JEE Mains (11th)

Sample Paper - V

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(2)
2.	(3)
3.	(2)
4.	(2)
5.	(4)
6.	(2)
7.	(1)
8.	(4)
9.	(4)
10.	(4)
11.	(3)
12.	(3)
13.	(3)
14.	(3)
15.	(1)
16.	(3)
17.	(1)
18.	(3)
19.	(1)
20.	(3)
21.	(100)
22.	(7.5)
23.	(10)
24.	(22.68)
25.	(2)
26.	(0.1)
27.	(1.5)
28.	(7)
29.	(30)
30.	(5000)

CHEMISTRY

31.	(1)
32.	(2)
33.	(4)
34.	(2)
35.	(1)
36.	(2)
37.	(4)
38.	(2)
39.	(3)
40.	(4)
41.	(3)
42.	(4)
43.	(4)
44.	(4)
45.	(3)
46.	(2)
47.	(1)
48.	(3)
49.	(4)
50.	(2)
51.	(9)
52.	(2)
53.	(3)
54.	(5)
55.	(3)
56.	(6)
57.	(7)
58.	(4)
59.	(3)
60.	(1)

MATHEMATICS

61.	(1)
62.	(3)
63.	(3)
64.	(3)
65.	(3)
66.	(3)
67.	(2)
68.	(2)
69.	(3)
70.	(3)
71.	(1)
72.	(2)
73.	(3)
74.	(3)
75.	(2)
76.	(4)
77.	(1)
78.	(2)
79.	(3)
80.	(3)
81.	(5)
82.	(22)
83.	(0)
84.	(6)
85.	(3)
86.	(3)
87.	(2)
88.	(26)
89.	(80)
90.	(3)

PHYSICS

1. **(2)**

Let l be the length of the pipes and v the speed of sound. Then frequency of open organ pipe of n^{th} overtone is,

$$f_1 = (n+1)\frac{v}{2l}$$

and frequency of closed organ pipe of n^{th} overtone is,

$$f_2 = (2n+1)\frac{v}{4l}$$

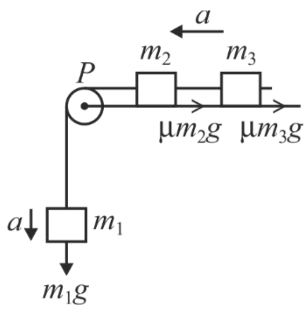
$\therefore$ The desired ratio is $\frac{f_1}{f_2} = \frac{2(n+1)}{(2n+1)}$.

2. **(3)**

Force of friction on mass $m_2 = \mu m_2 g$

Force of friction on mass $m_3 = \mu m_3 g$

Let a be common acceleration of the system.



$$\therefore \; a = \frac{m_1 g - m_2 g - \mu m_3 g}{m_1 + m_2 + m_3}$$

Here,

$$m_1 = m_2 = m_3 = m$$

$$\therefore \; a = \frac{mg - \mu mg - \mu mg}{m + m + m}$$

$$= \frac{mg - 2\mu mg}{3m}$$

$$= \frac{g(1-2\mu)}{3}$$

Hence, the downward acceleration of mass m_1 is $\frac{g(1-2\mu)}{3}$.

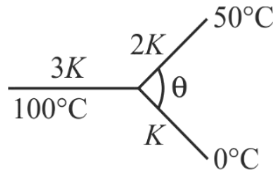
3. **(2)**

$$v_e = \sqrt{\frac{2GM}{R}}$$

i.e., $\frac{(v_e)_1}{(v_e)_2} = \sqrt{\frac{R_2}{R_1}}$ or $\frac{1}{100} = \frac{R_2}{R_1}$

or $R_2 = \frac{R_1}{100} = \frac{6400}{100} = 64\,\text{km}$.

4. **(2)**



$3K(100 - \theta) + 2K(50 - \theta) = K\theta$

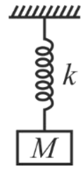
$300K - 3K\theta + 100K - 2K\theta = K\theta$

$400 = 6\theta$

$$\frac{400}{6} = \theta$$

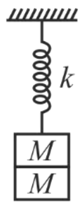
$$\frac{200}{3} = \theta$$

5. **(4)**



A mass M is suspended from a massless spring of spring constant k as shown in adjoining figure. Then, Time period of oscillation is,

$$T = 2\pi\sqrt{\frac{M}{k}} \quad \text{...(i)}$$



When an another mass M is also suspended with it as in adjoining figure. Then,

Time period of oscillation is,

$$T' = 2\pi\sqrt{\frac{M+M}{k}}$$

$$= 2\pi\sqrt{\frac{2M}{k}}$$

$$= \sqrt{2}\left(2\pi\sqrt{\frac{M}{k}}\right) = \sqrt{2}T$$

[Using eqn. (i)]

6. **(2)**

$$SL = 10\log\left(\frac{I}{I_0}\right) \text{ or } 60 = 10\log\left(\frac{I}{I_0}\right)$$

or $\log\left(\frac{I}{I_0}\right) = 6$ or $\left(\frac{I}{I_0}\right) = 10^6$ watt/m^2

