + ... + {}^8C_8$

$= ({}^8C_0 + {}^8C_1 + ... {}^8C_8) - ({}^8C_0 + {}^8C_1 + {}^8C_2)$

$= 2^8 - (1 + 8 + 28) = 219$

81. **(16)**

$A = \{-2, -1, 0, 1, 2\}$

$R = \{(-2, -2), (0, 0), (1, 1), (2, 2)\}$

As R has four elements, the power set of R contains 16 elements

82. **(40)**

$$a_r = 6\left(\frac{1}{r} - \frac{1}{r+1}\right)$$

$$\Rightarrow \sum_{r=1}^{20} a_r = 6\left(1 - \frac{1}{21}\right) = \frac{120}{21} = \frac{40}{7} = \frac{k}{7}$$

$\therefore\ k = 40$

83. **(5)**

We have $x + 2y + 3z + 4w = 50$

Using the fact A.M $\geq$ G.M., we get

$$\frac{2\left(\frac{x}{2}\right)+4\left(\frac{y}{2}\right)+3\left(\frac{z}{1}\right)+1\left(\frac{4w}{1}\right)}{2+4+3+1}$$

$$\geq\left[\left(\frac{x}{2}\right)^2\left(\frac{y}{2}\right)^4(z)^3(4w)\right]^{1/10}$$

$$\Rightarrow\ 5\geq\left[\left(\frac{x^2}{2^2}\right)\left(\frac{y^4}{2^4}\right)(z)^3(2^2w)\right]^{1/10}$$

$$\Rightarrow\ 5\geq\left(\frac{x^2y^4z^3w}{16}\right)^{1/10}$$

84. **(27)**

$$\cos\left(x-\frac{3\pi}{2}\right)+\sin\left(\frac{3\pi}{2}+x\right)+\sin(32\pi+x)$$
$$-18\cos(19\pi-x)+\cos(56\pi+x)-9\sin(x+17\pi)$$
$$=-\sin x-\cos x+\sin x+18\cos x+\cos x$$
$$+9\sin x=18\cos x+9\sin x$$

$a + b = 27$

85. **(3)**

Here $a = 1$. Any tangent having slope m is

$$y=mx+\frac{1}{m}$$

If passes through (–2, –1). Therefore,

$2m^2 - m - 1 = 0$

or $m = 1,\ -\frac{1}{2}$

or $\tan\alpha=\frac{1+(1/2)}{1-(1/2)}=3$

86. **(9)**

We have,

$x^2 + bx - 1 = 0$ ......(i)

$x^2 + x + b = 0$ .....(ii)

On subtracting (ii) from (i), we get

$$x(1-b)+1+b=0\ \Rightarrow x=\frac{b+1}{b-1}$$

On putting value of x in (ii), We get

$$\left(\frac{b+1}{b-1}\right)^2+\left(\frac{b+1}{b-1}\right)+b=0$$

$$\Rightarrow\ (b+1)^2+(b+1)(b-1)+b(b-1)^2=0$$

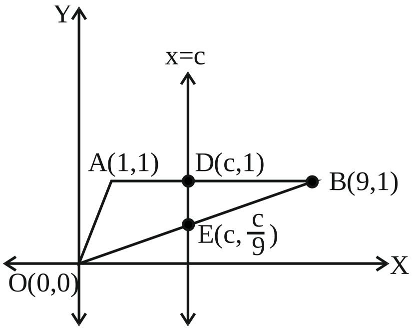
$$\Rightarrow\ b^3+3b=0\Rightarrow b(b^2+3)=0$$

But $b\neq 0$

$$\therefore\ b^2=-3\Rightarrow b^4=9$$

87. **(3)**

Area of $\Delta OAB=\frac{1}{2}(1)(8)$ = 4 sq units



The equation of OB is $y=\frac{1}{9}x$

Hence, the point E is $(c, c/9)$

Now, the area of ΔBDE is 2 sq units.

Therefore, $\frac{1}{2}\left(1-\frac{c}{9}\right)(9-c)=2$

$$\Rightarrow\ (9-c)^2=36$$

$$\Rightarrow\ 9-c=\pm 6\Rightarrow c=3$$

88. **(4)**

Radius of given circle

$x^2 + y^2 + 4x - 2\sqrt{2}y + c = 0$

is $\sqrt{4+2-c}=\sqrt{6-c}=a$ (let)

Now radius of circle $S_1 = \frac{a}{\sqrt{2}}$,

Radius of circle $S_2 = \frac{a}{2}$ and so on.

Now, $a+\frac{a}{\sqrt{2}}+\frac{a}{2}+.....\infty=2$ (given)

$$\Rightarrow\ a=2-\sqrt{2}=\sqrt{6-c}$$

$$\Rightarrow\ 4+2-4\sqrt{2}=6-c$$

$$\Rightarrow\ c=4\sqrt{2}$$

89. **(6)**

$$(1+ax)^n=1+n(ax)+\frac{n(n-1)}{1.2}(ax)^2+...$$

Equating coefficients of x and x^2

$na = 8$ .... (1) and

$$\frac{n(n-1)}{2}a^2=24 \quad .....(2)$$

$$\Rightarrow\ a=\frac{8}{n}$$

Substitute the value of a in (2)

$$\frac{n(n-1)}{2}\frac{64}{n^2}=24$$

$$\Rightarrow\ 4n-4=3n$$

$$\Rightarrow\ n=4$$

$$\therefore\ a=\frac{8}{4}=2$$

90. **(17)**

$$t_{r+1}={}^{100}C_r\,x^{\frac{(100-r)}{2}}\left(y^{1/3}\right)^r$$

so $\frac{100-r}{2}$ should be integer as well as $\frac{r}{3}$ should be integer. $r = 0, 6, 12, 18, ..., 96$

Thus r can assume total 17 terms.