$\therefore \; I = I_0 \times 10^6 = 10^{-12} \times 10^6 = 10^{-6}$ watt/m^2

$\therefore$ P = power = intensity × area $= 10^{-6} \times 2 = 2\mu W$

and E = energy = $P \times t$

$= 2 \times 10^{-6} \times 5 \times 60 \times 60$

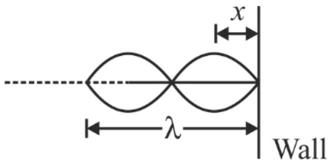
$= 36 \times 10^{-3}$ J

7. **(1)**

$\nu = 256$ Hz

$v = 336$ m/s

The incident and reflected waves interfer to form a displacement node at the wall.



$$\therefore\ x = \frac{\lambda}{4} = \frac{v}{\nu} \times \frac{1}{4}$$

$$= \frac{336 \times 100}{256 \times 4} = 32.8\,\text{cm}$$

8. **(4)**

$$y = \frac{a^4 b^2}{(cd^4)^{\frac{1}{3}}}$$

Taking log on both sides,

$$\log y = 4\log a + 2\log b - \frac{1}{3}\log c - \frac{4}{3}\log d$$

Differentiating,

$$\frac{\Delta y}{y} = 4\frac{\Delta a}{a} + 2\frac{\Delta b}{b} - \frac{1}{3}\frac{\Delta c}{c} - \frac{4}{3}\frac{\Delta d}{d}$$

Percentage error in y,

$$\frac{\Delta y}{y} \times 100 = 4\left(\frac{\Delta a}{a} \times 100\right) + 2\left(\frac{\Delta b}{b} \times 100\right)$$

$$+\frac{1}{3}\left(\frac{\Delta c}{c} \times 100\right) + \frac{4}{3}\left(\frac{\Delta d}{d} \times 100\right)$$

$$= \left[4 \times 2\% + 2 \times 3\% + \frac{1}{3} \times 4\% + \frac{4}{3} \times 5\%\right] = 22\%$$

9. **(4)**

When a particle undergoes normal collision with a floor or a wall, with coefficient of restitution e, the speed after collision is e times the speed before collision. Thus, in this case, the change in momentum for the first impact is $ep - (-p) = p(1 + e)$, for the second impact it is $e(ep) - (-ep) = ep(1 + e)$ and so on. The total change in momentum is $p(1 + e)\ [1 + e + e^2 + ...]$.

$$\Rightarrow p(1+e)\left(\frac{1}{1-e}\right) = p\frac{(1+e)}{(1-e)}$$

10. **(4)**

$$a = v\frac{dv}{dx} = \mu g = (\alpha x)g\,, \qquad \text{where } \alpha = \text{constant.}$$

$$\text{or } \int v\,dv = \int \alpha g x\,dx \qquad \text{or } \frac{1}{2}mv^2 = \frac{1}{2}m\alpha g x^2\,.$$

$$E \propto x^2.$$

11. **(3)**

The centre of mass of the 'block plus wedge' must move with speed $\dfrac{mu}{m + \eta m} = \dfrac{u}{1+\eta} = v_{CM}$.

$$\therefore\ \frac{1}{2}mu^2 - mgh = \frac{1}{2}(m + \eta m)v_{CM}^2\,.$$

$$\frac{1}{2}mu^2 - mgh = \frac{1}{2}m(1+\eta)\frac{u^2}{(1+\eta)^2}$$

$$u = \sqrt{2gh\left(1 + \frac{1}{\eta}\right)}$$

12. **(3)**

Maximum force of friction = kmg.

$\therefore$ maximum acceleration of insect $= a_1 = \dfrac{kmg}{m} = kg$

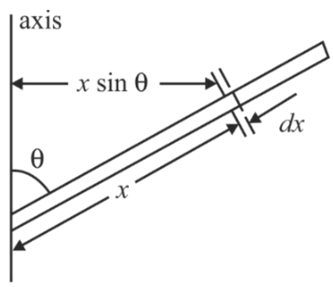
and maximum acceleration of stick $= a_2 = \dfrac{kmg}{M}$.

$\therefore$ acceleration of insect with respect to stick

$$= a = a_1 - (-a_2) = kg\left(1 + \frac{m}{M}\right).$$

$$\therefore\ L = \frac{1}{2}at^2 \text{ or } t^2 = \frac{2L}{a} = \frac{2ML}{kg(M+m)}.$$

13. **(3)**



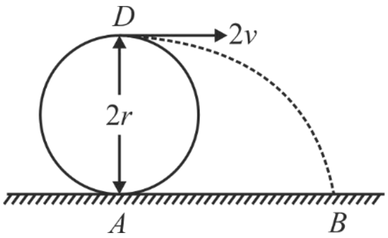
Mass of the element $= \left(\dfrac{m}{x}\right)dx$.

Moment of inertia of the element about the axis

$$= \left(\frac{m}{l}dx\right)(x\sin\theta)^2$$

$$I = \frac{m}{l}\sin^2\theta \cdot \int_0^l x^2 dx = \frac{ml^2}{3}\sin^2\theta$$

14. **(3)**



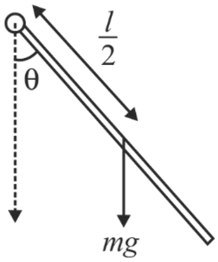
At the point of leaving the wheel, the blob of mud is at a height $2r$ above the road and has a horizontal velocity $2v$

Let t = time of travel from D to B. Then, $2r = \frac{1}{2}gt^2$

or $t = 2\sqrt{\frac{r}{g}}$ and $AB = (2v)t$

$AB = 2v \times 2\sqrt{\frac{r}{g}} = 4v\sqrt{\frac{r}{g}}$

15. **(1)**



$T = 2\pi\sqrt{\frac{l}{g}}$ where l = length of simple pendulum = length of rod.

$\tau = (mg)\frac{l}{2}\sin\theta$

For small $\theta, \tau = \frac{1}{2}mgl\theta = -I\alpha = -\left(\frac{ml^2}{3}\right)\alpha$

or $\alpha = -\left(\frac{3g}{2l}\right)\theta$

Time period $= 2\pi\sqrt{\frac{2l}{3g}} < T$.

16. **(3)**

Let M, R be the mass and radius of the planet, and g be the acceleration due to gravity on its surface.

Then, $V = \sqrt{2Rg}$ and $GM = R^2g$.

Gravitational potential at the surface is $-\frac{GM}{R}$ and at the centre is $-\frac{3GM}{2R}$. In going from the surface to the centre, loss in gravitational PE

$$= m\left[-\frac{GM}{R} - \left(-\frac{3}{2}\frac{GM}{R}\right)\right] = \frac{1}{2}\frac{GMm}{R} = \frac{1}{2}mv^2$$

or $v^2 = \frac{GM}{R} = Rg = \frac{V^2}{2}$ or $\frac{V}{\sqrt{2}}$.

17. **(1)**

$AB \rightarrow$ constant p, increasing V; $\therefore$ increasing T

$BC \rightarrow$ constant T, increasing V, decreasing p

$CD \rightarrow$ constant V, decreasing p; $\therefore$ decreasing T

$DA \rightarrow$ constant T, decreasing V, increasing p

Also, BC is at a higher temperature than AD.

18. **(3)**

Let p_A, p_B be the initial pressures in A and B respectively. When the gases double their volumes at constant temperature, their pressures fall to $\frac{p_A}{2}$ and $\frac{p_B}{2}$

$\therefore$ for $A, p_A - \frac{p_A}{2} = \Delta p$

or $p_A = 2\Delta p$

for $B, p_B - \frac{p_B}{2} = 1.5\Delta p$

or $p_B = 3\Delta p$

$\therefore \frac{p_A}{p_B} = \frac{2}{3}$

Also, $p_A V = \frac{m_A}{M}RT$ and $p_B V = \frac{m_B}{M}RT$.

$\therefore \frac{p_A}{p_B} = \frac{m_A}{m_B}$

$\therefore \frac{m_A}{m_B} = \frac{2}{3}$ or $3m_A = 2m_B$.

19. **(1)**

Area of spherical shell = $4\pi R^2$

Rate of heat flow $= P = k(4\pi R^2)\frac{T}{d}$, where d = thickness of shell.

20. **(3)**

Let a = initial amplitude due to S_1 and S_2 each.

$I_0 = k(4a^2)$, where k is a constant.

After reduction of power of S_1, amplitude due to S_1 = $0.6a$.

Due to superposition, $a_{max} = a + 0.6a = 1.6a$, and

$a_{min} = a - 0.6a = 0.4a$

$I_{max} / I_{min} = (a_{max} / a_{min})^2 = (1.6a / 0.4a) = 16$.

21. (100)

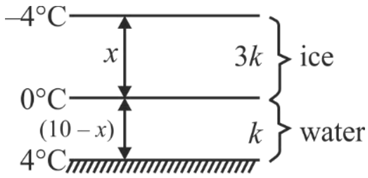
$$\gamma + 1 + \frac{2}{f} = 1 + \frac{2}{6} = \frac{4}{3}$$

$$\frac{\Delta W}{\Delta Q} = \frac{\Delta Q - \Delta U}{\Delta Q} = 1 - \frac{nC_V \Delta T}{nC_p \Delta T} = 1 - \frac{1}{\gamma} = 1 - \frac{3}{4} = \frac{1}{4}$$

$\therefore\ \Delta Q = 4\Delta W = 100\,\text{J}$.

22. (7.5)

The rate of heat flow is the same through water and ice in the steady state.



$$I = kA\frac{4-0}{10-x} = 3kA\frac{0-(-4)}{x}$$

$$x = (10-x)\cdot 3$$

or x = 7.5 m

23. (10)

Spring constant, k = 1960 N/m = 1960000 dyne/cm

Let x cm be the maximum compression of the spring.
Decrease in potential energy of the block

= increase in potential energy of the spring

$$mg[h+x] = \frac{1}{2}kx^2$$

$$2000 \times 980[40+x] = \frac{1}{2} \times 1960000x^2$$

or $40 + x = \frac{x^2}{2}$ or x = 10 cm.

24. (22.68)

$$\tau = I\alpha$$

or $\alpha = \frac{\tau}{I} = \frac{-1}{0.2} = -5\,\text{rad/s}^2$

$$\omega = \omega_0 + \alpha t = 2\pi \times \frac{360}{60} + (-5 \times 3)$$

= 2 × 3.14 × 6 – 15 = 22.68 rad/sec.

25. (2)

Force constant, $K = \frac{YA}{L}$ or $K \propto Y$

$$\therefore\ \frac{K_A}{K_B} = \frac{Y_A}{Y_B} = 2$$

26. (0.1)

$W = 200N$, y = 10^{-3} m

$\therefore$ Spring constant, $k = \frac{W}{y} = \frac{200}{10^{-3}}$

= 2×10^5 N/m

$$\therefore\ U = \frac{1}{2}ky^2$$

$$= \frac{1}{2} \times (2 \times 10^5) \times (10^{-3})^2 = 0.1J.$$

27. (1.5)

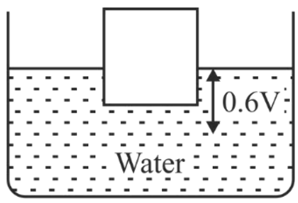
According to Archimedes' principle,

Weight of a body = Weight of the liquid displaced

Let V be volume of the body.

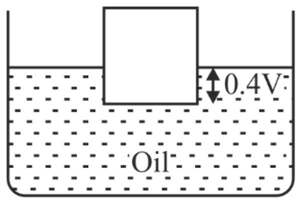
In water,

$$V\rho_{\text{body}}\, g = 0.6V\rho_{\text{water}}\, g \quad \text{...(i)}$$



In oil,

$$V\rho_{\text{body}}\, g = 0.4V\rho_{\text{oil}}\, g \quad \text{...(ii)}$$



Divide eqn. (i) by eqn. (ii), we get;

$$1 = \frac{0.6\rho_{\text{water}}}{0.4\rho_{\text{oil}}} \text{ or } \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} = \frac{6}{4} = \frac{3}{2}$$

Relative density of the oil $= \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} = \frac{3}{2} = 1.5$

28. (7)

η = 0.07 kg $\text{m}^{-1}\,\text{s}^{-1}$

dv = 1 m/s, dx = 1 mm = 1×10^{-3}m

A = 0.1 m^2

$$\therefore\ F = \eta A\frac{dv}{dx} = 0.07 \times 0.1 \times \frac{1}{1 \times 10^{-3}} = 7N.$$

29. **(30)**

Since, the block of ice at 0°C is large, the whole of ice will not melt, hence final temperature is 0°C.

∴ Q_1 = heat given by water in cooling upto 0°C

$= ms\Delta\theta = 80 \times 1 \times (30 - 0)$

= 2400 cal

If *m* gm be the mass of ice melted, then

$Q_2 = mL_F = m \times 80$

Now, $Q_2 = Q_1$

m × 80 = 2400 or *m* = 30 gm.

30. **(5000)**

$P = \frac{2}{3}E$ or $E = \frac{3}{2}P$

∴ Total energy $= EV = \frac{3}{2}PV$

For H_e : $1500 = \frac{3}{2}PV, PV = 1000$

For N_2 : $E'V = \frac{5}{2} \times 2PV = 5PV = 5 \times 1000$

= 5000 J

CHEMISTRY

31. **(1)**

	2 Mg +	$O_2 \longrightarrow$	2MgO
S.C.	2 mol	1 mol	2 mol
Given moles	0.041667	0.0175	–
L.R.	$\frac{0.041667}{2}$	$\frac{0.0175}{1}$	–
	0.02089	0.0175	–

Excess Reactant = Mg

1 mole of O_2 requires 2 mol Mg

0.0175 mol O_2 requires (2 × 0.0175) mol Mg

= 0.035 mol Mg

Remaining moles of Mg = 0.041667 – 0.035

= 0.006667

Amount of Mg remaining = 0.00667 × 24 = 0.16g

32. **(2)**

Acc. to de-Broglie, $\lambda = \frac{h}{mv}$

Now, λ = d = v × t

∴ λ = v × 1

Here, $v = \frac{h}{mv}$

$v^2 = \frac{h}{mv}$

$v = \sqrt{\frac{h}{m}}$

33. **(4)**

Gd is a f-block element

Gd : $[Xe]^{54}\ 4f^7\ 5d^1\ 6s^2$

4f is half-filled, extra stable

34. **(2)**

Bond length $\propto \frac{1}{\text{bond order}}$

Bond order = $\frac{\text{Total no. of bonds}}{\text{No. of side atom}}$

$SiO_4^{4-} = \frac{4}{4} = 1$

$PO_4^{3-} = \frac{5}{4} = 1.25$

$SO_4^{2-} = \frac{6}{4} = 1.5$

$ClO_4^{-} = \frac{7}{4} = 1.75$

35. **(1)**

Molecule	**Bond order**
CO	3
NO^-	2
NO^+	3
CN^-	3
N_2	3

36. **(2)**

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$\frac{V_2}{V_1} = \frac{P_1 \times T_2}{P_2 \times T_1} = \frac{1.5 \times 298}{288}$

$\frac{V_2}{V_1} = 1.55 \approx 1.6$

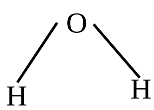
37. **(4)**

ΔG° = – RTlnK

ΔH° – TΔS° = – RTlnK

ln K = $\frac{T\Delta S° - \Delta H°}{RT}$

38. **(2)**



H_2O has 2(O–H) bonds. So, during formation of H_2O, energy released = (109 × 2) = 218 Kcal

39. (3)

$$K_{p1}=\frac{\alpha^2 p_1}{1-\alpha^2}, K_{p2}=\frac{4\alpha^2 p_2}{1-\alpha^2}$$

$$\frac{K_{p1}}{K_{p2}}=\frac{p_1}{4p_2}\Rightarrow\frac{p_1}{p_2}=\frac{36}{1}$$

40. (4)

HCO_3^- can give H^+ and hence act as bronsted acid

$$HCO_3^-+H_2O \rightleftharpoons CO_3^{2-}+H_3O^+$$

It can also gain H^+,

∴ behaves as bronsted base also

$$HCO_3^-+H^+ \rightleftharpoons H_2CO_3$$

41. (3)

$$AgIO_{3(s)} \rightleftharpoons Ag^+(aq)+IO_3^-(aq)$$

$$K_{sp}=[Ag^+]\left[IO_3^-\right]$$

$$s=\sqrt{K_{sp}}=\sqrt{10^{-8}}=10^{-4}\text{ M}$$

In 1L i.e. 1000 mL, moles of $AgIO_3 = 10^{-4}$

Moles of $AgIO_3$ in 100 ml = 10^{-5}

Mass of $AgIO_3$ in 100 ml = $10^{-5} \times 283$

= 2.83×10^{-3}g

42. (4)

Normality of $(FeSO_4).(NH_4)_2SO_4.6H_2O$

$$=\frac{3.92\times1000}{392\times100}$$

= 0.1 N

Eq. of $FeSO_4.(NH_4)_2SO_4.6H_2O$ = Eq. of $KMnO_4$

$N_1V_1 = N_2V_2$

$0.1\times20 = N_2 \times 18$

$N_2 = 0.11$ N

Normality of $KMnO_4$ = $\frac{\text{mass}\times5}{\text{M.M}\times1}$

$$\text{Mass}_{KMnO_4}=\frac{0.11\times158}{5}$$

= 3.5 g

43. (4)

Electrolysis of molten anhydrous $CaCl_2$

Cathode; $Ca^{2+} + 2e^- \rightarrow Ca(s)$

Anode; $2Cl^- \rightarrow Cl_{2(g)} + 2e^-$

44. (4)

$MnSO_4 + H_2O_2 + 2NaOH \rightarrow Na_2SO_4 + MnO_2 + 2H_2O$

$2KMnO_4+3H_2SO_4+5H_2O_2 \rightarrow K_2SO_4+2MnSO_4+8H_2O+5O_2$
(acidified)

$2KMnO_4+3H_2O_2 \rightarrow 2MnO_2+2KOH+3O_2+2H_2O$
(alkaline)

45. (3)

Acidic strength of oxides decreases down the group and increases along a period.

46. (2)

Follow rules to determine the stability of resonating structures

47. (1)

In case of Lassaigne's test of halogens, it is necessary to remove sodium cyanide and sodium sulphide from the sodium extract if nitrogen and sulphur are present. This is done by boiling the sodium extract with conc. HNO_3

48. (3)

Informative

49. (4)

Electron releasing group increases the reactivity of benzene toward electrophilic aromatic substitution.

50. (2)

Alcohols undergoes dehydration in presence of conc. H_2SO_4 to give more stable alkene.

51. (9)

$$\frac{\text{Strength of }HA_1}{\text{Strength of }HA_2}=\frac{\alpha_1}{\alpha_2}=\frac{9}{1}=9$$

52. (2)

Pyrosilicate **:** $Si_2O_7^{6-}$

53. (3)

Element	**charge**
Be	+2
Al	+3
Si	+4
O	–2

$(2\times n) + (2\times3) + (6\times4) + (18\times(-2)) = 0$

n = 3

54. (5)

$$n=\frac{PV}{RT}$$

$$=\frac{4.1\times30}{0.0821\times300}$$

= 4.99 ≈ 5.

55. **(3)**

Average title value

$= \frac{25.2 + 25.25 + 25.0}{3} = 25.15$

In case of division, the final answer will contain as many significant figures as there in a number with least significant numbers i.e. 3

56. **(6)**

for n = 4, $\ell = 0$, 1, 2, 3

for, $\ell = 0$, m = 0

for $\ell = 1$, m = –1, 0, +1

for $\ell = 2$, m = – 2, –1, 0, +1, +2

for $\ell = 3$, m = –3, –2, –1, 0, +1, +2, +3

Every value of m represents 1 orbital

$\therefore$ we have 6 orbitals with $|m_\ell| = 1$

Each orbital have 1 e^- with $+\frac{1}{2}$ value and

1 e^- with $-\frac{1}{2}$ value

$\therefore$ Total 6 electrons are present for

n = 4, $|m_\ell| = 1$ and $m_s = -\frac{1}{2}$

57. **(7)**

$0.68 = \left(\frac{n_x}{n_x + 0.1}\right) \times 1 \left(n_x = \text{moles of unknown compound present in vapour form}\right)$

$n_x = 0.212$

PV = nRT

V = (0.212 + 0.1) 0.0821 × 273

V = 7L

58. **(4)**

Follow structure of $CuSO_4.5H_2O$

59. **(3)**

Follow structure of each molecule

60. **(1)**

$C_4H_{10} + 6.5O_2 \longrightarrow 4CO_2 + 5H_2O$

$C_4H_{10} + 6O_2 \longrightarrow 3CO_2 + 5H_2O + CO$

MATHEMATICS

61. **(1)**

Let the roots be *mk* and *nk*.

$\therefore mk + nk = -\frac{b}{a}$ and $(mk)(nk) = \frac{c}{a}$

$\therefore k(m + n) = -\frac{b}{a}$ and $k^2 mn = \frac{c}{a}$

Eliminating *k*, we get

$\left(-\frac{b}{a(m+n)}\right)^2 = \frac{c}{amn} \Rightarrow mnb^2 = ac(m + n)^2$

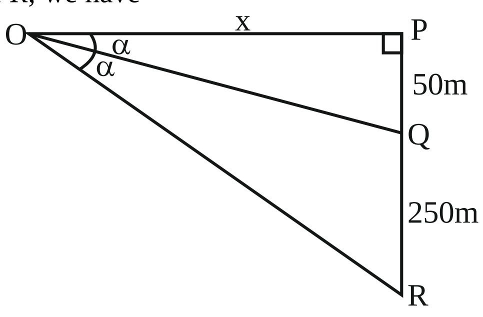
62. **(3)**

Let *PQ* be the pole on the building *QR* and *O* be the observer then *PQ* = 50m, *QR* = 250 m

$\therefore$ *PR* = 300

So the observer is at the same height as the top *P* of the pole.

Let *OP* = *x*, then from right angle triangle *OPQ* and *OPR*, we have



$\tan\alpha = \frac{50}{x}$ and $\tan 2\alpha = \frac{300}{x}$

so $\frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{300}{x}$

$\Rightarrow \frac{2 \times \frac{50}{x}}{1 - \frac{2500}{x^2}} = \frac{300}{x} \Rightarrow \frac{100}{x} \times \frac{x^2}{x^2 - 2500} = \frac{300}{x}$

$\Rightarrow x^2 = 3x^2 - 7500 \quad \Rightarrow x = \frac{50\sqrt{3}}{\sqrt{2}} = 25\sqrt{6}\, m$

63. **(3)**

If Z_1, Z_2, Z_3 form an equilateral triangle, then we know that,

$Z_1^2 + Z_2^2 + Z_3^2 = Z_1Z_2 + Z_2Z_3 + Z_3Z_1$

$\Rightarrow a^2 - 1 + 2ai + 1 - b^2 + 2bi - a + b - i - abi = 0$

$\Rightarrow (a - b)(a + b - 1) + (2a + 2b - ab - 1)i = 0$

Case-I:

$a = b$ & $2a + 2b - ab - 1 = 0$

$\Rightarrow a = b$ & $a^2 - 4a + 1 = 0$

$\Rightarrow a = b = 2 - \sqrt{3}$

Case-II:

$a + b - 1 = 0$ & $2a + 2b - ab - 1 = 0$

$\Rightarrow ab = 1$ (not possible because $a, b \in (0,1)$)

$\Rightarrow a = b = 2 - \sqrt{3}$ is the only solution

64. **(3)**

$3 - \cos x + 1 - \cos^2 x = -(\cos^2 x + \cos x) + 4$

$= -\left(\cos^2 x + \cos x + \frac{1}{4}\right) + 4 + \frac{1}{4}$

$= -\left(\cos x + \frac{1}{2}\right)^2 + \frac{17}{4}$

$= \frac{17}{4} - \left(\cos x + \frac{1}{2}\right)^2$

Maximum value $\left(\text{at } \cos x = -\frac{1}{2}\right) = \frac{17}{4}$

Minimum value $\left(\text{at } \cos x = 1\right) = \frac{17}{4} - \frac{9}{4} = 2$

Difference = $= \frac{17}{4} - 2 = \frac{9}{4}$

65. **(3)**

p : Amit is eating

q : Bipin is sleeping

$\sim(p \to q) \equiv p \wedge \sim q$

$\therefore$ Amit is eating and Bipin is not sleeping

66. **(3)**

$S = 1 + 1\cdot1! + 2\cdot2! + 3\cdot3! \ldots\ldots\ldots n\cdot n!$

$S - 1 = (2 - 1)1! + (3 - 1)2! + (4 - 1)3! \ldots\ldots\ldots\ldots(n + 1 - 1)n!$

$S - 1 = (2! - 1!) + (3! - 2!) + (4! - 3!) + \ldots\ldots\ (n + 1)! - n!$

$S - 1 = (n - 1)! - 1$

$S = (n + 1)!$

67. **(2)**

Let x-axis divides the join of line segment in k : 1. Coordinates of the point are

$\Rightarrow \left(\frac{-3k+3}{k+1}, \frac{2k+4}{k+1}, \frac{5k+10}{k+1}\right)$

Since, y and z-coordinates on x-axis is zero

$\Rightarrow \frac{2k+4}{k+1} = 0, \frac{5k+10}{k+1} = 0$

On solving the equation, we get. $k = -2$

Hence, the required ratio is 2 : 1 externally.

68. **(2)**

$$\frac{x + y + \frac{z}{2} + \frac{z}{2}}{4} \geq \left(\frac{xyz^2}{4}\right)^{1/4}$$

$$\frac{4}{4} \geq \left(\frac{xyz^2}{4}\right)^{1/4}$$

$xyz^2 \leq 4$

69. **(3)**

If each observation is increased by a same quantity the variance remains unchanged.

$\therefore\ \sigma_A : \sigma_B : \sigma_C = 1:1:1$

70. **(3)**

$$\frac{\frac{p}{2}(2a_1 + (p-1)d)}{\frac{q}{2}(2a_1 + (q-1)d)} = \frac{p^2 + p}{q^2 + q}$$

$$\Rightarrow \frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p+1}{q+1}$$

Now $\frac{p-1}{2} = 4 \quad \Rightarrow p = 9$ &

$\frac{q-1}{2} = 14 \ \Rightarrow q = 29$

$\Rightarrow \frac{a_1 + 4d}{a_1 + 14d} = \frac{10}{30} = \frac{1}{3}$

71. **(1)**

Given $\frac{x}{3} = \cos t + \sin t$ & $\frac{y}{4} = \cos t - \sin t$

Squaring these two,

$\Rightarrow \frac{x^2}{9} = 1 + 2\cos t \sin t$ ….(i)

$\frac{y^2}{16} = 1 - 2\sin t \cos t$ ….(ii)

Adding (i) & (ii)

$\frac{x^2}{9} + \frac{y^2}{16} = 2 \quad \Rightarrow \frac{x^2}{18} + \frac{y^2}{32} = 1$

72. **(2)**

${}^6C_3 \times 2 \times 1 = 40$

73. **(3)**

$a_2\ a_7\ a_{12} = (a + d)\ (a + 6d)\ (a + 11d)$

$= (15 - 5d).\ 15.\ (15 + 5d) = 15(225 - 25d^2)$

74. **(3)**

Case 1: When $x < 3/2$

$x^2 - (2x - 3) - 4 = 0$

$\Rightarrow\ x^2 - 2x + 1 = 2$

$\Rightarrow\ (x - 1)^2 = 2 \Rightarrow x - 1 = \pm\sqrt{2}$

$\Rightarrow\ x = 1 \pm\sqrt{2}$

As $x < 3/2$, we take, $x = 1 - \sqrt{2}$

Case 2 : When $x \geq 3/2$

In this case the equation become

$$x^2 + (2x-3) - 4 = 0$$

$$\Rightarrow \quad (x+1)^2 = 8$$

$$\Rightarrow \quad x+1 = \pm 2\sqrt{2}$$

As $x \geq 3/2$, $x = -1 + 2\sqrt{2}$

Sum of the root is

$$(1-\sqrt{2}) + (-1+2\sqrt{2}) = \sqrt{2}$$

75. **(2)**

$x + 2 = a$, $y - 3 = 0$

$x + 2 = 2$, $y - 3 = 0$

$x = 0$, $y = 3$

(0, 3) is focus

76. **(4)**

Centres (0, 0), (–3, 1) and (6, –2)

$$\frac{1-0}{-3-0} = \frac{-2-1}{6-(-3)} \Rightarrow -\frac{1}{3} = -\frac{1}{3}$$

Centres are collinear.

77. **(1)**

$$\frac{1}{1-\cos\alpha} = 2 - \sqrt{2}$$

$$\Rightarrow \quad 1 - \cos\alpha = \frac{1}{2-\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$

$$1 - \cos\alpha = 1 + \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos\alpha = -\frac{1}{\sqrt{2}} \quad \Rightarrow \alpha = \frac{5\pi}{4}$$

78. **(2)**

$$\frac{x+3}{2} = t \text{ and } \frac{y+1}{4} = t^2$$

$$\Rightarrow \frac{x^2 + 9 + 6x}{4} = \frac{y+1}{4} \Rightarrow x^2 + 6x - y + 8 = 0$$

79. **(3)**

For ellipse $16 - k > 0$ and $k - 9 > 0$

$k < 16$ and $k > 9$ $\Rightarrow 9 < k < 16$

But $16 - k \neq k - 9$ $\Rightarrow k \neq 12.5$

80. **(3)**

We have, $\dfrac{{}^{n+2}C_6}{{}^{n-2}P_2} = 11$

$$\Rightarrow \frac{(n+2)(n+1)n(n-1)}{6.5.4.3.2.1} = 11$$

$$\Rightarrow (n+2)(n+1)n(n-1) = 11.10.9.8$$

$$\Rightarrow n = 9$$

81. **(5)**

We know that, $5^4 = 625 = 52 \times 12 + 1$

$\Rightarrow 5^4 = 52\lambda + 1$, where λ is a positive integer,

$\Rightarrow (5^4)^{24} = (52\lambda + 1)^{24}$

$= {}^{24}C_0(52\lambda)^{24} + {}^{24}C_1(52\lambda)^{23} + {}^{24}C_2(52\lambda)^{22} + ...+ {}^{24}C_{23}(52\lambda) + {}^{24}C_{24}$

(by binomial theorem)

$\Rightarrow 5^{96}$ = (a multiple of 52) + 1

On multiplying both sides by 5, we get

$5^{97} = 5^{96}.5$ = 5 (a multiple of 52) + 5

Hence, the required remainder is 5.

82. **(22)**

$$\frac{1}{12!}\left(\frac{12!}{1!11!} + \frac{12!}{3!9!} + \frac{12!}{5!7!}\right) = \frac{2^n}{m!}$$

$$\Rightarrow \frac{1}{12!}\{{}^{12}C_1 + {}^{12}C_3 + {}^{12}C_5\} = \frac{2^n}{m!}$$

$$\Rightarrow \frac{1}{12!}\{{}^{12}C_1 + {}^{12}C_1 + {}^{12}C_3 + {}^{12}C_3 + {}^{12}C_5 + {}^{12}C_5\} = \frac{2^{n+1}}{m!}$$

$$\Rightarrow \frac{1}{12!}\{{}^{12}C_1 + {}^{12}C_{11} + {}^{12}C_3 + {}^{12}C_9 + {}^{12}C_5 + {}^{12}C_7\} = \frac{2^{n+1}}{m!}$$

$$\Rightarrow \frac{1}{12!}\{{}^{12}C_1 + {}^{12}C_3 + {}^{12}C_5 + {}^{12}C_7 + {}^{12}C_9 + {}^{12}C_{11}\} = \frac{2^{n+1}}{m!}$$

$$\Rightarrow \frac{2^{11}}{12!} = \frac{2^{n+1}}{m!} \Rightarrow n = 10, m = 12$$

$$\Rightarrow m + n = 22$$

83. **(0)**

We know that, ${}^{100}C_{50} = \dfrac{100!}{50!50!}$

The exponent of 7 in 50! is

$$\left[\frac{50}{7}\right] + \left[\frac{50}{7^2}\right] = 7 + 1 = 8$$

And the exponent of 7 in 100! is

$$\left[\frac{100}{7}\right] + \left[\frac{100}{7^2}\right] = 14 + 2 = 16$$

Thus, the exponent of 7 is ${}^{100}C_{50}$ is

$16 - 2 \times 8 = 0$

84. **(6)**

$y = 7\cos^2 x + 6\sin x\cos x - \sin^2 x$

$= 3\sin 2x + 4\cos 2x + 3$

The maximum value of $y = 8$

Also $\log_{\sqrt{2}} 8 = \log_{\sqrt{2}}(\sqrt{2})^6 = 6$

85. **(3)**

$y = \dfrac{x^{12}+x^6+1}{x^6+x^3+1} = \dfrac{x^{12}+2x^6+1-x^6}{x^6+x^3+1}$

$= \dfrac{(x^6+1)^2-(x^3)^2}{x^6+x^3+1}$

$\Rightarrow y = x^6 - x^3 + 1$

$\Rightarrow \dfrac{dy}{dx} = 6x^5 - 3x^2$

$\therefore$ $a = 6, b = -3$

$a + b = 6 - 3 = 3.$

86. **(3)**

$2^{2+4\sin^2 x} = 2^{6\sin x}$

$\Rightarrow 2 + 4\sin^2 x = 6\sin x$

$\Rightarrow$ $2\sin^2 x - 3\sin x + 1 = 0$

$\Rightarrow$ $(2\sin x - 1)(\sin x - 1) = 0$

$\therefore$ $\sin x = \dfrac{1}{2}$ or $\sin x = 1$

$\therefore$ $x = \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{\pi}{2}$

87. **(2)**

$x^2 - 3kx + 2k^2 - 1 = 0$

$2k^2 - 1 = 7$

$\Rightarrow k = \pm 2$

For roots to be real, $D \geq 0$

$\Rightarrow 9k^2 - 4(2k^2 - 1) \geq 0$

$\Rightarrow k^2 + 4 \geq 0$

Also log k is defined for $k > 0$

$\Rightarrow k = 2$

88. **(26)**

$x_1 + x_2 + x_3 + x_4 = 10$

Number of ways $= {}^{n+r-1}C_{r-1} = {}^{13}C_3 = 286$

89. **(80)**

$\sum_{r=16}^{30}(r+2)(r-3) = \sum_{r=16}^{30}(r^2 - r - 6)$

$\sum_{r=16}^{30} r^2 - \sum_{r=16}^{30} r - \sum_{r=16}^{30} 6$

$= \left(\sum_{r=1}^{30} r^2 - \sum_{r=1}^{15} r^2\right) - \left(\sum_{r=1}^{30} r - \sum_{r=1}^{15} r\right) - 6\times 15$

$= \left(\dfrac{30\times 31\times 61}{6} - \dfrac{15\times 16\times 31}{6}\right)$

$-\left(\dfrac{30\times 31}{2} - \dfrac{15\times 16}{2}\right) - 90$

$= (9455 - 1240) - (465 - 120) - 90 = 7780$

90. **(3)**

$2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha - 2 = 0$

$\Rightarrow$ $2\sin^2\alpha(\sin\alpha - 1) - 5\sin\alpha(\sin\alpha - 1)$

$+ 2(\sin\alpha - 1) = 0$

$\Rightarrow$ $(\sin\alpha - 1)(2\sin^2\alpha - 5\sin\alpha + 2) = 0$

$\Rightarrow$ $\sin\alpha - 1 = 0$ or $2\sin^2\alpha - 5\sin\alpha + 2 = 0$

$\sin\alpha = 1$ or $\sin\alpha = \dfrac{5\pm\sqrt{25-16}}{4} = \dfrac{5\pm 3}{4}$

$\alpha = \dfrac{\pi}{2}$ or $\sin\alpha = \dfrac{1}{2}, 2$

Now, $\sin\alpha \neq 2$

For, $\sin\alpha = \dfrac{1}{2}, \alpha = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$

There are three values of α between $[0, 2\pi]$